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Research Article

Positive l_1 State-Bounding Observer Design for Positive Interval Markovian Jump Systems

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This paper studies the problem of positive l_1 state-bounding observer design for a class of positive Markovian jump systems with interval parameter uncertainties by a linear programming approach. For the first, necessary and sufficient conditions are obtained for stochastic stability and l_1 performance of positive Markovian jump systems by an "equivalent" deterministic positive linear system. Furthermore, based on the results obtained in this paper, sufficient conditions for the existence of the positive l_1 state-bounding observer are derived. The conditions can be solved in terms of linear programming. Finally, a numerical example is used to illustrate the effectiveness of the results obtained.

1. Introduction

Positive systems whose state and output are nonnegative for any given nonnegative initial state and input have developed a new branch and play an important role in system theory. Positive systems are frequently used in communication, queue processes, and traffic modeling [1]. Recently, many contributions, such as realization, controllability, reachability and stability [2, 3], and positive filtering [4], have been highlighted by many researchers.

As we know, if systems have their parameters or structures changed abruptly, it is necessary and natural to describe them as Markovian jump systems. Markovian jump systems have two mechanisms simultaneously. The first one is the time-evolving mechanism and related to the state vector. The second one called system mode is event-driven mechanism and driven by a Markov process taking values in a finite set. Some achievements on Markovian jump systems are given; for instance, see [5–10]. The conditions of stochastic stability on this kind of system are reported in [5–8]. When this system is positive, stochastic stability is investigated in [9, 10]. Also, there are many other results proposed, such as stabilisation [11], l_1-gain performance analysis and positive filter design [12], H_∞ filtering [13, 14], and l_1 control [15]. Sometimes, it is not easy to obtain all the state variables in practical systems. It is necessary to design observer to estimate state variables. The observer design problems for positive systems have been considered in [16–20]. To the best of our knowledge, the observer design for positive Markovian jump systems has not been fully investigated, especially systems with interval parameter uncertainties. We know that the conventional observers estimate the state of the system in an asymptotic way. If we want to obtain the information of the transient state of positive interval Markovian jump systems, we need to design new observers, which motivate the current research.

In this paper, we investigate the positive l_1 state-bounding observer design problem for positive interval Markovian jump systems. The main contributions of this paper include the following. (1) By an "equivalent" deterministic positive linear system, necessary and sufficient conditions are obtained for stochastic stability and l_1 performance of positive Markovian jump systems. (2) For positive interval Markovian jump systems, we design a new observer which is different from the traditional observer. Based on the proposed results, sufficient conditions for the existence of the positive l_1 state-bounding observer are derived.

The rest of the paper is organized as follows. Preliminaries are presented in Section 2. Stochastic stability and l_1 performance analysis problem are discussed in Section 3. In Section 4, observer problem of positive interval Markovian jump systems is studied. A numerical example is provided in Section 5. Conclusions are presented in Section 6.

Notations. $\mathbb{R}$ is the set of real number. $\mathbb{R}^n(\mathbb{R}_+^n)$ is the n-dimensional real (nonnegative) vector space; $\mathbb{R}^{p\times m}$ is the set of $p \times m$ real matrix. $\mathbb{Z}^+$ is the set of positive integer. $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space where Ω is sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space, and $\mathcal{P}$ is the probability measure. $A \succeq 0$ ($A \succ 0$, $A \preceq 0$, $A \prec 0$) means that all entries of matrix A are nonnegative (positive, nonpositive, and negative). $A \succ B$ ($A \succeq B$) means $A - B \succ 0$ ($A - B \succeq 0$). $E\{\cdot\}$ means the mathematical expectation of $\{\cdot\}$. $A_i \in [\underline{A_i}, \overline{A_i}]$ means $\underline{A_i} \preceq A_i \preceq \overline{A_i}$. 1-norm of vector $x(t) \in \mathbb{R}^n$ is denoted by $\|x(t)\|_1 = \sum_{k=1}^{n} |x_k(t)|$, where $x_k(t)$ is the kth component of $x(t) \in \mathbb{R}^n$. The l_1-norm of a Lebesgue integrable function $x(t)$ is defined as $\|x(t)\|_{l_1} = \int_0^\infty \|x(t)\|_1 dt$. The space of all vector-valued functions defined on $\mathbb{R}_+^n$ with finite l_1-norm is denoted by $l_1(\mathbb{R}_+^n)$. I_r is the r-dimensional identity matrix. The transpose of a matrix or a vector is expressed as the superscript "T." A block diagonal matrix with diagonal block $A_1, A_2, \ldots, A_r$ will be denoted by $\mathrm{diag}\{A_1, A_2, \ldots, A_r\}$. The symbol $\mathbf{1}_n$ is the n-dimensional vector whose all entries are equal to 1. $\otimes$ denotes the Kronecker product.

2. Preliminaries

In the complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, we will consider a class of continuous-time Markovian jump systems described as follows:

$$\begin{aligned} \dot{x}(t) &= A(r_t)\,x(t) + B(r_t)\,w(t) \\ y(t) &= C(r_t)\,x(t) + D(r_t)\,w(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $w(t) \in \mathbb{R}^m$ is the input, and $y(t) \in \mathbb{R}^p$ is the output. For simplicity, when $r_t = i$, the system matrices $A(r_t)$, $B(r_t)$, $C(r_t)$, and $D(r_t)$ are expressed as A_i, B_i, C_i, and D_i. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{p\times n}$, and $D_i \in \mathbb{R}^{p\times m}$ belong to the following interval uncertainty domain:

$$\begin{aligned} A_i &\in \left[\underline{A_i}, \overline{A_i}\right], \\ B_i &\in \left[\underline{B_i}, \overline{B_i}\right], \\ C_i &\in \left[\underline{C_i}, \overline{C_i}\right], \\ D_i &\in \left[\underline{D_i}, \overline{D_i}\right]. \end{aligned} \tag{2}$$

The jump process $\{r_t, t \geq 0\}$ is a homogeneous Markov process taking values in a finite set $S = \{1, 2, 3, \ldots, N\}$. System (1) has the following mode transition probabilities:

$$\Pr\{r_{t+\Delta t} = j \mid r_t = i\} = \begin{cases} \lambda_{ij}\Delta t + o(\Delta t) & i \neq j \\ 1 + \lambda_{ij}\Delta t + o(\Delta t) & i = j, \end{cases} \tag{3}$$

where $\Delta t > 0$, $\lim_{\Delta t\to 0}(o(\Delta t)/\Delta t) = 0$, and $\lambda_{ij} \geq 0$ ($i, j \in S$, $i \neq j$) denotes the transition rates from mode i at time t to mode j at time $t+\Delta t$, and $\sum_{j=1, j\neq i}^{N} \lambda_{ij} = -\lambda_{ii}$. Furthermore, the transition rate matrix of the Markov process can be expressed as

$$\Pi = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N1} & \lambda_{N2} & \cdots & \lambda_{NN} \end{pmatrix}. \tag{4}$$

Definition 1 (see [1]). System (1) is said to be positive if and only if $x(t) \in \mathbb{R}_+^n$, $y(t) \in \mathbb{R}_+^p$, for any initial condition $x(0) \in \mathbb{R}_+^n$ and $w(t) \in \mathbb{R}_+^m$, $\forall t \geq 0$, $\forall r_0 \in S$.

Definition 2 (see [10]). System (1) is stochastically stable if the solution to system (1) for $w(t) = 0$ satisfies $E\{\int_0^\infty \|x(t)\|_1 dt \mid x(0), r_0\} < \infty$, where $x(0)$ is the initial condition and $r_0 \in S$.

Lemma 3 (see [1]). *System (1) is positive if and only if A_i is Metzler matrix, $B_i \succeq 0$, $C_i \succeq 0$, and $D_i \succeq 0$, $i \in S$.*

Remark 4. A_i, B_i, C_i, and D_i belong to the following interval uncertainty domain: $A_i \in [\underline{A_i}, \overline{A_i}]$, $B_i \in [\underline{B_i}, \overline{B_i}]$, $C_i \in [\underline{C_i}, \overline{C_i}]$, and $D_i \in [\underline{D_i}, \overline{D_i}]$. A_i, B_i, C_i, and D_i are uncertain, but $\underline{A_i}$, $\underline{B_i}$, $\underline{C_i}$, and $\underline{D_i}$ are known. Due to $\underline{A_i} \preceq A_i$, $\underline{B_i} \preceq B_i$, $\underline{C_i} \preceq C_i$, and $\underline{D_i} \preceq D_i$, if $\underline{A_i}$ is Metzler matrix, $\underline{B_i} \succeq 0$, $\underline{C_i} \succeq 0$, and $\underline{D_i} \succeq 0$ for any $i \in S$, it is natural that system (1) is positive.

Lemma 5 (see [21]). *Consider the following positive system:*

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bw(t) \\ y(t) &= Cx(t) + Dw(t). \end{aligned} \tag{5}$$

The following statements are equivalent for $w(t) = 0$:

(i) *This system is asymptotically stable.*

(ii) *A is a Hurwitz.*

(iii) *There exists a vector $p \succ 0$ such that $p^T A \prec 0$.*

Definition 6 (see [19]). Suppose that positive system (5) is stable; its l_1-induced norm is defined as

$$\|\mathfrak{I}\|_{(l_1-l_1)} \triangleq \sup_{w\neq 0, w\in l_1(\mathbb{R}_+^n)} \frac{\|y\|_{l_1}}{\|w\|_{l_1}}, \tag{6}$$

where $\mathfrak{I} : l_1 \to l_1$ denotes the convolution operator; that is, $y(t) = (\mathfrak{I} * w)(t)$. System (5) has l_1-induced performance at the level γ if, under zero initial conditions, $\|\mathfrak{I}\|_{(l_1-l_1)} < \gamma$, where γ is a given scalar.

Lemma 7 (see [4]). *Positive system (5) is asymptotically stable and satisfies $\|y\|_{l_1} \leq \gamma\|w\|_{l_1}$ if and only if there exists a vector $p \succ 0$ satisfying*

$$\begin{aligned} \mathbf{1}^T C + p^T A &\prec 0 \\ p^T B + \mathbf{1}^T D - \gamma \mathbf{1}^T &\prec 0. \end{aligned} \tag{7}$$

Definition 8. Suppose that positive system (1) is stable; its l_1-induced norm is defined as

$$\|\mathfrak{I}_M\|_{(l_1-l_1)} \triangleq \sup_{w\neq 0, w\in l_1(\mathbb{R}_+^n), r_0\in S} \frac{\|y\|_{l_1}}{\|w\|_{l_1}}, \tag{8}$$

where $\mathfrak{I} : l_1 \to l_1$ denotes the convolution operator; that is, $y(t) = (\mathfrak{I} * w)(t)$. System (1) has l_1-induced performance at the level γ if, under zero initial conditions, $\|\mathfrak{I}_M\|_{(l_1-l_1)} < \gamma$, where γ is a given scalar.

3. Stochastic Stability and l_1 Performance Analysis

In this section, we consider the stochastic stability and analyze the l_1-induced performance for positive Markovian jump systems.

Theorem 9. *The following statements are equivalent:*

(i) *Positive system (1) is stochastically stable.*

(ii) *There exist vectors $v_i \succ 0$, $i \in S$, such that*

$$A_i^T v_i + \sum_{j=1}^{N} \lambda_{ij} v_j \prec 0. \tag{9}$$

(iii) $\widetilde{A} = \Pi^T \otimes I_n + \operatorname{diag}\{A_1, A_2, \dots, A_N\}$ *is Hurwitz.*

Proof. (i)⇔(iii) Define the indicator function as (2.2) in [22]

$$\mathbf{1}_{\{r_t=i\}}(w) = \begin{cases} 1 & r_t(w) = i,\ i \in S \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Let $q(t) = E\{x(t)\}$, $q_i(t) = E\{x(t)\mathbf{1}_{\{r_t=i\}}\}$. Then

$$q(t) = E\{x(t)\} = \sum_{i=1}^{N} E\left\{x(t)\mathbf{1}_{\{r_t=i\}}\right\} = \sum_{i=1}^{N} q_i(t). \tag{11}$$

By [22], we have

$$\dot{q}_i(t) = A_i q_i(t) + \sum_{j=1}^{N} \lambda_{ji} q_j(t). \tag{12}$$

Note

$$\begin{gathered}\widehat{q}(t) = \left[q_1^T(t)\ \ q_2^T(t)\ \ \cdots\ \ q_N^T(t)\right]^T, \\ \widetilde{A} = \Pi^T \otimes I_n + \operatorname{diag}\{A_1, A_2, \dots, A_N\}.\end{gathered} \tag{13}$$

Then we obtain $\widehat{q}(t)$ that satisfies the following system:

$$\begin{gathered}\dot{\widehat{q}}(t) = \widetilde{A}\widehat{q}(t) \\ \widehat{q}(0) = \left[q_1^T(0)\ \ q_2^T(0)\ \ \cdots\ \ q_N^T(0)\right]^T \geq 0.\end{gathered} \tag{14}$$

Also we can conclude the following equation as (16) in [12]:

$$\begin{aligned}\|\widehat{q}(t)\|_1 &= \mathbf{1}_{Nn}^T \widehat{q}(t) = \mathbf{1}_n^T \sum_{i=1}^{N} q_i(t) = \mathbf{1}_n^T E\{x(t)\} \\ &= E\{\|x(t)\|_1\}.\end{aligned} \tag{15}$$

Then we have

$$\begin{aligned}&E\left\{\int_0^\infty \|x(t)\|_1\, dt \mid x(0), r_0\right\} \\ &\quad = \int_0^\infty E\{\|x(t)\|_1 \mid x(0), r_0\}\, dt = \int_0^\infty \|\widehat{q}(t)\|_1\, dt \\ &\quad < \infty\end{aligned} \tag{16}$$

which implies $\lim_{t\to\infty}\|\widehat{q}(t)\|_1 = 0$; that is, for every $\widehat{q}(0) \geq 0$, $\lim_{t\to\infty}\widehat{q}(t) = 0$. Therefore, the stochastic stability of system (1) is equivalent to asymptotic stability of system (14). By Lemma 5, system (14) being asymptotically stable is equivalent to the matrix $\widetilde{A} = \Pi^T \otimes I_n + \operatorname{diag}\{A_1, A_2, \dots, A_N\}$ being Hurwitz.

(ii)⇔(iii) By Lemma 5, the matrix $\widetilde{A} = \Pi^T \otimes I_n + \operatorname{diag}\{A_1, A_2, \dots, A_N\}$ being Hurwitz is equivalent to the fact that there exists a vector $p \succ 0$ satisfying $p^T\widetilde{A} \prec 0$. Let $p = \left[p_1^T\ \ p_2^T\ \ \cdots\ \ p_n^T\right]^T$, where $p_i \in \mathbb{R}^n$; then

$$\left(p_1^T\ \ p_2^T\ \ \cdots\ \ p_n^T\right)\begin{pmatrix} A_1 + \lambda_{11}I_n & \lambda_{21}I_n & \cdots & \lambda_{N1}I_n \\ \lambda_{12}I_n & A_2 + \lambda_{22}I_n & \cdots & \lambda_{N2}I_n \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{1N}I_n & \lambda_{2N}I_n & \cdots & A_N + \lambda_{NN}I_n \end{pmatrix} \prec 0 \tag{17}$$

which implies $p_i^T A_i + \sum_{j=1}^{N} \lambda_{ij} p_j^T \prec 0$; it is equivalent to $A_i^T p_i + \sum_{j=1}^{N} \lambda_{ij} p_j \prec 0$. The conclusion holds. The proof is completed. □

Theorem 10. *For positive system (1) and a given $\gamma > 0$, system (1) is stochastically stable and satisfies $\|y\|_{l_1} \leq \gamma\|w\|_{l_1}$ if and only if there exist vectors $p_i \succ 0$, $i \in S$, such that*

$$\mathbf{1}^T C_i + p_i^T A_i + \sum_{j=1}^{N} \lambda_{ij} p_j^T \prec 0 \tag{18}$$

$$p_i^T B_i + \mathbf{1}^T D_i - \gamma \mathbf{1}^T \prec 0. \tag{19}$$

Proof. Define the indicator function as Theorem 9. Let

$$\begin{aligned}
\widehat{z}(t) &= \left[z_1^T(t) \;\; z_2^T(t) \;\; \cdots \;\; z_N^T(t)\right]^T \\
\widehat{w}(t) &= \left[w_1^T(t) \;\; w_2^T(t) \;\; \cdots \;\; w_N^T(t)\right]^T \\
z_i(t) &= E\left\{y(t)\mathbf{1}_{\{r_t=i\}}\right\} \\
w_i(t) &= E\left\{w(t)\mathbf{1}_{\{r_t=i\}}\right\} \\
\widetilde{A} &= \Pi^T \otimes I_n + \operatorname{diag}\{A_1, A_2, \ldots, A_N\} \\
\widetilde{B} &= \operatorname{diag}\{B_1, B_2, \ldots, B_N\} \\
\widetilde{C} &= \operatorname{diag}\{C_1, C_2, \ldots, C_N\} \\
\widetilde{D} &= \operatorname{diag}\{D_1, D_2, \ldots, D_N\}.
\end{aligned} \tag{20}$$

By [22], it follows that

$$\begin{aligned}
dq_i(t) &= E\left\{dx(t)\mathbf{1}_{\{r_t=i\}} + x(t)\, d\mathbf{1}_{\{r_t=i\}}\right\} \\
&= E\left\{\left(A_i x(t) + B_i w(t)\right)\mathbf{1}_{\{r_t=i\}}\right\} dt \\
&\quad + E\left\{x(t)\, d\mathbf{1}_{\{r_t=i\}}\right\} \\
&= A_i E\left\{x(t)\mathbf{1}_{\{r_t=i\}}\right\} dt + B_i E\left\{w(t)\mathbf{1}_{\{r_t=i\}}\right\} dt \\
&\quad + \sum_{j=1}^{N} \lambda_{ji} q_j(t)\, dt \\
&= A_i q_i(t)\, dt + B_i w_i(t)\, dt + \sum_{j=1}^{N} \lambda_{ji} q_j(t)\, dt \\
z_i(t) &= E\left\{y(t)\mathbf{1}_{\{r_t=i\}}\right\} \\
&= E\left\{\left[C_i x(t) + D_i w(t)\right]\mathbf{1}_{\{r_t=i\}}\right\} \\
&= C_i E\left\{x(t)\mathbf{1}_{\{r_t=i\}}\right\} + D_i E\left\{w(t)\mathbf{1}_{\{r_t=i\}}\right\} \\
&= C_i q_i(t) + D_i w_i(t).
\end{aligned} \tag{21}$$

From (21), we obtain the following system:

$$\begin{aligned}
\dot{\widehat{q}}(t) &= \widetilde{A}\widehat{q}(t) + \widetilde{B}\widehat{w}(t) \\
\widehat{z}(t) &= \widetilde{C}\widehat{q}(t) + \widetilde{D}\widehat{w}(t).
\end{aligned} \tag{22}$$

Since the proof of Theorem 9, system (22) is stable if and only if system (1) is stochastically stable. Next, we want to show the relationship of l_1-induced performance between system (1) and system (22). Applying the similar way of (15), there are $\|\widehat{z}(t)\|_1 = E\{\|y(t)\|_1\}$ and $\|\widehat{w}(t)\|_1 = E\{\|w(t)\|_1\}$; then $\|y\|_{l_1} \le \gamma\|w\|_{l_1}$ is equivalent to $\|\widehat{z}\|_{l_1} \le \gamma\|\widehat{w}\|_{l_1}$. Therefore, system (1) is stochastically stable and satisfies $\|y\|_{l_1} \le \gamma\|w\|_{l_1}$ if and only if system (22) is stable and satisfies $\|\widehat{z}\|_{l_1} \le \gamma\|\widehat{w}\|_{l_1}$. By Lemma 7, there exists a vector $p = \left[p_1^T \;\; p_2^T \;\; \cdots \;\; p_n^T\right]^T \ge 0$ satisfying

$$\begin{aligned}
\mathbf{1}^T\widetilde{C} + p_i^T\widetilde{A} &\prec 0 \\
p_i^T\widetilde{B} + \mathbf{1}^T\widetilde{D} - \gamma\mathbf{1}^T &\prec 0.
\end{aligned} \tag{23}$$

Substitute $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$, and $\widetilde{D}$ to (23); the conclusion is proved. The proof is completed. □

4. Design of Observer

We know that the conventional observers estimate the state of the system in an asymptotic way. If we want to obtain the information of the transient state of positive interval Markovian jump systems, we need to design new observers. Therefore, we design a pair of positive l_1 state-bounding observers that can bound the state $x(t)$ all the time.

For system (1), observers are considered as follows:

$$\dot{\widehat{x}}(t) = \overline{F}_i\widehat{x}(t) + \overline{G}_i y(t) + \overline{K}_i w(t) \tag{24}$$

$$\dot{\check{x}}(t) = \underline{F}_i\check{x}(t) + \underline{G}_i y(t) + \underline{K}_i w(t), \tag{25}$$

where $i \in S$, $\widehat{x}(t) \in \mathbb{R}^n$, and $\check{x}(t) \in \mathbb{R}^n$ are the upper-bounding and lower-bounding estimated state of state $x(t)$; $\overline{F}_i \in \mathbb{R}^{n\times n}$, $\overline{G}_i \in \mathbb{R}^{n\times p}$, $\overline{K}_i \in \mathbb{R}^{n\times m}$, $\underline{F}_i \in \mathbb{R}^{n\times m}$, $\underline{G}_i \in \mathbb{R}^{n\times m}$, and $\underline{K}_i \in \mathbb{R}^{n\times m}$ are observer parameters to be determined.

Define $\widehat{e}(t) = \widehat{x}(t) - x(t)$. By systems (1) and (24), we have

$$\begin{aligned}
\dot{\widehat{e}}(t) &= \left(\overline{F}_i + \overline{G}_i C_i - A_i\right)x(t) + \overline{F}_i\widehat{e}(t) \\
&\quad + \left(\overline{G}_i D_i + \overline{K}_i - B_i\right)w(t).
\end{aligned} \tag{26}$$

We let $\widehat{z}_o(t) = L_i\widehat{e}(t)$, where $\widehat{z}_o(t)$ is the output of error state; $L_i \ge 0$ $(i \in S)$ are known.

Define

$$\begin{aligned}
\widehat{x}_o(t) &= \begin{pmatrix} x(t) \\ \widehat{e}(t) \end{pmatrix}, \\
\overline{A}_{oi} &= \begin{pmatrix} A_i & 0 \\ \overline{F}_i + \overline{G}_i C_i - A_i & \overline{F}_i \end{pmatrix} \\
\overline{B}_{oi} &= \begin{pmatrix} B_i \\ \overline{G}_i D_i + \overline{K}_i - B_i \end{pmatrix}, \\
\overline{C}_{oi} &= \begin{pmatrix} 0 & L_i \end{pmatrix}.
\end{aligned} \tag{27}$$

Then by (26) and (27), we have the system as follows:

$$\begin{aligned}
\dot{\widehat{x}}_o(t) &= \overline{A}_{oi}\widehat{x}_o(t) + \overline{B}_{oi}w(t) \\
\widehat{z}_o(t) &= \overline{C}_{oi}\widehat{x}_o(t).
\end{aligned} \tag{28}$$

Observer (24) is designed for positive system (1) to approximate $x(t)$ by $\widehat{x}(t)$. Therefore, the estimate $\widehat{x}(t)$ is required to be positive; that is, the observer (24) is positive. By Lemma 3, we know it needs that $\overline{F}_i$ is Metzler, $\overline{G}_i \succeq 0$ and $\overline{K}_i \succeq 0$.

Therefore, the upper-bounding observer problem can be stated as follows: design a positive observer in the form of (24) such that system (28) is positive and stochastically stable and satisfies the performance $\|\hat{z}_o\|_{l_1} \le \gamma\|w\|_{l_1}$ under zero initial conditions.

Similarly, define $\check{e}(t) = x(t) - \check{x}(t)$. By systems (1) and (25), we have

$$\begin{aligned}\dot{\check{e}}(t) &= (A_i - \underline{G}_i C_i - \underline{F}_i)\, x(t) + \underline{F}_i \check{e}(t) \\ &\quad + (B_i - \underline{G}_i D_i - \underline{K}_i)\, w(t).\end{aligned} \tag{29}$$

We let $\check{z}_o(t) = L_i\check{e}(t)$, where $z_o(t)$ is the output of error state; $L_i \succeq 0$ $(i \in S)$ are known. Define

$$\begin{aligned}\check{x}_o(t) &= \begin{pmatrix} x(t) \\ \check{e}(t)\end{pmatrix}, \\ \underline{A}_{oi} &= \begin{pmatrix} A_i & 0 \\ A_i - \underline{G}_i C_i - \underline{F}_i & \underline{F}_i\end{pmatrix} \\ \underline{B}_{oi} &= \begin{pmatrix} B_i \\ B_i - \underline{G}_i D_i - \underline{K}_i\end{pmatrix}, \\ \underline{C}_{oi} &= \begin{pmatrix} 0 & L_i\end{pmatrix}.\end{aligned} \tag{30}$$

Then by (29) and (30), we have the system as follows:

$$\begin{aligned}\dot{\check{x}}_o(t) &= \underline{A}_{oi} x_o(t) + \underline{B}_{oi} w(t) \\ \check{z}_o(t) &= \underline{C}_{oi} x_o(t).\end{aligned} \tag{31}$$

Therefore, the lower-bounding observer problem can be stated as follows: design a positive observer in the form of (25) such that system (31) is positive and stochastically stable and satisfies the performance $\|\check{z}_o\|_{l_1} \le \gamma\|w\|_{l_1}$ under zero initial conditions.

Next, we give the existence condition of the upper-bounding and lower-bounding observer. Before giving the condition, we denote system matrix as follows:

$$\begin{aligned}\overline{A}_i &= \begin{pmatrix}\overline{A}_{i1} & \overline{A}_{i2} & \cdots & \overline{A}_{in}\end{pmatrix}^T \\ \underline{A}_i &= \begin{pmatrix}\underline{A}_{i1} & \underline{A}_{i2} & \cdots & \underline{A}_{in}\end{pmatrix}^T \\ \overline{B}_i &= \begin{pmatrix}\overline{B}_{i1} & \overline{B}_{i2} & \cdots & \overline{B}_{in}\end{pmatrix}^T \\ \underline{B}_i &= \begin{pmatrix}\underline{B}_{i1} & \underline{B}_{i2} & \cdots & \underline{B}_{in}\end{pmatrix}^T \\ \overline{C}_i &= \begin{pmatrix}\overline{C}_{i1} & \overline{C}_{i2} & \cdots & \overline{C}_{in}\end{pmatrix} \\ \underline{C}_i &= \begin{pmatrix}\underline{C}_{i1} & \underline{C}_{i2} & \cdots & \underline{C}_{in}\end{pmatrix},\end{aligned} \tag{32}$$

where $\overline{A}_{it} \in \mathbb{R}^n$, $\overline{B}_{it} \in \mathbb{R}^m$, $\overline{C}_{it} \in \mathbb{R}^p$, $\underline{A}_{it} \in \mathbb{R}^n$, $\underline{B}_{it} \in \mathbb{R}^m$, $\underline{C}_{it} \in \mathbb{R}^p$, $i \in S$, $t = 1, 2, \ldots, n$, $s = 1, 2, \ldots, m$.

Theorem 11. *Consider positive system (1). For a given $\gamma > 0$, there exists positive upper-bounding observer (24) such that system (28) is positive and stochastically stable and satisfies $\|\hat{z}_o\|_{l_1} \le \gamma\|w\|_{l_1}$ if there exist Metzler matrix $\overline{\eta}_i = \begin{pmatrix}\overline{\eta}_{i1} & \overline{\eta}_{i2} & \cdots & \overline{\eta}_{in}\end{pmatrix}^T$, $\overline{\xi}_i = \begin{pmatrix}\overline{\xi}_{i1} & \overline{\xi}_{i2} & \cdots & \overline{\xi}_{in}\end{pmatrix}^T \succeq 0$, $\overline{\theta}_i = \begin{pmatrix}\overline{\theta}_{i1} & \overline{\theta}_{i2} & \cdots & \overline{\theta}_{in}\end{pmatrix}^T \succeq 0$, $\alpha_i \succ 0$, and $\beta_i = \begin{pmatrix}\beta_{i1} & \beta_{i2} & \cdots & \beta_{in}\end{pmatrix}^T \succ 0$ with $\alpha_i \in \mathbb{R}^n$, $\beta_i \in \mathbb{R}^n$, $\overline{\eta}_{it} \in \mathbb{R}^n$, $\overline{\xi}_{it} \in \mathbb{R}^p$, $\overline{\theta}_{it} \in \mathbb{R}^m$, $i \in S$, $t = 1, 2, \ldots, n$, such that*

$$\overline{\eta}_{it}^T + \overline{\xi}_{it}^T \underline{C}_i - \beta_{it}\overline{A}_{it}^T \succeq 0 \tag{33}$$

$$\overline{\xi}_{it}^T \underline{D}_i + \overline{\theta}_{it}^T - \beta_{it}\overline{B}_{it}^T \succeq 0 \tag{34}$$

$$\alpha_i^T \overline{A}_i + \sum_{j=1}^{n} \overline{\eta}_{ij}^T + \sum_{j=1}^{n} \overline{\xi}_{ij}^T \overline{C}_i - \beta_i^T \underline{A}_i + \sum_{j=1}^{N} \lambda_{ij}\alpha_j^T \prec 0 \tag{35}$$

$$\mathbf{1}^T L_i + \sum_{j=1}^{n} \overline{\eta}_{ij}^T + \sum_{j=1}^{N} \lambda_{ij}\beta_j^T \prec 0 \tag{36}$$

$$\alpha_i^T \overline{B}_i + \sum_{j=1}^{n} \overline{\xi}_{ij}^T \overline{D}_i + \sum_{j=1}^{n} \overline{\theta}_{ij}^T - \beta_i^T \underline{B}_i - \gamma\mathbf{1}^T \prec 0. \tag{37}$$

Then, the parameters of the observer are given by

$$\begin{aligned}\overline{G}_i &= \begin{pmatrix}\overline{G}_{i1} & \overline{G}_{i2} & \cdots & \overline{G}_{in}\end{pmatrix}^T \\ &= \begin{pmatrix}\beta_{i1}^{-1}\overline{\xi}_{i1} & \beta_{i2}^{-1}\overline{\xi}_{i2} & \cdots & \beta_{in}^{-1}\overline{\xi}_{in}\end{pmatrix}^T \\ \overline{F}_i &= \begin{pmatrix}\overline{F}_{i1} & \overline{F}_{i2} & \cdots & \overline{F}_{in}\end{pmatrix}^T \\ &= \begin{pmatrix}\beta_{i1}^{-1}\overline{\eta}_{i1} & \beta_{i2}^{-1}\overline{\eta}_{i2} & \cdots & \beta_{in}^{-1}\overline{\eta}_{in}\end{pmatrix}^T \\ \overline{K}_i &= \begin{pmatrix}\overline{K}_{i1} & \overline{K}_{i2} & \cdots & \overline{K}_{in}\end{pmatrix}^T \\ &= \begin{pmatrix}\beta_{i1}^{-1}\overline{\theta}_{i1} & \beta_{i2}^{-1}\overline{\theta}_{i2} & \cdots & \beta_{in}^{-1}\overline{\theta}_{in}\end{pmatrix}^T.\end{aligned} \tag{38}$$

Proof. Since $\beta_i \succ 0$, $\overline{\xi}_i \succeq 0$, and $\overline{\theta}_i \succeq 0$ and $\overline{\eta}_i$ is Metzler, it follows that $\overline{F}_i$ is Metzler, $\overline{G}_i \succeq 0$ and $\overline{K}_i \succeq 0$ from (38). Thus, observer (24) is positive.

From (33), (34), and $\beta_i \succ 0$, we obtain

$$\begin{aligned}\beta_{it}^{-1}\overline{\eta}_{it}^T + \beta_{it}^{-1}\overline{\xi}_{it}^T \underline{C}_i - \overline{A}_{it}^T &\succeq 0 \\ \beta_{it}^{-1}\overline{\xi}_{it}^T \underline{D}_i + \beta_{it}^{-1}\overline{\theta}_{it}^T - \overline{B}_{it}^T &\succeq 0.\end{aligned} \tag{39}$$

By (38), we have

$$\begin{aligned}\overline{F}_{it}^T + \overline{G}_{it}^T \underline{C}_i - \overline{A}_{it}^T &\succeq 0 \\ \overline{G}_{it}^T \underline{D}_i + \overline{K}_{it}^T - \overline{B}_{it}^T &\succeq 0.\end{aligned} \tag{40}$$

Further, we obtain

$$\begin{aligned}\overline{F}_i + \overline{G}_i \underline{C}_i - \overline{A}_i &\succeq 0 \\ \overline{G}_i \underline{D}_i + \overline{K}_i - \overline{B}_i &\succeq 0.\end{aligned} \tag{41}$$

From (41) and $\overline{G}_i \geq 0$, it follows that, for any $A_i \in [\underline{A}_i, \overline{A}_i]$, $B_i \in [\underline{B}_i, \overline{B}_i]$, $C_i \in [\underline{C}_i, \overline{C}_i]$, and $D_i \in [\underline{D}_i, \overline{D}_i]$,

$$\begin{aligned} &\overline{F}_i + \overline{G}_i C_i - A_i \geq \overline{F}_i + \overline{G}_i \underline{C}_i - \overline{A}_i \geq 0 \\ &\overline{G}_i D_i + \overline{K}_i - B_i \geq \overline{G}_i \underline{D}_i + \overline{K}_i - \overline{B}_i \geq 0 \end{aligned} \tag{42}$$

which imply $\overline{A}_{oi}$ is Metzler and $\overline{B}_{oi} \geq 0$ in (27); therefore, system (28) is positive.

From (38), we have

$$\begin{aligned} &\sum_{j=1}^{n} \overline{\xi}_{ij}^T = \sum_{j=1}^{n} \beta_{ij} \overline{G}_{ij}^T = \beta_i^T \overline{G}_i \\ &\sum_{j=1}^{n} \overline{\eta}_{ij}^T = \sum_{j=1}^{n} \beta_{ij} \overline{F}_{ij}^T = \beta_i^T \overline{F}_i \\ &\sum_{j=1}^{n} \overline{\theta}_{ij}^T = \sum_{j=1}^{n} \beta_{ij} \overline{K}_{ij}^T = \beta_i^T \overline{K}_i. \end{aligned} \tag{43}$$

According to (43), (35)–(37) become

$$\begin{aligned} &\alpha_i^T \overline{A}_i + \beta_i^T \overline{F}_i + \beta_i^T \overline{G}_i \overline{C}_i - \beta_i^T \underline{A}_i + \sum_{j=1}^{N} \lambda_{ij} \alpha_j^T \prec 0 \\ &\mathbf{1}^T L_i + \beta_i^T \overline{F}_i + \sum_{j=1}^{N} \lambda_{ij} \beta_j^T \prec 0 \\ &\alpha_i^T \overline{B}_i + \beta_i^T \overline{G}_i \overline{D}_i + \beta_i^T \overline{K}_i - \beta_i^T \underline{B}_i - \gamma \mathbf{1}^T \prec 0 \end{aligned} \tag{44}$$

which imply that

$$\begin{aligned} &\mathbf{1}^T \begin{pmatrix} 0 & L_i \end{pmatrix} + p_i^T \begin{pmatrix} \overline{A}_i & 0 \\ \overline{F}_i + \overline{G}_i \overline{C}_i - \underline{A}_i & \overline{F}_i \end{pmatrix} + \sum_{j=1}^{N} \lambda_{ij} p_j^T \prec 0 \\ &p_i^T \begin{pmatrix} \overline{B}_i \\ \overline{G}_i \overline{D}_i + \overline{K}_i - \underline{B}_i \end{pmatrix} - \gamma \mathbf{1}^T \prec 0, \end{aligned} \tag{45}$$

where $p_i^T = \begin{pmatrix} \alpha_i^T & \beta_i^T \end{pmatrix}$. For any $A_i \in [\underline{A}_i, \overline{A}_i]$, $B_i \in [\underline{B}_i, \overline{B}_i]$, $C_i \in [\underline{C}_i, \overline{C}_i]$, and $D_i \in [\underline{D}_i, \overline{D}_i]$, we obtain

$$\begin{aligned} &\overline{F}_i + \overline{G}_i C_i - A_i \leq \overline{F}_i + \overline{G}_i \overline{C}_i - \underline{A}_i \\ &\overline{G}_i D_i + \overline{K}_i - B_i \leq \overline{G}_i \overline{D}_i + \overline{K}_i - \underline{B}_i. \end{aligned} \tag{46}$$

Further, we have

$$\begin{aligned} &\mathbf{1}^T \begin{pmatrix} 0 & L_i \end{pmatrix} + p_i^T \begin{pmatrix} A_i & 0 \\ \overline{F}_i + \overline{G}_i C_i - A_i & \overline{F}_i \end{pmatrix} + \sum_{j=1}^{N} \lambda_{ij} p_j^T \prec 0 \\ &p_i^T \begin{pmatrix} B_i \\ \overline{G}_i \mathrm{D}_i + \overline{K}_i - B_i \end{pmatrix} - \gamma \mathbf{1}^T \prec 0. \end{aligned} \tag{47}$$

By Theorem 10, system (28) is stochastically stable and satisfies $\|y\|_{l_1} \leq \gamma \|w\|_{l_1}$. The proof is completed. □

Similarly, we give the existence condition of the lower-bounding observer.

Theorem 12. *Consider positive system (1). For a given $\gamma > 0$, there exists positive lower-bounding observer (25) such that system (31) is positive and stochastically stable and satisfies $\|\breve{z}_o\|_{l_1} \leq \gamma \|w\|_{l_1}$ if there exist Metzler matrix $\underline{\eta}_i = \begin{pmatrix} \underline{\eta}_{i1} & \underline{\eta}_{i2} & \cdots & \underline{\eta}_{in} \end{pmatrix}^T$, $\underline{\xi}_i = \begin{pmatrix} \underline{\xi}_{i1} & \underline{\xi}_{i2} & \cdots & \underline{\xi}_{in} \end{pmatrix}^T \geq 0$, $\underline{\theta}_i = \begin{pmatrix} \underline{\theta}_{i1} & \underline{\theta}_{i2} & \cdots & \underline{\theta}_{in} \end{pmatrix}^T \geq 0$, and $\alpha_i \succ 0$, $\beta_i = \begin{pmatrix} \beta_{i1} & \beta_{i2} & \cdots & \beta_{in} \end{pmatrix}^T \succ 0$ with $\alpha_i \in \mathbb{R}^n$, $\beta_i \in \mathbb{R}^n$, $\underline{\eta}_{it} \in \mathbb{R}^n$, $\underline{\xi}_{it} \in \mathbb{R}^p$, $\underline{\theta}_{it} \in \mathbb{R}^m$, $i \in S$, $t = 1, 2, \ldots, n$, such that*

$$\begin{aligned} &\beta_{it} \underline{A}_{it}^T - \underline{\xi}_{it}^T \overline{C}_i - \underline{\eta}_{it}^T \geq 0 \\ &\beta_{it} \underline{B}_{it}^T - \underline{\xi}_{it}^T \overline{D}_i - \underline{\theta}_{it}^T \geq 0 \\ &\alpha_i^T \overline{A}_i + \beta_i^T \overline{A}_i - \sum_{j=1}^{n} \underline{\xi}_{ij}^T \underline{C}_i - \sum_{j=1}^{n} \underline{\eta}_{ij}^T + \sum_{j=1}^{N} \lambda_{ij} \alpha_j^T \prec 0 \\ &\sum_{j=1}^{n} \underline{\eta}_{ij}^T + \sum_{j=1}^{N} \lambda_{ij} \beta_j^T + \mathbf{1}^T L_i \prec 0 \\ &\alpha_i^T \overline{B}_i + \beta_i^T \overline{B}_i - \sum_{j=1}^{n} \underline{\xi}_{ij}^T \underline{D}_i - \sum_{j=1}^{n} \underline{\theta}_{ij}^T - \gamma \mathbf{1}^T \prec 0. \end{aligned} \tag{48}$$

Then, the parameters of the observer are given by

$$\begin{aligned} \underline{G}_i &= \begin{pmatrix} \underline{G}_{i1} & \underline{G}_{i2} & \cdots & \underline{G}_{in} \end{pmatrix}^T \\ &= \begin{pmatrix} \beta_{i1}^{-1} \underline{\xi}_{i1} & \beta_{i2}^{-1} \underline{\xi}_{i2} & \cdots & \beta_{in}^{-1} \underline{\xi}_{in} \end{pmatrix}^T \\ \underline{F}_i &= \begin{pmatrix} \underline{F}_{i1} & \underline{F}_{i2} & \cdots & \underline{F}_{in} \end{pmatrix}^T \\ &= \begin{pmatrix} \beta_{i1}^{-1} \underline{\eta}_{i1} & \beta_{i2}^{-1} \underline{\eta}_{i2} & \cdots & \beta_{in}^{-1} \underline{\eta}_{in} \end{pmatrix}^T \\ \underline{K}_i &= \begin{pmatrix} \underline{K}_{i1} & \underline{K}_{i2} & \cdots & \underline{K}_{in} \end{pmatrix}^T \\ &= \begin{pmatrix} \beta_{i1}^{-1} \underline{\theta}_{i1} & \beta_{i2}^{-1} \underline{\theta}_{i2} & \cdots & \beta_{in}^{-1} \underline{\theta}_{in} \end{pmatrix}^T. \end{aligned} \tag{49}$$

5. Numerical Examples

Consider a three-dimensional continuous-time uncertain Markovian jump system of form (1) with $r_t \in S = \{1, 2\}$, and its parameters are given by

$$A_1 = \begin{pmatrix} -1.5 \pm 0.01 & 0.1 & 0.5 \\ 0.5 & -2 \pm 0.01 & 0.2 \\ 0.1 & 0.7 \pm 0.02 & -0.9 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -1.4 \pm 0.02 & 0.2 & 0.4 \pm 0.01 \\ 0.7 & -1.5 & 0.3 \\ 0.2 & 0.5 & -0.8 \pm 0.03 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0.2 \\ 0.5 \\ 0.3 \end{pmatrix},$$
$$B_2 = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \pm 0.01 \end{pmatrix},$$
$$C_1 = \begin{pmatrix} 0.9 & 0.8 & 1 \pm 0.1 \end{pmatrix},$$
$$C_2 = \begin{pmatrix} 0.8 \pm 0.03 & 0.9 & 0.9 \end{pmatrix},$$
$$D_1 = 1,$$
$$D_2 = 1.2 \pm 0.02. \tag{50}$$

The transition rate matrix is given as

$$\Pi = \begin{pmatrix} -0.5 & 0.5 \\ 0.4 & -0.4 \end{pmatrix}. \tag{51}$$

Here, we choose $L_1 = \begin{pmatrix} 0.2 & 0.3 & 0.4 \end{pmatrix}$, $L_2 = \begin{pmatrix} 0.3 & 0.2 & 0.4 \end{pmatrix}$ and assume that $\gamma = 0.1$. Solving the LP problem in Theorems 11 and 12, the parameters of the positive upper-bounding observer and lower-bounding observer are given by

$$\underline{G}_1 = \begin{pmatrix} 0.0242 & 0.0173 & 0.0080 \end{pmatrix}^T,$$
$$\underline{G}_2 = \begin{pmatrix} 0.0233 & 0.0130 & 0.0057 \end{pmatrix}^T$$
$$\underline{F}_1 = \begin{pmatrix} -1.5315 & 0.0807 & 0.4733 \\ 0.4066 & -2.0237 & 0.1810 \\ 0.0722 & 0.6736 & -0.9088 \end{pmatrix}$$
$$\underline{F}_2 = \begin{pmatrix} -1.4393 & 0.1791 & 0.3691 \\ 0.6892 & -1.5117 & 0.2883 \\ 0.1953 & 0.4949 & -0.8051 \end{pmatrix}$$
$$\underline{K}_1 = \begin{pmatrix} 0.1757 & 0.4826 & 0.2920 \end{pmatrix}^T,$$
$$\underline{K}_2 = \begin{pmatrix} 0.0716 & 0.1842 & 0.2831 \end{pmatrix}^T$$
$$\overline{G}_1 = \begin{pmatrix} 0.0017 & 0.0011 & 0.0007 \end{pmatrix}^T,$$
$$\overline{G}_2 = \begin{pmatrix} 0.1244 & 0.0005 & 0.0100 \end{pmatrix}^T$$
$$\overline{F}_1 = \begin{pmatrix} -1.4914 & 0.0987 & 0.4984 \\ 0.4990 & -1.9890 & 0.1990 \\ 0.1006 & 0.7206 & -0.9006 \end{pmatrix}$$
$$\overline{F}_2 = \begin{pmatrix} -1.4758 & 0.1230 & 0.2979 \\ 0.7010 & -1.4912 & 0.2995 \\ 0.2230 & 0.7160 & -0.7790 \end{pmatrix}$$
$$\overline{K}_1 = \begin{pmatrix} 0.1984 & 0.4989 & 0.2993 \end{pmatrix}^T,$$
$$\overline{K}_2 = \begin{pmatrix} 0 & 0.1993 & 0.2980 \end{pmatrix}^T. \tag{52}$$

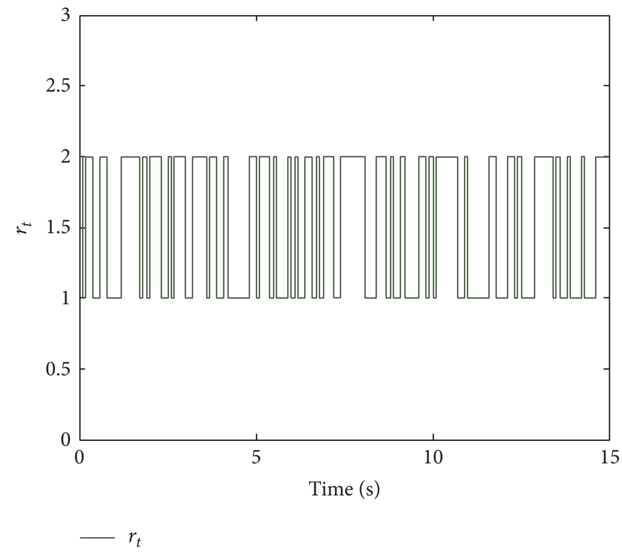

Figure 1: The simulation of system mode r_t.

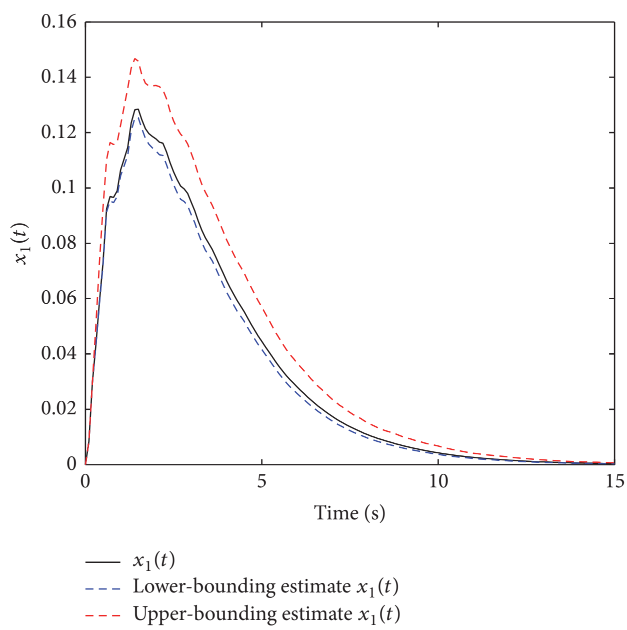

Figure 2: System state and estimated system state $x_1(t)$.

With input $w(t) = 2.3e^{-t}|\sin 4t|$ and the initial conditions $x(0) = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$, we have the simulation of system mode shown in Figure 1. The system state, upper-bounding, and lower-bounding estimated states are showed in Figures 2, 3, and 4.

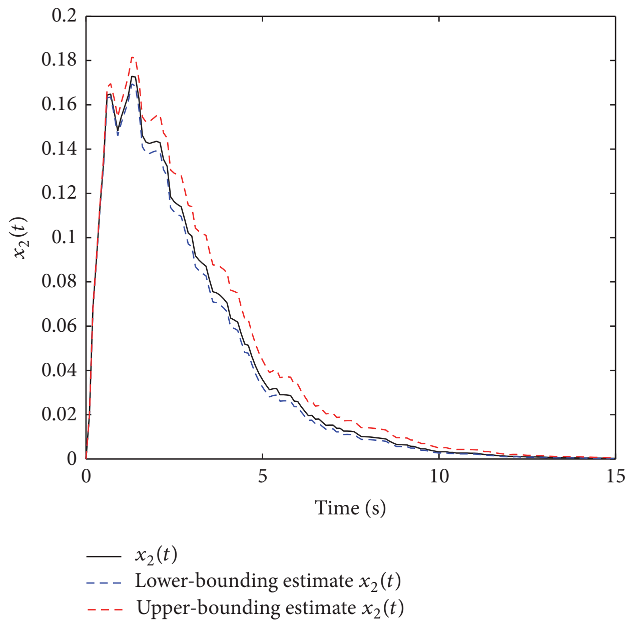

FIGURE 3: System state and estimated system state $x_2(t)$.

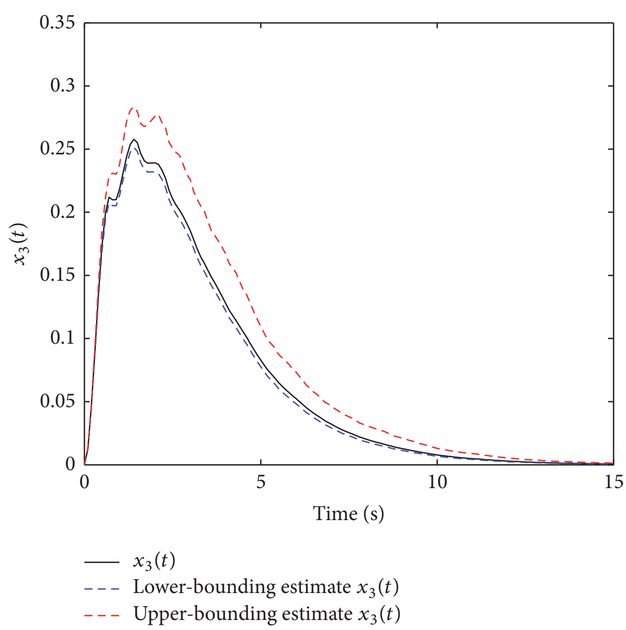

FIGURE 4: System state and estimated system state $x_3(t)$.

6. Conclusions

In this paper, positive l_1 state-bounding observer design for a class of positive Markovian jump systems with interval parameter uncertainties is investigated. First, necessary and sufficient conditions are obtained for stochastic stability and l_1 performance of positive Markovian jump systems by an "equivalent" deterministic positive linear system. Then based on the proposed results, sufficient conditions for existence of the positive l_1 state-bounding observer are derived. The conditions can be solved in terms of linear programming. Finally, a numerical example is used to demonstrate the effectiveness of the proposed results.

So far, many results on Markov jump systems have been applied to networked control systems, such as [23–27]. However, the state of networked control systems may need to be nonnegative. Therefore, the results of positive Markov jump systems can also be applied to networked control systems, which have been proposed in a few papers, such as [20]. In the future, we can try our best to extend our results to deal with the problem of networked control systems.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper

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