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Research Article

The Nondifferentiable Solution for Local Fractional Tricomi Equation Arising in Fractal Transonic Flow by Local Fractional Variational Iteration Method

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We present the nondifferentiable approximate solution for local fractional Tricomi equation arising in fractal transonic flow by local fractional variational iteration method. Some illustrative examples are shown and graphs are also given.

1. Introduction

In this paper, we study the local fractional Tricomi equation given as follows:

$$\frac{y^{\alpha}}{\Gamma(1+\alpha)}\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}u(x,y)}{\partial y^{2\alpha}}=0, \quad (1)$$

where the quantity $u(x,y)$ is the nondifferentiable function and the operator is local fractional operator suggested as follows [1–3]:

$$\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}=\frac{\Delta^{\alpha}(u(x,t)-u(x,t_0))}{(t-t_0)^{\alpha}}, \quad (2)$$

where

$$\begin{aligned}\Delta^{\alpha}&(u(x,t)-u(x,t_0))\\&\cong\Gamma(1+\alpha)\left[u(x,t)-u(x,t_0)\right].\end{aligned} \quad (3)$$

Local fractional derivative was applied to deal with nondifferentiable phenomena arising in mathematical physics [4–9]. When the fractal dimension α is equal to 1, we obtain the following differential equation:

$$y\frac{\partial^{2}u(x,y)}{\partial x^{2}}+\frac{\partial^{2}u(x,y)}{\partial y^{2}}=0, \quad (4)$$

which is structured by Tricomi [10]. The Tricomi equation was used to describe the transonic flow [10–22].

Local fractional variational iteration method first structured in [4] was an efficient tool to solve the local fractional differential equations, such as the fractal heat equation [4], the damped and dissipative wave equation in fractal strings [5], the wave equation on Cantor sets [6], the local fractional Poisson equation [7], the local fractional Laplace equation [8], and the local fractional Helmholtz equation [9]. The aim of this paper is to use the local fractional variational iteration method to deal with the local fractional Tricomi equation which arises in fractal transonic flow. The paper is organized as follows. In Section 2, the local fractional calculus theory is introduced. In Section 3, the local fractional variational iteration method is presented. In Section 4, the local fractional Tricomi equation is discussed. Finally, the conclusions are presented in Section 5.

2. Local Fractional Calculus Theory

In this section, we present the local fractional calculus theory, which is used in the present paper.

Definition 1 (see [1, 2]). One has the function $f(x)\in C_{\alpha}(a,b)$, if

$$\left|f(x)-f(x_0)\right|<\varepsilon^{\alpha}, \quad 0<\alpha\le 1, \quad (5)$$

is valid, where $|x-x_0|<\delta$, for $\varepsilon,\delta>0$ and $\varepsilon\in R$.

Definition 2 (see [1, 4–9]). Let $f(x)$ satisfy condition (5). The local fractional integral of $f(x)$ of order α in the interval $[a,b]$ is defined through

$$
\begin{aligned}
{}_aI_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^\alpha \\
&= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{j=N-1} f(t_j)(\Delta t_j)^\alpha,
\end{aligned}
\tag{6}
$$

where the partitions of the interval $[a,b]$ are (t_j, t_{j+1}), $j = 0,\ldots,N-1$, $t_0 = a$, and $t_N = b$ with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_0, \Delta t_1, \Delta t_j, \ldots\}$.

Definition 3 (see [1, 4–9]). Let $f(x)$ satisfy condition (5). The inverse formula of (6) is given as follows:

$$
\frac{d^\alpha f(x_0)}{dx^\alpha} = D_x^{(\alpha)} f(x_0) = \frac{\Delta^\alpha (f(x) - f(x_0))}{(x-x_0)^\alpha}, \tag{7}
$$

where

$$
\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(1+\alpha)[f(x) - f(x_0)]. \tag{8}
$$

The formulas of local fractional derivative and integral used in the paper are presented as follows [1, 6, 7]:

$$
\begin{gathered}
\frac{d^\alpha}{dx^\alpha} \frac{x^{n\alpha}}{\Gamma(1+n\alpha)} = \frac{x^{(n-1)\alpha}}{\Gamma(1+(n-1)\alpha)}, \quad n \in N, \\
D_x^{(\alpha)} a = 0, \\
D_x^{(\alpha)} a g(x) = a D_x^{(\alpha)} g(x), \\
D_x^{(\alpha)} \left[D_x^{(\alpha)} f(x)\right] = D_x^{(2\alpha)} f(x), \\
{}_0I_x^{(\alpha)} a g(x) = a\, {}_0I_x^{(\alpha)} g(x), \\
{}_0I_t^{(\alpha)} \left(\frac{(t-s)^\alpha t^{n\alpha}}{\Gamma(1+\alpha)\Gamma(1+n\alpha)} \right) = \frac{t^{(n+2)\alpha}}{\Gamma(1+(n+2)\alpha)}, \\
{}_0I_x^{(\alpha)} \frac{x^{(n-1)\alpha}}{\Gamma(1+(n-1)\alpha)} = \frac{x^{n\alpha}}{\Gamma(1+n\alpha)}, \quad n \in N,
\end{gathered}
\tag{9}
$$

where $g(x)$ is a local fractional continuous function, a is a constant, and N is a set of positive integers.

3. Local Fractional Variational Iteration Method

In this section, we introduce the local fractional variational iteration method. In order to show it, we consider the following local fractional operator equation:

$$
L_\alpha^{(2)} u + R_\alpha u = 0, \tag{10}
$$

where $L_\alpha^{(2)}$ denotes the linear local fractional differential operator and R_α denotes the linear local fractional differential operators of order less than $L_\alpha^{(2)}$.

According to the local fractional variational iteration method [4–9], we have a local fractional correction functional. Consider

$$
\begin{aligned}
u_{n+1}(x) &= u_n(x) + \frac{1}{\Gamma(1+\alpha)} \\
&\quad \times \int_0^x \frac{\lambda^\alpha}{\Gamma(1+\alpha)} \left\{ L_\alpha^{(2)} u_n(s) + R_\alpha \tilde{u}_n(s) \right\} (ds)^\alpha
\end{aligned}
\tag{11}
$$

so that the local fractional variational iteration algorithm can be written as follows:

$$
\begin{aligned}
u_{n+1}(x) &= u_n(x) + \frac{1}{\Gamma(1+\alpha)} \\
&\quad \times \int_0^x \frac{\lambda^\alpha}{\Gamma(1+\alpha)} \left\{ L_\alpha^{(2)} u_n(s) + R_\alpha u_n(s) \right\} (ds)^\alpha,
\end{aligned}
\tag{12}
$$

where $\tilde{u}_n$ is as a restricted local fractional variation [1]; that is, $\delta^\alpha \tilde{u}_n = 0$.

Therefore, for $n \in N$, we give

$$
\begin{aligned}
\delta^\alpha u_{n+1} &= \left[1 - \left(\frac{\lambda^\alpha}{\Gamma(1+\alpha)} \right)^{(\alpha)} \bigg|_{s=x} \right] \delta^\alpha u_n \\
&\quad + \frac{\lambda^\alpha}{\Gamma(1+\alpha)} \bigg|_{s=x} \delta^\alpha \frac{\partial^\alpha u_n}{\partial x^\alpha} \\
&\quad + {}_0I_x^{(\alpha)} \left\{ \left(\frac{\lambda^\alpha}{\Gamma(1+\alpha)} \right)^{(2\alpha)} \bigg|_{s=x} \delta^\alpha u_n \right\}.
\end{aligned}
\tag{13}
$$

From (13), we obtain the stationary condition as follows:

$$
\begin{gathered}
1 - \left(\frac{\lambda^\alpha}{\Gamma(1+\alpha)} \right)^{(\alpha)} \bigg|_{s=x} = 0, \qquad \frac{\lambda^\alpha}{\Gamma(1+\alpha)} \bigg|_{s=x} = 0, \\
\left(\frac{\lambda^\alpha}{\Gamma(1+\alpha)} \right)^{(2\alpha)} \bigg|_{s=x} = 0.
\end{gathered}
\tag{14}
$$

Then, the fractal Lagrange multiplier is

$$
\frac{\lambda^\alpha}{\Gamma(1+\alpha)} = \frac{(s-x)^\alpha}{\Gamma(1+\alpha)}. \tag{15}
$$

Making use of (12) and (15), we have the local fractional interaction formula as follows:

$$
\begin{aligned}
u_{n+1}(x) &= u_n(x) + \frac{1}{\Gamma(1+\alpha)} \\
&\quad \times \int_0^x \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left\{ L_\alpha^{(2)} u_n(s) + R_\alpha u_n(s) \right\} (ds)^\alpha.
\end{aligned}
\tag{16}
$$

Therefore, from (16), we get the solution given by

$$
u(x) = \lim_{n \to \infty} u_n(x). \tag{17}
$$

4. The Initial-Boundary Value Problems for Local Fractional Tricomi Equation

In this section, we discuss the initial-boundary value problems for local fractional Tricomi equation.

Example 1. Let us consider the initial-boundary value conditions for the local fractional Tricomi equation as follows:

$$u(0,y)=0, \tag{18}$$

$$u(l,y)=0, \tag{19}$$

$$u(x,0)=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}, \tag{20}$$

$$\frac{\partial^{\alpha}u(x,0)}{\partial x^{\alpha}}=0. \tag{21}$$

From (18), (20), and (21), we have

$$\begin{aligned} u_{n+1}(x,y) &= u_n(x,y) + {}_0I_y^{(\alpha)} \\ &\quad\times\left\{\frac{(s-y)^{\alpha}}{\Gamma(1+\alpha)}\right. \\ &\quad\left.\times\left(\frac{s^{\alpha}}{\Gamma(1+\alpha)}\frac{\partial^{2\alpha}u_n(x,s)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}u_n(x,s)}{\partial y^{2\alpha}}\right)\right\}, \end{aligned} \tag{22}$$

where the initial value is given by

$$u_0(x,y)=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{23}$$

From (22), we present the first approximate formula as follows:

$$\begin{aligned} u_1(x,y) &= u_0(x,y) + {}_0I_y^{(\alpha)} \\ &\quad\times\left\{\frac{(s-y)^{\alpha}}{\Gamma(1+\alpha)}\right. \\ &\quad\left.\times\left(\frac{s^{\alpha}}{\Gamma(1+\alpha)}\frac{\partial^{2\alpha}u_0(x,s)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}u_0(x,s)}{\partial y^{2\alpha}}\right)\right\} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{y^{3\alpha}}{\Gamma(1+3\alpha)} \end{aligned} \tag{24}$$

and its graph is shown in Figure 1.

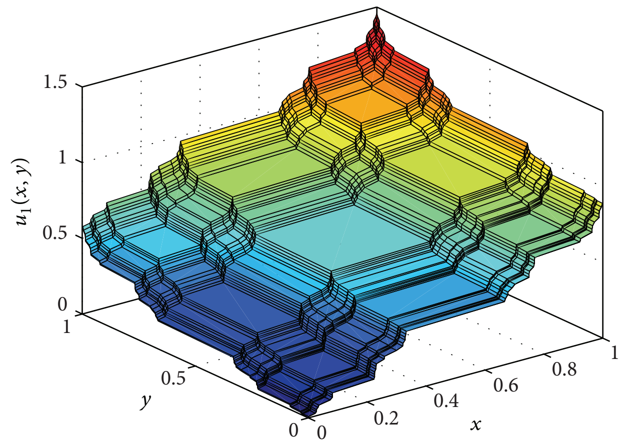

Figure 1: The plot of $u_1(x,y)$ with the parameter $\alpha=\ln 2/\ln 3$.

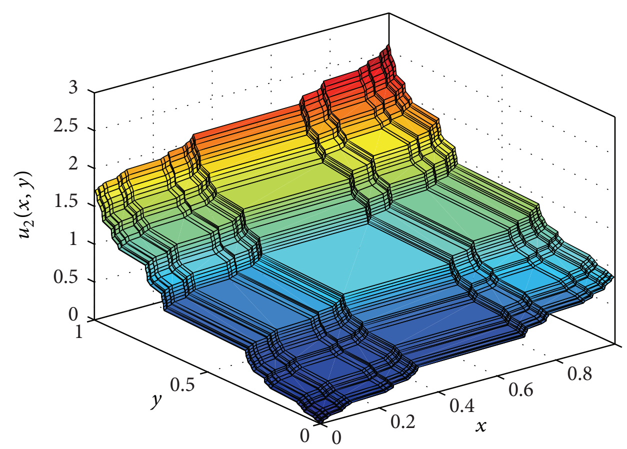

Figure 2: The plot of $u_2(x,y)$ with the parameter $\alpha=\ln 2/\ln 3$.

The second approximate term is

$$\begin{aligned} u_2(x,y) &= u_1(x,y) + {}_0I_y^{(\alpha)} \\ &\quad\times\left\{\frac{(s-y)^{\alpha}}{\Gamma(1+\alpha)}\right. \\ &\quad\left.\times\left(\frac{s^{\alpha}}{\Gamma(1+\alpha)}\frac{\partial^{2\alpha}u_1(x,s)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}u_1(x,s)}{\partial y^{2\alpha}}\right)\right\} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{y^{3\alpha}}{\Gamma(1+3\alpha)} \\ &\quad+ {}_0I_y^{(\alpha)}\left\{\frac{(s-y)^{\alpha}}{\Gamma(1+\alpha)}\left(\frac{s^{\alpha}}{\Gamma(1+\alpha)}+\frac{s^{\alpha}}{\Gamma(1+\alpha)}\right)\right\} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{3y^{3\alpha}}{\Gamma(1+3\alpha)} \end{aligned} \tag{25}$$

and its graph is given in Figure 2.

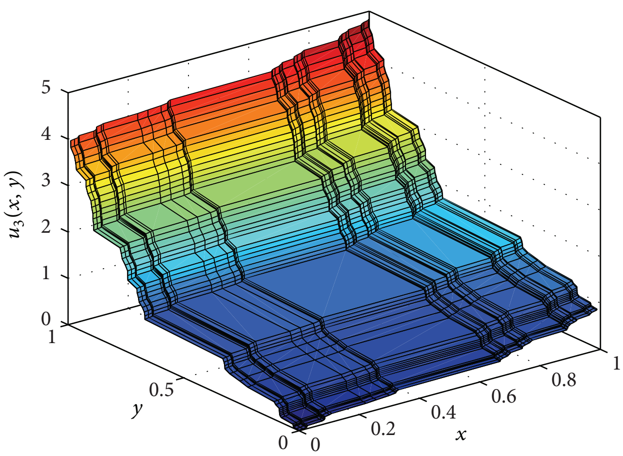

FIGURE 3: The plot of $u_3(x, y)$ with the parameter $\alpha = \ln 2/\ln 3$.

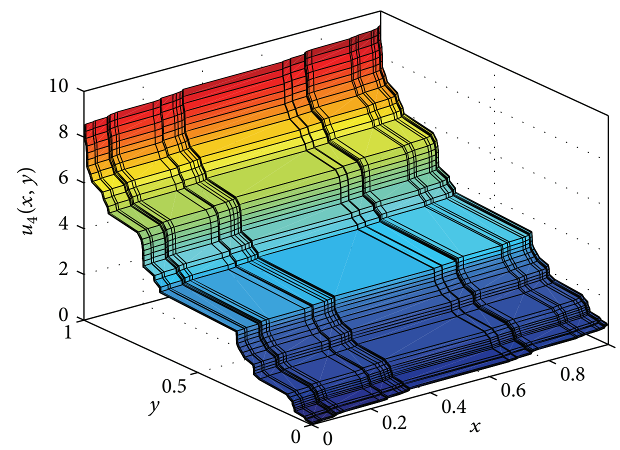

FIGURE 4: The plot of $u_4(x, y)$ with the parameter $\alpha = \ln 2/\ln 3$.

The third approximation is presented as follows:

$$\begin{aligned} u_3(x, y) &= u_2(x, y) + {}_0I_y^{(\alpha)} \times \left\{ \frac{(s-y)^{\alpha}}{\Gamma(1+\alpha)} \times \left(\frac{s^{\alpha}}{\Gamma(1+\alpha)} \frac{\partial^{2\alpha} u_2(x, s)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_2(x, s)}{\partial y^{2\alpha}} \right) \right\} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{3y^{3\alpha}}{\Gamma(1+3\alpha)} + {}_0I_y^{(\alpha)} \left\{ \frac{(s-y)^{\alpha}}{\Gamma(1+\alpha)} \left(\frac{s^{\alpha}}{\Gamma(1+\alpha)} + \frac{3s^{\alpha}}{\Gamma(1+\alpha)} \right) \right\} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{7y^{3\alpha}}{\Gamma(1+3\alpha)} \end{aligned} \tag{26}$$

and its graph is illustrated in Figure 3.

The fourth approximation reads as follows:

$$\begin{aligned} u_4(x, y) &= u_3(x, y) + {}_0I_y^{(\alpha)} \times \left\{ \frac{(s-y)^{\alpha}}{\Gamma(1+\alpha)} \times \left(s^{\alpha} \frac{\partial^{2\alpha} u_3(x, s)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_3(x, s)}{\partial y^{2\alpha}} \right) \right\} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{7y^{3\alpha}}{\Gamma(1+3\alpha)} + {}_0I_y^{(\alpha)} \left\{ \frac{(s-y)^{\alpha}}{\Gamma(1+\alpha)} \left(\frac{s^{\alpha}}{\Gamma(1+\alpha)} + \frac{7s^{\alpha}}{\Gamma(1+\alpha)} \right) \right\} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{15y^{3\alpha}}{\Gamma(1+3\alpha)} \end{aligned} \tag{27}$$

and its graph is presented in Figure 4.

The fifth approximation is as follows:

$$\begin{aligned} u_5(x, y) &= u_4(x, y) + {}_0I_y^{(\alpha)} \times \left\{ \frac{(s-y)^{\alpha}}{\Gamma(1+\alpha)} \times \left(\frac{s^{\alpha}}{\Gamma(1+\alpha)} \frac{\partial^{2\alpha} u_4(x, s)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_4(x, s)}{\partial y^{2\alpha}} \right) \right\} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{15y^{3\alpha}}{\Gamma(1+3\alpha)} + {}_0I_y^{(\alpha)} \left\{ \frac{(s-y)^{\alpha}}{\Gamma(1+\alpha)} \left(\frac{s^{\alpha}}{\Gamma(1+\alpha)} + \frac{15s^{\alpha}}{\Gamma(1+\alpha)} \right) \right\} \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{31y^{3\alpha}}{\Gamma(1+3\alpha)} \end{aligned} \tag{28}$$

and its graph is shown in Figure 5.

After successive iterative processes, we obtain the nondifferentiable series solution as follows:

$$\begin{aligned} u(x, y) &= \lim_{i \to \infty} u_n(x, y) \\ &= \lim_{n \to \infty} \left\{ \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + (2^n - 1) \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} \right\}, \end{aligned} \tag{29}$$

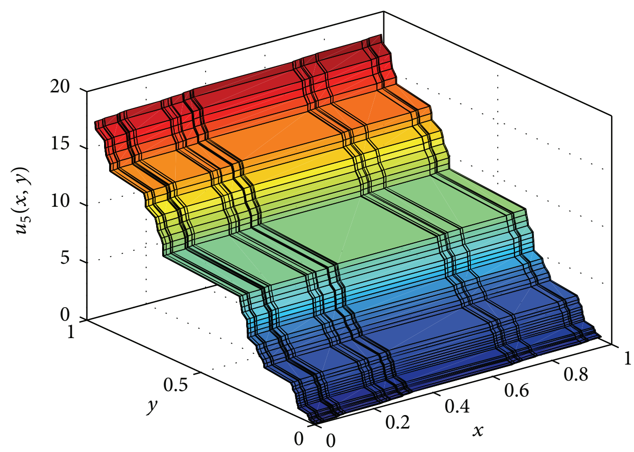

Figure 5: The plot of $u_5(x, y)$ with the parameter $\alpha = \ln 2/\ln 3$.

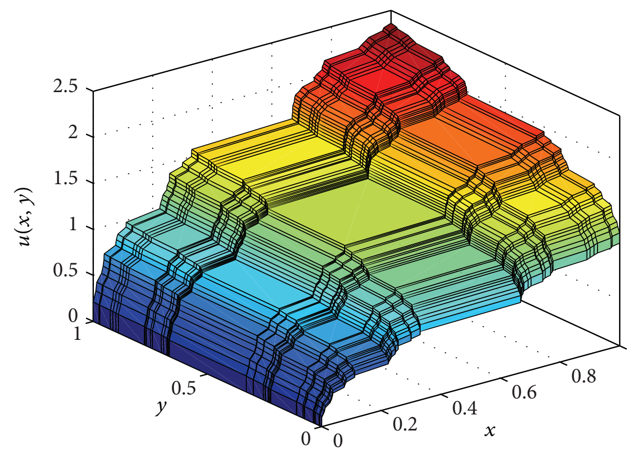

Figure 6: The plot of $u(x, y)$ with the parameter $\alpha = \ln 2/\ln 3$.

which is the local fractional divergent series. Therefore, we can obtain the approximate solution.

Example 2. The initial-boundary value conditions for the local fractional Tricomi equation are presented as follows:

$$u(0, y) = 0, \tag{30}$$

$$u(l, y) = 0, \tag{31}$$

$$u(x, 0) = \frac{x^\alpha}{\Gamma(1+\alpha)}, \tag{32}$$

$$\frac{\partial^\alpha u(x, 0)}{\partial x^\alpha} = \frac{x^\alpha}{\Gamma(1+\alpha)}. \tag{33}$$

In view of (16), (32), and (33), we obtain the local fractional iterative formula as follows:

$$\begin{aligned} u_{n+1}(x, y) &= u_n(x, y) + {}_0I_y^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-y)^\alpha}{\Gamma(1+\alpha)} \right. \\ &\quad \left. \times \left(\frac{s^\alpha}{\Gamma(1+\alpha)} \frac{\partial^{2\alpha} u_n(x, s)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_n(x, s)}{\partial y^{2\alpha}} \right) \right\}, \end{aligned} \tag{34}$$

with the initial value suggested as follows:

$$u_0(x, y) = \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{x^\alpha}{\Gamma(1+\alpha)} \frac{y^\alpha}{\Gamma(1+\alpha)}. \tag{35}$$

From (34) and (35), we give the first approximation as follows:

$$\begin{aligned} u_1(x, y) &= u_0(x, y) + {}_0I_y^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-y)^\alpha}{\Gamma(1+\alpha)} \right. \\ &\quad \left. \times \left(\frac{s^\alpha}{\Gamma(1+\alpha)} \frac{\partial^{2\alpha} u_0(x, s)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_0(x, s)}{\partial y^{2\alpha}} \right) \right\} \\ &= \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{x^\alpha}{\Gamma(1+\alpha)} \frac{y^\alpha}{\Gamma(1+\alpha)}. \end{aligned} \tag{36}$$

Hence, from (36), we arrive at the following results:

$$u_0(x, y) = u_1(x, y) = u_2(x, y) = \cdots = u_n(x, y). \tag{37}$$

Therefore, we get the exact solution with nondifferential term as follows:

$$\begin{aligned} u(x, y) &= \lim_{i\to\infty} u_n(x, y) \\ &= \lim_{n\to\infty} \left\{ \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{x^\alpha}{\Gamma(1+\alpha)} \frac{y^\alpha}{\Gamma(1+\alpha)} \right\} \\ &= \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{x^\alpha}{\Gamma(1+\alpha)} \frac{y^\alpha}{\Gamma(1+\alpha)} \end{aligned} \tag{38}$$

and its graph is shown in Figure 6.

5. Conclusions

The initial-boundary value problems for local fractional Tricomi equation arising in fractal transonic flow based upon the local fractional derivatives are discussed. The solutions with nondifferentiable terms are obtained by using the local fractional variational iteration method and their graphs are also given to show the implement of the present method.

Conflict of Interests

The authors declare that they have no competing interests in this paper.

Acknowledgments

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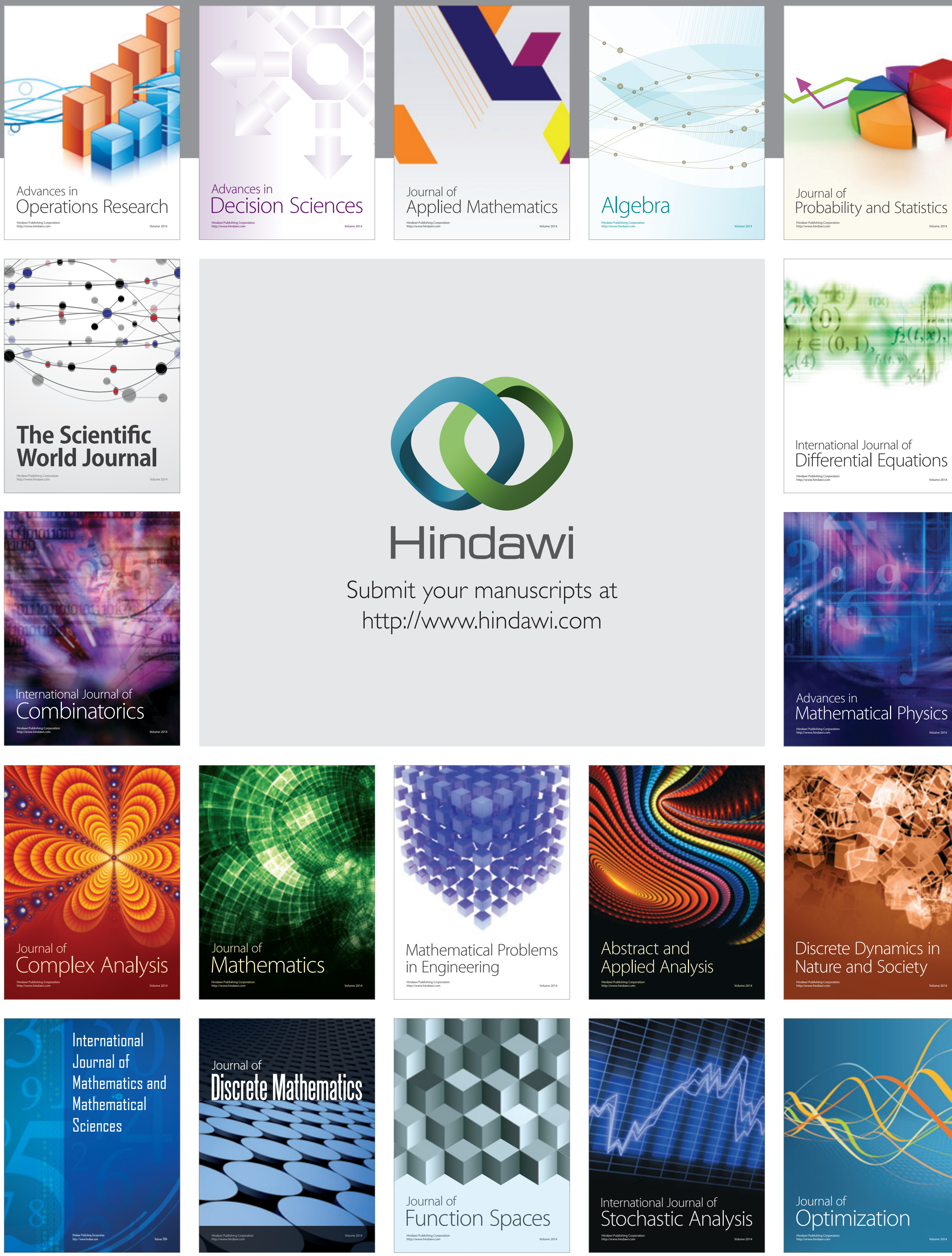

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