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Research Article

Service and Price Decisions of a Supply Chain with Optional After-Sale Service

Xiaochen Sun,[1] Qingshuai Zhang,[1] and Yancong Zhou[2]

[1] *Department of Mathematics, Tianjin University, Tianjin 300072, China*
[2] *School of Information Engineering, Tianjin University of Commerce, Tianjin 300134, China*

Correspondence should be addressed to Xiaochen Sun; sxc722@163.com

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For durable products, the high quality after-sales service has been playing an increasingly important role in consumers' purchase behaviors. We mainly study a supply chain composed of a manufacturer and a retailer. In a process of products sales, the manufacturer will provide a basic free quality assurance service. On this basis, the retailer provides paid optional quality assurance service to consumers to promote sales. Users are divided into two categories in this paper: users with no optional service and users with optional services. We derive the equilibrium decisions between the manufacturer and the retailer under the following two cases: (i) the optional after-sales service level and the wholesale price determined by the manufacturer and the retail price determined by the retailer; (ii) the wholesale price determined by the manufacturer and the optional after-sales service level and the retail price determined by the retailer.

1. Introduction

In recent years, as the living standard of people is improving with the rapid development of economy, people become more and more sensitive to nonprice factors they could enjoy rather than a price attribute. In 2007 the global consumer electronics products consumer research showed that service has become the second important factor that affects consumer's purchase behaviors. More and more enterprises have realized this point in time. Relying solely on price advantage, it is difficult to maintain a lasting competitive edge, such as Lenovo, WAL-MART, SONY, General Electric, and DELL. These enterprises also have to provide better services to customers so that they can maintain their market shares.

For the same product, providing different services in daily life is also becoming more common. When we buy a computer, we will encounter this kind of situation. Basic quality assurance services are to be purchased, but not all consumers are satisfied with this basic service; there is a part of the consumers who want to get more and better service. Therefore, for retailers and manufacturers, it is necessary to set up a reasonable and optional service policy to meet more consumers.

Kameshwaran et al. [1] considered the pricing problem in the following three cases: (i) only selling product, not providing service, (ii) offering product and service independently, and (iii) offering product and service bundled. Lu et al. [2] studied the supply chain decision-making problem of two competing manufacturers and one retailer. It emphasizes the importance of service provided by the manufacturer when customers are more sensitive to the price and service level of the product. Xiao and Yang [3] constructed a model that considers both price and service competition, to study the optimal decision of each member when the demand is uncertain.

In the model of Cohen and Whang [4], a customer can obtain after-sales service only from either the manufacturer or an independent service shop. Li et al. [5] considered a supply chain comprising a manufacturer and a retailer. The manufacturer supplies a product to the retailer, while the retailer sells the product bundled with after-sales service to consumers in a fully competitive market. He finds that the manufacturer's sharing of the cost with the retailer to build service capacity improves the profits of both parties. In the model of Kurata and Nam [6], a customer can obtain simultaneously two after-sales services, one from the manufacturer and the other from the retailer. And formulating five analytical models, they find that the service level maximizing profits is not the service level that can satisfy consumers the most. As an extension of Kurata and Nam [6], Kurata and Nam [7] explored the effect of uncertainty on after-sales service decisions by comparing several information structures in a two-stage supply chain.

In fact, the optional service to a certain extent to meet the needs of customers is a stimulating factor, so we assume that the demand is subject to the impact of optional services. This is consistent with the study of Xu et al. [8]. In this case, all users are divided into two categories: users with no optional service and users with an optional service. We will focus on the equilibrium decisions between the manufacturer and the retailer with two categories of users, in which a basic service is provided by the manufacturer and an optional service is provided by the retailer or the manufacturer.

Our problem is most relevant to Kurata and Nam [6]. However, we consider a model with an optional after-sale service, in which the customers need to pay for selecting the optional service, and the price of the optional service is a decision variable. And we study the decision problems under the following two cases: (i) the optional after-sales service level and the wholesale price determined by the manufacturer and the retail price determined by the retailer; (ii) the wholesale price determined by the manufacturer and the optional after-sales service level and the retail price determined by the retailer.

The remainder of this paper is organized as follows. The second part is the problem description and modeling. In the third part, we establish the model where the optional service level is decided by the manufacturer. The fourth part is the model analysis and the solution. In the fifth part, we establish the model where the optional service level is decided by the retailer. The sixth part is the model analysis and the solution. The seventh part is the comparisons of the two kinds of equilibrium decisions by numerical results. Finally, conclusions and the future research directions are addressed in Section 8.

2. Problem Description and Models

We consider a supply chain composed of a manufacturer and a retailer, in which the manufacturer sells the product to the retailer at the wholesale price w, and the retailer sells the product to the customer at the price p.

There are two categories of users in the market. The first category of users only use basic service items, such as the basic quality assurance services; the second category of users are not satisfied with the basic services but also need enterprises to provide more optional services, such as extended warranty service. We use a_1 to represent the base-case potential market size for the first category of users. We use a_2 to represent the base-case potential market size for the second category of users. We assume that the basic service and the retail price are bound; that is, after the payment of the cost of the sales, the basic service does not require an additional payment, so the two categories of users are similarly sensitive to the basic service. But the optional service will attract a part of the first category of users and other product users to transfer; therefore, the optional service is negative for the first category of users' demand and is positive for the second category of users' demand, and the impact of the second category of users is more. Let d_1 and d_2, respectively, represent the first and second categories of user's demand function for the product. As popularly used in other literatures such as [9–11] on price and service competition, here d_1 and d_2 are thought of as linear functions about retail price and optional service level. Thus, the demand functions could be described by price and optional service as follows:

$$\begin{aligned} d_1(p, y) &= a_1 - \beta p - \mu_1 y, \\ d_2(p, y) &= a_2 - \beta p + \mu_2 y, \end{aligned} \tag{1}$$

where y indicates the optional service level and β indicates the price sensitive coefficient. μ_1 measures the responsiveness of market demand for the first category of users to the optional service level. When the optional service is high enough, the customer insensitive to the service level will change the attitude, so the first category of customers will be less. Therefore, the optional service level has a negative effect on d_1, where $\mu_1 > 0$. Corresponding to the first category of customers, the second category of customers are sensitive to the optional service level, so the optional service level has a positive effect on d_2. μ_2 measures the responsiveness of market demand for the second category of users to the optional service level, so $\mu_2 > 0$. With the optional service level increasing, the total demand of two categories should be increased, so there should be $\mu_1 < \mu_2$, meaning that, with the optional service level improving, the overall demand for the product will increase $(\mu_2 - \mu_1)y$.

If one user chooses the optional service, she/he needs to pay a certain service charge, which we call the optional service price. Of course, the optional service price changes with the optional service level. Let y indicate the optional service level; then the optional service price is $r(y)$, and it is a concave function. For convenience, make $r(y) = \alpha y$.

In order to maximize their own profits, the manufacturer and the retailer need to make decisions based on their own pieces of information. We assume that the basic service level is known as well as the optional service price given by the retailer. We consider the decision-making problem of supply chain members under the condition of determined environment.

3. The Optional Service Level Decided by the Manufacturer

In this part, the optional service level y is determined by the manufacturer, the manufacturer also determines the wholesale price w, and the retail price p is determined by the retailer.

Hence, the manufacturer's profit function is

$$\Pi_M(w, y) = (w - c)(d_1(p, y) + d_2(p, y)) - \frac{\eta y^2}{2}. \quad (2)$$

In the above equation, the first item is the sales revenue, and the second is the cost of the optional service, as Tsay and Agrawal [10] does.

Then, the retailer's profit function can be formulated by the following:

$$\Pi_R(p) = (p - w) d_1(p, y) + (p + \alpha y - w) d_2(p, y). \quad (3)$$

Based on (1)–(3), the general optimization model can be described as follows:

$$\max \Pi_M(w, y) = (w - c)(a_1 + a_2 - 2\beta p - \mu_1 y + \mu_2 y) - \frac{\eta y^2}{2} \quad (4)$$

subject to

$$\max \Pi_R(p) = (p - w)(a_1 - \beta p - \mu_1 y) + (p + \alpha y - w)(a_2 - \beta p + \mu_2 y). \quad (5)$$

We can see that, based on the analysis stated above, in this model the manufacturer determines the wholesale price w and the optional service level y, and the retailer determines the retail price p, respectively.

4. Results and Discussions for the Manufacturer Case

We now derive chain members' optimal strategies by maximizing their profits in the above general optimization model.

Theorem 1. *For any given wholesale price w and optional service level y, the profit function $\Pi_R(p)$ is concave with respect to the retail price p.*

Proof. Consider the following derivatives:

$$\frac{\partial \Pi_R(p)}{\partial p} = a_1 + a_2 - \mu_1 y + \mu_2 y + 2\beta w - 4\beta p - \alpha\beta y,$$
$$\frac{\partial^2 \Pi_R(p)}{\partial p^2} = -4\beta. \quad (6)$$

Because $\partial^2 \Pi_R(p)/\partial p^2 = -4\beta < 0$, $\Pi_R(p)$ is concave with respect to the retail price p; that is to say, there does exist retail price p' maximizing $\Pi_R(p)$. □

In the following, all best response functions are denoted by the superscript "$'$" and let the superscript "$*$" denote the equilibrium decisions.

Proposition 2. *For any given w and y, the retailer best response function satisfies the following:*

$$p' = \frac{a_1 + a_2 - \mu_1 y + \mu_2 y + 2\beta w - \alpha\beta y}{4\beta}. \quad (7)$$

Proof. It can be easily proved by solving the first condition $\partial \Pi_R(p)/\partial p = 0$. □

Paying attention to the impact of wholesale price and optional service level on the retailer's best response function, we derive the first derivatives of p' with respect to wholesale price and optional service level; that is, $\partial p/\partial w = 0.5 > 0$ and $\partial p/\partial w = (-\mu_1 + \mu_2 - \alpha\beta)/4\beta$. We can easily see that the higher the wholesale price, the higher the retail price. However, the effect of optional service level on the retail price is not very evident.

Obviously, if the manufacturer increases the wholesale price w of the product, the retailer tends to increase the sales price p of the product, which is very consistent with our intuition. And for the optional service level y, because the optional service price is linear with optional service level y, when the correlation coefficient α is relatively large, the retailer, tending to decrease retail price, increases sales to increase his profit.

Lemma 3. *The inequality $8\beta(\eta - 2\alpha\mu_2) > (\alpha\beta - \mu_1 + \mu_2)^2$ holds for any practical problem.*

Proof. As the retailer's operation problem, its profit must have a maximum value in any practical case.

Bringing the formulation of p' into the manufacturer profit function $\Pi_M(w, y)$, we can get

$$\Pi_M = (w - c)\left(a_1 + a_2 - 2\beta\left(\frac{a_1 + a_2 - \mu_1 y + \mu_2 y + 2\beta w - \alpha\beta y}{4\beta}\right) - \mu_1 y + \mu_2 y\right) - \frac{\eta y^2}{2}. \quad (8)$$

Consider the following derivatives:

$$\frac{\partial \Pi_M}{\partial w} = \frac{a_1 + a_2 - \mu_1 y + \mu_2 y - 4\beta w + \alpha\beta y}{2} + \beta c,$$
$$\frac{\partial \Pi_M}{\partial y} = (w - c)\frac{-\mu_1 + \mu_2 + \alpha\beta}{2} - \eta y,$$
$$\begin{pmatrix} \dfrac{\partial^2 \Pi_M}{\partial w^2} & \dfrac{\partial^2 \Pi_M}{\partial w \partial y} \\ \dfrac{\partial^2 \Pi_M}{\partial y \partial w} & \dfrac{\partial^2 \Pi_M}{\partial y^2} \end{pmatrix} = \begin{pmatrix} -2\beta & \dfrac{\alpha\beta - \mu_1 + \mu_2}{2} \\ \dfrac{\alpha\beta - \mu_1 + \mu_2}{2} & -\eta \end{pmatrix}. \quad (9)$$

From the existence condition of extreme function for multivariable function, there must be

$$\Delta_2 = 2\beta\eta - \left(\frac{\alpha\beta - \mu_1 + \mu_2}{2}\right)^2 > 0. \quad (10)$$

Therefore, the lemma holds. □

Proposition 4. *The manufacturer equilibrium decisions satisfy*

$$w^* = \frac{2(a_1+a_2)\eta + 4\beta\eta c - c(-\mu_1+\mu_2+\alpha\beta)^2}{8\beta\eta - (-\mu_1+\mu_2+\alpha\beta)^2},$$
$$y^* = \frac{(a_1+a_2-2\beta c)(-\mu_1+\mu_2+\alpha\beta)}{8\beta\eta - (-\mu_1+\mu_2+\alpha\beta)^2}. \tag{11}$$

Proof. With the best response function for retail price p' in (7), the manufacturer's best response function for the product can be achieved by solving $w^*, y^* \in \arg\max \Pi_M(w, y)$.

Let $\partial\Pi_M(x,y)/\partial y = (w-c)((-\mu_1+\mu_2+\alpha\beta)/2) - \eta y = 0$; we can get

$$y = \frac{(w-c)(-\mu_1+\mu_2+\alpha\beta)}{2\eta}. \tag{12}$$

Bring it into $\partial\Pi_M/\partial w = (a_1+a_2-\mu_1 y+\mu_2 y - 4\beta w + \alpha\beta y)/2 + \beta c$. Making it zero, we can get the manufacturer equilibrium decisions:

$$w^* = \frac{2(a_1+a_2)\eta + 4\beta\eta c - c(-\mu_1+\mu_2+\alpha\beta)^2}{8\beta\eta - (-\mu_1+\mu_2+\alpha\beta)^2}, \tag{13}$$

$$y^* = \frac{(a_1+a_2-2\beta c)(-\mu_1+\mu_2+\alpha\beta)}{8\beta\eta - (-\mu_1+\mu_2+\alpha\beta)^2}. \tag{14}$$

Bringing w^*, y^* into $p' = (a_1+a_2-\mu_1 y+\mu_2 y + 2\beta w - \alpha\beta y)/4\beta$, we can get the retailer equilibrium decisions:

$$p^* = \frac{6\eta(a_1+a_2) - 2c(-\mu_1+\mu_2+\alpha\beta)(-\mu_1+\mu_2) - \alpha(a_1+a_2)(-\mu_1+\mu_2+\alpha\beta) + 4\beta\eta c}{2\left(8\beta\eta - (-\mu_1+\mu_2+\alpha\beta)^2\right)}. \tag{15}$$

□

Proposition 5. *The equilibrium wholesale price w^* is increasing in a_1, a_2, μ_2, and α but decreasing in μ_1.*

Proof. From (13), we get the following derivatives:

$$\frac{\partial w^*}{\partial a_1} = \frac{2\eta}{8\beta\eta - (-\mu_1+\mu_2+\alpha\beta)^2} > 0,$$
$$\frac{\partial w^*}{\partial a_2} = \frac{2\eta}{8\beta\eta - (-\mu_1+\mu_2+\alpha\beta)^2} > 0. \tag{16}$$

From (16), we can see that the equilibrium wholesale price w^* is increasing in a_1, a_2.

From (13), it is easily got that

$$w^* = \frac{2(a_1+a_2)\eta - 4\beta\eta c}{8\beta\eta - (-\mu_1+\mu_2+\alpha\beta)^2} + c. \tag{17}$$

From (17), we can see that when μ_1 is increasing, the equilibrium wholesale price w^* is decreasing in μ_1. And the equilibrium wholesale price w^* is increasing in μ_2 and α. □

Proposition 6. *If $c > 4(a_1+a_2)$, the equilibrium wholesale price w^* is increasing in η; otherwise, the equilibrium wholesale price w^* is decreasing in η.*

Proof. From (13), we get the following derivative:

$$\frac{\partial w^*}{\partial \eta} = \frac{(c - 4(a_1+a_2))(-\mu_1+\mu_2+\alpha\beta)^2}{2\left(8\beta\eta - (-\mu_1+\mu_2+\alpha\beta)^2\right)^2}. \tag{18}$$

It is easy to know that if $c > 4(a_1+a_2)$, the equilibrium wholesale price w^* is increasing in η; otherwise, the equilibrium wholesale price w^* is decreasing in η. □

Proposition 7. *If $4\beta\eta > (-\mu_1+\mu_2+\alpha\beta)^2$, the equilibrium wholesale price w^* is increasing in c; otherwise, the equilibrium wholesale price w^* is decreasing in c.*

Proof. From (13), we get the following derivative:

$$\frac{\partial w^*}{\partial c} = \frac{4\beta\eta - (-\mu_1+\mu_2+\alpha\beta)^2}{8\beta\eta - (-\mu_1+\mu_2+\alpha\beta)^2}. \tag{19}$$

It is easy to know that if $4\beta\eta > (-\mu_1+\mu_2+\alpha\beta)^2$, the equilibrium wholesale price w^* is increasing in c; otherwise, the equilibrium wholesale price w^* is decreasing in c. □

Proposition 8. *The equilibrium optional service level y^* is increasing in a_1, a_2, μ_2, and α but decreasing in μ_1, η, and c.*

Proof. From (12), it is easily got that

$$y^* = \frac{(w^*-c)(-\mu_1+\mu_2+\alpha\beta)}{2\eta}. \tag{20}$$

From (20), we can see that when μ_1 is increasing, $-\mu_1+\mu_2+\alpha\beta$ is decreasing and is greater than zero. And because w^* is decreasing in μ_1, when μ_1 is increasing, w^*-c is decreasing and is greater than zero. Hence, the equilibrium optional service level y^* is decreasing in μ_1. And the equilibrium wholesale price w^* is increasing in μ_2 and α.

From (20), we can see that when a_1 is increasing, w^* is increasing, so $w^* - c$ is increasing and is greater than zero. Hence, the equilibrium optional service level y^* is decreasing in a_1. And the equilibrium wholesale price w^* is increasing in a_2.

From (14), we can see that when η is increasing, the equilibrium optional service level y^* is decreasing in η. □

Proposition 9. *(i) If $6\eta > \alpha(-\mu_1 + \mu_2 + \alpha\beta)$, the equilibrium retail price p^* is increasing in a_1 and a_2; otherwise, the equilibrium retail price p^* is decreasing in a_1 and a_2. (ii) If $2\beta\eta > (-\mu_1 + \mu_2)(-\mu_1 + \mu_2 + \alpha\beta)$, the equilibrium retail price p^* is increasing in c; otherwise, the equilibrium retail price p^* is decreasing in c.*

Proof. From (15), it is easily got that

$$\frac{\partial p^*}{\partial a_1} = \frac{6\eta - \alpha(-\mu_1 + \mu_2 + \alpha\beta)}{2\left(8\beta\eta - (-\mu_1 + \mu_2 + \alpha\beta)^2\right)},$$
$$\frac{\partial p^*}{\partial a_2} = \frac{6\eta - \alpha(-\mu_1 + \mu_2 + \alpha\beta)}{2\left(8\beta\eta - (-\mu_1 + \mu_2 + \alpha\beta)^2\right)}. \quad (21)$$

It is easy to know that if $6\eta > \alpha(-\mu_1 + \mu_2 + \alpha\beta)$, the equilibrium retail price p^* is increasing in a_1 and a_2; otherwise, the equilibrium retail price p^* is decreasing in a_1 and a_2.

From (15), it is easily got that

$$\frac{\partial p^*}{\partial c} = \frac{2\beta\eta - (-\mu_1 + \mu_2)(-\mu_1 + \mu_2 + \alpha\beta)}{8\beta\eta - (-\mu_1 + \mu_2 + \alpha\beta)^2}. \quad (22)$$

It is easy to know that if $2\beta\eta > (-\mu_1 + \mu_2)(-\mu_1 + \mu_2 + \alpha\beta)$, the equilibrium retail price p^* is increasing in c; otherwise, the equilibrium retail price p^* is decreasing in c. □

5. The Optional Service Level Decided by the Retailer

In this part, the optional service level y is determined by the retailer, the retailer also determines the retail price p, and the wholesale price w is determined by the manufacturer.

Hence, the manufacturer profit function is

$$\Pi_M = (w - c)(d_1(p, y) + d_2(p, y)). \quad (23)$$

Then, the profit function of retailer can be formulated by the following:

$$\Pi_R = (p - w) d_1(p, y) + (p + \alpha y - w) d_2(p, y) - \frac{\eta y^2}{2}. \quad (24)$$

In the above equation, the first item is sales revenue, and the second is the cost of the optional service, as Tsay and Agrawal [10] does.

Based on (1), (23), and (24), the general optimization model can be described as follows:

$$\max \Pi_M = (w - c)(a_1 + a_2 - 2\beta p - \mu_1 y + \mu_2 y) \quad (25)$$

subject to

$$\max \Pi_R = (p - w)(a_1 - \beta p - \mu_1 y) + (p + \alpha y - w)(a_2 - \beta p + \mu_2 y) - \frac{\eta y^2}{2}. \quad (26)$$

We can see that, based on the analysis stated above, in this model the manufacturer determines the wholesale price w, and the retailer determines the retail price p and the optional service level y, respectively.

6. Results and Discussions for the Retailer Case

We now derive chain members' optimal strategies by maximizing their profits in the above general optimization model.

Firstly, the following parameters hold.

Lemma 10. *The inequalities $4\beta(\eta - 2\alpha\mu_2) > (-\alpha\beta - \mu_1 + \mu_2)^2$ and $2\alpha\mu_2 < \eta$ hold for any practical problem.*

Proof. As the retailer's operation problem, its profit must have a maximum value in any practical case. Consider the following derivatives:

$$\frac{\partial \Pi_R}{\partial p} = a_1 + a_2 - 4\beta p + 2\beta w - \alpha\beta y - \mu_1 y + \mu_2 y,$$
$$\frac{\partial \Pi_R}{\partial y} = \alpha a_2 - \alpha\beta p + \mu_1 w - \mu_2 w - \mu_1 p + \mu_2 p + 2\alpha\mu_2 y - \eta y,$$
$$\begin{pmatrix} \frac{\partial^2 \Pi_R}{\partial p^2} & \frac{\partial^2 \Pi_R}{\partial p \partial y} \\ \frac{\partial^2 \Pi_R}{\partial y \partial p} & \frac{\partial^2 \Pi_R}{\partial y^2} \end{pmatrix} = \begin{pmatrix} -4\beta & -\alpha\beta - \mu_1 + \mu_2 \\ -\alpha\beta - \mu_1 + \mu_2 & 2\alpha\mu_2 - \eta \end{pmatrix}. \quad (27)$$

From the existence condition of extreme function for multivariable function, there must be

$$\Delta_3 = 2\alpha\mu_2 - \eta < 0,$$
$$\Delta_4 = 4\beta(\eta - 2\alpha\mu_2) - (-\alpha\beta - \mu_1 + \mu_2)^2 > 0. \quad (28)$$

Therefore, the lemma holds. □

Proposition 11. *For any given w, the retailer best response functions satisfy*

$$p' = \frac{\zeta(A+2\beta w)+\lambda(\alpha a_2+\mu_1 w-\mu_2 w)}{\Delta_4},$$
$$y' = \frac{\lambda\zeta(A+2\beta w)+4\beta\zeta(\alpha a_2+\mu_1 w-\mu_2 w)}{\zeta\Delta_4}. \quad (29)$$

Proof. Let $\partial\Pi_R/\partial y = \alpha a_2 - \alpha\beta p + \mu_1 w - \mu_2 w - \mu_1 p + \mu_2 p + 2\alpha\mu_2 y - \eta y = 0$; we can get

$$y = \frac{\alpha a_2 - \alpha\beta p + \mu_1 w - \mu_2 w - \mu_1 p + \mu_2 p}{\eta - 2\alpha\mu_2}. \quad (30)$$

Bring it into $\partial\Pi_R/\partial p = a_1 + a_2 - 4\beta p + 2\beta w - \alpha\beta y - \mu_1 y + \mu_2 y$. Making it zero, we can get the retailer's best response decisions:

$$p = \frac{\zeta(A+2\beta w)+\lambda(\alpha a_2+\mu_1 w-\mu_2 w)}{\Delta_4}. \quad (31)$$

It is easy to get that

$$y = \frac{\lambda\zeta(A+2\beta w)+4\beta\zeta(\alpha a_2+\mu_1 w-\mu_2 w)}{\zeta\Delta_4}. \quad (32)$$

Among them, $A = a_1 + a_2$, $\sigma = \mu_1 - \mu_2 - \alpha\beta$, $\xi = -\mu_1 + \mu_2$, and $\Delta_4 = 4\beta\zeta - \lambda^2$; that is, $\zeta = \eta - 2\alpha\mu_2$, $\lambda = -\alpha\beta - \mu_1 + \mu_2$. □

Paying attention to the impact of wholesale price on the retailer's best response function, we derive the first derivatives of p' with respect to wholesale price and the first derivatives of y' with respect to wholesale price; that is, $\partial p/\partial w = (2\beta\zeta + \lambda(\mu_1 - \mu_2))/\Delta_4$ and $\partial y/\partial w = 2\beta\zeta(\mu_1 - \mu_2 - \alpha\beta)/\zeta\Delta_4 < 0$. We can easily see that the higher the wholesale price, the higher the optional service level. However, the effect of the wholesale price on the retail price is not very evident.

Theorem 12. *The profit function $\Pi_M(w)$ is concave with respect to the wholesale price w.*

Proof. Bringing the retailer best response functions into the manufacturer profit function $\Pi_M(w)$, we can get

$$\Pi_M = (w-c)\,\frac{A\zeta\Delta_4 + \left(\lambda\zeta(-\mu_1+\mu_2) - 2\beta\zeta^2\right)(A+2\beta w) + \left(4\beta\zeta(-\mu_1+\mu_2) - 2\beta\zeta\lambda\right)(\alpha a_2 + \mu_1 w - \mu_2 w)}{\zeta\Delta_4}. \quad (33)$$

Consider the following derivatives:

$$\frac{\partial\Pi_M}{\partial w} = \frac{A\zeta\Delta_4 + \left(\lambda\zeta(-\mu_1+\mu_2) - 2\beta\zeta^2\right)(A+2\beta w) + \left(4\beta\zeta(-\mu_1+\mu_2) - 2\beta\zeta\lambda\right)(\alpha a_2 + \mu_1 w - \mu_2 w)}{\zeta\Delta_4} + (w-c)\,\frac{2\beta\left(\lambda\zeta(-\mu_1+\mu_2) - 2\beta\zeta^2\right) + (\mu_1-\mu_2)\left(4\beta\zeta(-\mu_1+\mu_2) - 2\beta\zeta\lambda\right)}{\zeta\Delta_4}, \quad (34)$$

$$\frac{\partial^2\Pi_M}{\partial w^2} = \frac{-8\beta^2\zeta^2 - 8\alpha\zeta\beta^2(-\mu_1+\mu_2)}{\zeta\Delta_4} < 0. \quad (35)$$

Because $\partial^2\Pi_M/\partial w^2 = (-8\beta^2\zeta^2 - 8\alpha\zeta\beta^2(-\mu_1+\mu_2))/\zeta\Delta_4 < 0$, $\Pi_M(w)$ is concave with respect to the wholesale price w; that is to say, there does exist wholesale price w^* maximizing $\Pi_M(w)$. □

Proposition 13. *The manufacturer equilibrium decisions satisfy*

$$w^* = \frac{A\zeta\Delta_4 + \left(\lambda\zeta\xi - 2\beta\zeta^2\right)(A-2\beta c) + 2\beta\zeta\sigma(\alpha a_2 + \xi c)}{8\beta^2\zeta^2 + 8\alpha\zeta\beta^2\xi}. \quad (36)$$

Proof. Let (34) be equal to zero; we can get

$$w^* = \frac{A\zeta\Delta_4 + \left(\lambda\zeta\xi - 2\beta\zeta^2\right)(A-2\beta c) + 2\beta\zeta\sigma(\alpha a_2 + \xi c)}{8\beta^2\zeta^2 + 8\alpha\zeta\beta^2\xi}. \quad (37)$$

□

Bringing w^* into (31) and (32), we can get the retailer equilibrium decisions:

$$p^* = \frac{8\beta^2\zeta(\zeta+\alpha\xi)(\zeta A + \lambda\alpha a_2) + (2\beta\zeta - \lambda\xi)\left(A\Delta_4\zeta + \left(\lambda\zeta\xi - 2\beta\zeta^2\right)(A-2\beta c) + 2\beta\zeta\sigma(\alpha a_2 + \xi c)\right)}{8\Delta_4(\beta^2\zeta^2 + \alpha\zeta\beta^2\xi)},$$
$$y^* = \frac{8\beta^2\zeta^2(\zeta+\alpha\xi)(\lambda A + 4\alpha\beta a_2) + 2\beta A\Delta_4\zeta^2\sigma + 2\beta\zeta\sigma\left(\lambda\zeta\xi - 2\beta\zeta^2\right)(A-2\beta c) + 4\beta^2\zeta^2\xi\sigma^2(\alpha a_2 + \xi c)}{8\zeta\Delta_4(\beta^2\zeta^2 + \alpha\zeta\beta^2\xi)}. \quad (38)$$

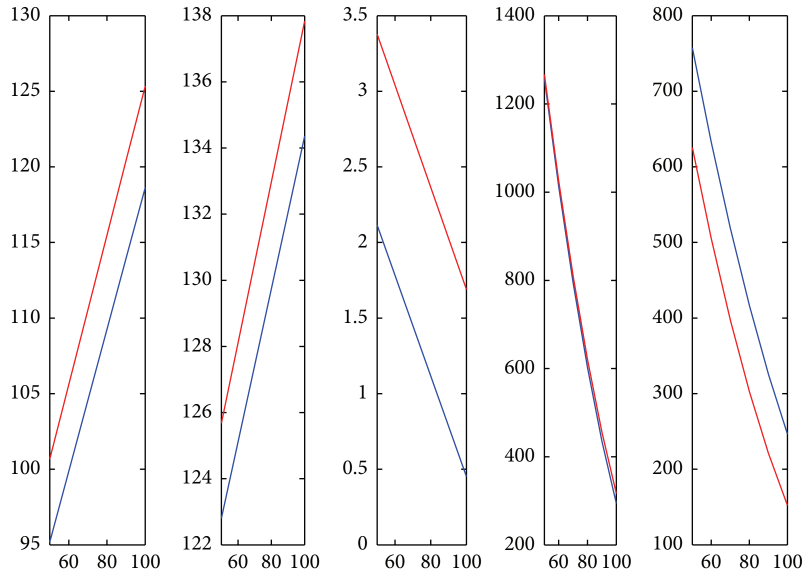

Figure 1: Effects of unit production cost.

7. Numerical Simulation

In this section, we will analyze the changes of different parameters of supply chain members by numerical simulation. The objective of numerical analyses is to examine how the equilibrium profit functions change when the values of parameters change. And we compare the equilibrium decisions and the equilibrium profits under two models. The baseline parameters are set as follows: $c = 80$, $a_1 = 100$, $a_2 = 50$, $\alpha = 0.2$, $\mu_1 = 0.1$, $\mu_2 = 0.4$, $\eta = 3$, and $\beta = 0.5$. In order to compare directly, we will use a diagram showing the curves of two models. The red curve represents the equilibrium decisions and the equilibrium profits where the optional service level is determined by the manufacturer and the blue curve represents the equilibrium decisions and the equilibrium profits where the optional service level is determined by the retailer.

The ordinate axes in Figures 1–8, from left to right, respectively, represent the equilibrium wholesale price, the equilibrium retail price, the equilibrium optional service level, the manufacturer's profit, and the retailer's profit.

Figure 1 indicates how the equilibrium decisions and the profits of all chain members change with respect to c. It is easy to see that as c increases, the retail price and the wholesale price are linearly increasing, but the optimal optional service level and the profits of all chain members are linearly decreasing. In Figure 1, c is from 50 to 100. We can see that the equilibrium decisions and the manufacturer profit where the optional service level is determined by the manufacturer are greater than the equilibrium decisions and the manufacturer profit where the optional service level is determined by the retailer. But the retailer profit where the optional service level is determined by the manufacturer is less than the retailer profit where the optional service level is determined by the retailer.

Figure 2 indicates how the equilibrium decisions and the profits of all chain members change with respect to η. It is easy to see that as η increases, in the model where the optional service level is determined by manufacturer, the retail price and the wholesale price and the optional service level are decreasing, but in the model where the optional service level is determined by the retailer, the retail price and the wholesale price are increasing and the optional service level is decreasing. In Figure 2, η is from 2 to 6. We can see that the equilibrium decisions and the manufacturer profit where the optional service level is determined by the manufacturer are greater than the equilibrium decisions and the manufacturer profit where the optional service level is determined by the retailer. But the retailer profit where the optional service level is determined by the manufacturer is less than the retailer profit where the optional service level is determined by the retailer. For μ_1, there are similar results. It is represented in Figure 3. The only difference is that the retailer's profit in the model where the optional service level is determined by the retailer firstly decreases and then increases.

Figure 4 indicates how the equilibrium decisions and the profits of all chain members change with respect to a_1. It is easy to see that as a_1 increases, all of the equilibrium decisions and the profits of all chain members are increasing. In Figure 4, a_1 is from 50 to 200. We can see that the optimal retail price and the optimal wholesale price and the manufacturer profit in the model where the optional service level is determined by manufacturer are greater than the optimal retail price and the optimal wholesale price and the manufacturer profit in the model where the optional service level is determined by the retailer. But the retailer's profit in the model where the optional service level is determined by the manufacturer is less than the retailer's profit in the model where the optional service level is determined by the

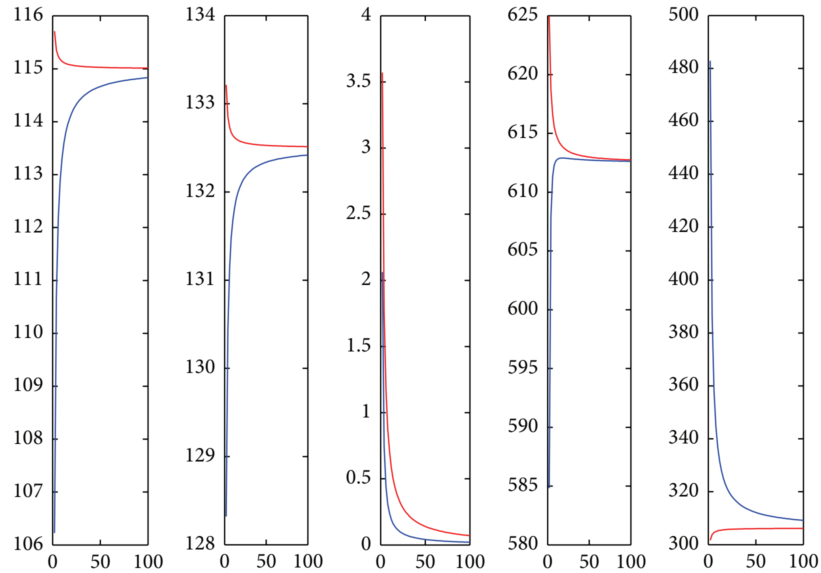

FIGURE 2: Effects of optional service cost coefficient.

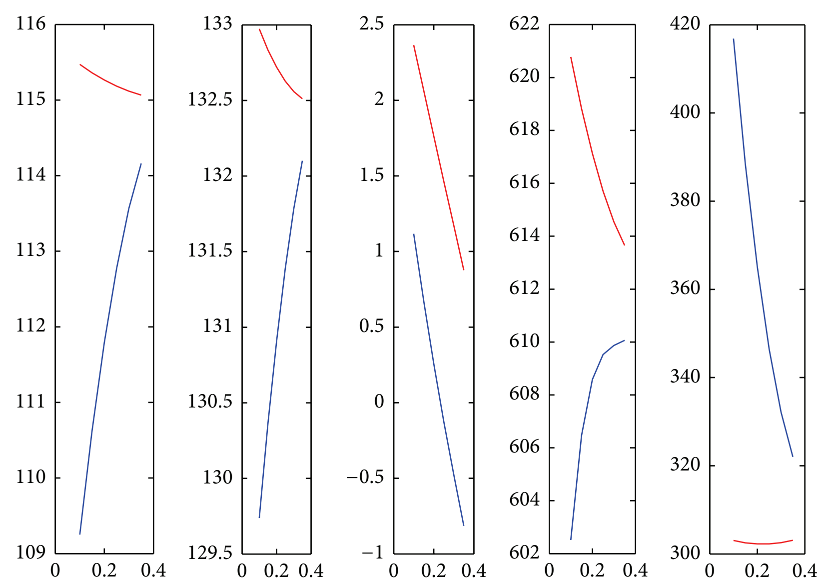

FIGURE 3: Effects of service elastic coefficient of the first category of customers.

retailer. Moreover, the optimal optional service level in the model where the optional service level is determined by the manufacturer is firstly less and then greater than the optimal optional service level in the model where the optional service level is determined by the retailer. For a_2, there are similar results. It is represented in Figure 5. The only difference is that the optimal optional service level in the model where the optional service level is determined by the manufacturer is firstly greater and then less than the optimal optional service level in the model where the optional service level is determined by the retailer.

Figure 6 indicates how the equilibrium decisions and the profits of all chain members change with respect to β. It is easy to see that as β increases, all of the equilibrium decisions and the profits of all chain members are decreasing. In Figure 6, β is from 0.1 to 0.5. We can see that the optimal retail price and the optimal wholesale price and the manufacturer profit in the model where the optional service level is determined by the manufacturer are greater than the optimal retail price and the optimal wholesale price and the manufacturer profit in the model where the optional service level is determined by the retailer. But the retailer's profit in the model where the

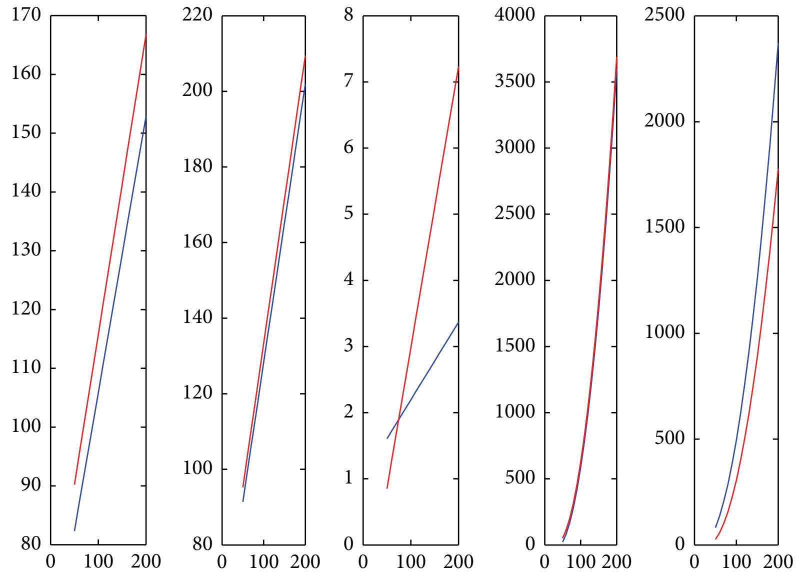

Figure 4: Effects of market base of the first category of customers.

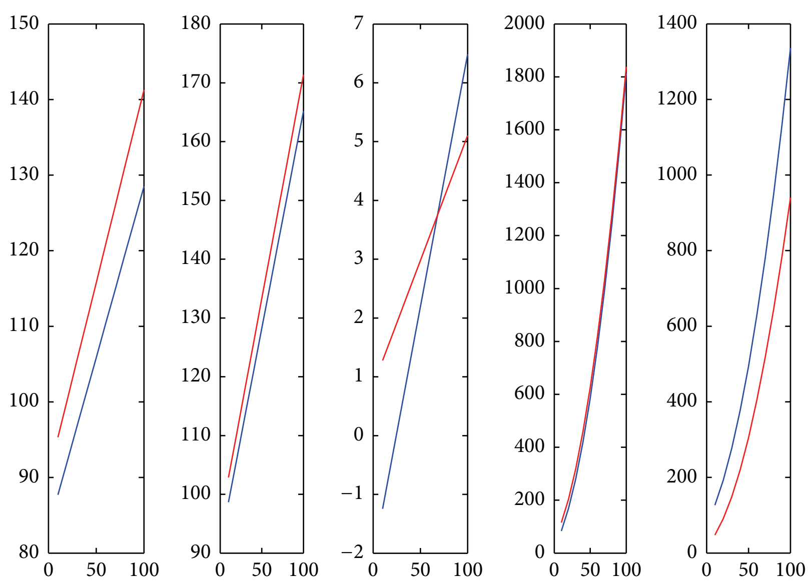

Figure 5: Effects of market base of the second category of customers.

optional service level is determined by the manufacturer is less than the retailer's profit in the model where the optional service level is determined by the retailer. Moreover, the optimal optional service level in the model where the optional service level is determined by the manufacturer is firstly greater and then less than the optimal optional service level in the model where the optional service level is determined by the retailer.

Figure 7 indicates how the equilibrium decisions and the profits of all chain members change with respect to μ_2. It is easy to see that as μ_2 increases, all of the equilibrium decisions and the profits of all chain members in the model where the optional service level is determined by the manufacturer are increasing, and the optimal optional service level and the retailer's profit in the model where the optional service level is determined by the retailer are increasing. In Figure 7, μ_2 is from 0.3 to 0.8. We can see that the optimal retail price and the optimal wholesale price and the manufacturer's profit in the model where the optional service level is determined by the manufacturer are greater than the optimal retail price

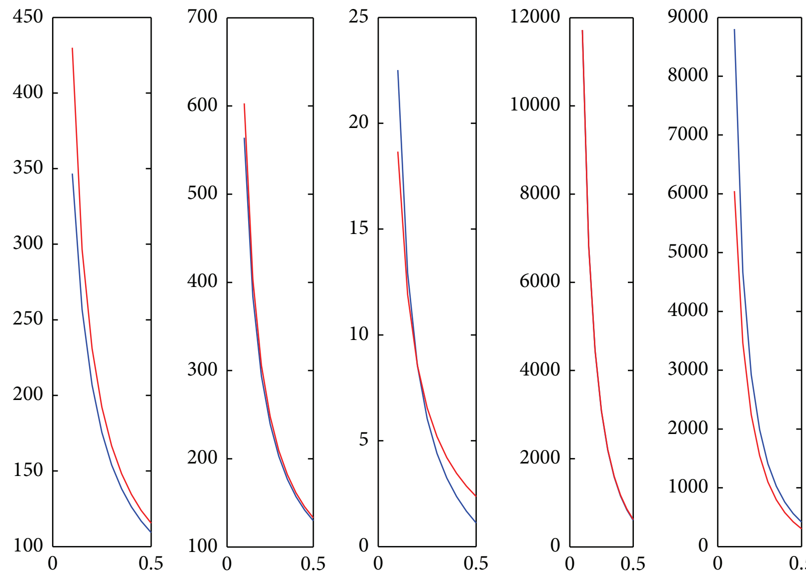

FIGURE 6: Effects of price sensitive coefficient.

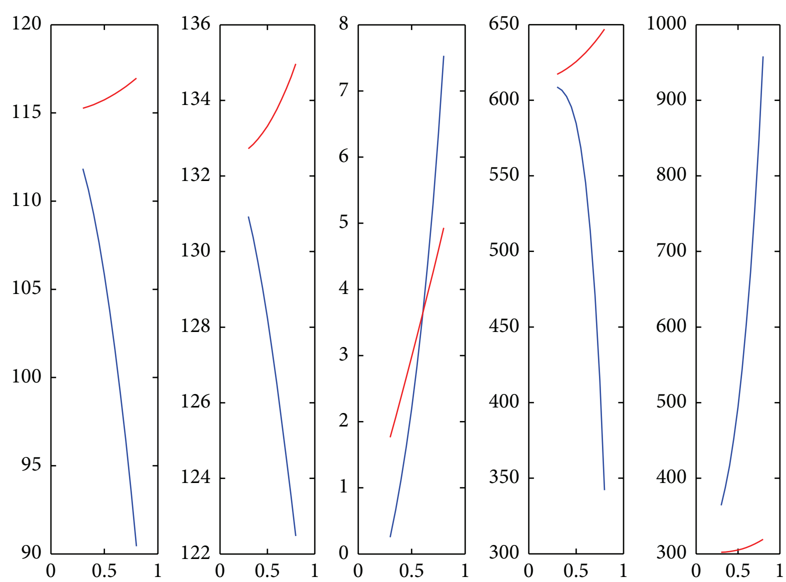

FIGURE 7: Effects of service elastic coefficient of the second category of customers.

and the optimal wholesale price and the manufacturer profit in the model where the optional service level is determined by the retailer. But the retailer's profit in the model where the optional service level is determined by the manufacturer is less than the retailer's profit in the model where the optional service level is determined by the retailer. Moreover, the optimal optional service level in the model where the optional service level is determined by the manufacturer is firstly greater and then less than the optimal optional service level in the model where the optional service level is determined by the retailer. For α, there are similar results. It is represented in Figure 8. The difference is that the optimal optional service level in the model where the optional service level is determined by the retailer and the retailer's profit in the model where the optional service level is determined by the manufacturer are decreasing and the optimal optional service level in the model where the optional service level is determined by the manufacturer is firstly greater than the optimal optional service level in the model where the optional service level is determined by the retailer.

8. Conclusions and the Future Research Directions

Considering the existence of price and service competition, the supply chain equilibrium decision problem has become an important problem in the field of management science.

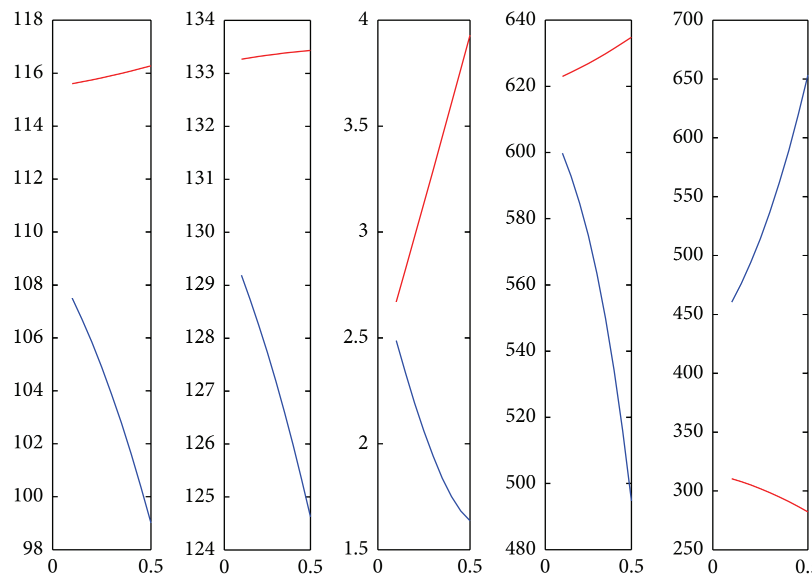

Figure 8: Effects of service elastic coefficient.

Optional service is one aspect of the service and also becomes more and more common in real life. In this paper, we assume that the basic service level is known and we use a general demand function to describe the impact of product price and optional service level on market demand in such a competitive environment. Moreover, a Stackelberg model structure is proposed between the retailer and the manufacturer in a two-echelon supply chain. We obtain the optimal wholesale price, the optimal retail price, and the optimal optional service level when the optional service level is, respectively, determined by the manufacturer and the retailer. Finally, in order to compare the two models in the optimal equilibrium decision, the profit of the members, and the whole supply chain, we carried out numerical simulation, which has certain guiding significance to the enterprise management.

However, our research leaves several unanswered questions for future research. In this paper, we assume that the demand function is determined, while, in real life, the market demand is often random. Assuming that the demand function is random is a good extension of this paper. We also add some constraints to some known parameters. The other limitation of the model is that we cannot compare the equilibrium decisions of the two models in theory because of the complexity of the equilibrium decision. Hence, how to simplify the model is another important research direction.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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