Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2014, Article ID 921767, 9 pages
http://dx.doi.org/10.1155/2014/921767

Research Article

Spillways Scheduling for Flood Control of Three Gorges Reservoir Using Mixed Integer Linear Programming Model

Maoyuan Feng[1,2] and Pan Liu[1,2]

[1] *State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan 430072, China*
[2] *Hubei Provincial Collaborative Innovation Center for Water Resources Security, Wuhan 430072, China*

Correspondence should be addressed to Pan Liu; liupan@whu.edu.cn

Received 11 May 2014; Revised 26 July 2014; Accepted 9 August 2014; Published 28 August 2014

Academic Editor: Erik Cuevas

This study proposes a mixed integer linear programming (MILP) model to optimize the spillways scheduling for reservoir flood control. Unlike the conventional reservoir operation model, the proposed MILP model specifies the spillways status (including the number of spillways to be open and the degree of the spillway opened) instead of reservoir release, since the release is actually controlled by using the spillway. The piecewise linear approximation is used to formulate the relationship between the reservoir storage and water release for a spillway, which should be open/closed with a status depicted by a binary variable. The control order and symmetry rules of spillways are described and incorporated into the constraints for meeting the practical demand. Thus, a MILP model is set up to minimize the maximum reservoir storage. The General Algebraic Modeling System (GAMS) and IBM ILOG CPLEX Optimization Studio (CPLEX) software are used to find the optimal solution for the proposed MILP model. The China's Three Gorges Reservoir, whose spillways are of five types with the total number of 80, is selected as the case study. It is shown that the proposed model decreases the flood risk compared with the conventional operation and makes the operation more practical by specifying the spillways status directly.

1. Introduction

Flood disasters, accounting for about one-third of all natural catastrophes throughout the world, have been extremely severe in recent decades [1]. For example, flood disasters have caused the loss of 30 billion dollars per year in China [1–3]. As a result, reservoirs have been built and served for one of the most useful measurements for flood control.

Reservoir operations are complex, nonlinear control processes and significantly affected by hydrological conditions and constraints, which are not predictable beforehand [4, 5]. Great effort has been made to determine the optimal scheduling of the reservoirs with various methods and techniques, including linear programming, nonlinear programming, dynamic programming, and genetic algorithm [3–28]. Karaboga et al. [6] proposed a control method to derive reservoir operating rules based on the fuzzy logic with optimum rule number and tabu search. Wei and Hsu [7] presented the tree-based rules which were used to determine the optimal real-time releases for a multipurpose multireservoir system during flood periods. Bagis and Karaboga [8] developed an evolutionary algorithm-based fuzzy proportional derivative-type controller for reservoir operation. Chang [9] proposed a penalty-type genetic algorithm to find a rational reservoir release hydrograph for flood control. Li et al. [10] developed a dynamic control operation model that considers inflow uncertainty. Fu [11] presented a fuzzy optimization method based on the concept of ideal and anti-ideal solutions. Hashemi et al. [12] presented a multiple attribute group decision-making model based on the compromise ratio method. Karbowski et al. [13] presented a hybrid analytic/rule-based approach to reservoir system management during flood seasons. Liu et al. [28] proposed three methods to derive the multiple near-optimal solutions to deterministic reservoir operation problems.

Based on the above methods and techniques, the reservoir water release hydrograph can be obtained. However, the reservoir operation is a control process that essentially manages the spillway gates of dams to increase or decrease the released water [29]. In practice, two basic issues associated with spillway gates should be determined: (1) the number of various spillways to be open or used and (2) the degree of the spillway opened (full or scale open). Most of the solutions proposed so far address the release scheduling problem leaving the allocation problem as a secondary one, performed by trial and error methods. This study deals with the spillways scheduling, instead of release scheduling, for the flood control reservoir, which has seldom been addressed in the literature. The most popular reservoir operation method, dynamic programming, becomes difficulty for this specified issue, owing to the large number of discrete states (say reservoir storage) and heavy computation for accuracy.

The mixed integer linear programming (MILP) model ensures a global optimal solution, which hence is widely used in optimization fields [14–18, 30, 31]. For example, Needham et al. [14] presented a MILP model for a reservoir system analysis of three projects on the Iowa and Des Moines rivers. Norouzi et al. [15] proposed a MILP model for short term unit commitment for hydro and thermal generation units with security-constrained commitment. Liu et al. [17] used a MILP model for the optimal load distribution, which reaches the global optimum, to validate the proposed algorithm in a hydropower station. Ashouri et al. [30] developed a MILP model to obtain the optimal design and operation of building services. Luathep et al. [31] proposed a MILP model for solving a mixed transportation network design problem.

This study aims at developing a MILP model to operate reservoir by scheduling spillways. In Section 2, the MILP model is set up with (1) transforming the objective function into a linear form and (2) formulating the constraints of potential maximum water release as a piecewise linear function. Section 3 describes a case study application to China's Three Gorges Reservoir (TGR), where the optimal scheduling is compared with the conventional scheduling method. Finally, conclusions are given in Section 4.

2. Mathematical Model

2.1. Reservoir Flood Control Model. The commonly used reservoir flood control model is as follows (e.g., [4, 5]).

2.1.1. Objective Function. For the reservoir flood control operation, maximum water storage should be minimized, that is,

$$\min \max \left(V_1, V_2, \ldots, V_t, \ldots, V_T\right), \tag{1}$$

where V_t is the reservoir water storage at time t and T is the number of time periods.

2.1.2. Constraints

(1) Reservoir water balance equation:

$$V_{t+1} = V_t + \left(\frac{I_t + I_{t+1}}{2} - \frac{O_t + O_{t+1}}{2}\right)\Delta t, \quad t = 1, 2, \ldots, T-1, \tag{2}$$

where I_t and O_t are the reservoir inflow and release at time t, respectively. Δt is the time step length. It should be noted that the water losses from the reservoir in the form of seepage and evaporation are omitted in this study.

(2) Water storage capacity constraint:

$$\underline{V} \le V_t \le \overline{V}, \quad t = 1, 2, \ldots, T, \tag{3}$$

where $\underline{V}$ and $\overline{V}$ denote the minimum and maximum reservoir storages, respectively.

(3) Reservoir potential maximum water release constraint:

$$O_t \le f\left(V_t\right), \quad t = 1, 2, \ldots, T, \tag{4}$$

where $f(\cdot)$ is the functional relationship between the reservoir storage and potential maximum water release.

(4) Water release constraint for the downstream safety:

$$O_t \le O_{\max}^{\text{down}}, \quad t = 1, 2, \ldots, T, \tag{5}$$

where $O_{\max}^{\text{down}}$, often a constant, is the water release for the downstream safety.

2.2. Linearization. The objective function and all the constraints should be in a linear form for a MILP model. However, the objective function (1) and potential maximum water release constraint (4) are unsatisfied with this assumption. Consequently, transformations have been proposed as follows.

2.2.1. Objective Function. A new variable V_m is introduced to represent the maximum value of V_t; that is, $V_m = \max(V_1, V_2, \ldots, V_t, \ldots, V_T)$. Then the objective function can be transferred as follows:

$$\min V_m \tag{6}$$

with an additional constraint:

$$V_m \ge V_t, \quad t = 1, 2, \ldots, T. \tag{7}$$

2.2.2. Potential Maximum Water Release Constraint. Recalling (4), the potential maximum water release depends on the functional relationship $f(\cdot)$ and the current water storage, while the relationship is determined based on all spillways (including turbines). Since the reservoir release is the sum of all spillways, we have

$$O_t = \sum_{i=1}^{n} q_{i,t} = \sum_{i=1}^{n} g_i\left(V_t, S_i\right), \quad t = 1, 2, \ldots, T, \tag{8}$$

where $q_{i,t}$, namely, $g_i(V_t, S_i)$, is the release for spillway i, which can be described with the reservoir storage V_t and status S_i (closed, full open, or scale open). n is the number of spillways. It should be noted that the spillway of scale open is always limited to several specific degrees, which are denoted as $S_{i,t}^1, S_{i,t}^2, \ldots, S_{i,t}^{m_i}$, where m_i is the number of possible statuses for spillway i.

(1) Piecewise Linear Approximation of Relationship between Water Release and Reservoir Storage for Individual Spillway. A nonlinear function can be linearized with additional binary variables [14–17], which is very common for the interpolation of the relationship between reservoir storage and water release. As shown in Figure 1, for a specific spillway i with the status $S_{i,t}^j$, the water release $q_{i,t}^j$ is a function of the reservoir storage V_t, and this relationship is often nonlinear. Assuming that the nonlinear function is approximated with k_i^j breakpoints (Figure 1), the water release can be expressed as a piecewise linear function as follows:

$$0 \le w_{i,t,p}^j \le 1, \quad p = 1, 2, \ldots, k_i^j, \tag{9}$$

$$\sum_{p=1}^{k_i^j} w_{i,t,p}^j = 1, \tag{10}$$

$$\sum_{p=1}^{k_i^j - 1} r_{i,t,p}^j = 1, \tag{11}$$

$$w_{i,t,p}^j \le \begin{cases} r_{i,t,p}^j & (p = 1), \\ r_{i,t,p}^j + r_{i,t,p-1}^j & \left(1 < p < k_i^j\right), \\ r_{i,t,p-1}^j & \left(p = k_i^j\right), \end{cases} \tag{12}$$

$$V_t = \sum_{p=1}^{k_i^j} \left(w_{i,t,p}^j V_{i,p}^j\right), \tag{13}$$

$$q_{i,t}^j = \sum_{p=1}^{k_i^j} \left(w_{i,t,p}^j Q_{i,p}^j\right), \tag{14}$$

where $w_{i,t,p}^j$ is the weight of breakpoint p for the spillway i with status $S_{i,t}^j$ at time t and $r_{i,t,p}^j$ is the binary variable to ensure at most two adjacent breakpoints are greater than zero. $V_{i,p}^j$ and $Q_{i,p}^j$ are the reservoir storage and water release for breakpoint p of spillway i with status $S_{i,t}^j$.

Equation (11) implies that only one of the binary variables $r_{i,t,p}^j$ is equal to one, and (12) ensures that two adjacent $w_{i,t,p}^j$ can be nonzero, which makes a linear interpolation between these two breakpoints. Equations (13) and (14) are the linear combinations of the reservoir storage and water release, respectively. Therefore, (10) to (14) transfer the nonlinear relationship between water release and reservoir storage into a piecewise linear function.

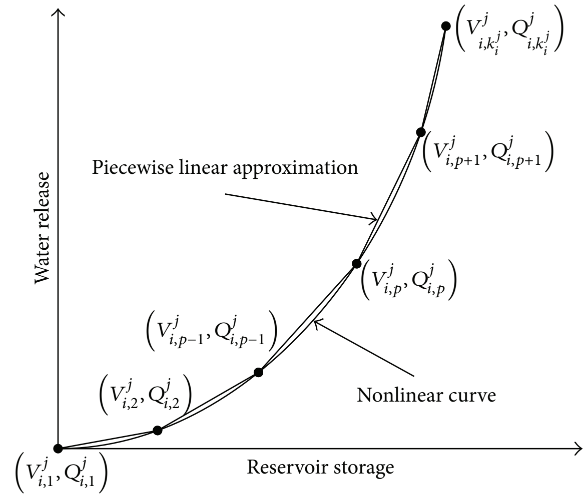

Figure 1: Piecewise linear approximation of relationship between water release and reservoir.

(2) Water Release for Individual Spillway. Based upon the above piecewise linear relationship, the water release of each individual spillway can be expressed as follows:

$$q_{i,t} \ge q_{i,t}^j - \left(1 - s_{i,t}^j\right) O_{\max}, \tag{15}$$

$$q_{i,t} \le q_{i,t}^j + \left(1 - s_{i,t}^j\right) O_{\max}, \tag{16}$$

$$q_{i,t} \ge -s_{i,t}^j O_{\max}, \tag{17}$$

$$q_{i,t} \le s_{i,t}^j O_{\max}, \tag{18}$$

$$\sum_{j=1}^{m_i} s_{i,t}^j \le 1, \tag{19}$$

where $s_{i,t}^j$ is the binary variable to describe the status of spillway i at time t; that is, zero means that the spillway is not used with status $S_{i,t}^j$; otherwise this spillway opens with status $S_{i,t}^j$. Equations (15) to (18) form an if-then statement; that is, $s_{i,t}^j = 1$ means $q_{i,t} = q_{i,t}^j$ and $s_{i,t}^j = 0$ means $q_{i,t} = 0$. Equation (19) ensures that only one status could be used, including the zero for closed status. Therefore, the binary variable $s_{i,t}^j$ can be used to indicate the status of the spillway.

(3) Control Order of Spillway. Two common control rules for the spillway are as follows.

(1) Symmetry rules: it is very popular for the spillways to open/close symmetrically, which ensures the safety of dam. For example, the spillways i_1 and i_2 should be open/closed at the same time, and this rule can be described as follows:

$$\sum_{j=1}^{m_{i_2}} s_{i_2,t}^j = \sum_{j=1}^{m_{i_1}} s_{i_1,t}^j. \tag{20}$$

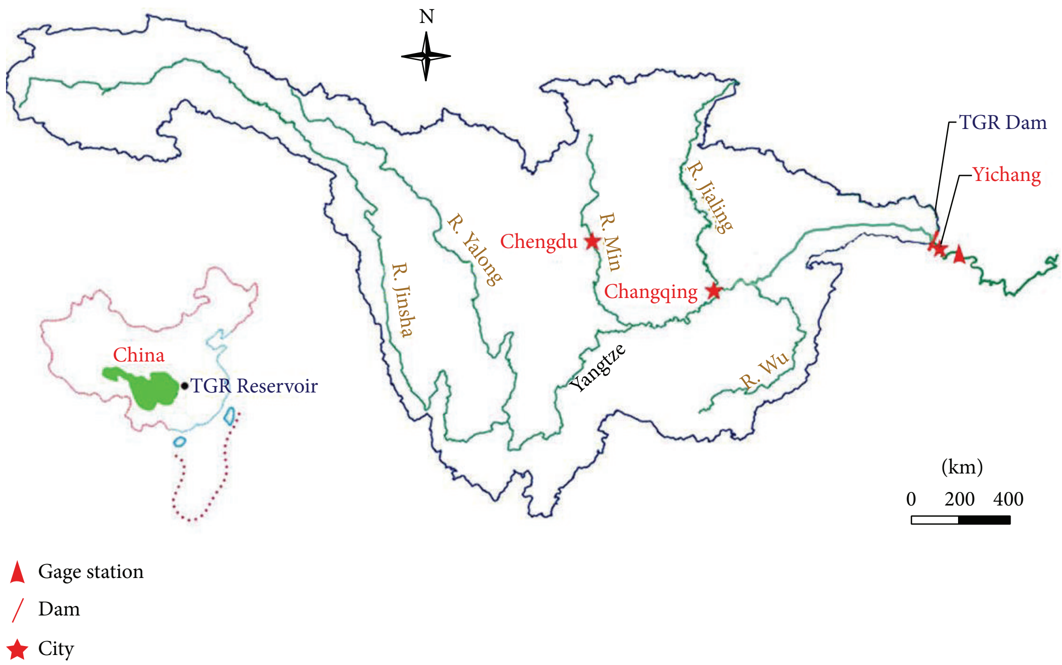

Figure 2: Location of the Three Gorges Reservoir Basin in China.

Table 1: Numbers of spillways for various types and the code for the MILP model.

Spillway gate	Turbines	Deep outlets	Floats outlets	Desilting outlets	Surface outlets
Number	26	23	2	7	22
Code	1	2–13	14-15	16–22	23–33

(2) Control order: if the spillways i_1 should be used prior to both of i_2 and i_3; this rule can be described as follows:

$$\sum_{j=1}^{m_{i_2}} s_{i_2,t}^{j} + \sum_{j=1}^{m_{i_3}} s_{i_3,t}^{j} \leq M \sum_{j=1}^{m_{i_1}} s_{i_1,t}^{j}, \tag{21}$$

where M is a large positive value.

Equation (4) is then reformulated with linear equations from (8) to (21) for the consideration of the spillway rules.

3. Case Study

3.1. Three Gorges Reservoir. The Three Gorges Reservoir (TGR) is a vital project for water resources development of China's largest river, the Yangtze River (Figure 2). The TGR receives inflow from a 4.5×10^3 km long channel with a contributing drainage area of 10^6 km^2. The mean annual runoff at the dam site is 451 billion m^3. With a flood storage capacity of 22.15 billion m^3, the TGR plays the most important role in flood control of the Yangtze River.

Several big floods in the Yangtze River basin, including the flood in 1981, have caused serious disasters. Based on the Chinese guidelines for design flood, the flood in 1981 is used as the typical flood to design flood hydrographs of 20-year return period flood (the flood prevention standard for the Yangtze River). Finally, the design flood hydrograph of the TGR, with a return period of 20-year, is used to test the proposed method. The optimal scheduling of the proposed MILP model is compared with the conventional method.

3.1.1. TGR Spillways. For the TGR, there are five types of spillways: turbines, deep outlets, floats outlets, desilting outlets, and surface outlets. Note that the turbines are taken as spillways owing to its capability of releasing flood, and they should be fully open to generate hydropower during the flood events. The numbers of various types of spillways and the code for the formulation are shown in Table 1.

Note that the paired deep outlets are denoted as the codes number from 2 to 12, for the consideration of symmetry. For example, code 2 denotes the symmetric deep outlets, 1 and 23. Similarly, the codes from 23 to 33 denote the symmetry paired surface outlets.

The spillways must be fully open or closed for the safety and life span of facilities. Two kinds of spillway constraints should be taken into consideration when scheduling the reservoir system: (1) the potential maximum release for each individual spillway corresponding to specific reservoir storage and (2) the control order of the spillways.

(1) Relationship between Water Release and Reservoir Storage. The water release of each individual spillway depends on its type and status (open, closed, and scale open). Spillway

Table 2: Water release relationships of spillways.

Water level (m)	Reservoir storage (billion m^3)	Water releases (m^3/s) Deep outlets (23 outlets)	Floats outlets (2 outlets)	Desilting outlets (7 outlets)	Surface outlets (22 outlets)	Turbines (26 units)	Total release
135	12.40	33500	100	2200	/	22100	57900
140	14.70	35800	700	2300	/	23000	61800
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
155	22.80	41800	3500	/	/	25200	70500
158	24.81	42900	3900	/	0	25600	72400
160	26.20	43600	4100	/	800	25900	74400
165	30.02	45400	4600	/	5800	25800	81600
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

relationships between reservoir storage and water release are given in Table 2.

(2) Control Order of Spillways. The spillways should be operated with a specific order, which are described as follows.

(1) The spillways should be opened in the following order: turbines; deep outlets; floats outlets; desilting outlets; surface outlets. The spillways should be closed in the reversed order.

(2) The deep outlets, floats outlets, and desilting outlets should be either fully open or fully closed. The partial open is not allowed in the operation.

(3) The deep outlets and surface outlets should be evenly and symmetrically used, in order that the water release can be distributed evenly along the dams. The spillways should be closed in a reversed order and the concentrated water release at the same location must be prohibited.

(4) For the floats outlets, floats outlet 2 should be used before the use of the floats outlet 1.

(5) The desilting outlets are mainly responsible for the sediment releasing and the water release should be avoided in the operations. The water level in the reservoir should be kept below 150 meters if the desilting outlets have to be used for the water releasing.

(6) Desilting outlets 2 to 6 should be opened earlier and the desilting outlets 1 and 7 can be followed.

3.2. The Conventional Operation. Based on the conventional operating rules, the reservoir water release should be kept below 56700 m^3/s. That is, the reservoir release is equal to the inflow when the water level is lower than 145 m and the inflow is less than 56700 m^3/s; otherwise the water release is equal to 56700 m^3/s. When the water level is higher than the maximum flood level (175 m), the water release is equal to the potential maximum water release for the consideration of dam safety. It should be noted that the conventional operating rules are the optimal solution for the model that consists of (1) to (5).

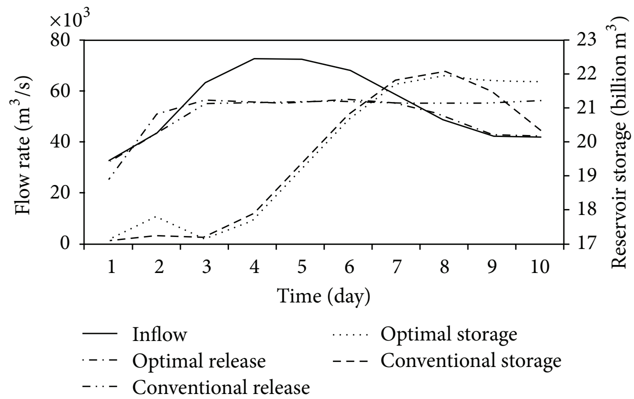

Figure 3: TGR operation for 20-year flood with 1981 type.

However, this release should be specified to the spillway to satisfy their operation constraints. The allocation is performed by trial and error method and the result is shown in Figure 3. The maximum reservoir storage is 22.09 billion m^3.

3.3. Optimal Operation

3.3.1. MILP Model. Since there are five types of spillways, including turbines, deep outlets, floats outlets, desilting outlets, and surface outlets, for the TGR. With the assistance of the binary variables, the water release of each individual spillway can be formulated in piecewise linear relationship between reservoir storage and water release in Section 2. With the objective function of (6) and the spillways constraints, the MILP model has been set up for the TGR finally (see Appendix). In the model, $O_{\max}$ and M are set as 100000 m^3/s and 100, respectively, and they are proper for the TGR case.

3.3.2. MILP Solver. The MILP model is resolved by using IBM ILOG CPLEX Optimization Studio (CPLEX) [32], with the interface of the General Algebraic Modeling System (GAMS) [33]. The GAMS is specifically designed for modeling linear, nonlinear, and mixed integer optimization

Table 3: Optimal scheduling of 20-year flood with 1981 type.

Time (day)	Inflow (m^3/s)	Storage (billion m^3)	Release (m^3/s)	Turbine	Deep outlets	Floats outlets	Desilting outlets	Surface outlets
1	32875	17.15	25347	26	1	0	0	0
2	43250	17.80	50626	26	16	0	0	0
3	62875	17.16	**56669**	26	20	0	0	0
4	72550	17.70	55538	26	19	0	0	0
5	72525	19.17	55240	26	18	0	0	0
6	68475	20.66	56409	26	18	0	0	0
7	58200	21.71	55354	26	17	0	0	0
8	48500	**21.95**	50136	26	14	0	0	0
9	42400	21.81	42887	26	10	0	0	0
10	42000	21.77	42867	26	10	0	0	0

Table 4: Status of the deep outlets of 20-year flood with 1981 type, where ● and ○ imply that the deep outlet is open and closed, respectively.

Time	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	○	○	○	○	○	○	○	○	○	○	○	●	○	○	○	○	○	○	○	○	○	○	○
2	○	○	○	●	●	●	●	●	●	●	●	○	●	●	●	●	●	●	●	●	○	○	○
3	○	●	●	●	●	●	●	●	●	●	●	○	●	●	●	●	●	●	●	●	●	●	○
4	○	○	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	○	○
5	○	○	●	●	●	●	●	●	●	●	●	○	●	●	●	●	●	●	●	●	●	○	○
6	○	○	●	●	●	●	●	●	●	●	●	○	●	●	●	●	●	●	●	●	●	○	○
7	○	○	○	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	○	○	○
8	○	○	○	○	●	●	●	●	●	●	●	○	●	●	●	●	●	●	●	○	○	○	○
9	○	○	○	○	○	○	●	●	●	●	●	○	●	●	●	●	●	○	○	○	○	○	○
10	○	○	○	○	○	○	●	●	●	●	●	○	●	●	●	●	●	○	○	○	○	○	○

problems. CPLEX is an optimization software package, solving integer programming, linear programming, convex, and nonconvex quadratic programming and so on problems. The CPLEX is accessible through GAMS in this study.

3.3.3. Results of Optimal Scheduling. The optimal scheduling has been found by using the CPLEX solving the MILP model. Table 3 lists the results of the optimal scheduling of 20-year flood. As shown in Table 3, the numbers of spillways of different types opened in different time intervals have already been determined. Furthermore, the status of each individual spillway can also been determined.

Since all the turbines are opened and all the floats outlets, desilting outlets, and surface outlets are closed during the whole process of the flood, the descriptions of the statuses of the turbines, floats outlets, desilting outlets, and surface outlets are relatively meaningless. The optimal status of the deep outlets is listed in Table 4.

As shown in the Table 4, the filled circle implies that the deep outlet is open and the empty circle implies that the deep outlet is closed. It demonstrates the whole process of scheduling of the 20-year flood.

As shown in Figure 3, the optimal scheduling is compared with the conventional method. With the comparison of the results of 20-year flood, the following findings can be observed.

(1) The maximum reservoir storage, 21.95 billion m^3 in the optimal scheduling, is lower than that in the conventional scheduling 22.09 billion m^3, indicating that the optimization is effective. The proposed MILP model provides more available reservoir storage for potential floods. Indeed, it is able to find the global optimum.

(2) The maximum water releases of the optimal and conventional method are 56669 m^3/s and 56517 m^3/s, respectively. These releases are feasible for the downstream safety. Since the optimal operation prereleases more water before the flood peak occurs, it outperforms the conventional operation.

(3) Compared with the conventional method, the proposed model specifies the spillways status directly without the allocation using trial and error methods, making the operation more objective. The spillway gates can be easily operated according to the optimal results (Table 4).

4. Conclusion

This paper proposes a MILP model to determine the optimal reservoir spillways scheduling. The piecewise linear approximation is used to formulate the relationship between the reservoir storage and water releases for spillways. The control order and symmetry rules of the spillways are described and incorporated into the constraints. Conclusions can be drawn as follows.

(1) The optimal scheduling obtained with the MILP model is better than the conventional scheduling in terms of objective function.

(2) The optimal scheduling is more advantageous than the conventional scheduling in that the spillways status can be specified directly from the MILP model without water release allocation based on trial and error methods and that the global optimum is ensured.

However, the MILP model is time consuming and the extension of multireservoir systems operation needs further research.

Appendix

TGR MILP Model

Objective function is as follows:

$$\min V_m \tag{A.1}$$

subject to

$$
\begin{gathered}
V_m \geq V_t, \quad t = 1, 2, \ldots, T, \\
V_{t+1} = V_t + \left(\frac{I_t + I_{t+1}}{2} - \frac{O_t + O_{t+1}}{2} \right) \Delta t, \quad t = 1, 2, \ldots, T-1, \\
\underline{V} \leq V_t \leq \overline{V}, \quad t = 1, 2, \ldots, T, \\
O_t \leq O_{\max}^{\text{down}}, \quad t = 1, 2, \ldots, T, \\
O_t = \sum_{i=1}^{n} q_{i,t}, \quad t = 1, 2, \ldots, T, \\
0 \leq w_{i,t,p}^1 \leq 1, \quad p = 1, 2, \ldots, k_i^1, \\
\sum_{p=1}^{k_i^1 - 1} r_{i,t,p}^1 = 1, \\
w_{i,t,p}^1 \leq \begin{cases} r_{i,t,p}^1 & (p = 1), \\ r_{i,t,p}^1 + r_{i,t,p-1}^1 & (1 < p < k_i^1), \\ r_{i,t,p-1}^1 & (p = k_i^1), \end{cases} \\
V_t = \sum_{p=1}^{k_i^1} \left(w_{i,t,p}^1 V_{i,p}^1 \right), \\
q_{i,t} = \sum_{p=1}^{k_i^1} \left(w_{i,t,p}^1 Q_{i,p}^1 \right), \\
q_{i,t} \geq q_{i,t}^1 - \left(1 - s_{i,t}^1 \right) O_{\max}, \\
q_{i,t} \leq q_{i,t}^1 + \left(1 - s_{i,t}^1 \right) O_{\max}, \\
q_{i,t} \geq -s_{i,t}^1 O_{\max}, \\
q_{i,t} \leq s_{i,t}^1 O_{\max}, \\
2 \sum_{i=2}^{12} s_{i,t}^1 + s_{13,t}^1 + \sum_{i=14}^{15} s_{i,t}^1 + \sum_{i=16}^{22} s_{i,t}^1 + 2 \sum_{i=23}^{33} s_{i,t}^1 \leq M s_{1,t}^1, \\
\sum_{i=14}^{15} s_{i,t}^1 + \sum_{i=16}^{22} s_{i,t}^1 + 2 \sum_{i=23}^{33} s_{i,t}^1 \leq M s_{j,t}^1, \quad j \in 2, 3, \ldots, 13, \\
\sum_{i=16}^{22} s_{i,t}^1 + 2 \sum_{i=23}^{33} s_{i,t}^1 \leq M s_{j,t}^1, \quad j \in 14, 15, \\
2 \sum_{i=23}^{33} s_{i,t}^1 \leq M s_{j,t}^1, \quad j \in 16, 17, \ldots, 22, \\
s_{14,t}^1 \leq s_{15,t}^1, \\
s_{16,t}^1 + s_{22,t}^1 \leq M s_{j,t}^1, \quad j \in 17, 18, \ldots, 21.
\end{gathered}
\tag{A.2}
$$

The parameters used above are as follows:

- $\underline{V}$: the minimum reservoir storage;
- $\overline{V}$: the maximum reservoir storage;
- $O_{\max}^{\text{down}}$: the discharge for the downstream safety;
- $O_{\max}$: a large water release value;
- $V_{t,p}^1$: the storage of breakpoint p at time t with the status of full open;
- $Q_{t,p}^1$: the water release of breakpoint p at time t with the status of full open;
- M: a large positive value;
- k_i^1: the number of breakpoints for spillway i with the status of full open.

The variables are as follows:

- V_m: the maximum storage during the flood;
- O_t: the water release at time t;
- $q_{i,t}$: the water release of spillway i at time t;
- $q_{i,t}^1$: the potential maximum water release of spillway i at time t with the status of full open;
- $w_{i,t,p}^1$: the weight of breakpoint p for spillway i at time t with the status of full open;
- $r_{i,t,p}^1$: the binary variable to ensure at most two adjacent breakpoints are greater than zero with the status of full open;
- $s_{i,t}^1$: the binary status variable of spillway i at time t with the status of full open.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This study was supported by the Program for New Century Excellent Talents in University (NCET-11-0401), the Non-Profit Industry Financial Program of Ministry of Water Resources (201201051), the Central Water Resources Allocation Fee Program (1261430210028), and the National Natural Science Foundation of China (51190094).

References

[1] S. Guo, H. Zhang, H. Chen, D. Peng, P. Liu, and B. Pang, "A reservoir flood forecasting and control system for China," *Hydrological Sciences Journal*, vol. 49, no. 6, pp. 959–972, 2004.

[2] C. Cheng and K. W. Chau, "Flood control management system for reservoirs," *Environmental Modelling and Software*, vol. 19, no. 12, pp. 1141–1150, 2004.

[3] S. Wang and G. H. Huang, "A two-stage mixed-integer fuzzy programming with interval-valued membership functions approach for flood-diversion planning," *Journal of Environmental Management*, vol. 117, pp. 208–218, 2013.

[4] W. W.-. Yeh, "Reservoir management and operations models: a state-of-the-art review," *Water Resources Research*, vol. 21, no. 12, pp. 1797–1818, 1985.

[5] J. W. Labadie, "Optimal operation of multireservoir systems: state-of-the-art review," *Journal of Water Resources Planning and Management*, vol. 130, no. 2, pp. 93–111, 2004.

[6] D. Karaboga, A. Bagis, and T. Haktanir, "Controlling spillway gates of dams by using fuzzy logic controller with optimum rule number," *Applied Soft Computing Journal*, vol. 8, no. 1, pp. 232–238, 2008.

[7] C. Wei and N. Hsu, "Optimal tree-based release rules for real-time flood control operations on a multipurpose multireservoir system," *Journal of Hydrology*, vol. 365, no. 3-4, pp. 213–224, 2009.

[8] A. Bagis and D. Karaboga, "Evolutionary algorithm-based fuzzy PD control of spillway gates of dams," *Journal of the Franklin Institute*, vol. 344, no. 8, pp. 1039–1055, 2007.

[9] L. Chang, "Guiding rational reservoir flood operation using penalty-type genetic algorithm," *Journal of Hydrology*, vol. 354, no. 1–4, pp. 65–74, 2008.

[10] X. Li, S. Guo, P. Liu, and G. Chen, "Dynamic control of flood limited water level for reservoir operation by considering inflow uncertainty," *Journal of Hydrology*, vol. 391, no. 1-2, pp. 124–132, 2010.

[11] G. Fu, "A fuzzy optimization method for multicriteria decision making: an application to reservoir flood control operation," *Expert Systems with Applications*, vol. 34, no. 1, pp. 145–149, 2008.

[12] H. Hashemi, J. Bazargan, S. M. Mousavi, and B. Vahdani, "An extended compromise ratio model with an application to reservoir flood control operation under an interval-valued intuitionistic fuzzy environment," *Applied Mathematical Modelling*, vol. 38, no. 14, pp. 3495–3511, 2014.

[13] A. Karbowski, K. Malinowski, and E. Niewiadomska-Szynkiewicz, "A hybrid analytic/rule-based approach to reservoir system management during flood," *Decision Support Systems*, vol. 38, no. 4, pp. 599–610, 2005.

[14] J. T. Needham, D. W. Watkins Jr., J. R. Lund, and S. K. Nanda, "Linear programming for flood control in the Iowa and Des Moines Rivers," *Journal of Water Resources Planning and Management*, vol. 126, no. 3, pp. 118–127, 2000.

[15] M. R. Norouzi, A. Ahmadi, A. E. Nezhad, and A. Ghaedi, "Mixed integer programming of multi-objective security-constrained hydro/thermal unit commitment," *Renewable and Sustainable Energy Reviews*, vol. 29, pp. 911–923, 2014.

[16] D. W. Watkins, D. J. Jones, and D. T. Ford, "Flood control optimization using mixed-integer programming," in *Proceedings of the Flood Control Optimization Using Mixed-Integer Programming Conference*, Reston, Va, USA, 1999.

[17] P. Liu, T. Nguyen, X. Cai, and X. Jiang, "Finding multiple optimal solutions to optimal load distribution problem in hydropower plant," *Energies*, vol. 5, no. 5, pp. 1413–1432, 2012.

[18] P. Guo, G. H. Huang, and Y. P. Li, "An inexact fuzzy-chance-constrained two-stage mixed-integer linear programming approach for flood diversion planning under multiple uncertainties," *Advances in Water Resources*, vol. 33, no. 1, pp. 81–91, 2010.

[19] C. Ma, J. Lian, and J. Wang, "Short-term optimal operation of Three-gorge and Gezhouba cascade hydropower stations in non-flood season with operation rules from data mining," *Energy Conversion and Management*, vol. 65, pp. 616–627, 2013.

[20] M. Breckpot, O. M. Agudelo, P. Meert, P. Willems, and B. D. Moor, "Flood control of the demer by using model predictive control," *Control Engineering Practice*, vol. 21, no. 12, pp. 1776–1787, 2013.

[21] N. Hsu and C. Wei, "A multipurpose reservoir real-time operation model for flood control during typhoon invasion," *Journal of Hydrology*, vol. 336, no. 3-4, pp. 282–293, 2007.

[22] E. C. Özelkan, Á. Galambosi, E. Fernández-Gaucherand, and L. Duckstein, "Linear quadratic dynamic programming for water reservoir management," *Applied Mathematical Modelling*, vol. 21, no. 9, pp. 591–598, 1997.

[23] X. Fu, A. Li, L. Wang, and C. Ji, "Short-term scheduling of cascade reservoirs using an immune algorithm-based particle swarm optimization," *Computers and Mathematics with Applications*, vol. 62, no. 6, pp. 2463–2471, 2011.

[24] Q. Zou, J. Zhou, C. Zhou et al., "Fuzzy risk analysis of flood disasters based on diffused-interior-outer-set model," *Expert Systems with Applications*, vol. 39, no. 6, pp. 6213–6220, 2012.

[25] E. Danso-Amoako, M. Scholz, N. Kalimeris, Q. Yang, and J. Shao, "Predicting dam failure risk for sustainable flood retention basins: a generic case study for the wider Greater Manchester area," *Computers, Environment and Urban Systems*, vol. 36, no. 5, pp. 423–433, 2012.

[26] Y. Ding and S. S. Y. Wang, "Optimal control of flood diversion in watershed using nonlinear optimization," *Advances in Water Resources*, vol. 44, pp. 30–48, 2012.

[27] C. Wei and N. Hsu, "Multireservoir real-time operations for flood control using balanced water level index method," *Journal of Environmental Management*, vol. 88, no. 4, pp. 1624–1639, 2008.

[28] P. Liu, X. Cai, and S. Guo, "Deriving multiple near-optimal solutions to deterministic reservoir operation problems," *Water Resources Research*, vol. 47, no. 8, article 7208, 2011.

[29] *Design of Small Dams*, United States Department of the Interior, Bureau of Reclamation, 3rd edition, 1987.

[30] A. Ashouri, S. S. Fux, M. J. Benz, and L. Guzzella, "Optimal design and operation of building services using mixed-integer linear programming techniques," *Energy*, vol. 59, pp. 365–376, 2013.

[31] P. Luathep, A. Sumalee, W. H. K. Lam, Z. Li, and H. K. Lo, "Global optimization method for mixed transportation network design problem: a mixed-integer linear programming approach," *Transportation Research B: Methodological*, vol. 45, no. 5, pp. 808–827, 2011.

[32] "CPLEX User's Manual," IBM ILOG CPLEX Optimization Studio, ftp://ftp.software.ibm.com/software/websphere/ilog/docs/optimization/cplex/ps_usrmancplex.pdf.

[33] *GAMS—A User's Guide*, GAMS Development Corporation, Washington, DC, USA, http://www.gams.com/dd/docs/bigdocs/GAMSUsersGuide.pdf.