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Research Article

Stochastic Synchronization of Neutral-Type Neural Networks with Multidelays Based on M-Matrix

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The problem of stochastic synchronization of neutral-type neural networks with multidelays based on M-matrix is researched. Firstly, we designed a control law of stochastic synchronization of the neural-type and multiple time-delays neural network. Secondly, by making use of Lyapunov functional and M-matrix method, we obtained a criterion under which the drive and response neutral-type multiple time-delays neural networks with stochastic disturbance and Markovian switching are stochastic synchronization. The synchronization condition is expressed as linear matrix inequality which can be easily solved by MATLAB. Finally, we introduced a numerical example to illustrate the effectiveness of the method and result obtained in this paper.

1. Introduction

In recent years, neutral-type systems have been intensively studied due to the cause that many practical processes can be modeled as general neutral-type descriptor systems, such as computer aided design, circuit analysis, chemical process simulation, power systems, real time simulation of mechanical systems, population dynamics, and automatic control (see, e.g., [1–6] and the references therein). For example, in [1], the author studied the stability and asymptotic properties of a class of neutral-type functional differential equations based on the pattern equation method. In [2], the author investigated the asymptotic stability properties of neutral-type systems in Hilbert space.

On the one hand, time-delays as a source of instability and oscillators always appear in various aspects of neural networks. Recently, the stability of neural networks with time-delays has received lots of attention, such as [7, 8], and many methods, such as the linear matrix inequality (LMI) approach and M-matrix approach, have been adopted by the scholars; see, for example, [9, 10].

On the other hand, systems with Markov jump parameters, driven by continuous-time Markov chain, have been widely used to model many practical systems where they may experience abrupt changes in their structure and parameters. For example, in paper [11], a general model of an array of N linearly coupled delayed neural networks with Markovian jumping hybrid coupling is researched. In paper [12], the author researched the feedback control problem for a class of linear systems with Markovian jump parameters. The stabilization of stochastic delayed neural networks with Markovian switching was discussed in paper [13–16].

Meanwhile, the stability and synchronization of neutral-type systems which depend on the delays of state and state derivative have attracted a lot of attention (see [17–22] and the references therein) due to the fact that some physical systems in the real world can be described by neutral-type models. Besides the above these, according to [23, 24], M-matrix approach can not only design feed controller and trace all information of Markovian switching parameters but also has lower complexity than that of LMIs technology.

Inspired by the above discussions, in this paper, we are concerned with the analysis issue for the problem of stochastic synchronization of neutral-type neural networks with multidelays and Markovian switching. By using M-matrix approach and the stochastic analysis method, some synchronization criteria are obtained to ensure the stochastic synchronization for the neutral-type neural networks with multidelays. A numerical example is provided to illustrate the effectiveness of the results obtained in this paper. The main contributions of this paper are twofold: (1) Stochastic synchronization for a new class of neutral-type neural networks with multidelays and Markovian switching is considered. (2) The theory results which are obtained by M-method approach are more practical than that of LMIs technology.

2. System and Problem Description and Preliminaries

Consider n-dimensional multiple time-delays neutral-type neural network with Markovian switching of the form

$$d[x(t)-D(r(t))x(t-\tau)] = \Big[-A(r(t))x(t) + W(r(t))\varphi(x(t)) + \sum_{k=1}^{l} W_{dk}(r(t))\varphi(x(t-\tau_k(t))) + W_{d0}(r(t))\int_{t-\tau_0(t)}^{t}\varphi(x(s))\,ds\Big]dt, \tag{1}$$

where $x(t) = [x_1(t) \ \cdots \ x_n(t)]^T \in \mathbb{R}^n$ is the state vector associated with n neurons. $\varphi(x(t)) = [\varphi_1(x_1(t)) \ \cdots \ \varphi_n(x_n(t))]^T$ denotes the neuron activation function; τ denotes the constant time-delay. $\tau_k(t)$, $k = 0,1,\ldots,l$, are time-varying delays satisfying $0 < \tau_k(t) < \overline{\tau}$ and $\dot{\tau}_k(t) < \widehat{\tau} < 1$, $k = 0,1,\ldots,l$.

$\{r(t),\ t \geq 0\}$ is a right-continuous Markov chain on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). The Markov chain takes values in a finite state space $\mathbb{S} = \{1,2,\ldots,S\}$ with generator $\Gamma = (\gamma_{ij})_{S\times S}$ given by

$$\mathbb{P}\{r(t+\Delta)=j \mid r(t)=i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & \text{if } i \neq j \\ 1 + \gamma_{ii}\Delta + o(\Delta) & \text{if } i = j, \end{cases} \tag{2}$$

where $\Delta > 0$. Here $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$ while $\gamma_{ii} = -\sum_{j=1,j\neq i}^{S}\gamma_{ij}$.

$A(r(t)) \in \mathbb{R}^{n\times n}$ (A_i, $r(t) = i$, for short) is a positive diagonal matrix with its diagonal elements a_j, $j = 1,2,\ldots,n$. $W(r(t)) \in \mathbb{R}^{n\times n}$ and $W_{dk}(r(t)) \in \mathbb{R}^{n\times n}$ ($k = 0,1,\ldots,l$) are the connection weight matrix and the time-delay connection weight matrix, respectively. $D(r(t)) \in \mathbb{R}^{n\times n}$ is external input matrix.

For system (1) (called the drive system), the response system is of the following form:

$$d[y(t)-D(r(t))y(t-\tau)] = \Big[-A(r(t))y(t) + W(r(t))\varphi(y(t)) + \sum_{k=1}^{l} W_{dk}(r(t))\varphi(y(t-\tau_k(t))) + W_{d0}(r(t))\int_{t-\tau_0(t)}^{t}\varphi(y(s))\,ds + U(r(t))\Big]dt + g\big(t,r(t),e(t),e_{\tau_0}(t),e_{\tau_1}(t),\ldots,e_{\tau_l}(t)\big)\,d\omega(t), \tag{3}$$

where $y(t) = [y_1(t) \ \cdots \ y_n(t)]^T \in \mathbb{R}^n$ is the state vector of the drive system, $U(r(t)) \in \mathbb{R}^n$ is the control input vector, $e(t) = y(t) - x(t)$, $e_{\tau_k}(t) = y(t-\tau_k(t)) - x(t-\tau_k(t))$, $k = 0,1,\ldots,l$, and g is the noise intensity function satisfying $g : \mathbb{R}\times\mathbb{S}\times R^n\times\cdots\times\mathbb{R}^n \to \mathbb{R}^{n\times m}$.

$\omega(t) = [\omega_1(t),\omega_2(t),\ldots,\omega_m(t)]^T$ is m-dimensional $\mathcal{F}_t$-adapted Brownian motion. It is assumed that $\{r(t)\}$ and $\omega(t)$ in system (3) are independent.

For drive system (1) and response system (3), we can obtain the error system as follows:

$$d[e(t)-D(r(t))e(t-\tau)] = \Big[-A(r(t))e(t) + W(r(t))\psi(t) + \sum_{k=1}^{l} W_{dk}(r(t))\psi_{\tau_k}(t) + W_{d0}(r(t))\int_{t-\tau_0(t)}^{t}\psi(s)\,ds + U(r(t))\Big]dt + g\big(t,r(t),e(t),e_{\tau_0}(t),e_{\tau_1}(t),\ldots,e_{\tau_l}(t)\big)\,d\omega(t), \tag{4}$$

where $\psi(t) = \varphi(y(t)) - \varphi(x(t))$ and $\psi_{\tau_k}(t) = \varphi(y(t-\tau_k(t))) - \varphi(x(t-\tau_k(t)))$.

The initial data is given by $\{e(\theta) : -\widetilde{\tau} \leq \theta \leq 0\} = \xi(\theta) \in L^2_{\mathcal{F}_0}([-\widetilde{\tau},0];\mathbb{R}^n])$, $r(0) = r_0$, $\xi(0) = 0$, where $\widetilde{\tau} = \max\{\tau,\overline{\tau}\}$.

For error system (4), we impose the following assumptions.

Assumption 1. Each function $\varphi_j : \mathbb{R} \to \mathbb{R}$ is nondecreasing and there exists a positive constant Φ such that

$$\left|\varphi_j(u) - \varphi_j(v)\right| \leq \Phi|u-v| \quad \forall u,v \in \mathbb{R},\ j = 1,2,\ldots,n. \tag{5}$$

Assumption 2. $\forall i \in \mathbb{S}$, there exist some positive constants G and G_i, $i = 0,1,\ldots,l$, such that

$$\operatorname{trace}\left(g^T(\cdot)\,g(\cdot)\right) \leq G|e(t)|^2 + \sum_{k=0}^{l} G_k\left|e_{\tau_k}(t)\right|^2, \tag{6}$$

and $g(t,i,0,\ldots,0) = 0$.

Assumption 3. For the external input matrix D_i $(i \in \mathbb{S})$, there exists positive constant $\kappa_i \in (0,1)$, such that

$$\rho(D_i) = \kappa_i \le \kappa \in (0,1), \tag{7}$$

where $\kappa = \max_{i\in\mathbb{S}}\kappa_i$ and $\rho(D_i)$ is the spectral radius of matrix D_i.

We now begin with the following concept of stochastic synchronization.

Definition 4. Neutral-type response neural networks (3) are said to be stochastic synchronized with drive neural network (1) if, for any $\xi(t) \in L^2_{\mathscr{F}_0}([-\widetilde{\tau},0];\mathbb{R}^n])$ and $r_0 \in \mathbb{S}$,

$$\int_0^\infty \mathbb{E}\left|x(t;i_0,\xi(t))\right|^2 < \infty, \tag{8}$$

where $x(t;i_0,\xi(t))$ is the solution of system (4) for the initial condition $\xi(t)$.

Now, we describe the problem to solve in this paper as follows.

Target Description. For neutral-type and multiple time-delays neural networks (1) and (3) with stochastic disturbance and Markovian switching, by using Lyapunov functional, general Itô's formula, and M-matrix method, this paper will design a control law and obtain some criteria of stochastic synchronization.

The following lemmas are useful for obtaining the main result.

Lemma 5 (see [25]). *Let $x, y \in \mathbb{R}^n$; then the inequality $x^Ty + y^Tx \le \epsilon x^Tx + \epsilon^{-1}y^Ty$ holds for any $\epsilon > 0$.*

Lemma 6 (see [26]). *For any positive definite matrix $M \in \mathfrak{R}^{n\times n}$, a scalar $\gamma > 0$, vector function $\omega : [0,\gamma] \to \mathfrak{R}^n$ such that the integration concerned is well defined; then*

$$\begin{aligned}\left(\int_0^\gamma \omega(s)\,ds\right)^T M \left(\int_0^\gamma \omega(s)\,ds\right) \\ \le \gamma \int_0^\gamma \omega^T(s) M\omega(s)\,ds.\end{aligned} \tag{9}$$

Lemma 7 (see [27]). *If $M = (m_{ij})_{n\times n} \in \mathbb{R}^{n\times n}$ with $m_{ij} < 0$ $(i \ne j)$, then the following statements are equivalent:*

(i) *M is a nonsingular M-matrix.*

(ii) *Every real eigenvalue of M is positive.*

(iii) *M is positive stable. That is, M^{-1} exists and $M^{-1} > 0$ (i.e., $M^{-1} \ge 0$ and at least one element of M^{-1} is positive).*

3. Main Results

We are now in a position to derive a condition under which neutral-type multiple time-delay neural networks (3) with stochastic disturbance and Markovian switching are stochastic synchronized with drive system (1). We have the following result.

Theorem 8. *Let Assumptions 1, 2, and 3 hold. Assume that $M := -\operatorname{diag}\{\underbrace{\eta,\eta,\ldots,\eta}_{S}\} - \Gamma$ is a nonsingular M-matrix, where*

$$\eta = -2\varsigma + \beta^2 + \beta_d^2 + \sum_{k=1}^{l}\beta_{dk}^2, \tag{10}$$

with ς being a nonnegative real number, and $\beta = \max_{i\in\mathbb{S}}\rho(W_i)$, $\beta_d = \max_{i\in\mathbb{S}}\rho(W_{di})$, and $\beta_{dk} = \max_{i\in\mathbb{S}}\rho(W_{dki})$.

Let $m > 0$, $\vec{m} = [\underbrace{m,m,\ldots,m}_{S}]^T$ and $[q_1,q_2,\ldots,q_S]^T := M^{-1}\vec{m}$.

Assume also that

$$\begin{aligned}a = 2\underline{\alpha}q - \overline{\alpha}^2 q - \Phi^2 q - G_0 q - Q_\tau - \sum_{k=0}^{l} Q_k - \overline{\tau}^2\Phi^2 \\ > 0,\end{aligned} \tag{11}$$

where $\underline{\alpha} = \min_{i\in\mathbb{S}}\min_{1\le j\le n}a_j^i$, $\overline{\alpha} = \max_{i\in\mathbb{S}}\max_{1\le j\le n}a_j^i$, $q = \min_{i\in\mathbb{S}}q_i$, $Q_\tau = q\kappa^2$, and $Q_k = q(\Phi^2 + G_k)/(1-\widehat{\tau})$.

We choose the feedback control U_i with the update law as

$$U_i = \left(\operatorname{diag}\{k_1,k_2,\ldots,k_n\} - \varsigma I\right)\left(e(t) - D_i e_\tau(t)\right) \tag{12}$$

with

$$\dot{k}_j = -\alpha_j q\left(e_j - D_i\left(e_\tau\right)_j\right)^2, \tag{13}$$

where $\alpha_j > 0$ $(j = 1,2,\ldots,n)$ are arbitrary constants.

Then neutral-type multiple time-delays neural networks (3) with stochastic disturbance and Markovian switching can be stochastic synchronized with drive system (1).

Proof. Fix any $(i_0,\xi(t)) \in \mathbb{S}\times\mathbb{R}^n$ write $e(t;i_0,\xi(t)) = e(t)$ for simplicity. For neutral-type multiple time-delays neural networks (4), consider the following Lyapunov functional $V \in \mathbb{C}^{2,1}(\mathbb{R}_+\times\mathbb{S}\times\mathbb{R}^n;\mathbb{R}_+)$:

$$\begin{aligned}V(t,i,e(t)) &= q\left|e(t)\right|^2 + \sum_{j=1}^{n}\frac{1}{\alpha_j}k_j^2 \\ &\quad + \sum_{k=0}^{l}\int_{t-\tau_k(t)}^{t} e^T(s)Q_k e(s)\,ds \\ &\quad + \int_{t-\tau}^{t} e^T(s)Q_\tau e(s)\,ds \\ &\quad + \overline{\tau}\int_{-\tau_0(t)}^{0}\int_{t+\alpha}^{t}\psi^T(s)\psi(s)\,ds\,d\alpha.\end{aligned} \tag{14}$$

Computing $\mathscr{L}V$ along neural networks (4), we can obtain

$$\begin{aligned}&\mathscr{L}V(t,i,e(t)) \\ &\quad = V_t\left(t,i,e-D_ie_\tau\right) + V_e\left(t,i,e-D_ie_\tau\right)f\left(t,i,e,e_\tau\right) \\ &\qquad + \frac{1}{2}\operatorname{trace}\left[g^T(\cdot)V_{ee}\left(t,i,e-D_ie_\tau\right)g(\cdot)\right] \\ &\qquad + \sum_{k=1}^{S}\gamma_{ik}V\left(t,k,e-D_ie_\tau\right),\end{aligned} \tag{15}$$

where $f(t,i,e,e_\tau) = -A(r(t))e(t) + W(r(t))\psi(t) + \sum_{k=1}^{l} W_{dk}(r(t))\psi_{\tau_k}(t) + W_{d0}(r(t))\int_{t-\tau_0(t)}^{t}\psi(s)ds + U(r(t))$. Now,

$$\begin{aligned}
&V_e\left(t,i,e-D_ie_\tau\right) = 2q\left(e-D_ie_\tau\right)^T,\\
&V_{ee}\left(t,i,e-D_ie_\tau\right) = 2q,
\end{aligned}\tag{16}$$

$$\begin{aligned}
&V_t\left(t,i,e-D_ie_\tau\right)\\
&\quad=\sum_{j=1}^{n}\frac{2}{\alpha_j}k_j\dot{k}_j+\sum_{k=1}^{l}e^T(t)\,Q_ke(t)\\
&\quad-\sum_{k=1}^{l}\left(1-\dot{\tau}_k(t)\right)e^T\left(t-\tau_k(t)\right)Q_ke\left(t-\tau_k(t)\right)\\
&\quad+e^T(t)\,Q_\tau e(t)-e^T(t-\tau)\,Q_\tau e(t-\tau)\\
&\quad+\overline{\tau}\tau_0(t)\,\psi^T(t)\,\psi(t)-\overline{\tau}\int_{t-\tau_0(t)}^{t}\psi^T(s)\,\psi(s)\,ds\\
&\quad-2q\sum_{j=1}^{n}k_j\left(x_j-D_i\left(x_\tau\right)_j\right)^2+\sum_{k=1}^{l}e^T(t)\,Q_ke(t)\\
&\quad-\sum_{k=1}^{l}\left(1-\widehat{\tau}\right)e^T\left(t-\tau_k(t)\right)Q_ke\left(t-\tau_k(t)\right)\\
&\quad+e^T(t)\,Q_\tau e(t)-e^T(t-\tau)\,Q_\tau e(t-\tau)\\
&\quad+\overline{\tau}^2\Phi^2e^T(t)\,e(t)-\overline{\tau}\int_{t-\tau_0(t)}^{t}\psi^T(s)\,\psi(s)\,ds.
\end{aligned}\tag{17}$$

From Lemma 5, we get

$$\begin{aligned}
-2q\left(e-D_ie_\tau\right)^TA_ie&\le-2\min\rho\left(A_i\right)q\left(e^Te\right)\\
&\quad+2qe_\tau^TD_i^TA_ie\\
&\le-2\underline{\alpha}q\,|e|^2\\
&\quad+q\left(e^TA_i^TA_ie+e_\tau^TD_i^TD_ie_\tau\right)\\
&\le-2\underline{\alpha}q\,|e|^2+q\overline{\alpha}^2\,|e|^2+q\kappa^2\,\left|e_\tau\right|^2\\
&=q\left(-2\underline{\alpha}+\overline{\alpha}^2\right)|e|^2+q\kappa^2\left|e_\tau\right|^2.
\end{aligned}\tag{18}$$

From Assumption 1 and Lemma 5, it can be computed that

$$\begin{aligned}
&2q\left(e-D_ie_\tau\right)^TW_i\psi(t)\\
&\quad\le q\left(\left(e-D_ie_\tau\right)^TW_iW_i^T\left(e-D_ie_\tau\right)+\psi^T(t)\,\psi(t)\right)\\
&\quad\le q\left(\max_{i\in\mathbb{S}}\rho\left(W_i\right)\right)^2\left|e-D_ie_\tau\right|^2+q\Phi^2\,|e|^2\\
&\quad=q\Phi^2\,|e|^2+q\beta^2\left|e-D_ie_\tau\right|^2.
\end{aligned}\tag{19}$$

Similarly,

$$\begin{aligned}
&2q\left(e-D_ie_\tau\right)^T\sum_{k=1}^{l}W_{dki}\psi\left(t-\tau_k(t)\right)=q\sum_{k=1}^{l}2\,(e\\
&\quad-D_ie_\tau)^TW_{dki}\psi\left(t-\tau_k(t)\right)\\
&\quad\le q\sum_{k=1}^{l}\left(\left(e-D_ie_\tau\right)^TW_{dki}W_{dki}^T\left(e-D_ie_\tau\right)\right.\\
&\quad\left.+\psi^T\left(t-\tau_k(t)\right)\psi\left(t-\tau_k(t)\right)\right)\\
&\quad\le q\sum_{k=1}^{l}\left(\Phi^2\left|e\left(t-\tau_k(t)\right)\right|^2+\beta_{dk}^2\left|e-D_ie_\tau\right|^2\right).
\end{aligned}\tag{20}$$

From Lemmas 5 and 6, we have

$$\begin{aligned}
&2q\left(e-D_ie_\tau\right)^TW_{di}\int_{t-\tau_0(t)}^{t}\psi(s)\,ds\\
&\quad\le q\left(W_{di}W_{di}^T\left|e-D_ie_\tau\right|^2\right.\\
&\quad\left.+\left(\int_{t-\tau_0(t)}^{t}\psi(s)\,ds\right)^T\int_{t-\tau_0(t)}^{t}\psi(s)\,ds\right)\\
&\quad\le q\left(\beta_d^2\left|e-D_ie_\tau\right|^2+\overline{\tau}\int_{t-\tau_0(t)}^{t}\psi^T(s)\,\psi(s)\,ds\right).
\end{aligned}\tag{21}$$

From control law (12), one can obtain

$$\begin{aligned}
&2q\left(e-D_ie_\tau\right)^TU_i=2q\left(e-D_ie_\tau\right)^T\\
&\quad\cdot\operatorname{diag}\left\{k_1-\varsigma,k_2-\varsigma,\ldots,k_n-\varsigma\right\}\left(e-D_ie_\tau\right)\\
&\quad=2q\sum_{j=1}^{n}k_j\left(x_j-D_i\left(x_\tau\right)_j\right)^2-2q\varsigma\left|e-D_ie_\tau\right|^2.
\end{aligned}\tag{22}$$

From Assumption 2, we have

$$\begin{aligned}
\left(\frac{1}{2}\right)\operatorname{trace}\left(g^Tg\right)&=q\operatorname{trace}\left(g^Tg\right)\\
&\le q\left(G\,|e|^2+\sum_{k=0}^{l}G_k\left|e\left(t-\tau_k(t)\right)\right|^2\right).
\end{aligned}\tag{23}$$

Finally,

$$\sum_{k=1}^{\mathcal{S}}\gamma_{ik}V\left(t,k,e-D_ie_\tau\right)=\sum_{k=1}^{\mathcal{S}}\gamma_{ik}q_k\left|e-D_ie_\tau\right|^2.\tag{24}$$

Substituting (17)–(24) into (15), we obtain that

$$\mathscr{L}V \leq -a|e|^2 - m\left|e - D_i e_\tau\right|^2 < -a|e|^2, \quad (25)$$

where $m = -[\eta q + \sum_{k=1}^{S} \gamma_{ik} q_k]$ by $(q_1, q_2, \ldots, q_S)^T = M^{-1}\vec{m}$.

Integrating and taking the mathematical expectation on both sides of (25), one can get that

$$\int_0^\infty \mathbb{E}\left|x\left(t; i_0, \xi(t)\right)\right|^2 < \left(\frac{1}{a}\right)\mathbb{E}V\left(0, r(0), e(0)\right) < \infty. \quad (26)$$

Therefore, it can be concluded from Definition 4 that response system (3) synchronizes stochastically drive system (1). This completes the proof. □

Remark 9. In Theorem 8, we have assumed that $M := -\operatorname{diag}\{\underbrace{\eta, \eta, \ldots, \eta}_{S}\} - \Gamma$ is a nonsingular M-matrix. For given Markovian rate generator Γ, networks parameters W_i and W_{dki}, $k = 0, 1, \ldots, l$, according to M-matrix properties (see Lemma 7), we can obtain parameter ς. Thus, we can design feedback control update law (12). This technique is different from those, such as linear matrix inequality method [28]. From the process of proving Theorem 8, if M-matrix approach is not used, and LMIs technology is only adopted to solve stochastic synchronization of addressed system (1), (25) cannot be produced. In order to make $\int_0^\infty \mathbb{E}|x(t; i_0, \xi(t))|^2 < (1/a) < \infty$, the theory result in Theorem 8 will become more complex.

Remark 10. From the analysis of Remark 9, we can also obtain that if feedback control update law (12) has been designed, because $M := -\operatorname{diag}\{\underbrace{\eta, \eta, \ldots, \eta}_{S}\} - \Gamma$, using M-matrix method, Markovian rate generator Γ can be checked. This also reflects that all information of Markovian switching parameters is traced.

When the multidelays turn to single delay and the neutral term disappears in the neural networks, respectively, we have the following two special cases of system (4).

Special Case 1 (single delay). Consider

$$\begin{aligned} d\left[e(t) - D(r(t))e(t-\tau)\right] = \Bigg[&-A(r(t))e(t) \\ &+ W(r(t))\psi(t) + W_{d1}(r(t))\psi_{\tau_1}(t) \\ &+ W_{d0}(r(t))\int_{t-\tau_0(t)}^{t}\psi(s)\,ds + U(r(t))\Bigg]dt \\ &+ g\left(t, r(t), e(t), e_{\tau_0}(t), e_{\tau_1}(t), \ldots, e_{\tau_l}(t)\right)d\omega(t). \end{aligned} \quad (27)$$

Special Case 2 (without neutral term). Consider

$$\begin{aligned} de(t) = \Bigg[&-A(r(t))e(t) + W(r(t))\psi(t) \\ &+ \sum_{k=1}^{l} W_{dk}(r(t))\psi_{\tau_k}(t) + W_{d0}(r(t))\int_{t-\tau_0(t)}^{t}\psi(s)\,ds \\ &+ U(r(t))\Bigg]dt \\ &+ g\left(t, r(t), e(t), e_{\tau_0}(t), e_{\tau_1}(t), \ldots, e_{\tau_l}(t)\right)d\omega(t). \end{aligned} \quad (28)$$

Accordingly we can derive the two corollaries of Theorem 8 below.

Corollary 11. *Let Assumptions 1, 2, and 3 hold. Assume that* $M := -\operatorname{diag}\{\underbrace{\eta, \eta, \ldots, \eta}_{S}\} - \Gamma$ *is a nonsingular* M*-matrix, where*

$$\eta = -2\varsigma + \beta^2 + \beta_d^2 + \beta_{d1}^2, \quad (29)$$

with ς *being a nonnegative real number, and* $\beta = \max_{i\in\mathbb{S}}\rho(W_i)$, $\beta_d = \max_{i\in\mathbb{S}}\rho(W_{di})$, *and* $\beta_{d1} = \max_{i\in\mathbb{S}}\rho(W_{d1i})$.

Let $m > 0$, $\vec{m} = [\underbrace{m, m, \ldots, m}_{S}]^T$ *and* $[q_1, q_2, \ldots, q_S]^T := M^{-1}\vec{m}$.

Assume also that

$$a = 2\underline{\alpha}q - \overline{\alpha}^2 q - \Phi^2 q - G_0 q - Q_\tau - \sum_{k=0}^{1} Q_k - \overline{\tau}^2\Phi^2 > 0, \quad (30)$$

where $\underline{\alpha} = \min_{i\in\mathbb{S}}\min_{1\leq j\leq n} a_j^i$, $\overline{\alpha} = \max_{i\in\mathbb{S}}\max_{1\leq j\leq n} a_j^i$, $q = \min_{i\in\mathbb{S}} q_i$, $Q_\tau = q\kappa^2$, *and* $Q_k = q(\Phi^2 + G_k)/(1-\widehat{\tau})$.

We choose the feedback control U_i *with the update law as*

$$U_i = \left(\operatorname{diag}\{k_1, k_2, \ldots, k_n\} - \varsigma I\right)\left(e(t) - D_i e_\tau(t)\right) \quad (31)$$

with

$$\dot{k}_j = -\alpha_j q\left(e_j - D_i\left(e_\tau\right)_j\right)^2, \quad (32)$$

where $\alpha_j > 0$ $(j = 1, 2, \ldots, n)$ *are arbitrary constants.*

Then the response system can synchronize with the drive system of neutral-type.

Corollary 12. *Let Assumptions 1, 2, and 3 hold. Assume that* $M := -\operatorname{diag}\{\underbrace{\eta, \eta, \ldots, \eta}_{S}\} - \Gamma$ *is a nonsingular* M*-matrix, where*

$$\eta = -2\varsigma + \beta^2 + \beta_d^2 + \sum_{k=1}^{l}\beta_{dk}^2, \quad (33)$$

with ς being a nonnegative real number, and $\beta = \max_{i\in\mathbb{S}}\rho(W_i)$, $\beta_d = \max_{i\in\mathbb{S}}\rho(W_{di})$, *and* $\beta_{dk} = \max_{i\in\mathbb{S}}\rho(W_{dki})$.

Let $m > 0$, $\vec{m} = [\underbrace{m, m, \ldots, m}_{S}]^T$ *and* $[q_1, q_2, \ldots, q_S]^T := M^{-1}\vec{m}$.

Assume also that

$$a = 2\underline{\alpha}q - \overline{\alpha}^2 q - \Phi^2 q - G_0 q - \sum_{k=0}^{l} Q_k - \overline{\tau}^2\Phi^2 > 0, \tag{34}$$

where $\underline{\alpha} = \min_{i\in\mathbb{S}}\min_{1\le j\le n} a_j^i$, $\overline{\alpha} = \max_{i\in\mathbb{S}}\max_{1\le j\le n} a_j^i$, $q = \min_{i\in\mathbb{S}} q_i$, *and* $Q_k = q(\Phi^2 + G_k)/(1 - \widehat{\tau})$.

We choose the feedback control U_i *with the update law as*

$$U_i = \left(\operatorname{diag}\{k_1, k_2, \ldots, k_n\} - \varsigma I\right) e(t) \tag{35}$$

with

$$\dot{k}_j = -\alpha_j q e_j^2, \tag{36}$$

where $\alpha_j > 0$ $(j = 1, 2, \ldots, n)$ *are arbitrary constants.*

Then the response system can synchronize with the drive system of multiple time-delays.

4. Numerical Simulation

One example is presented here in order to show the usefulness of our results. Our aim is to examine the stochastic synchronization for the given neutral-type multiple time-delays neural networks with stochastic noise and Markovian switching.

Consider the error system of two-neuron delayed neural network (4) with 2-state Markovian switching and one-dimensional noise $B(t)$. We set the constant time-delay $\tau = 1$ and $\tau_0(t) = 1, \tau_1(t) = 0.8\sin(t), \tau_2(t) = 0.8\cos(t)$, which means $\overline{\tau} = 1$ and $\widehat{\tau} = 0.8$.

The transition rate matrix of Markovian switching is given by

$$\Gamma = \begin{bmatrix} -4 & 4 \\ 3 & -3 \end{bmatrix}, \quad \varphi(\cdot) = \frac{\tanh(\cdot)}{4}, \quad g(\cdot) = \frac{e(t)}{2}. \tag{37}$$

Then we choose $\Phi = 0.25$, $G = 0.25$, and $G_k = 0$ so that Assumptions 1 and 2 can be satisfied.

The other parameters are given as follows:

$$\begin{aligned} D_1 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} 0.9 & 0 \\ 0 & 1 \end{bmatrix}, \\ W_1 &= \begin{bmatrix} 0.02 & -0.03 \\ 0.05 & 0.04 \end{bmatrix}, \\ W_{d01} &= \begin{bmatrix} 0.04 & 0.03 \\ -0.03 & 0.02 \end{bmatrix}, \\ W_{d11} &= \begin{bmatrix} 0.05 & -0.02 \\ 0.03 & -0.05 \end{bmatrix}, \\ W_{d21} &= \begin{bmatrix} 0.04 & -0.02 \\ 0.01 & 0.02 \end{bmatrix}, \\ D_2 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.05 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 0.9 \end{bmatrix}, \\ W_2 &= \begin{bmatrix} -0.02 & 0.01 \\ 0.02 & -0.05 \end{bmatrix}, \\ W_{d02} &= \begin{bmatrix} 0.03 & -0.01 \\ 0.02 & -0.04 \end{bmatrix}, \\ W_{d12} &= \begin{bmatrix} 0.03 & -0.06 \\ 0.02 & -0.01 \end{bmatrix}, \\ W_{d22} &= \begin{bmatrix} 0.03 & 0.01 \\ -0.04 & 0.02 \end{bmatrix}. \end{aligned} \tag{38}$$

Given $\varsigma = 0.5$, $m = 1$, we compute

$$\eta = -0.9923, \quad M = \begin{bmatrix} 1.4923 & -0.5 \\ -0.8 & 1.7923 \end{bmatrix}. \tag{39}$$

It can be easily verified that M is M-matrix. We then derived $q = 1.0078$ and $\alpha = 0.0509 > 0$. Hence it follows from Theorem 8 that drive system (1) and response system (3) are stochastic synchronization. Figures 1–4 show the 2-state Markov chain, the state trajectory of the drive system and response system, the state trajectory and evolution of the error system, and the update law of the control gain matrix, respectively. We can see from Figure 3 that the system state tends to zero with the increase of t, which verifies the synchronization of the drive system and response system.

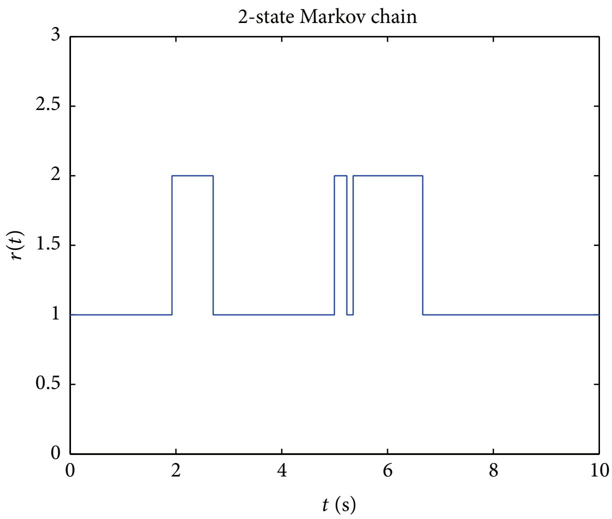

Figure 1: 2-state Markov chain.

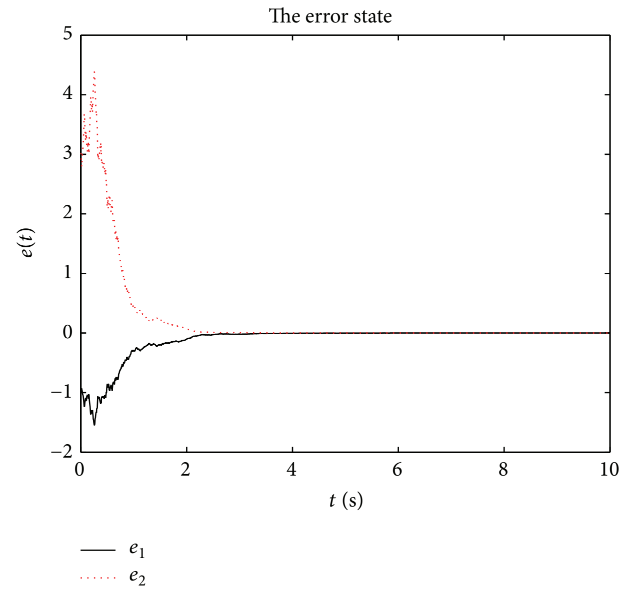

Figure 3: State trajectory of the error system.

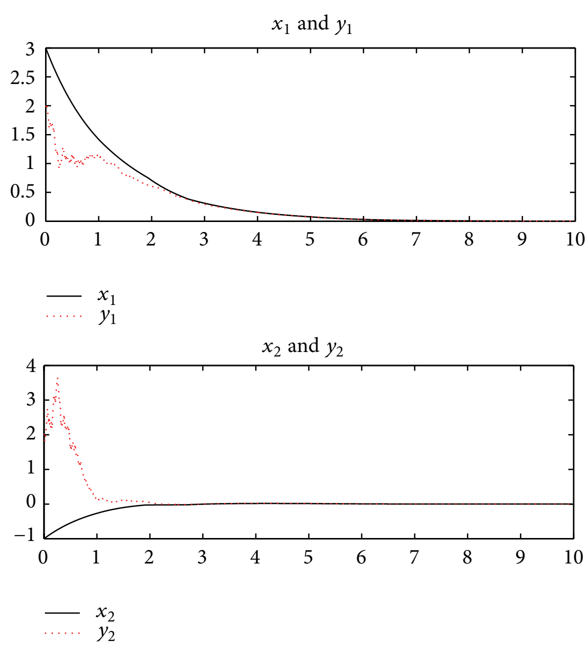

Figure 2: State trajectory of two systems.

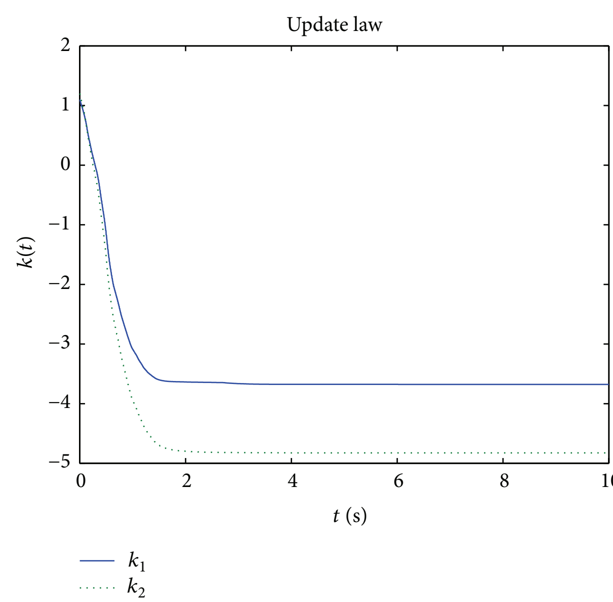

Figure 4: Update law.

Remark 13. For Markovian switching with known transition rate, we can choose reasonable parameter η to realize $M := -\operatorname{diag}\{\underbrace{\eta, \eta, \dots, \eta}_{S}\} - \Gamma$ to become M-matrix. In simulation results, choosing $\varsigma = 0.5$, it is easily observed that M is M-matrix. Furthermore, we can get q and α. From the above result, we can see that the analysis of M-matrix approach in Remarks 9 and 10 is reasonable.

5. Conclusion

In this paper, we have dealt with the problem of stochastic synchronization of neutral-type neural networks with multidelays and Markovian switching. By using M-matrix approach and the stochastic analysis method, some synchronization criteria are obtained to ensure the stochastic synchronization for the neutral-type neural networks with multidelays. A numerical example is provided to illustrate the effectiveness of the result obtained in this paper.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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