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Research Article

Periodic Solutions Generated by Impulses for State-Dependent Impulsive Differential Equation

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We study the existence of periodic solutions for a class of state-dependent impulsive differential systems via geometrical analysis methods. Our results show that these periodic solutions are generated by impulses. Moreover, numerical simulations are used to examine the existence of the periodic solutions.

1. Introduction

It is known that many evolutionary processes are characterized by the fact that at certain moments of time the states change abruptly. Such processes often occur in biology, control theory, optimization theory, physics, and mechanics problems (e.g., [1–6]). It is natural to assume that these perturbations act instantaneously, that is, in the form of impulses.

The theory of impulsive differential equations (IDEs) is rather rich, especially for impulse at fixed time. There are many classical methods to study impulsive differential equations. For example, Chen et al. [7] obtained some new results concerning the existence of solutions to an impulsive first-order, nonlinear ordinary differential equation with periodic boundary conditions via differential inequalities and Schaefer's fixed-point theorem. Wang et al. [8] got the existence of extreme solutions of a periodic boundary value problem for a second-order functional differential equation by using upper and lower solutions. Based on a nonlinear alternative principle of Leray-Schauder, together with a truncation technique, Chu and Nieto [9] studied the impulsive periodic solutions of first-order singular ordinary differential equations. By using a variational method and a variant fountain theorem, Dai and Zhang [10] considered the existence and multiplicity of solutions for a class of nonlinear impulsive problem on the half-line. For more related work, the reader is referred to [11–13] and the references therein. As we know, state-dependent IDEs have become a hot topic in recent years due to their extensive application space, but it is also a difficult research field because of their essential properties: uncertainties for impulsive time and collision times. Very recently, many papers have been devoted to the analysis of IDEs with state-dependent impulsive effect. By using differential equation geometry theory and the method of successor functions, the existence and stability of periodic solution for pest management model with state feedback control strategy were discussed in [14, 15] and the homoclinic cycle and homoclinic bifurcation were analyzed for predator-prey model with state-dependent impulsive harvesting in [16, 17]. On the basis of rotated vector fields theory, Dai et al. [18] discussed the order-1 positive periodic solution and homoclinic cycles and homoclinic bifurcations for a general semicontinuous dynamic system. Considering the influence of Allee effect on prey species, the authors in [19, 20] investigated a prey-predator model with Allee effect and state-dependent impulsive harvesting and got the sufficient conditions for the existence of order-1 periodic solution and heteroclinic bifurcation via the geometry theory of semicontinuous dynamic systems. Some other related studies can be seen in [21–23] and the references therein.

The aforementioned papers all assumed that the predator just lived on the prey. However, in practice, it is very likely that many enemies have some other food sources. Motivated by this, in this paper, we consider the following state-dependent predator-prey model in which the predator species display the logistic growth in the absence of prey species:

$$\begin{aligned} x'(t) &= x(r_1 - a_{11}x - a_{12}y) \\ y'(t) &= y(r_2 + a_{21}x - a_{22}y) \\ x &\neq h_1, h_2 \quad \text{or} \quad x = h_1, \quad y > \tilde{y}, \end{aligned}$$

$$\Delta x(t) = 0, \quad \Delta y(t) = \tau_1, \quad x = h_1, \; y \le \tilde{y},$$
$$\Delta x(t) = -\alpha x, \qquad \Delta y(t) = -\beta y + \tau_2,$$
$$x = h_2, \tag{1}$$

where $x(t)$ and $y(t)$ denote population densities of prey and predator at time t, respectively. All the parameters are positive constants, in addition, $\alpha, \beta \in (0,1)$, $h_1 < h_2$, and $(h_1, \tilde{y})$ is the point of intersection of $x' = 0$ and $x = h_1$.

This paper is organized as follows. In Section 2, we present some preliminaries. Then in Section 3, we discuss the existence of positive periodic solution of system (1) for different cases. At last, in Section 4, some numerical simulations and conclusions are presented.

2. Preliminaries

For Model (1), if there is no impulsive effect, we have the following subsystem:

$$x'(t) = x(r_1 - a_{11}x - a_{12}y),$$
$$y'(t) = y(r_2 + a_{21}x - a_{22}y). \tag{2}$$

Followed by [24], the following results can be concluded.

Lemma 1. *Consider Model* (2)*, there are one trivial equilibrium $S_0 = (0,0)$ and two boundary equilibria $S_1 = (r_1/a_{11}, 0)$ and $S_2 = (0, r_2/a_{22})$. S_0 is always unstable; S_1 is a saddle. Moreover, if $r_1a_{22} - r_2a_{12} > 0$, then S_2 is a saddle and there exists a unique positive equilibrium*

$$S_+ = (x^*, y^*) = \left(\frac{r_1a_{22} - r_2a_{12}}{a_{11}a_{22} + a_{12}a_{21}}, \frac{r_2a_{11} + r_1a_{21}}{a_{11}a_{22} + a_{12}a_{21}}\right), \tag{3}$$

which is globally asymptotically stable.

Throughout this paper, we always assume that the condition $r_1a_{22} - r_2a_{12} > 0$ holds true. Considering the biological background, we only discuss Model (1) in the region $\{(x,y) : x \ge 0, y \ge 0\}$. Obviously, due to Lakshmikantham et al. [25] and Bainov and Simeonov [26], the global existence and uniqueness of solution for Model (1) are guaranteed by the smoothness properties of right-side functions.

To discuss the dynamics of Model (1), we define three cross sections and two regions:

$$\Sigma_0 = \{(x,y) : x = (1-\alpha)h_2, \; y > 0\};$$
$$\Sigma_1 = \{(x,y) : x = h_1, \; y > 0\};$$
$$\Sigma_2 = \{(x,y) : x = h_2, \; y > 0\}; \tag{4}$$
$$\Omega_1 = \{(x,y) : x \le h_1, \; y > 0\};$$
$$\Omega_2 = \{(x,y) : x \le h_2, \; y > 0\}.$$

Definition 2 (see [27]). Suppose that the impulse set M and its phase set N are both lines, as shown in Figure 1. Assume

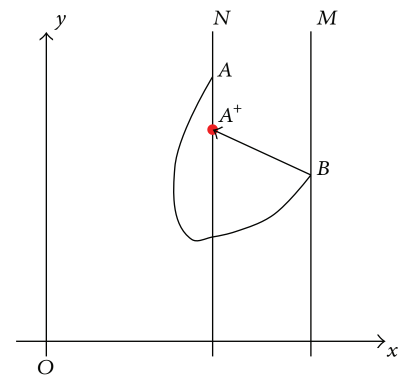

FIGURE 1: Illustration of the successor function.

that the trajectory starting from A in N firstly intersects M at point B and then jumps to A^+ in N due to the impulsive effect. Then, one defines A^+ as the successor point of A, and the corresponding successor function of point A is that $f(A) = y_{A^+} - y_A$; here y_A and y_{A^+} are the ordinates of A and A^+.

Lemma 3 (see [27]). *Successor function $f(A)$ is continuous.*

Lemma 4 (see [27]). *For Model* (1)*, if there exist $\widetilde{A} \in N$, $\widehat{A} \in N$ satisfying successor function $f(\widetilde{A})f(\widehat{A}) < 0$, then there must exist a positive periodic solution.*

3. Existence of Positive Periodic Solution for System (1)

Considering the biological meaning, here we always assume that $h_1 < x^*$. Therefore, we have four cases to discuss: $(1-\alpha)h_2 < h_1 < h_2 < x^*$, $h_1 < (1-\alpha)h_2 < h_2 < x^*$, $h_1 < (1-\alpha)h_2 < x^* < h_2$, and $(1-\alpha)h_2 < h_1 < x^* < h_2$.

3.1. The Case of $(1-\alpha)h_2 < h_1 < h_2 < x^*$. About the existence of positive periodic solution, we have the following graph illustrations.

Take a point $P(h_1, \tilde{y} + \varepsilon)$ on $\sum_1$, where ε is small sufficiently. Assuming that the trajectory of Model (1) starting from P firstly intersects $\sum_1$ at point P_1 and then jumps to P_1^+, obviously, P_1^+ is above P; that is to say,

$$f(P) = y_{P_1^+} - y_P > 0. \tag{5}$$

On the other hand, assume that the trajectory starting from P_1^+ intersects $\sum_1$ at P_2 and then jumps to P_2^+. For P_2^+, if $\tilde{y} < y_{P_2^+} \le y_{P_1^+}$, then

$$f(P_1^+) = y_{P_2^+} - y_{P_1^+} \le 0; \tag{6}$$

thus, Model (1) exists as a positive periodic solution whose initial point is between P and P_1^+; see Figure 2(a). If $y_{P_2^+} \le \tilde{y}$, then Model (1) will keep on impulse until $y_{P_2^{k+}} > \tilde{y}$ ($y_{P_2^{k+}} = y_{P_2} + k\tau_2$); this returns to the situation of $\tilde{y} < y_{P_2^+} \le y_{P_1^+}$ and the positive periodic solution can be seen in Figure 2(b).

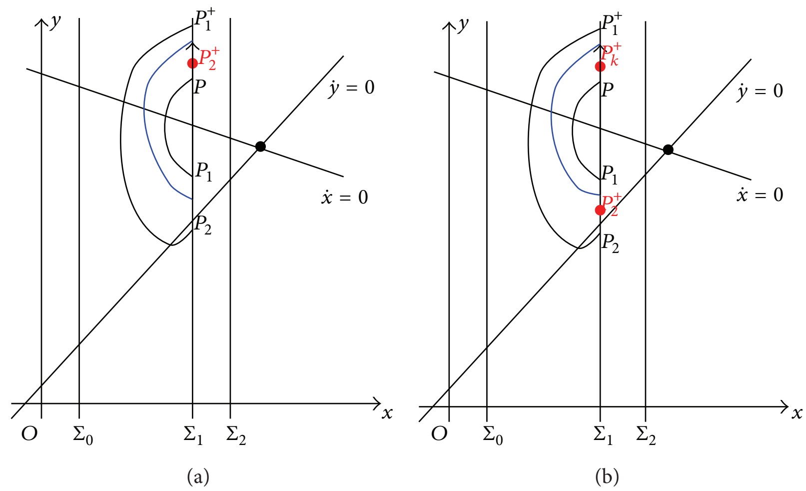

FIGURE 2: The possible trajectories in the case of $(1-\alpha)h_2 < h_1 < h_2 < x^*$.

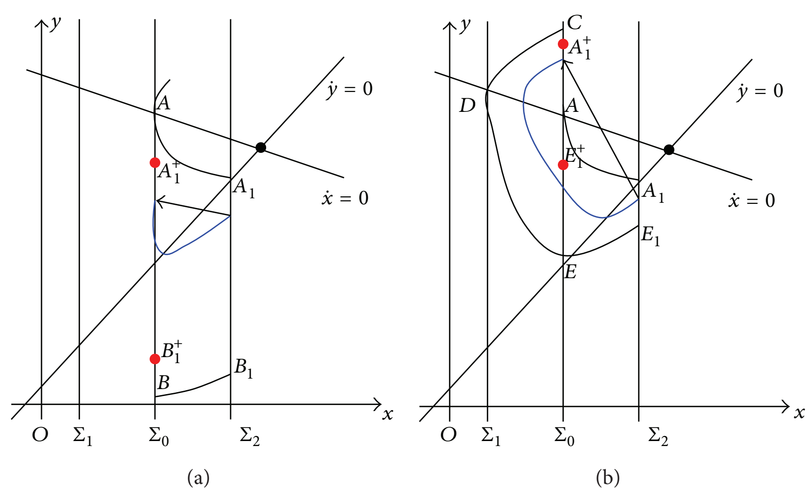

FIGURE 3: The possible trajectories in the case of $h_1 < (1-\alpha)h_2 < h_2 < x^*$.

Theorem 5. *Assume that* $(1-\alpha)h_2 < h_1 < h_2 < x^*$. *Then Model* (1) *exists as a positive periodic solution whose initial point is located between P and* P_1^+.

3.2. The Case of $h_1 < (1-\alpha)h_2 < h_2 < x^*$. For this case, we have the following graph illustrations.

Assuming that the trajectory starting from $A((1-\alpha)h_2, \tilde{y})$ firstly intersects $\sum_2$ at A_1 and then jumps to A_1^+, here A is the intersection point of $x' = 0$ and Σ_0. For A_1^+, we have the following two situations.

(i) If $y_{A_1^+} \le y_A$, then

$$f(A) = y_{A_1^+} - y_A \le 0. \quad (7)$$

On the other hand, choosing a point B next to x-axis on $\sum_0$, the trajectory starting from B firstly intersects $\sum_2$ at point B_1 and then jumps to B_1^+ on $\sum_0$; obviously, B_1^+ is above B; thus,

$$f(B) = y_{B_1^+} - y_B > 0. \quad (8)$$

Therefore, Model (1) exists as a positive periodic solution whose initial point is between A and B; this is shown in Figure 3(a).

(ii) If $y_{A_1^+} > y_A$, then

$$f(A) = y_{A_1^+} - y_A > 0. \quad (9)$$

On the other hand, there must exist a trajectory starting from C on $\sum_0$ that tangents $\sum_1$ at point $D(h_1, \tilde{y})$ and then intersects $\sum_0, \sum_2$ at points E, E_1, respectively; due to impulsive effect, E_1 jumps to E_1^+. For E_1^+, if $y_{E_1^+} \le y_C$, then

$$f(C) = y_{E_1^+} - y_C \le 0. \quad (10)$$

Therefore, Model (1) exists as a positive periodic solution whose initial point is between A and C; this can be seen in Figure 3(b). If $y_{E_1^+} > y_C$, then the trajectory starting from Ω_2 will ultimately stay in Ω_1. (Here we always assume the impulsive phase set with initial point on $\sum_0$ will ultimately exceed point E after one or finite times impulses. In fact, the assumption is reasonable as β should be very small in practical problem.)

Theorem 6. *Assume that* $h_1 < (1-\alpha)h_2 < h_2 < x^*$. *If* $y_{A_1^+} \le y_A$ *or* $y_{A_1^+} > y_A$, $y_{E_1^+} \le y_C$, *then Model* (1) *exists as a positive periodic solution.*

3.3. The Case of $h_1 < (1-\alpha)h_2 < x^* < h_2$. For this case, there must exist a trajectory Γ starting from A at Σ_2 that firstly intersects $\dot{x} = 0$, $\dot{y} = 0$ at B, C, respectively, and then tangents Σ_2 at point D. For x_B, there are three possible situations.

3.3.1. $(1-\alpha)h_2 < x_B < h_2$. Obviously, there must exist a trajectory Γ_1 starting from A_1 at Σ_2 that tangents Σ_0 at point B_1 and then intersects $y' = 0$, Σ_2 at C_1, D_1, respectively. Due to impulsive effect, D_1 jumps to D_1^+. For D_1^+, there are two cases.

(i) If $y_{D_1^+} \le y_{B_1}$, obviously, this situation returns to (i) of case of Section 3.2; here we omit it.

(ii) If $y_{D_1^+} > y_{B_1}$, assume that the trajectory starting from D_1^+ intersects $x' = 0$, $y' = 0$, and Σ_2 at B_2, C_2, and D_2, respectively. Obviously, for the orbit $D_1^+\widehat{B_2C_2}D_2$, x_{B_2} is the smallest abscissa. For x_{B_2}, there are two possible positives.

(a) If $x_{B_2} \ge h_1$, then Model (1) exists as a positive periodic solution whose initial point is between B_1 and D_1^+; this is shown in Figure 4(a).

(b) If $x_{B_2} < h_1$, then there must exist a trajectory Γ' starting from P at Σ_0 that tangents Σ_1 at point P_1 and then intersects Σ_0 and Σ_2 at points P_2 and P_3. For P_3, it also has two possibilities:

(a)′ if $y_{P_3^+} \le y_P$, then Model (1) exists as a positive periodic solution whose initial point is between B_1 and P; this can be seen in Figure 4(b);

(b)′ if $y_{P_3^+} > y_P$, then the trajectory starting from $\Omega_0 = \{(x,y) : (x,y) \in \widehat{ABCDA}\}$ will tend to equilibrium and the trajectory starting from $\Omega_2 \setminus \Omega_0$ will ultimately stay in Ω_1. (In this case, we still assume the impulsive phase set with initial point on $\sum_0$ will ultimately exceed point P_2 after one or finite impulses.)

Theorem 7. *Assume that* $h_1 < (1-\alpha)h_2 < x_B < x^* < h_2$. *If* $y_{D_1^+} \le y_{B_1}$ *or* $y_{D_1^+} > y_{B_1}$, $x_{B_2} \ge h_1$ *or* $y_{D_1^+} > y_{B_1}$, $x_{B_2} < h_1$, $y_{P_3^+} \le y_P$, *then Model* (1) *exists as a positive periodic solution.*

3.3.2. $h_1 < x_B < (1-\alpha)h_2$. Assuming that the trajectory Γ intersects Σ_0 at points P_1, P_2 with $y_{P_1} > y_{P_2}$, due to impulsive effect, D jumps to D^+. For D^+, one has the following four situations to discuss.

(i) If $y_{D^+} = y_{P_1}$ or $y_{D^+} = y_{P_2}$, then $P_1\widehat{BCDP}_1$ or $\widehat{P_2DP_2}$ is an order-1 periodic solution.

(ii) If $y_{D^+} < y_{P_2}$, then it is easy to get that there exists an order-1 periodic solution; here we omit the details.

(iii) If $y_{D^+} > y_{P_1}$, assume that the trajectory starting from D^+ intersects $\dot{x} = 0$ at point E. For x_E, there exist two cases.

(a) If $x_E \ge h_1$, then Model (1) exists as a positive periodic solution whose initial point is between P_1 and D^+ (see Figure 5(a)).

(b) If $x_E < h_1$, then there must exist a trajectory Γ_1 starting from P at Σ_2 that firstly intersects Σ_0 at point A_1 and then tangents to Σ_1 at point B_1, and intersects $y' = 0$, Σ_2 at points C_1, D_1, respectively. Due to impulsive effect, D_1 jumps to D_1^+. For D_1^+, we have the following two cases to discuss:

(a)′ if $y_{D_1^+} \le y_{A_1}$, then Model (1) exists as a positive periodic solution whose initial point is between P_1 and A_1 (see Figure 5(b));

(b)′ if $y_{D_1^+} > y_{A_1}$, then the trajectory starting from $\Omega_0 = \{(x,y) : (x,y) \in AP_1\widehat{BCDA}\}$ will tend to equilibrium and the trajectory starting from $\Omega_2 \setminus \Omega_0$ will ultimately stay in Ω_1.

(iv) If $y_{P_2} < y_{D^+} < y_{P_1}$, then the trajectory starting from $\Omega_0 = \{(x,y) : (x,y) \in PA_1\widehat{B_1O_1}O_2P\}$ $(O_i(h_i, 0)$ $(i = 1, 2))$ will tend to equilibrium and the trajectory starting from $\Omega_2 \setminus \Omega_0$ will ultimately stay in Ω_1.

Theorem 8. *Assume that* $h_1 < x_B < (1-\alpha)h_2 < x^* < h_2$. *If* $y_{D^+} \le y_{P_2}$ *or* $y_{D^+} \ge y_{P_1}$, $x_E \ge h_1$ *or* $y_{D^+} > y_{P_1}$, $x_E < h_1$, $y_{D_1^+} \le y_{A_1}$, *then Model* (1) *exists as a positive periodic solution.*

Similarly, for the case $h_1 < (1-\alpha)h_2 < x^* < h_2$ with $x_B < h_1$ or $(1-\alpha)h_2 < h_1 < x^* < h_2$, one can prove there is no positive periodic solution; here we omit it.

Remark 9. The positive periodic solutions for Model (1) obtained in Theorem 5~Theorem 8 are generated by impulses. Here, we say that a solution is generated by impulses if this solution is nontrivial when impulsive effect exists, but it is trivial when there does not exist impulsive effect. For example, when $r_1a_{22} - r_2a_{12} > 0$, by Lemma 1 we know that Model (2) does not possess any positive periodic solution; then positive periodic solutions of Model (1) under state-dependent impulsive conditions are called positive periodic solution generated by impulses.

Remark 10. As we know, the previous papers concerning state-dependent impulsive effect all assumed that the predator just lived on the prey; here we point out that the predator has some other food resources; this is more practical. On the other hand, the existing state-dependent impulsive differential systems mainly discussed the properties of solutions, including existence, uniqueness, and orbitally asymptotical stability. Here, not aiming at the properties of solutions, we

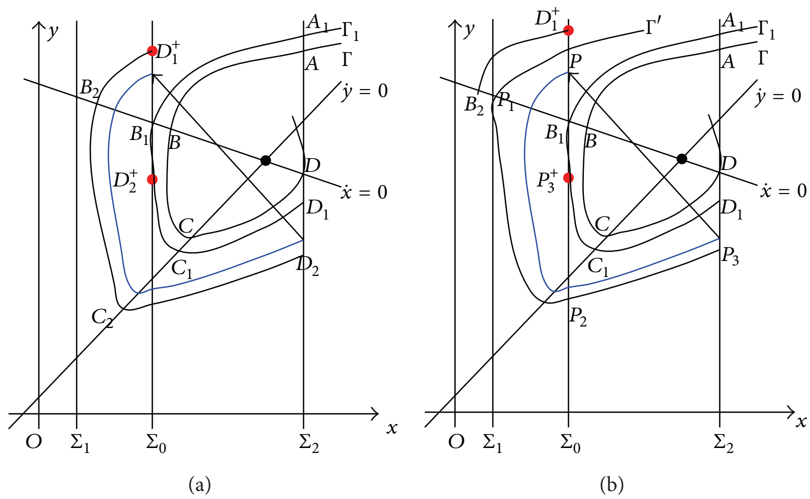

Figure 4: The possible trajectories in the case of $h_1 < (1-\alpha)h_2 < x_B < x^* < h_2$.

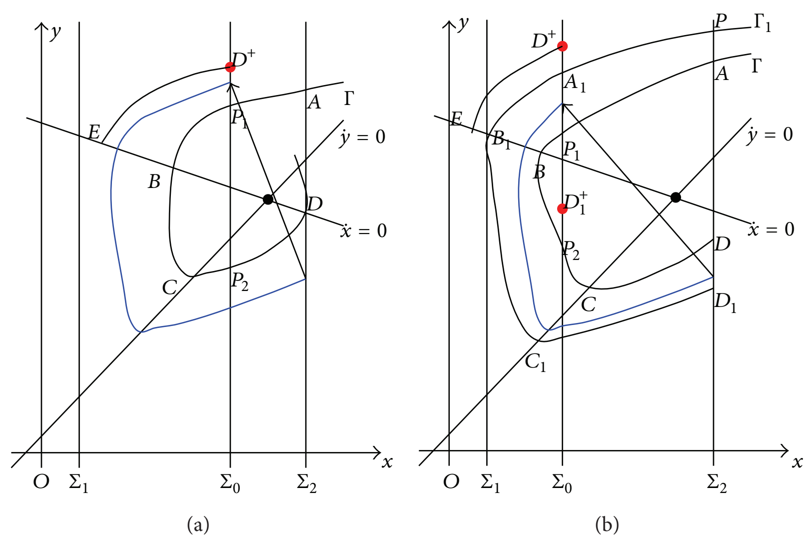

Figure 5: The possible trajectories in the case of $h_1 < x_B < (1-\alpha)h_2 < x^* < h_2$.

are focused on considering the influence of impulsive effect on the system itself. The theoretical results imply that if impulses do not exist, then the predator and prey species will tend to a point; if impulsive effect occurs, then the predator and prey species will be maintained at a periodic oscillation; that is, both the densities of these two species can change periodically. Therefore, our results demonstrate that impulsive effect takes an important role in ecological system.

4. Simulations and Conclusions

In this paper, we propose and analyse a state-dependent impulsive predator-prey model in which the predator species display a logistic growth. By using geometrical analysis methods, the existence of positive periodic solutions of Model (1) is given. Here we should point out that the positive periodic solutions are generated by impulses. For system (2), which does not exist as impulsive effect, the interior equilibrium is globally asymptotically stable, the phase trajectory and time series chart can be seen in Figures 6 and 7; therefore, system (2) does not exist as positive periodic solution and all the phase trajectories will tend to the interior equilibrium. When the impulsive effects are operated, system (1) can be gotten, the theoretical results demonstrate that system (1) exists as positive periodic solutions for some cases, and the numerical simulations also illustrate the existence of the periodic solutions; please see Figures 8, 9, and 10. Here $r_1 = 0.4$, $r_2 = 0.1$, $a_{11} = 0.6$, $a_{22} = 0.6$, $a_{12} = 0.6$, and $a_{21} = 0.4$. Therefore, the positive periodic solutions are generated by impulses.

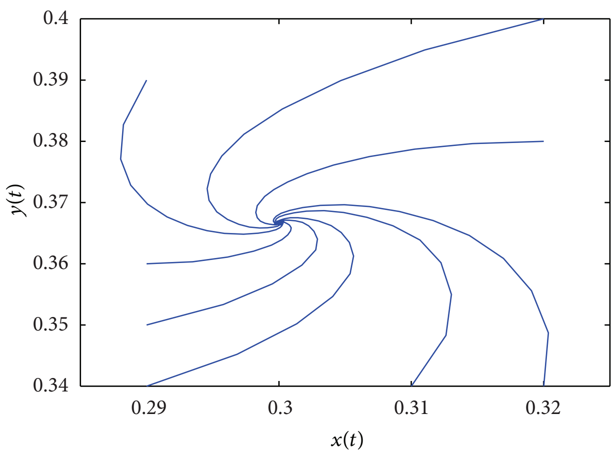

Figure 6: The phase trajectory without impulse.

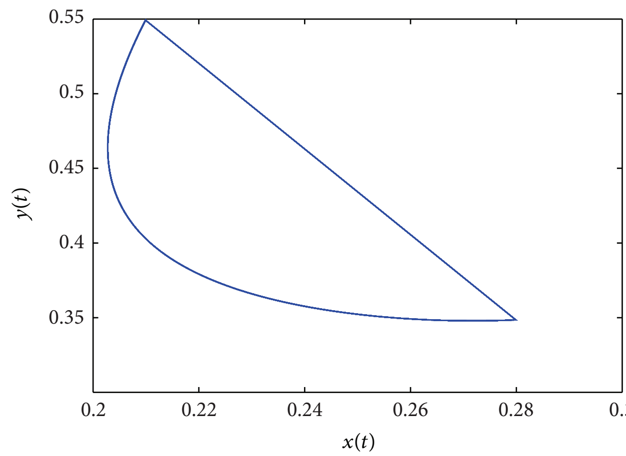

Figure 9: Plot of the positive periodic solution when $h_1 < (1-\alpha)h_2 < h_2 < x^*$ with $h_1 = 0.2$, $h_2 = 0.28$, $\alpha = 0.25$, $\beta = 0.2$, and $\tau_2 = 0.27$.

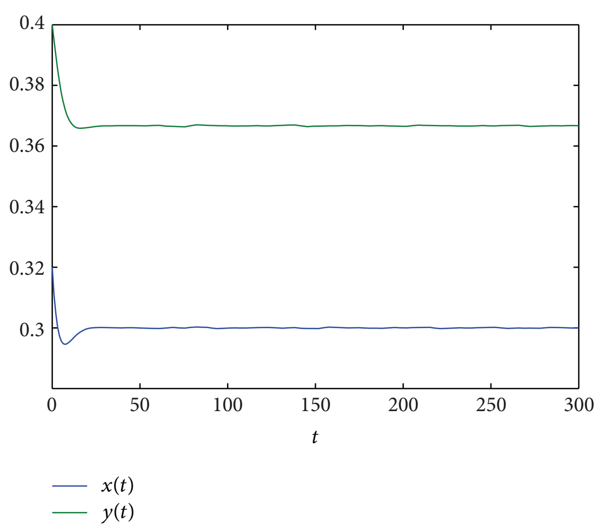

Figure 7: The time series chart for x and y without impulse.

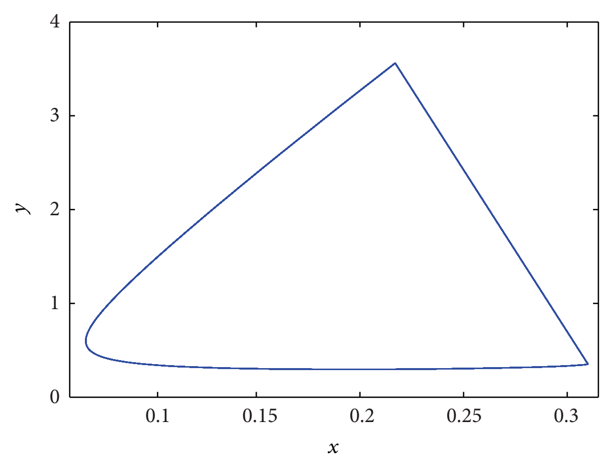

Figure 10: Plot of the positive periodic solution when $h_1 < (1-\alpha)h_2 < x^* < h_2$ with $h_1 = 0.05$, $h_2 = 0.31$, $\alpha = 0.3$, $\beta = 0.2$, and $\tau_2 = 3.28$.

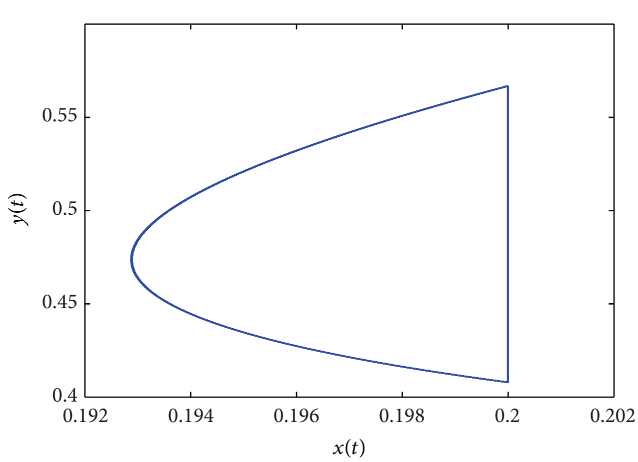

Figure 8: Plots of the positive periodic solution when $(1-\alpha)h_2 < h_1 < h_2 < x^*$ with $h_1 = 0.2$, $h_2 = 0.28$, $\alpha = 0.4$, $\beta = 0.2$, and $\tau_1 = 0.16$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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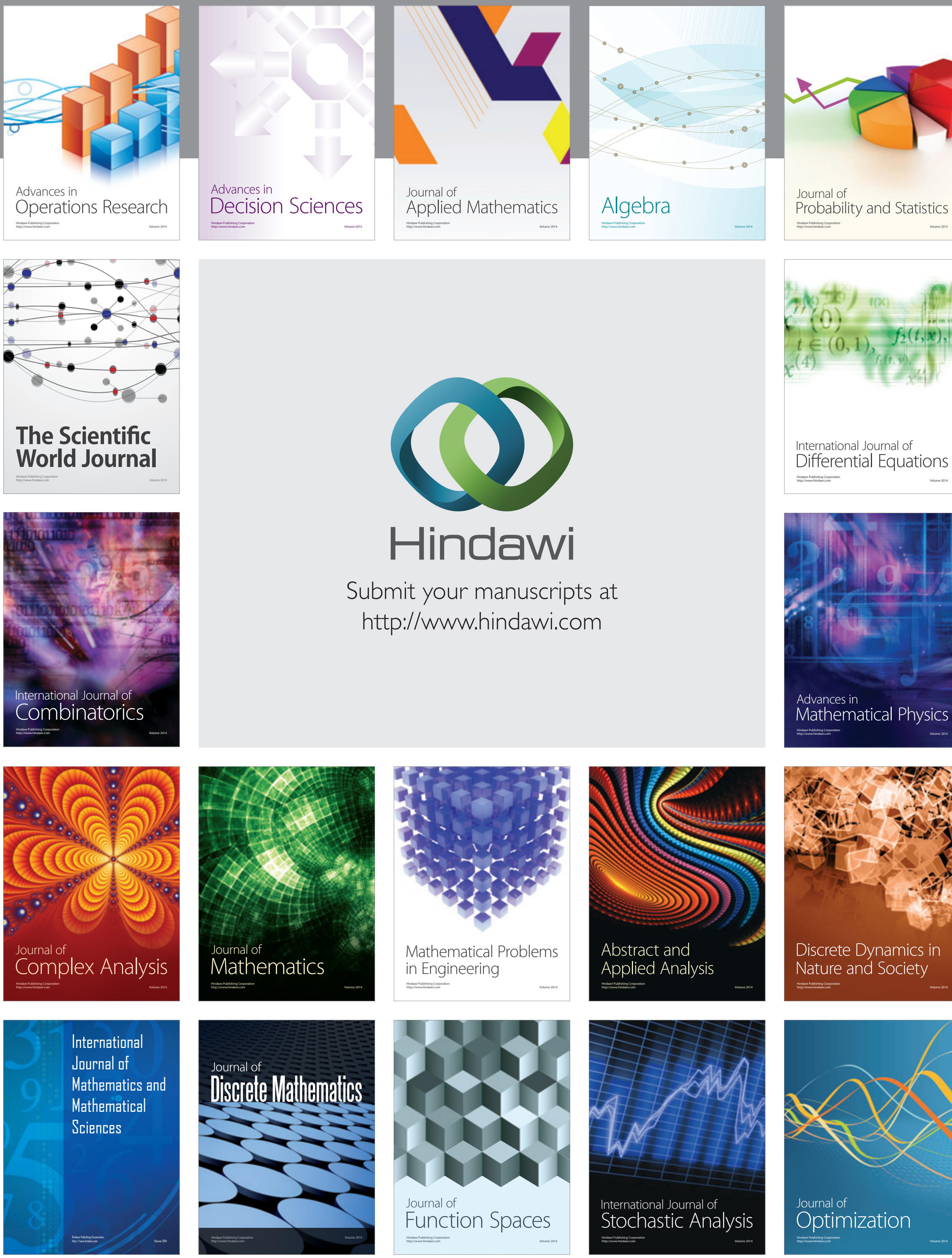
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