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Research Article

A New Class of Meromorphic Functions Associated with Spirallike Functions

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We introduce a new class of meromorphic functions associated with spirallike functions. Such results as subordination property, integral representation, convolution property, and coefficient inequalities are proved.

1. Introduction

Let Σ denote the class of functions f of the form

$$f(z) = \frac{1}{z} + \sum_{k=0}^{\infty} a_k z^k, \quad (1.1)$$

which are *analytic* in the *punctured* open unit disk

$$\mathbb{U}^* := \{z : z \in \mathbb{C}, 0 < |z| < 1\} =: \mathbb{U} \setminus \{0\}. \quad (1.2)$$

Let \mathcal{P} denote the class of functions p given by

$$p(z) = 1 + \sum_{k=1}^{\infty} p_k z^k \quad (z \in \mathbb{U}), \quad (1.3)$$

which are analytic in \mathbb{U} and satisfy the condition

$$\operatorname{Re}(p(z)) > 0 \quad (z \in \mathbb{U}). \quad (1.4)$$

Let $f, g \in \Sigma$, where f is given by (1.1) and g is defined by

$$g(z) = \frac{1}{z} + \sum_{k=0}^{\infty} b_k z^k, \quad (1.5)$$

then the Hadamard product (or convolution) $f * g$ is defined by

$$(f * g)(z) := \frac{1}{z} + \sum_{k=0}^{\infty} a_k b_k z^k =: (g * f)(z). \quad (1.6)$$

For two functions f and g , analytic in \mathbb{U} , we say that the function f is subordinate to g in \mathbb{U} and write

$$f(z) \prec g(z) \quad (z \in \mathbb{U}), \quad (1.7)$$

if there exists a Schwarz function ω , which is analytic in \mathbb{U} with

$$\omega(0) = 0, \quad |\omega(z)| < 1 \quad (z \in \mathbb{U}), \quad (1.8)$$

such that

$$f(z) = g(\omega(z)) \quad (z \in \mathbb{U}). \quad (1.9)$$

Indeed, it is known that

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \implies f(0) = g(0), \quad f(\mathbb{U}) \subset g(\mathbb{U}). \quad (1.10)$$

Furthermore, if the function g is univalent in \mathbb{U} , then we have the following equivalence:

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \iff f(0) = g(0), \quad f(\mathbb{U}) \subset g(\mathbb{U}). \quad (1.11)$$

A function $f \in \Sigma$ is said to be in the class $\mathcal{MS}^*(\beta)$ of meromorphic starlike functions of order β if it satisfies the inequality

$$\operatorname{Re} \left(\frac{zf'(z)}{f(z)} \right) < -\beta \quad (z \in \mathbb{U}; 0 \leq \beta < 1). \quad (1.12)$$

For the real number β ($0 < \beta < 1$), we know that

$$\left| \frac{f(z)}{zf'(z)} + \frac{1}{2\beta} \right| < \frac{1}{2\beta} \iff \operatorname{Re} \left(\frac{zf'(z)}{f(z)} \right) < -\beta. \quad (1.13)$$

If the complex number α satisfies the condition

$$\left| \alpha - \frac{1}{2} \right| < \frac{1}{2}, \quad (1.14)$$

it can be easily verified that

$$\left| \frac{f(z)}{zf'(z)} + \frac{1}{2\alpha} \right| < \frac{1}{2|\alpha|} \iff \operatorname{Re} \left(-\frac{1}{\alpha} \frac{zf'(z)}{f(z)} \right) > 1. \quad (1.15)$$

We now introduce and investigate the following class of meromorphic functions.

Definition 1.1. A function $f \in \Sigma$ is said to be in the class \mathcal{MS}_α if it satisfies the inequality

$$\operatorname{Re} \left(-\frac{1}{\alpha} \frac{zf'(z)}{f(z)} \right) > 1 \quad \left(z \in \mathbb{U}; \left| \alpha - \frac{1}{2} \right| < \frac{1}{2} \right). \quad (1.16)$$

Remark 1.2. For $0 < \alpha < 1$, the class \mathcal{MS}_α is the familiar class of meromorphic starlike functions of order α .

Remark 1.3. If $\alpha = |\alpha|e^{i\psi}$ ($-\pi/2 < \psi < \pi/2$), then the condition (1.16) is equivalent to

$$\operatorname{Re} \left(e^{-i\psi} \frac{zf'(z)}{f(z)} \right) < -|\alpha| \quad (z \in \mathbb{U}), \quad (1.17)$$

which implies that f belongs to the class of meromorphic spirallike functions. Thus, the class of meromorphic spirallike functions is a special case of the class \mathcal{MS}_α .

For some recent investigations on spirallike functions and related functions, see, for example, the earlier works [1–9] and the references cited in each of these earlier investigations.

Remark 1.4. The function

$$f(z) = z^{-1}(1-z)^{2\alpha[\operatorname{Re}(1/\alpha)-1]} \quad \left(z \in \mathbb{U}^*; \left| \alpha - \frac{1}{2} \right| < \frac{1}{2} \right) \quad (1.18)$$

belongs to the class \mathcal{MS}_α .

It is clear that

$$\operatorname{Re} \left(\frac{1}{\alpha} \right) > 1 \quad \left(\left| \alpha - \frac{1}{2} \right| < \frac{1}{2} \right). \quad (1.19)$$

Then, for the function f given by (1.18), we know that

$$\begin{aligned} \operatorname{Re}\left(-\frac{1}{\alpha} \frac{zf'(z)}{f(z)}\right) &= \operatorname{Re}\left(\frac{1}{\alpha} + 2\left[\operatorname{Re}\left(\frac{1}{\alpha}\right) - 1\right] \frac{z}{1-z}\right) \\ &> \operatorname{Re}\left(\frac{1}{\alpha}\right) - \operatorname{Re}\left(\frac{1}{\alpha}\right) + 1 = 1, \end{aligned} \quad (1.20)$$

which implies that $f \in \mathcal{MS}_\alpha$.

In this paper, we aim at deriving the subordination property, integral representation, convolution property, and coefficient inequalities of the function class \mathcal{MS}_α .

2. Preliminary Results

In order to derive our main results, we need the following lemmas.

Lemma 2.1. *Let λ be a complex number. Suppose also that the sequence $\{A_k\}_{k=0}^\infty$ is defined by*

$$A_0 = 2\lambda, \quad A_{k+1} = \frac{2\lambda}{k+2} \left(1 + \sum_{l=0}^k A_l\right) \quad (k \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}). \quad (2.1)$$

Then

$$A_k = \frac{1}{(k+1)!} \prod_{j=0}^k (2\lambda + j) \quad (k \in \mathbb{N}_0). \quad (2.2)$$

Proof. From (2.1), we know that

$$\begin{aligned} (k+2)A_{k+1} &= 2\lambda \left(1 + \sum_{l=0}^k A_l\right), \\ (k+1)A_k &= 2\lambda \left(1 + \sum_{l=0}^{k-1} A_l\right). \end{aligned} \quad (2.3)$$

By virtue of (2.3), we find that

$$\frac{A_{k+1}}{A_k} = \frac{k+1+2\lambda}{k+2} \quad (k \in \mathbb{N}_0). \quad (2.4)$$

Thus, for $k \geq 1$, we deduce from (2.4) that

$$A_k = \frac{A_k}{A_{k-1}} \cdots \frac{A_3}{A_2} \cdot \frac{A_2}{A_1} \cdot \frac{A_1}{A_0} \cdot A_0 = \frac{1}{(k+1)!} \prod_{j=0}^k (2\lambda + j). \quad (2.5)$$

By virtue of (2.1) and (2.5), we get the desired assertion (2.2) of Lemma 2.1. \square

Lemma 2.2 (Jack's Lemma [10]). *Let ϕ be a nonconstant regular function in \mathbb{U} . If $|\phi|$ attains its maximum value on the circle $|z| = r < 1$ at z_0 , then*

$$z_0\phi'(z_0) = t\phi(z_0), \quad (2.6)$$

for some real number t ($t \geq 1$).

3. Main Results

We begin by deriving the following subordination property of functions belonging to the class \mathcal{MS}_α .

Theorem 3.1. *A function $f \in \mathcal{MS}_\alpha$ if and only if*

$$-\frac{zf'(z)}{f(z)} < 1 + 2\alpha \left[\operatorname{Re}\left(\frac{1}{\alpha}\right) - 1 \right] \frac{z}{1-z} \quad \left(z \in \mathbb{U}^*; \left| \alpha - \frac{1}{2} \right| < \frac{1}{2} \right). \quad (3.1)$$

Proof. Suppose that

$$h(z) := \frac{-(1/\alpha)(zf'(z)/f(z)) - 1 - i \operatorname{Im}(1/\alpha)}{\operatorname{Re}(1/\alpha) - 1} \quad (z \in \mathbb{U}; f \in \mathcal{MS}_\alpha). \quad (3.2)$$

We easily know that $h \in \mathcal{P}$, which implies that

$$\frac{-(1/\alpha)(zf'(z)/f(z)) - 1 - i \operatorname{Im}(1/\alpha)}{\operatorname{Re}(1/\alpha) - 1} = \frac{1 + \omega(z)}{1 - \omega(z)} \quad (z \in \mathbb{U}; f \in \mathcal{MS}_\alpha), \quad (3.3)$$

where ω is analytic in \mathbb{U} with $\omega(0) = 0$ and $|\omega(z)| < 1$ ($z \in \mathbb{U}$).

It follows from (3.3) that

$$-\frac{zf'(z)}{f(z)} = 1 + 2\alpha \left[\operatorname{Re}\left(\frac{1}{\alpha}\right) - 1 \right] \frac{\omega(z)}{1 - \omega(z)} \quad (z \in \mathbb{U}), \quad (3.4)$$

which is equivalent to the subordination relationship (3.1).

On the other hand, the above deductive process can be converse. The proof of Theorem 3.1 is thus completed. \square

Theorem 3.2. *Let $f \in \mathcal{MS}_\alpha$. Then*

$$f(z) = \frac{1}{z} \cdot \exp\left(-2\alpha \left[\operatorname{Re}\left(\frac{1}{\alpha}\right) - 1 \right] \int_0^z \frac{\omega(t)}{t(1 - \omega(t))} dt\right) \quad (z \in \mathbb{U}^*), \quad (3.5)$$

where ω is analytic in \mathbb{U} with $\omega(0) = 0$ and $|\omega(z)| < 1$ ($z \in \mathbb{U}$).

Proof. For $f \in \mathcal{MS}_\alpha$, by Theorem 3.1, we know that (3.1) holds true. It follows that

$$-\frac{zf'(z)}{f(z)} = 1 + 2\alpha \left[\operatorname{Re}\left(\frac{1}{\alpha}\right) - 1 \right] \frac{\omega(z)}{1 - \omega(z)} \quad (z \in \mathbb{U}), \quad (3.6)$$

where ω is analytic in \mathbb{U} with $\omega(0) = 0$ and $|\omega(z)| < 1$ ($z \in \mathbb{U}$).

We now find from (3.6) that

$$\frac{f'(z)}{f(z)} + \frac{1}{z} = -2\alpha \left[\operatorname{Re}\left(\frac{1}{\alpha}\right) - 1 \right] \frac{\omega(z)}{z(1 - \omega(z))} \quad (z \in \mathbb{U}^*), \quad (3.7)$$

which, upon integration, yields

$$\log(zf(z)) = -2\alpha \left[\operatorname{Re}\left(\frac{1}{\alpha}\right) - 1 \right] \int_0^z \frac{\omega(t)}{t(1 - \omega(t))} dt \quad (z \in \mathbb{U}). \quad (3.8)$$

The assertion (3.5) of Theorem 3.2 can be easily derived from (3.8). \square

Theorem 3.3. *Let $f \in \mathcal{MS}_\alpha$. Then*

$$f(z) * \frac{(1 - e^{i\theta})z + 2\alpha[\operatorname{Re}(1/\alpha) - 1]e^{i\theta}(1 - z)}{z(1 - z)^2} \neq 0 \quad (z \in \mathbb{U}^*; 0 < \theta < 2\pi). \quad (3.9)$$

Proof. Assume that $f \in \mathcal{MS}_\alpha$. By Theorem 3.1, we know that (3.1) holds, which implies that

$$-\frac{zf'(z)}{f(z)} \neq 1 + 2\alpha \left[\operatorname{Re}\left(\frac{1}{\alpha}\right) - 1 \right] \frac{e^{i\theta}}{1 - e^{i\theta}} \quad (z \in \mathbb{U}^*; 0 < \theta < 2\pi). \quad (3.10)$$

It is easy to see that the condition (3.10) can be written as follows:

$$\left(1 - e^{i\theta}\right)zf'(z) + \left(1 - e^{i\theta} + 2\alpha \left[\operatorname{Re}\left(\frac{1}{\alpha}\right) - 1 \right] e^{i\theta}\right)f(z) \neq 0. \quad (3.11)$$

We note that

$$\begin{aligned} f(z) &= f(z) * \left(\frac{1}{z} + 1 + \frac{z}{1 - z} \right) = f(z) * \frac{1}{z(1 - z)}, \\ -zf'(z) &= f(z) * \left(\frac{1}{z} - \frac{z}{(1 - z)^2} \right) = f(z) * \frac{1 - 2z}{z(1 - z)^2}. \end{aligned} \quad (3.12)$$

Thus, by substituting (3.12) into (3.11), we get the desired assertion (3.9) of Theorem 3.3. \square

Theorem 3.4. Let $\lambda = [\operatorname{Re}(1/\alpha) - 1]|\alpha|$. If $f \in \mathcal{MS}_\alpha$, then

$$|a_k| \leq \frac{1}{(k+1)!} \prod_{j=0}^k (2\lambda + j) \quad (k \in \mathbb{N}_0). \quad (3.13)$$

The inequality (3.13) is sharp for the function given by

$$f(z) = \frac{1}{z(1-z)^{2-2\alpha}} \quad (0 < \alpha < 1). \quad (3.14)$$

Proof. Suppose that

$$h(z) := \frac{-(1/\alpha)(zf'(z)/f(z)) - 1 - i \operatorname{Im}(1/\alpha)}{\operatorname{Re}(1/\alpha) - 1}. \quad (3.15)$$

We easily know that $h \in \mathcal{P}$.

If we put

$$h(z) = 1 + h_1z + h_2z^2 + \dots, \quad (3.16)$$

it is known that

$$|h_k| \leq 2 \quad (k \in \mathbb{N}). \quad (3.17)$$

From (3.15), we have

$$-\frac{1}{\alpha} \frac{zf'(z)}{f(z)} - 1 - i \operatorname{Im}\left(\frac{1}{\alpha}\right) = \left[\operatorname{Re}\left(\frac{1}{\alpha}\right) - 1 \right] h(z). \quad (3.18)$$

We now set

$$\begin{aligned} A &:= 1 + i \operatorname{Im}\left(\frac{1}{\alpha}\right), \\ B &:= \operatorname{Re}\left(\frac{1}{\alpha}\right) - 1. \end{aligned} \quad (3.19)$$

It follows from (3.18) that

$$-zf'(z) = [\alpha A + \alpha B h(z)]f(z). \quad (3.20)$$

Combining (1.1), (3.16), and (3.20), we obtain

$$\begin{aligned} & -z \left(-\frac{1}{z^2} + a_1 + 2a_2z + \cdots + ka_kz^{k-1} + \cdots \right) \\ & = \left(1 + \alpha Bh_1z + \cdots + \alpha Bh_kz^k + \cdots \right) \left(\frac{1}{z} + a_0 + a_1z + \cdots + a_kz^k + \cdots \right). \end{aligned} \quad (3.21)$$

In view of (3.21), we get

$$a_0 + \alpha Bh_1 = 0, \quad (3.22)$$

$$-ka_k = a_k + \alpha B(a_{k-1}h_1 + a_{k-2}h_2 + \cdots + a_0h_k + h_{k+1}) \quad (k \in \mathbb{N}). \quad (3.23)$$

From (3.17) and (3.22), we obtain

$$|a_0| \leq 2|\alpha|B = 2\lambda. \quad (3.24)$$

Moreover, we deduce from (3.17) and (3.23) that

$$|a_k| \leq \frac{2|\alpha|B}{k+1} \left(1 + \sum_{l=0}^{k-1} |a_l| \right) = \frac{2\lambda}{k+1} \left(1 + \sum_{l=0}^{k-1} |a_l| \right) \quad (k \in \mathbb{N}). \quad (3.25)$$

Next, we define the sequence $\{A_k\}_{k=0}^{\infty}$ as follows:

$$A_0 = 2\lambda, \quad A_{k+1} = \frac{2\lambda}{k+2} \left(1 + \sum_{l=0}^k A_l \right) \quad (k \in \mathbb{N}_0). \quad (3.26)$$

In order to prove that

$$|a_k| \leq A_k, \quad (3.27)$$

we make use of the principle of mathematical induction. By noting that

$$|a_0| \leq 2\lambda = A_0. \quad (3.28)$$

Therefore, assuming that

$$|a_l| \leq A_l \quad (l = 0, 1, 2, \dots, k; k \in \mathbb{N}_0). \quad (3.29)$$

Combining (3.25) and (3.26), we get

$$|a_{k+1}| \leq \frac{2\lambda}{k+2} \left(1 + \sum_{l=0}^k |a_l| \right) \leq \frac{2\lambda}{k+2} \left(1 + \sum_{l=0}^k A_l \right) = A_{k+1}. \quad (3.30)$$

Hence, by the principle of mathematical induction, we have

$$|a_k| \leq A_k \quad (k \in \mathbb{N}_0) \quad (3.31)$$

as desired.

By means of Lemma 2.1 and (3.26), we know that

$$A_k = \frac{1}{(k+1)!} \prod_{j=0}^k (2\lambda + j) \quad (k \in \mathbb{N}_0). \quad (3.32)$$

Combining (3.31) and (3.32), we readily get the coefficient estimates asserted by Theorem 3.4.

For the sharpness, we consider the function f given by (3.14). A simple calculation shows that

$$\operatorname{Re}\left(-\frac{zf'(z)}{f(z)}\right) = \operatorname{Re}\left(\frac{1 + (2\alpha - 3)z}{1 - z}\right) \geq 2 - \alpha > \alpha. \quad (3.33)$$

Thus, the function f belongs to the class \mathcal{MS}_α . Since $0 < \alpha < 1$, we have

$$\lambda = 1 - \alpha. \quad (3.34)$$

Then f becomes

$$f(z) = z^{-1}(1 - z)^{-2\lambda} = z^{-1} \left(\sum_{n=0}^{\infty} \binom{-2\lambda}{n} (-z)^n \right) = \frac{1}{z} + \sum_{n=0}^{\infty} \frac{2\lambda(2\lambda + 1) \cdots (2\lambda + n)}{(n + 1)!} z^n. \quad (3.35)$$

This completes the proof of Theorem 3.4. \square

Theorem 3.5. *If $f \in \Sigma$ satisfies the inequality*

$$\sum_{k=0}^{\infty} (k + |k + 2\alpha|) |a_k| \leq 1 - |2\alpha - 1| \quad \left(\left| \alpha - \frac{1}{2} \right| < \frac{1}{2} \right), \quad (3.36)$$

then $f \in \mathcal{MS}_\alpha$.

Proof. To prove $f \in \mathcal{MS}_\alpha$, it suffices to show that

$$\left| \frac{f(z)}{zf'(z)} + \frac{1}{2\alpha} \right| < \frac{1}{2|\alpha|} \quad (z \in \mathbb{U}), \quad (3.37)$$

which is equivalent to

$$\left| \frac{zf'(z) + 2\alpha f(z)}{zf'(z)} \right| < 1 \quad (z \in \mathbb{U}^*). \quad (3.38)$$

From (3.36), we know that

$$1 - \sum_{k=0}^{\infty} k|a_k| \geq |2\alpha - 1| + \sum_{k=0}^{\infty} |k + 2\alpha||a_k| > 0. \quad (3.39)$$

Now, by the maximum modulus principle, we deduce from (1.1) and (3.39) that

$$\begin{aligned} \left| \frac{zf'(z) + 2\alpha f(z)}{zf'(z)} \right| &= \left| \frac{(2\alpha - 1) + \sum_{k=0}^{\infty} (k + 2\alpha)a_k z^{k+1}}{-1 + \sum_{k=0}^{\infty} k a_k z^{k+1}} \right| \\ &\leq \frac{|2\alpha - 1| + \sum_{k=0}^{\infty} |k + 2\alpha||a_k||z|^{k+1}}{1 - \sum_{k=0}^{\infty} k|a_k||z|^{k+1}} \\ &< \frac{|2\alpha - 1| + \sum_{k=0}^{\infty} |k + 2\alpha||a_k|}{1 - \sum_{k=0}^{\infty} k|a_k|} \\ &\leq 1, \end{aligned} \quad (3.40)$$

which implies that the assertion of Theorem 3.5 holds. \square

Theorem 3.6. *If $f \in \Sigma$ satisfies the condition*

$$\left| 1 + \frac{zf''(z)}{f'(z)} - \frac{zf'(z)}{f(z)} \right| < \frac{1 - \alpha}{2\alpha} \quad \left(\frac{1}{2} < \alpha < 1 \right), \quad (3.41)$$

then $f \in \mathcal{MS}_\alpha$.

Proof. Define the function φ by

$$\varphi(z) := \frac{(zf'(z)/f(z)) + 1}{(zf'(z)/f(z)) + 2\alpha - 1} \quad (z \in \mathbb{U}). \quad (3.42)$$

Then we see that φ is analytic in \mathbb{U} with $\varphi(0) = 0$.

It follows from (3.42) that

$$-\frac{zf'(z)}{f(z)} = \frac{1 + (1 - 2\alpha)\varphi(z)}{1 - \varphi(z)}. \quad (3.43)$$

By differentiating both sides of (3.43) logarithmically, we obtain

$$1 + \frac{zf''(z)}{f'(z)} - \frac{zf'(z)}{f(z)} = \frac{2(1 - \alpha)z\varphi'(z)}{[1 + (1 - 2\alpha)\varphi(z)](1 - \varphi(z))}. \quad (3.44)$$

From (3.41) and (3.44), we find that

$$\left| 1 + \frac{zf''(z)}{f'(z)} - \frac{zf'(z)}{f(z)} \right| = \left| \frac{2(1-\alpha)z\varphi'(z)}{[1+(1-2\alpha)\varphi(z)](1-\varphi(z))} \right| < \frac{1-\alpha}{2\alpha}. \quad (3.45)$$

Next, we claim that $|\varphi(z)| < 1$. Indeed, if not, there exists a point $z_0 \in \mathbb{U}$ such that

$$\max_{|z| \leq |z_0|} |\varphi(z)| = |\varphi(z_0)| = 1. \quad (3.46)$$

By Lemma 2.2, we have

$$\varphi(z_0) = e^{i\theta}, \quad z_0\varphi'(z_0) = te^{i\theta} \quad (t \geq 1). \quad (3.47)$$

Moreover, for $z = z_0$, we find from (3.44) and (3.47) that

$$\begin{aligned} & \left| 1 + \frac{z_0 f''(z_0)}{f'(z_0)} - \frac{z_0 f'(z_0)}{f(z_0)} \right| \\ &= \left| \frac{2(1-\alpha)te^{i\theta}}{(1+(1-2\alpha)e^{i\theta})(1-e^{i\theta})} \right| \\ &= \frac{2(1-\alpha)t}{\sqrt{1+2(1-2\alpha)\cos\theta+(1-2\alpha)^2} \cdot \sqrt{2-2\cos\theta}} \\ &\geq \frac{1-\alpha}{2\alpha} \quad \left(\frac{1}{2} < \alpha < 1 \right). \end{aligned} \quad (3.48)$$

But (3.48) contradicts to (3.45). Therefore, we conclude that $|\varphi(z)| < 1$, that is

$$\left| \frac{(zf'(z)/f(z)) + 1}{(zf'(z)/f(z)) + 2\alpha - 1} \right| < 1, \quad (3.49)$$

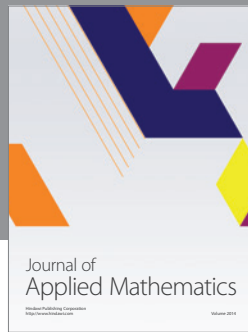
which shows that $f \in \mathcal{MS}_\alpha$. □

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