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Research Article

Cluster Synchronization of Stochastic Complex Networks with Markovian Switching and Time-Varying Delay via Impulsive Pinning Control

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This paper studies the cluster synchronization of a kind of complex networks by means of impulsive pinning control scheme. These networks are subject to stochastic noise perturbations and Markovian switching, as well as internal and outer time-varying delays. Using the Lyapunov-Krasovskii functional, Itö's formula, and some linear matrix inequalities (LMI), several novel sufficient conditions are obtained to guarantee the desired cluster synchronization. At the end of this writing, a numerical simulation is given to demonstrate the effectiveness of those theoretical results.

1. Introduction and Model Description

Complex networks can be used to describe properly many biological, social, and communication systems and are made up of a great number of nodes representing individuals or organizations and links that are employed to mimic the interactions among them [1]. In particular, interesting examples are the spatiotemporal chaos [2], the Internet [3, 4], spiral waves [5], and the World Wide Web [6]. Synchronization is a universal phenomenon in nature, has potential applications significantly in real-world dynamical systems, and can be understood as the adjustment of coherence or rhythms of all states through interaction [7, 8]. Hitherto, a lot of different synchronization patterns have been investigated such as complete synchronization [9], phase synchronization [10], partial synchronization [11], and cluster synchronization [12]. Actually, the synchronization patterns can be nearly contained by a uniform definition.

In brief, the cluster synchronization can be understood as the dynamical nodes synchronize each other in each group, while there is no synchronization between any two different groups. A large number of real-world networks exhibit community structure, for example, communication engineering and biological science [13]. Cluster synchronization in two-dimensional and three-dimensional lattices of diffusively coupled chaotic oscillators was discussed in [14]. Cluster synchronization in a kind of strictly semipassive complex networks by means of diffusive coupling was focused on in [15]. In [16], Ma et al., under the help of a special coupling matrix for some connected chaotic networks, expressed that the randomly selected cluster synchronization manifolds could be stabilized. A novel and effective approach was provided to reach cluster synchronization of complex network and ensure their stability for a given nearest neighborhood network with zero-flux or periodic boundary conditions. In a few words, cluster synchronization, due to its significant practical implications, was considered widely and deeply by many researchers from various fields.

Moreover, owning to uncertain interferences caused by man-made and particularly natural factors, perturbation is a very universal and vital property in many real systems. There are lots of previous works contributing to studying stochastic phenomena of complex networks. For instance, in [17], the authors considered the case where only a single node was affected by stochastic perturbations. Reference [18] talked over an array of networks with scalar Wiener processes, which implied that each node is influenced by the same noise. Apparently, this assumption might be a little unrealistic for the real-world networks which were commonly influenced by different multidimensional perturbations. In [19], the writers discussed a class of coupled neural networks with stochastic noise and intermittent control. Also, their Wiener processes were of a vector form that can be viewed as an improvement of [18]. Consequently, the complex networks should be and must be brought in stochastic perturbations.

Recently, more and more researchers turn their attention to switched system in control theory domain. Generally speaking, a switched system includes two basic elements at least: one is several dynamical subsystems and the other one is a switching law used to specify some active subsystem at each instant of time. In view of different characteristics of complex networks, switching law also has several distinct patterns. For example, [20] probed into a blinking network whose switching couplings were according to certain probability. The writing pointed out that this kind of networks could synchronize for nearly all instances of the fast random switching process. As a matter of fact, from the mathematical point of view, switching signals between different network models can be governed by a Markovian chain. Recurring to linear matrix inequality, [21] considered the synchronization of a class of complex networks with Markovian jumping parameters whose coupling configuration was not dependent on mode switching. In addition, the synchronization issue in an array of neural networks with mixed delays and stochastic hybrid jumping couplings was investigated in [22] by adaptive control scheme. Nevertheless, if all nodes placed restriction on sharing common time-delay in a Markovian chain, this will lead to unpractical results. However, as far as I am concerned, the result on synchronization of stochastic networks with Markovian switching and time-varying delay coupling is seldom.

In view of the preceding discussion, this paper will focus on the issue of the synchronization of stochastic complex networks with Markovian switching and time-varying delay coupling by impulsive pinning control method. By the aid of the Lyapunov-Krasovskii functional method and some common linear matrix inequality in this field, several valid sufficient conditions will be obtained to ensure the cluster synchronization.

This paper is organized as follows. In Section 2, we introduce the generally considered model for a stochastic complex network. Moreover, some preliminary definitions and theorems needed for the rest of the paper are also provided. Section 3 shows some synchronization criteria for the discussed complex networks by logic and pleasing proof. In Section 4, three numerical simulations are given to illustrate our theoretical results. Ultimately, the paper concludes in Section 5.

Notations. In this paper, the superscript T will denote the transpose of a matrix or a vector. $\mathbb{R}^n$ will denote the n-dimensional Euclidean space and $\mathbb{R}^{n\times n}$ the set of all $n\times n$ real matrices. $\mathbf{1}_n = (1,1,\ldots,1)^T \in \mathbb{R}^n$, and I_n is an n-dimensional identity matrix. $\mathrm{Tr}(\cdot)$ stands for the trace of the corresponding matrix. For square matrix M, the notation $M>0$ (<0) will mean that M is a positive-definite (negative-definite) matrix. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ will denote the greatest and least eigenvalues of a symmetric matrix, respectively, and $\check{p}=\max\{p_1,p_2,\ldots,p_n\}$, $\hat{p}=\min\{p_1,p_2,\ldots,p_n\}$.

2. Preliminaries

Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ that is right continuous with $\mathcal{F}_0$ including all the $\mathcal{P}$-null sets. $C([-\tau,0];\mathbb{R}^n)$ will denote the family of continuous functions ϕ from $[-\tau,0]$ to $\mathbb{R}^n$ with the uniform norm $\|\phi\|^2 = \sup_{-\tau\leq s\leq 0}\phi(s)^T\phi(s)$. And $C^2_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ denote the family of all $\mathcal{F}_0$ measurable, $C([-\tau,0];\mathbb{R}^n)$-valued stochastic variables $\xi = \{\xi(t) : -\tau \leq t \leq 0\}$ such that $\int_{-\tau}^{0}\mathbb{E}|\xi(t)|^2 dt \leq \infty$, where $\mathbb{E}$ stands for the correspondent expectation operator with respect to the given probability measure $\mathcal{P}$.

As follows, some definitions, lemmas, and notations are revealed which will be used throughout this paper.

Definition 1. If $A=[a_{ij}]\in R^{m\times m}$ is an irreducible matrix such that $a_{ij}=a_{ji}\geq 0$ for all $i\neq j$ and $\sum_{j=1}^{m}a_{ij}=0$ for all $i=1,2,\ldots,m$, one says that $A\in\mathbf{A_1}$.

Definition 2 (see [16, 23]). Let

$$A=\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1d}\\ A_{21} & A_{22} & \cdots & A_{2d}\\ \vdots & \vdots & \ddots & \vdots\\ A_{d1} & A_{d2} & \cdots & A_{dd}\end{bmatrix} \quad (1)$$

be an $m\times m$ symmetric matrix such that $A_{uv}\in R^{k_u\times k_v}$, $u,v=1,2,\ldots,d$. If each block A_{uv} is a zero-row-sum matrix, one says that $A\in\mathbf{M_1}$. Furthermore, if $A_{uu}\in\mathbf{A_1}$, $u=1,2,\ldots,d$, one says that $A\in\mathbf{M_2}$. It also follows from the symmetry of A that $A_{uv}=A_{vu}^T$, A_{uv}, A_{vu} are zero-row-sum matrices and A_{uv} is a zero-column-sum matrix.

Remark 3. Definition 2 defines a diffusively coupled matrix; if $a_{ij}>0$ for $i\neq j$, then the coupling between i and j is called cooperative coupling; if $a_{ij}<0$ for $i\neq j$, then the coupling between i and j is called competitive coupling [16].

Before discussing the synchronization of the considered complex networks, we will introduce the mathematical definition of cluster synchronization.

Definition 4 (see [24]). Let $G_1=\{1,2,\ldots,k_1\}$, $G_2=\{k_1+1,k_1+2,\ldots,k_1+k_2\},\ldots,G_d=\{k_1+k_2+\cdots+k_{d-1}+1,\ldots,k_1+k_2+\cdots+k_d\}$ be a partition of the set $G=\{1,2,\ldots,m\}$ for $1<d<m$, $1<k_l<m$, and $\sum_{l=1}^{d}k_l=m$. Moreover for every $i\in G$, let $\check{i}$ be the counting index of the subset in which the number is i; that is, $i\in G_{\check{i}}$. A network with m nodes is said to realize cluster synchronization with partition $\{G_1,G_2,\ldots,G_d\}$, if the state variables of the nodes satisfy $\lim_{t\to+\infty}\|x_i(t)-x_j(t)\|=0$ for $\check{i}=\check{j}$ and $\lim_{t\to+\infty}\|x_i(t)-x_j(t)\|\neq 0$ for $\check{i}\neq\check{j}$ for all initial values.

The state equation of the switched network consisting of N identical nodes without delay and time-varying delay coupling and Markovian jumping are given as follows:

$$\begin{aligned} dx_i(t) = \Bigg\{ & f_{\check{i}}(t, x_i(t), x_i(t-\tau(t))) \\ & + \sum_{j=1, i\neq j}^{N} a_{ij}^{(r(t))} \Sigma \left(x_j(t) - x_i(t) \right) \\ & + \sum_{j=1, i\neq j}^{N} b_{ij}^{(r(t))} \Sigma \left(x_j(t-\tau_c(t)) \right. \\ & \left. - x_i(t-\tau_c(t)) \right) \Bigg\} dt \\ & + \sigma_i(t, x(t), x(t-\tau(t)), x(t-\tau_c(t)), r(t)) \\ & \times dw_i(t), \quad i = 1, 2, \ldots, N, \end{aligned} \tag{2}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ is the state vector of the ith node of the network, $f(t, x_i(t), x_i(t-\tau(t))) = [f_1(t, x_i(t), x_i(t-\tau(t))), f_2(t, x_i(t), x_i(t-\tau(t))), \ldots, f_n(t, x_i(t), x_i(t-\tau(t)))]^T$ is a continuous vector-valued function, and $r(t)$ are the continuous-time Markov processes that describe the evolution of the modes at time t. $\Sigma = \mathrm{diag}(\varrho_1, \varrho_2, \ldots, \varrho_n)$ is an inner coupling matrix of the networks that satisfies $\varrho_j > 0$, $j = 1, 2, \ldots, n$. Here, $A^{(r(t))} = [a_{ij}^{(r(t))}] \in \mathbf{M_1}$ and $B^{(r(t))} = [b_{ij}^{(r(t))}] \in \mathbf{M_1}$ are the outer coupling matrices of the network at time t. $\tau(t)$ is the inner time-varying delay satisfying $\tau \geq \tau(t) \geq 0$ and $\tau_c(t)$ is the coupling time-varying delay satisfying $\tau_c \geq \tau_c(t) \geq 0$. Finally, $\sigma_i(t, x(t), x(t-\tau(t)), x(t-\tau_c(t)), r(t)) = \sigma_i(t, x_1(t), \ldots, x_n(t), x_1(t-\tau(t)), \ldots, x_n(t-\tau(t)), x_1(t-\tau_c(t)), \ldots, x_n(t-\tau_c(t)), r(t)) \in \mathbb{R}^{n\times n}$ and $w_i(t) = (w_{i1}(t), w_{i2}(t), \ldots, w_{in}(t))^T \in \mathbb{R}^n$ is a bounded vector-form Wiener process, satisfying

$$\begin{gathered} \mathbb{E}\left[w_{ij}(t)\right] = 0, \qquad \mathbb{E}\left[w_{ij}^2(t)\right] = 1, \\ \mathbb{E}\left[w_{ij}(t) w_{ij}(s)\right] = 0 \quad (s \neq t). \end{gathered} \tag{3}$$

Suppose that $r(t)$ $(t > 0)$ is a right-continuous Markov chain on a probability space that takes on values in a finite space $S = 1, 2, \ldots, M$ whose generator $\Gamma = [\gamma_{ij}] \in \mathbb{R}^{M\times M}$ is given by

$$P\{r(t+\Delta) = j \mid r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & \text{if } i \neq j, \\ 1 + \gamma_{ii}\Delta + o(\Delta) & \text{if } i = j, \end{cases} \tag{4}$$

for some $\Delta > 0$, where $\gamma_{ij} = 0$ is the transition speed from i to j if $i \neq j$ and $\gamma_{ii} = -\sum_{i\neq j} \gamma_{ij}$. By the way, this writing requires that $A^{(r(t))}$ is irreducible.

The following are some initial conditions for (2) described by

$$x_i(s) = \xi_i(s), \quad -\check{\tau} \leq s \leq 0, \ i = 1, 2, \ldots, N, \tag{5}$$

where $\check{\tau} = \max\{\tau(t), \tau_c(t)\}$, $\xi_i \in C_{\mathscr{F}_0}^b([-\check{\tau}, 0], \mathbb{R}^n)$ with the norm $\|\xi_i\|^2 = \sup_{-\check{\tau}\leq s\leq 0} \xi_i(s)^T \xi_i(s)$.

In this paper, we will make use of the impulse pinning controllers as follows:

$$\begin{gathered} \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = \epsilon_{ik} x_i(t_k^-) - s(t_k), \\ t = t_k, \qquad k \in Z^+, \qquad i = 1, 2, \ldots, N, \end{gathered} \tag{6}$$

where ϵ_{ik} are constants and $l < N$.

It is worth noting that system (2) achieves synchronization, that is, $x_i(t) = s_{\check{i}}(t)$, which implies that we have the following synchronized state equation:

$$ds_{\check{i}}(t) = f_{\check{i}}(t, s_{\check{i}}(t), s_{\check{i}}(t-\tau(t))) \, dt. \tag{7}$$

Let $e_i(t) = x_i(t) - s_{\check{i}}(t)$ $(i = 1, 2, \ldots, N)$ be the synchronization errors. Then, the error system according to controller (6) can be written as

$$\begin{aligned} de_i(t) = \Bigg\{ & f_{\check{i}}(t, x_i(t), x_i(t-\tau(t))) - f_{\check{i}}(t, s_{\check{i}}(t), s_{\check{i}}(t-\tau(t))) \\ & + \sum_{j=1, i\neq j}^{N} a_{ij}^{(r(t))} \Sigma e_j(t) + \sum_{j=1}^{N} b_{ij}^{(r(t))} \Sigma e_j(t-\tau_c(t)) \Bigg\} dt \\ & + \sigma_i^{r(t)}(t, e(t), e(t-\tau(t)), e(t-\tau_c(t))) \, dw_i(t), \\ & t \neq t_k, \qquad k \in Z^+, \qquad i = 1, 2, \ldots, N. \\ \Delta e_i(t_k) = {} & \epsilon_{ik} e_i(t_k^-), \quad t = t_k, \ k \in Z^+, \ i = 1, 2, \ldots, N. \end{aligned} \tag{8}$$

Definition 5. The complex network (2) is said to reach cluster synchronization when the trivial solution of system (8) satisfies the inequality

$$\sum_{i=1}^{N} \mathbb{E}\|e_i(t, t_0, \xi_i)\|^2 \leq K e^{-\kappa t}, \tag{9}$$

for some positive constants K and κ under any initial data $\xi_i \in \mathscr{C}_{\mathscr{F}_0}^b([-\tau, 0]; \mathbb{R}^n)$.

Definition 6 (see [21, 25]). A continuous function $f_{\check{i}}(t, x, y) : [0, +\infty] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is said to belong to the function class QUAD. denoted by $f \in \mathrm{QUAD}(P, \Delta, \eta_{\check{i}}, \zeta_{\check{i}})$, for some given matrix $\Sigma = \mathrm{diag}\{\varrho_1, \varrho_2, \ldots, \varrho_n\}$, if there exist a positive-definite diagonal matrix $P = \mathrm{diag}\{p_1, p_2, \ldots, p_n\}$, a diagonal matrix $\Delta = \mathrm{diag}\{\delta_1, \delta_2, \ldots, \delta_n\}$, and constants $\eta_{\check{i}} > 0$, $\zeta_{\check{i}} > 0$ such that $f_{\check{i}}(\cdot)$ satisfies the condition

$$\begin{aligned} & (x-y)^T P \left((f_{\check{i}}(t, x, z) - f_{\check{i}}(t, y, w)) - \Delta\Sigma(x-y) \right) \\ & \quad \leq -\eta_{\check{i}} (x-y)^T (x-y) + \zeta_{\check{i}} (z-w)^T (z-w) \end{aligned} \tag{10}$$

for all $x, y, z, w \in \mathbb{R}^n$.

Remark 7 (see [25]). The function class QUAD includes almost all the well-known chaotic systems with or without delays such as the Lorenz system, the Rössler system, the Chen system, the delayed Chua circuit, the logistic delayed differential system, the delayed Hopfield neural network, and the delayed CNNs.

The following assumptions will be used to establish the synchronization conditions.

(H1) $\tau(t)$ and $\tau_c(t)$ are bounded and continuously differentiable functions such that $0 < \tau(t) \leq \tau$, $\dot{\tau}(t) < \overline{\tau} < 1$, $0 < \tau_c(t) \leq \tau_c$, and $\dot{\tau}_c(t) < \overline{\tau}_c < 1$. Let $\check{\overline{\tau}} = \max\{\overline{\tau}, \overline{\tau}_c\}$.

(H2) Suppose that $\sigma(t, e(t), e(t-\tau(t)), e(t-\tau_c(t)), r) = \sigma(t, e_1(t), \ldots, e_N(t), e_1(t-\tau(t)), \ldots, e_N(t-\tau(t)), e_1(t-\tau_c(t)), \ldots, e_N(t-\tau_c(t)), r)$. Thereby, there are some positive-definite constant matrices Υ_{i1}^r, Υ_{i2}^r, and Υ_{i3}^r for $i = 1, 2, \ldots, N$ and $r = 1, 2, \ldots, M$ such that

$$\begin{aligned} \operatorname{Tr}\Big[&\sigma_i(t, e(t), e(t-\tau(t)), e(t-\tau_c(t)), r)^T \\ &\times \sigma_i(t, e(t), e(t-\tau(t)), e(t-\tau_c(t)), r)\Big] \\ \leq &\sum_{j=1}^{N} e_j(t)^T \Upsilon_{i1}^r e_j(t) + \sum_{j=1}^{N} e_j(t-\tau(t))^T \Upsilon_{i2}^r e_j(t-\tau(t)) \\ &+ \sum_{j=1}^{N} e_j(t-\tau_c(t))^T \Upsilon_{i3}^r e_j(t-\tau_c(t)). \end{aligned} \tag{11}$$

Lemma 8 (see [26]). *Consider a stochastic delayed differential equation with Markovian switching expressed as*

$$\begin{aligned} dx(t) = {} & f(t, x(t), x(t-\tau), r(t))\,dt \\ & + \sigma(t, x(t), x(t-\tau), r(t))\,d\omega(t) \end{aligned} \tag{12}$$

on $t \geq 0$ with initial value $x_0 = \xi \in C_{F_0}^b([-\tau, 0]; \mathbb{R}^n)$, where

$$f : \mathbb{R}^n \times \mathbb{R}_+ \times S \longrightarrow \mathbb{R}^n, \qquad \sigma : \mathbb{R}^n \times \mathbb{R}_+ \times S \longrightarrow \mathbb{R}^{n\times m}. \tag{13}$$

Let $C^{2,1}(\mathbb{R}_+ \times \mathbb{R}^n; \mathbb{R}_+)$ be the family of all the nonnegative functions $V(t, x, r)$ on $\mathbb{R}_+ \times \mathbb{R}^n \times S$ which are twice continuously differentiable in x and once differentiable in t. Let $V \in C^{2,1}(\mathbb{R}_+ \times \mathbb{R}^n \times S; \mathbb{R}_+)$. Define an operator $\mathscr{L}V$ from $\mathbb{R}^n \times \mathbb{R}_+ \times S$ to $\mathbb{R}^n$ by

$$\begin{aligned} \mathscr{L}V(t, x, r) = {} & V_t(t, x, r) + V_x(t, x, r) f(t, x, r) \\ & + \frac{1}{2} \operatorname{Tr}\left[\sigma(t, x, r)^T V_{xx} \sigma(t, x, r)\right] \\ & + \sum_{j=1}^{M} \gamma_{ij} V(t, x, j), \end{aligned} \tag{14}$$

where $V_t(t, x, r) = \partial V(t, x, r)/\partial t$, $V_x(t, x, r) = (\partial V(t, x, r)/\partial x_1, \ldots, \partial V(t, x, r)/\partial x_n)$, and $V_{xx}(t, x, r) = (\partial^2 V(t, x, r)/\partial x_i x_j)_{n\times n}$. If $V \in C^{2,1}(\mathbb{R}_+ \times \mathbb{R}^n \times S; \mathbb{R}_+)$, then

$$\mathbb{E}V(t, x(t), r) = \mathbb{E}V(t_0, x(t_0), r) + \mathbb{E}\int_{t_0}^{t} \mathscr{L}V(s, x(s), r)\,ds \tag{15}$$

for all $\infty > t > t_0 \geq 0$ as long as the expectations of the integrals exist.

3. Main Result

This section will show our main results with rigorous proof.

Theorem 9. *Suppose that (H1) and (H2) are true and $f_{\check{i}} \in \text{QUAD}(P, \Delta, \eta_{\check{i}}, \zeta_{\check{i}})$. If there are some positive constants θ, α_r, and β_r such that*

$$\begin{bmatrix} A(r)^s + \check{\delta} I_N - \alpha_r I_N & \dfrac{B(r)}{2} \\ \dfrac{B(r)^T}{2} & -\beta_r I_N \end{bmatrix} \leq 0, \quad \text{for } r = 1, 2, \ldots, M, \tag{16}$$

$$0 \leq \check{\overline{\tau}} \leq 1 - \frac{\check{q}(\check{b} + \check{c})}{1 + \theta}, \tag{17}$$

$$\varphi(\check{\tau} + T) + 2 \ln \frac{\check{q}}{\hat{q}} \left|1 + \epsilon_v\right| - \gamma T < 0, \tag{18}$$

$$\left(\frac{1}{b_1 + c_1}, \frac{1}{b_2 + c_2}, \ldots, \frac{1}{b_M + c_M}\right)^T > \widetilde{\Gamma}^{-1} \mathbf{1}_M, \tag{19}$$

where

$$\begin{aligned} &\varphi = 1 + \theta + \gamma \check{q}_r + \frac{\check{b}\check{q}}{1 - \overline{\tau}} e^{\gamma\tau} + \frac{\check{c}\check{q}}{1 - \overline{\tau}_c} e^{\gamma\tau_c}, \\ &\widetilde{\Gamma} = \operatorname{diag}\{a_1, a_2, \ldots, a_M\} + \Gamma, \\ &a_r = \frac{\lambda_{\max}\left(-2\eta_{\check{i}} I_N + \check{p} \sum_{j=1}^{N} \Upsilon_{j1}^r + 2\alpha_r P\Sigma\right)}{\check{p}}, \quad \check{a} = \max_{r\in S, i=1,\ldots,N} a_r, \\ &b_r = \frac{\lambda_{\max}\left(\sum_{j=1}^{N} P\Upsilon_{j2}^r + 2\zeta_{\check{i}} I_N\right)}{\hat{p}}, \quad \check{b} = \max_{r\in S, i=1,\ldots,N} b_r, \\ &c_r = \frac{\lambda_{\max}\left(\sum_{j=1}^{N} P\Upsilon_{j3}^r + 2\beta_r P\Sigma\right)}{\hat{p}}, \quad \check{c} = \max_{r\in S} c_r, \end{aligned} \tag{20}$$

then the global and exponential synchronization of the stochastic complex network (8) can be achieved.

Proof. Inequality (19) indicates that there is a sufficiently small constant $\theta > 0$ such that

$$\left(\frac{1}{b_1 + c_1}, \frac{1}{b_2 + c_2}, \ldots, \frac{1}{b_M + c_M}\right)^T \geq (1 + \theta) \widetilde{\Gamma}^{-1} \mathbf{1}. \tag{21}$$

Let $(1 + \theta)\widetilde{\Gamma}^{-1}\mathbf{1} = q = (q_1, q_2, \ldots, q_M)^T$. Thus

$$\widetilde{\Gamma} q = (1 + \theta) \mathbf{1}_M; \tag{22}$$

namely,

$$(b_r + c_r) q_r \leq 1 \quad \text{while } a_r q_r + \sum_{s=1}^{M} \gamma_{rs} q_s = 1 + \theta. \tag{23}$$

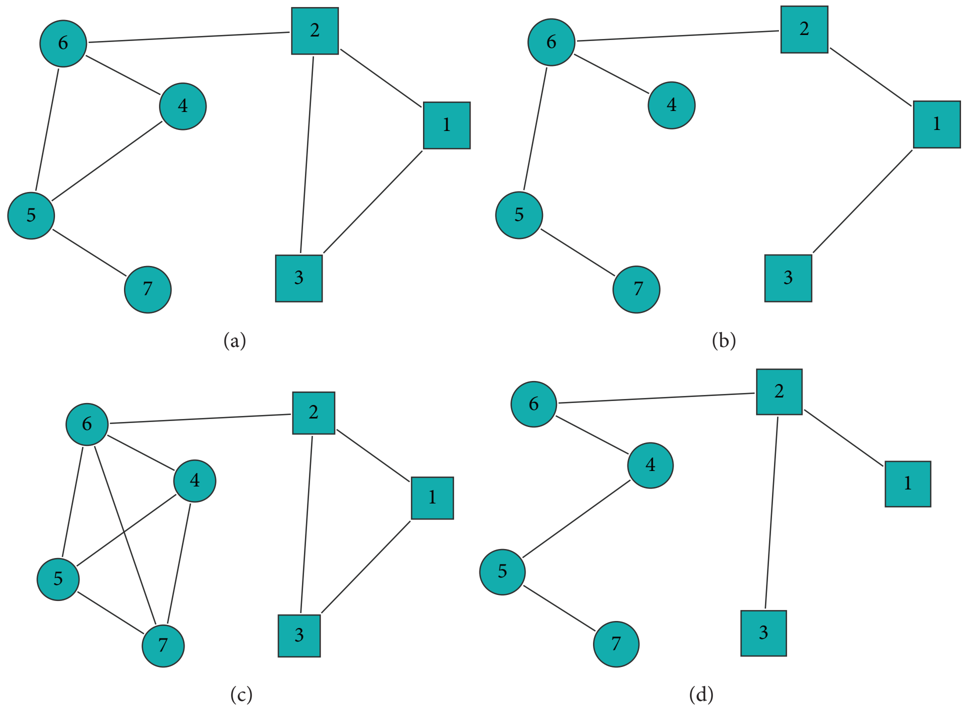


FIGURE 1: The topology structures of the switching networks that have 7 nodes divided into 2 clusters (nodes $\{1, 2, 3\}$ and nodes $\{4, 5, 6, 7\}$). (a) and (c) show the topology structures of the coupling matrices $A(1)$ and $A(2)$, respectively; (b) and (d) express the topology structures of the coupling matrices $B(1)$ and $B(2)$, respectively.

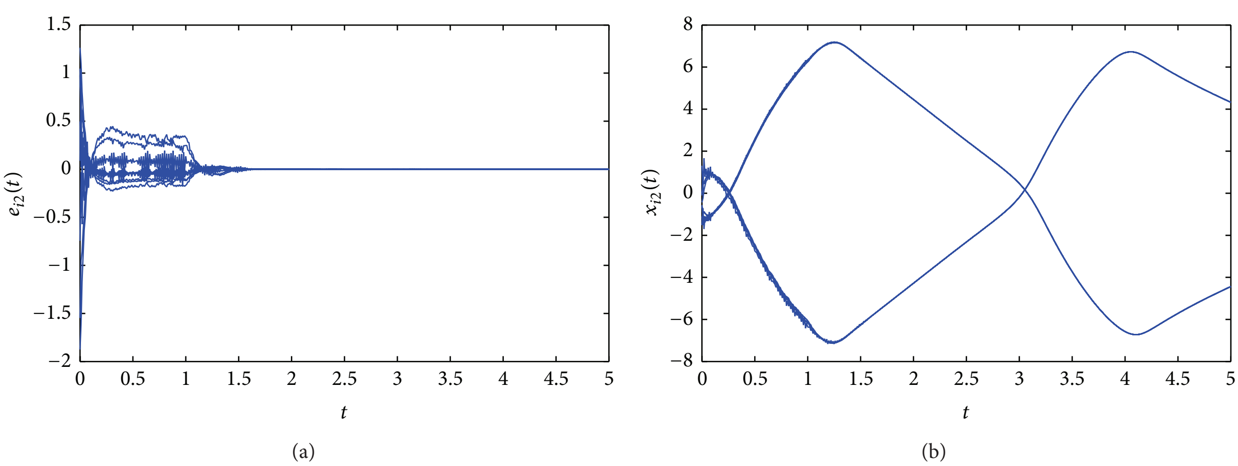


FIGURE 2: The trajectories of the state variables of x_{i1} and x_{i2} $(i = 1, 2, \ldots, 7)$ in system (46) by impulse control.

For $1 \le r \le M$, define the Lyapunov-Krasovskii function

$$V(t, e(t), r) = \frac{1}{2} q_r \sum_{i=1}^{N} e_i(t)^T P e_i(t) \tag{24}$$

and let $\tilde{e}^k(t) = (e_{1k}(t), e_{2k}(t), \ldots, e_{Nk}(t))^T$, $k = 1, 2, \ldots, n$. By Lemma 8, for any $t \in [t_{k-1}, t_k)$, $k = 1, 2, \ldots,$ we have

$$\begin{aligned}
&\mathscr{L}V(t, e(t), r) \\
&= q_r \sum_{i=1}^{N} e_i(t)^T P \Bigg\{ f_{\check{i}}(t, x_i(t), x_i(t - \tau(t))) \\
&\quad - f_{\check{i}}(t, s_{\check{i}}(t), s_{\check{i}}(t - \tau(t))) \\
&\quad + \sum_{j=1}^{N} a_{ij}(r) \Sigma e_j(t) \\
&\quad + \sum_{j=1}^{N} b_{ij}(r) \Sigma e_j(t - \tau_c(t)) \Bigg\} \\
&\quad + \frac{1}{2} q_r \sum_{i=1}^{N} \operatorname{Tr} \{\sigma_i(t, x(t), x(t - \tau(t)), x(t - \tau_c(t)), r)^T
\end{aligned}$$

$$
\begin{aligned}
&\times P\sigma_i\left(t, x(t), x(t-\tau(t)), x\left(t-\tau_c(t)\right), r\right)\Big\}\\
&+\sum_{s=1}^{M}\gamma_{rs}q_s\frac{1}{2}\sum_{i=1}^{N}e_i(t)^T Pe_i(t)\\
&\le q_r\Bigg\{-\sum_{i=1}^{N}\eta_{\check{i}}e_i(t)^T e_i(t)+\sum_{i=1}^{N}\zeta_{\check{i}}e_i(t-\tau(t))^T e_i(t-\tau(t))\\
&\quad+\sum_{k=1}^{n}p_k\varrho_k\delta_k\vec{e}^k(t)^T\vec{e}^k(t)+\sum_{k=1}^{n}p_k\varrho_k\vec{e}^k(t)^T A(r)\vec{e}^k(t)\\
&\quad+\sum_{k=1}^{n}p_k\varrho_k\vec{e}^k(t)^T B(r)\vec{e}^k\left(t-\tau_c(t)\right)\\
&\quad+\frac{1}{2}\check{p}\sum_{j=1}^{N}\Bigg[\sum_{i=1}^{N}e_i(t)^T\Upsilon_{j1}^r e_i(t)\\
&\qquad+\sum_{i=1}^{N}e_i(t-\tau(t))^T\Upsilon_{j2}^r e_i(t-\tau(t))\\
&\qquad+\sum_{i=1}^{N}e_i\left(t-\tau_c(t)\right)^T\Upsilon_{j3}^r e_i\left(t-\tau_c(t)\right)\Bigg]\Bigg\}\\
&+\sum_{s=1}^{M}\gamma_{rs}q_s\frac{1}{2}\sum_{i=1}^{N}e_i(t)^T Pe_i(t)\\
&=q_r\Bigg\{\sum_{i=1}^{N}e_i(t)^T\left(-\eta_{\check{i}}I_N+\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j1}^r+\alpha_r P\Sigma\right)e_i(t)\\
&\quad+\sum_{i=1}^{N}e_i(t-\tau(t))^T\left(\zeta_{\check{i}I_N}+\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j2}^r\right)e_i(t-\tau(t))\\
&\quad+\sum_{i=1}^{N}e_i\left(t-\tau_c(t)\right)^T\left(\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j3}^r+\beta_r P\Sigma\right)\\
&\qquad\times e_i\left(t-\tau_c(t)\right)\Bigg\}\\
&+\sum_{s=1}^{M}\gamma_{rs}q_s\frac{1}{2}\sum_{i=1}^{N}e_i(t)^T Pe_i(t)\\
&+q_r\Bigg\{\sum_{k=1}^{n}p_k\varrho_k\vec{e}^k(t)^T\left[A(r)+\left(\check{\delta}-\alpha_r\right)I_N\right]\vec{e}^k(t)\\
&\quad+\sum_{k=1}^{n}p_k\varrho_k\vec{e}^k(t)^T B(r)\vec{e}^k\left(t-\tau_c(t)\right)\\
&\quad-\sum_{k=1}^{n}p_k\varrho_k\vec{e}^k\left(t-\tau_c(t)\right)^T\beta_r\vec{e}^k\left(t-\tau_c(t)\right)\Bigg\}\\
&\le q_r\Bigg\{\sum_{i=1}^{N}e_i(t)^T\left(-\eta_{\check{i}}I_N+\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j1}^r+\alpha_r P\Sigma\right)e_i(t)\\
&\quad+\sum_{i=1}^{N}e_i(t-\tau(t))^T\left(\zeta_{\check{i}}I_N+\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j2}^r\right)e_i(t-\tau(t))\\
&\quad+\sum_{i=1}^{N}e_i\left(t-\tau_c(t)\right)^T\left(\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j3}^r+\beta_r P\Sigma\right)\\
&\qquad\times e_i\left(t-\tau_c(t)\right)\Bigg\}\\
&+\sum_{s=1}^{M}\gamma_{rs}q_s\frac{1}{2}\sum_{i=1}^{N}e_i(t)^T Pe_i(t).
\end{aligned}
\tag{25}
$$

Set

$$E(t)=\frac{1}{2}\sum_{i=1}^{N}e_i(t)^T Pe_i(t). \tag{26}$$

And then we have

$$
\begin{aligned}
\mathscr{L}V(t)&\le a_r q_r E(t)+b_r q_r E(t-\tau(t))+c_r q_r E\left(t-\tau_c(t)\right)\\
&\quad+\sum_{s=1}^{M}\gamma_{rs}q_s E(t)
\end{aligned}
\tag{27}
$$

and (23) deduces that

$$\mathscr{L}V(t)\le(1+\theta)E(t)+\check{b}\check{q}E(t-\tau(t))+\check{c}\check{q}E\left(t-\tau_c(t)\right). \tag{28}$$

Define

$$W(t)=e^{\gamma t}V(t). \tag{29}$$

By the aid of (28), we will deal with the operator

$$
\begin{aligned}
\mathscr{L}W(t)&=e^{\gamma t}\left[\gamma V(t)+\mathscr{L}V(t)\right]\\
&\le e^{\gamma t}\left[\gamma\check{q}E(t)+(1+\theta)E(t)+\check{b}\check{q}E(t-\tau(t))\right.\\
&\qquad\left.+\check{c}\check{q}E\left(t-\tau_c(t)\right)\right].
\end{aligned}
\tag{30}
$$

The generalized Itô formula shows that

$$e^{\gamma t}\mathbb{E}V(t)=e^{\gamma t_0}\mathbb{E}V\left(t_0\right)+\mathbb{E}\int_{t_0}^{t}\mathscr{L}W(s)\,ds \tag{31}$$

for any $t_k>t>t_0>t_{k-1}\ge 0$. Hence we have

$$
\begin{aligned}
e^{\gamma t}\mathbb{E}V(t)&\le e^{\gamma t_0}\mathbb{E}V\left(t_0\right)\\
&\quad+\mathbb{E}\int_{t_0}^{t}e^{\gamma s}\left[\gamma\check{q}E(s)+(1+\theta)E(s)\right.\\
&\qquad\left.+\check{b}\check{q}E(s-\tau(s))+\check{c}\check{q}E\left(s-\tau_c(s)\right)\right]ds
\end{aligned}
$$

$$\begin{aligned}&\leq \check{q}e^{\gamma t_0}\mathbb{E}E(t_0)+(\gamma\check{q}+1+\theta)\int_{t_0}^{t}e^{\gamma s}\mathbb{E}E(s)\,ds\\&\quad+\check{b}\check{q}e^{\gamma\tau}\int_{t_0}^{t}e^{\gamma(s-\tau(s))}\mathbb{E}E(s-\tau(s))\,ds\\&\quad+\check{c}\check{q}e^{\gamma\tau_c}\int_{t_0}^{t}e^{\gamma(s-\tau_c(s))}\mathbb{E}E(s-\tau_c(s))\,ds.\end{aligned}\tag{32}$$

Let $s-\tau(s)=u$; we have

$$\begin{aligned}\int_{t_0}^{t}e^{\gamma(s-\tau(s))}\mathbb{E}E(s-\tau(s))\,ds&=\int_{t_0-\tau(t_0)}^{t-\tau(t)}e^{\gamma u}\mathbb{E}E(u)\frac{du}{1-\dot{\tau}(t)}\\&\leq\frac{1}{1-\overline{\tau}}\int_{t_0-\tau}^{t}e^{\gamma u}\mathbb{E}E(u)\,du.\end{aligned}\tag{33}$$

Similarly, it can be obtained that

$$\int_{t_0}^{t}e^{\gamma(s-\tau_c(s))}\mathbb{E}E(s-\tau(s))\,ds\leq\frac{1}{1-\overline{\tau}_c}\int_{t_0-\tau_c}^{t}e^{\gamma u}\mathbb{E}E(u)\,du.\tag{34}$$

Substituting (33) and (34) into (32) can result in

$$e^{\gamma t}\mathbb{E}V(t)\leq q_r e^{\gamma t_0}\mathbb{E}V(t_0)+\varphi\int_{t_0-\check{\tau}}^{t}e^{\gamma u}\mathbb{E}E(u)\,du.\tag{35}$$

With help of Gronwall inequality, the following inequality can be achieved:

$$\mathbb{E}E(t)\leq\frac{\check{q}}{\widehat{q}}\mathbb{E}E(t_0)\,e^{\varphi(t-t_0+\check{\tau})+\gamma(t_0-t)}.\tag{36}$$

On the other hand, in view of the construction of $E(t)$, we have

$$E(t_k)\leq(1+\epsilon_k)^2E(t_k^-),\tag{37}$$

where $|1+\epsilon_k|=\max_{i=1,2,\dots,N}|1+\epsilon_{ik}|$.

Let $k=\lfloor(t-t_0)/T\rfloor$; according to (36) and (37), for any $t\in[t_{k-1},t_k)$, one has

$$\begin{aligned}\mathbb{E}[V(t)]&\leq\frac{\check{q}}{\widehat{q}}\mathbb{E}E(t_k)\,e^{\varphi(t-t_k+\check{\tau})+\gamma(t_k-t)}\\&\leq\frac{\check{q}}{\widehat{q}}\mathbb{E}V(t_{k-1}^-)\,e^{\varphi(t-t_{k-1}+\check{\tau})+\gamma(t_k-t)+2\ln|1+\epsilon_{k-1}|}\\&\leq\cdots\leq\left(\frac{\check{q}}{\widehat{q}}\right)^{k-1}\mathbb{E}E(0)\,e^{\varphi(t+k\check{\tau})-\gamma t+\sum_{v=1}^{k-1}2\ln|1+\epsilon_v|}.\end{aligned}\tag{38}$$

Let $|1+\epsilon|=\max_{v\in Z^+}|1+\epsilon_v|$; we have

$$\mathbb{E}[E(t)]\leq\mathbb{E}E(0)\,e^{\varphi(t+k\check{\tau})-\gamma t+2(k-1)\ln(\check{q}/\widehat{q})|1+\epsilon|}.\tag{39}$$

By means of condition (18) in Theorem 9, there is a number η such that $\mathbb{E}E(t)\leq\mathbb{E}E(t_0)e^{-\eta t}$. Consequently, $\mathbb{E}\|e_i(t)\|\leq(E(t_0)/\check{p})^{1/2}e^{-(\eta/2)(t-t_0)}$. The proof of Theorem 9 is completed. □

When the time-varying delays are constant (i.e., $\tau(t)=\tau$, $\tau_c(t)=\tau_c$), we obtain the following corollary.

Corollary 10. *Suppose that (H1) and (H2) are true and $f_{\check{i}}\in$ QUAD $(P,\Delta,\eta_{\check{i}},\zeta_{\check{i}})$. If there are some positive constants θ, α_r, and β_r such that*

$$\begin{gathered}\begin{bmatrix}A(r)^s+\check{\delta}I_N-\alpha_rI_N & \dfrac{B(r)}{2}\\ \dfrac{B(r)^T}{2} & -\beta_rI_N\end{bmatrix}\leq 0,\quad \text{for } r=1,2,\dots,M,\\ \varphi(\check{\tau}+T)+2\ln\frac{\check{q}}{\widehat{q}}|1+\epsilon_v|-\gamma T<0,\\ \left(\frac{1}{b_1+c_1},\frac{1}{b_2+c_2},\dots,\frac{1}{b_M+c_M}\right)^T>\widetilde{\Gamma}^{-1}\mathbf{1}_M,\end{gathered}\tag{40}$$

where

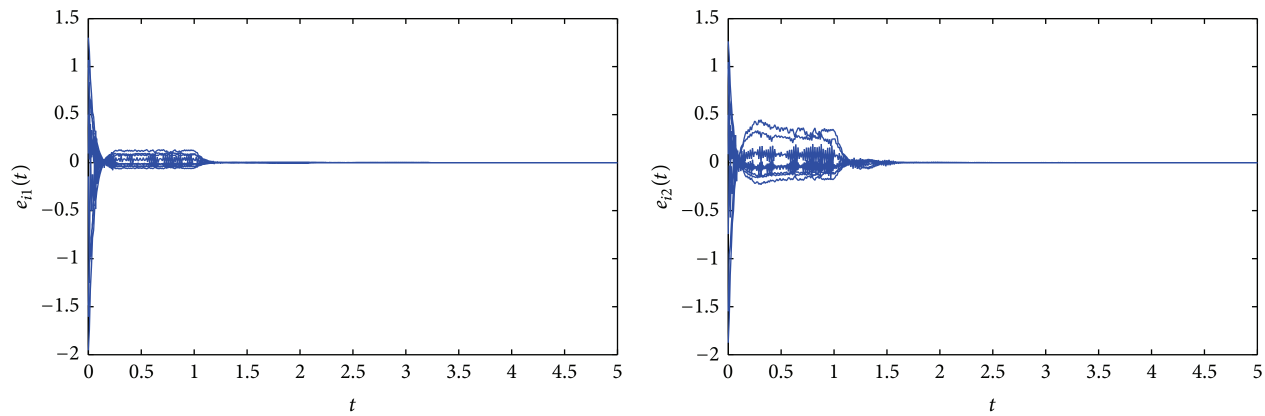
$$\begin{gathered}\varphi=1+\theta+\gamma\check{q}_r+\check{b}\check{q}e^{\gamma\tau}+\check{c}\check{q}e^{\gamma\tau_c},\\ \widetilde{\Gamma}=\operatorname{diag}\{a_1,a_2,\dots,a_M\}+\Gamma,\\ a_r=\frac{\lambda_{\max}\left(-2\eta_{\check{i}}I_N+\check{p}\sum_{j=1}^{N}\Upsilon_{j1}^r+2\alpha_rP\Sigma\right)}{\check{p}},\quad \check{a}=\max_{r\in S,i=1,\dots,N}a_r,\\ b_r=\frac{\lambda_{\max}\left(\sum_{j=1}^{N}P\Upsilon_{j2}^r+2\zeta_{\check{i}}I_N\right)}{\widehat{p}},\quad \check{b}=\max_{r\in S,i=1,\dots,N}b_r,\\ c_r=\frac{\lambda_{\max}\left(\sum_{j=1}^{N}P\Upsilon_{j3}^r+2\beta_rP\Sigma\right)}{\widehat{p}},\quad \check{c}=\max_{r\in S}c_r,\end{gathered}\tag{41}$$

then the global and exponential synchronization of the stochastic complex network (8) *can be achieved.*

When $A(r)$ and $B(r)$ are symmetric matrices, for $r=1,2,\dots,M$, and $\sigma(\cdot)=\mathbf{0}$, we can get the following corollary.

Corollary 11. *Suppose that (H1) and (H2) are true and $f_{\check{i}}\in$ QUAD $(P,\Delta,\eta_{\check{i}},\zeta_{\check{i}})$. If there are some positive constants θ, α_r, and β_r such that*

$$\begin{gathered}\begin{bmatrix}A(r)+\check{\delta}I_N-\alpha_rI_N & \dfrac{B(r)}{2}\\ \dfrac{B(r)^T}{2} & -\beta_rI_N\end{bmatrix}\leq 0,\quad \text{for } r=1,2,\dots,M,\\ 0\leq\check{\overline{\tau}}\leq 1-\frac{\check{q}(\check{b}+\check{c})}{1+\theta},\\ \varphi(\check{\tau}+T)+2\ln\frac{\check{q}}{\widehat{q}}|1+\epsilon_v|-\gamma T<0,\\ \left(\frac{1}{b_1+c_1},\frac{1}{b_2+c_2},\dots,\frac{1}{b_M+c_M}\right)^T>\widetilde{\Gamma}^{-1}\mathbf{1}_M,\end{gathered}\tag{42}$$



(a) (b)

FIGURE 3: The time evolution of e_{i1} and e_{i2} ($i = 1, 2, \ldots, 7$) in system (46) by impulse control.

where

$$
\begin{aligned}
\varphi &= 1 + \theta + \gamma \check{q}_r + \frac{\check{b}\check{q}}{1-\overline{\tau}} e^{\gamma\tau} + \frac{\check{c}\check{q}}{1-\overline{\tau}_c} e^{\gamma\tau_c},\\
\widetilde{\Gamma} &= \operatorname{diag}\{a_1, a_2, \ldots, a_M\} + \Gamma,\\
a_r &= \frac{\lambda_{\max}\left(-2\eta_{\check{i}} I_N + 2\alpha_r P\Sigma\right)}{\check{p}}, \quad \check{a} = \max_{r\in S, i=1,\ldots,N} a_r,\\
b_r &= \frac{2\zeta_{\check{i}}}{\widehat{p}}, \quad \check{b} = \max_{r\in S, i=1,\ldots,N} b_r,\\
c_r &= \frac{\lambda_{\max}\left(2\beta_r P\Sigma\right)}{\widehat{p}}, \quad \check{c} = \max_{r\in S} c_r,
\end{aligned}
\tag{43}
$$

then the global and exponential synchronization of the stochastic complex network (8) *can be achieved.*

4. Numerical Simulation

This section will employ some numerical examples to illustrate the effectiveness of the previous theoretical results.

Consider the following stochastic delayed neural network:

$$
\begin{aligned}
ds(t) = {} & \{-Cs(t) + Dhf(s(t)) + Eh(s(t-\tau(t)))\}\, dt\\
& + \sigma(s(t), s(t-\tau(t)))\, dw(t),
\end{aligned}
\tag{44}
$$

where $h(s) = \tanh(s)$, $\tau(t) = 1$, $\sigma(s(t), s(t-\tau(t))) = \operatorname{diag}\{s_1(t), s_2(t)\}$,

$$
C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad D = \begin{bmatrix} 2 & -0.1 \\ -5 & 4.5 \end{bmatrix},
\qquad
E = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -4 \end{bmatrix}.
\tag{45}
$$

Set $P = \operatorname{diag}\{1, 1\}$, $\Delta = \operatorname{diag}\{5, 11.5\}$, and let $\eta = 0.15$, $\zeta = 3.25$ ensure that condition (10) is satisfied. And then

$$
\begin{aligned}
dx_i(t) = {} & \Big\{ f(t, x_i(t), x_i(t-\tau(t)))\\
& + \sum_{j=1}^{7} a_{ij}^r \Sigma x_j(t) + \sum_{j=1}^{7} b_{ij}^r \Sigma x_j(t-\tau_c(t)) \Big\}\, dt\\
& + \sigma_i^r(t, x(t), x(t-\tau(t)), x(t-\tau_c(t)))\, dw_i(t),\\
& i = 1, 2, \ldots, 5, \quad r = 1, 2,
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
\Gamma &= \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}, \qquad \tau_c(t) = 0.1 \frac{e^t}{1+e^t},\\
&\sigma_i^1(t, x(t), x(t-\tau(t)), x(t-\tau_c(t)))\\
&\quad = 0.1 \operatorname{diag}\{x_{i1}(t) - x_{i+1,1}(t), x_{i2}(t) - x_{i+1,2}(t)\},\\
&\sigma_i^2(t, x(t), x(t-\tau(t)), x(t-\tau_c(t)))\\
&\quad = 0.1 \operatorname{diag}\{x_{i1}(t-\tau(t)) - x_{i+1,1}(t-\tau(t)),\\
&\qquad x_{i2}(t-\tau(t)) - x_{i+1,2}(t-\tau(t))\}.
\end{aligned}
\tag{47}
$$

We here consider a network that has 7 nodes divided into 2 clusters as shown in Figure 1. Some computations result in $\tau = 1$, $\overline{\tau} = 0$, $\tau_c = 0.025$, $\overline{\tau}_c = 0.1$, $\Upsilon_{ij} = 0.01 I_2$ for $i = 1, 2, \ldots, N$. Thus, the solutions of inequalities can be obtained as follows (16)–(19): $\alpha_1 = 3.150$, $\beta_1 = 0.001$, $a_1 = 2.903$, $b_1 = 4.207$, and $c_1 = 0.089$; $\alpha_2 = 3.806$, $\beta_2 = 0.012$, $a_2 = 3.105$, $b_2 = 2.103$, and $c_2 = 0.105$.

In this simulation, the initial conditions, $x_{ij}(t_0)$ for $i = 1, 2, \ldots, 7$, $j = 1, 2$, are all constants. Figure 2 provides the trajectories of the impulse pinning control gains. By Figure 3, we show the time evolution of the cluster synchronization errors with impulse control.

5. Conclusion

In this writing, we focused on the cluster synchronization issue of an array of stochastic complex networks with Markovian switching and time-varying delayed couplings. By means of an impulsive pinning control method imposed on a small fraction of the nodes, the desired cluster synchronization was reached, while a novel sufficient condition was derived to ensure the stability of the considered stochastic networks. At the end of this paper, a numerical simulation was given to show the validity of the theoretical analysis.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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