

## Research Article

# Adaptive Control of a Class of Switched Nonlinear System with Partial State Constraints Using a Barrier Lyapunov Function

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This paper discusses partial state constraint adaptive tracking control problem of switched nonlinear systems with uncertain parameters. In order to ensure boundedness of the outputs and prevent the states from violating the constraints, a barrier Lyapunov function (BLF) is employed. Based on backstepping method, an adaptive controller for the switched system is designed. Furthermore, the state-constrained asymptotic tracking under arbitrary switching is performed. The closed-loop signals keep bounded when the initial states and control parameters are given. Finally, examples and simulation results are reported to illustrate the effectiveness of the proposed controller.

## 1. Introduction

There is a strong industrial background of switched system in various fields. And for exactly that reason, many researchers have discussed the theoretical and applied research of switched system, and some of them have achieved commendable results in the last decade [1–3]. Since nonlinearity is the nature of the universe, more and more attention of switched nonlinear system has been drawn by control field [4–6]. It is also worth mentioning that a class of switched nonlinear system in lower triangular form is considered in [7], of which the backstepping method is used and a common virtual control is constructed to achieve the aim. Nevertheless, relatively accurate parameters are required by most control strategy for switched nonlinear system. Apparently, these desirable demands could not be met in practical terms.

Constraints are important issues in the study of physical systems, the authors in [8, 9] emphasized this problem in nonlinear saturation and performance and safety specifications, respectively. Taking a practical example for illustrative purposes, the attitude-control mass expulsion system of electrostatic microactuators is confronted with constraints, owing to the necessity of both the position and the speed of the movable electrode to be controlled. However, the development of constrained control is restrained by meeting

the practical requirement. In this scenario, the extensive attention of the design and the analysis of constrained control has been increased. Recently, barrier Lyapunov functions (BLFs) have been proposed to solve the control problem of nonlinear systems. The state of the system will be not contradictory to the constraints [10–13] utilizing BLFs. Although the BLF is proven to settle constrained control problems [14], the parameters of switched nonlinear systems are expected to be deterministic.

Moreover, parameter uncertainties are widespread in realistic systems; hereon it has already been reported that adaptive control is an effective method to deal with such uncertain. On the one hand, remarkable achievements have been obtained from the research on adaptive control of nonlinear system. In [15–17], for instance, the control method is proposed and the preferable control performances are provided aiming at nonlinear time-delay systems. On the other hand, many researchers study the adaptive control problem of switched systems with uncertain parameters [18–20] with the development of adaptive control, and preliminary results have been obtained [21, 22]. In [23], an adaptive tracking controller is designed for switched stochastic nonlinear systems with unknown actuator dead-zone; the satisfying control performance is obtained as well. Among the aforementioned works, few authors have addressed the

important issue that considering the uncertainty and the constraint together. Hence, the approach to design constraint adaptive controllers for switched nonlinear systems has been not reported to the best of our knowledge.

In the present paper, the adaptive tracking control problem of a class of switched nonlinear system with partial state constraints is solved. The progressive state tracking would not violate constraint conditions and all signals would be bounded when the parameter increases to infinitely great and approaches a certain value based on BLF. The control effectiveness of BLFs is verified by the comparative simulation results with quadratic Lyapunov functions (QLFs).

The remainder of this paper is organized as follows. Section 2 formulates the control problem. The partial state constraint problems of barrier Lyapunov functions are elucidated. Section 3 develops the adaptive controllers with state constraints; the stability analysis and the proof process are presented as well. Section 4 corroborates the expected effectiveness of the proposed controller by means of selected simulation results. Finally, Section 5 concludes this paper.

## 2. Problem Statements and Preliminaries

**2.1. Problem Formulation.** Consider a class of the switched nonlinear systems with uncertain parameters in the following form:

$$\begin{aligned}\dot{x}_i &= F_i^{\sigma(t)}(x_i) + \theta_b^{\sigma(t)} g_i(x_i) x_{i+1} \\ \dot{x}_n &= F_n^{\sigma(t)}(x_n) + \theta_b^{\sigma(t)} g_n(x_n) u \\ y &= x_1,\end{aligned}\quad (1)$$

where  $\theta_b^i \in R^n$  are the unknown piecewise constant parameters.  $\sigma(t) \in P = \{1, 2, \dots, p_s\}$  is a non-Zeno switching signal which is right continuous.  $u \in R$  is the control input.  $F_k^i$  and  $g_k$  are smooth vector fields with  $F_k^i(0) = 0$  and  $g_k(x) \neq 0, \forall x \in R^n$ .

In addition,  $F_i^j(x_i), \forall j \in P$  is unknown switched nonlinear function which can be linearly parameterized as

$$F_i^{\sigma(t)}(x_i) = \sum_{k=1}^p f_{ki}(x_i) \theta_{f_{ki}}^{\sigma(t)}, \quad (2)$$

where  $f_{ki}(x_i)$  is smooth function and  $\theta_{f_{ki}}^i \in R^n$  is a vector of uncertain parameters satisfying  $\theta \in \Omega_\theta$  with known compact set  $\Omega_\theta$ . The parameters  $\theta_b^i$  and  $\theta_{f_{ki}}^i, \forall i \in P$ , the switching time instants  $T_k, k = 1, 2, \dots$ , and the switching index  $\sigma(t) \in P = \{1, 2, \dots, p_s\}$  are all unknown. There exist positive constants  $P_i$  such that  $\|f_{ki}(x_i)\| \leq P_i$  for all  $|x_i| \leq b, i = 1, 2, \dots, n$  due to smoothness property.

According to (1) and (2), system (1) can be transformed in to the following form:

$$\begin{aligned}\dot{x}_i &= F_i^{\sigma(t)}(x_i) + \theta_b^{\sigma(t)} g_i(x_i) x_{i+1} \\ &= \sum_{k=1}^p f_{ki}(x_i) \theta_{f_{ki}}^{\sigma(t)} + \theta_b^{\sigma(t)} g_i(x_i) x_{i+1}, \\ i &= \{1, 2, \dots, n\}.\end{aligned}\quad (3)$$

The control target is to design an adaptive controller for system (1) such that  $y$  tracks a desired trajectory  $y_d$  asymptotically; that is,  $\lim_{t \rightarrow \infty} (y - y_d) = \lim_{t \rightarrow \infty} z_1 = 0$  and  $|x_i| \leq b, b$  is a positive constant. Moreover, in order to make the problem here more tractable, we give the following assumption, which is common but practical.

**Assumption 1** (see [24]).  $\theta_b^i$  for all  $i$ 's have the same sign and whose common lower bound is known, that is,  $0 < \bar{\theta}_b < |\theta_b^i|$ .

### 2.2. Preliminaries

**Definition 2** (see [12]). A barrier Lyapunov function (BLF) is a scalar function  $V(x)$ , defined with respect to the system  $\dot{x} = f(x)$  on an open region  $D$  containing the origin, that is, continuous, positive definite, has continuous first-order partial derivatives at every point of  $D$ , has the property  $V(x) \rightarrow \infty$  as  $x$  approaches the boundary of  $D$ , and satisfies  $V(x(t)) \leq b \forall t \geq 0$  along the solution of  $\dot{x} = f(x)$  for  $x(0) \in D$  and some positive constant  $b$ .

According to previous description, we can choose a BLF candidate as follows:

$$V_1 = \frac{1}{2} \left( \log \frac{k_{b1}^2}{k_{b1}^2 - z_1^2} \right), \quad (4)$$

where  $\log(\cdot)$  denotes the natural logarithm.  $k_{b1}$  is the constraint on  $z_1$  and satisfies the condition  $|z_1| < k_{b1}$ . Thus it can be seen that  $V_1$  is positive definite, which grows to infinity when its argument approaches to its finite limit  $k_{b1}$ .

**Lemma 3** (see [24]). For function  $\sum_{k=1}^p \bar{f}_{k1}(x_1) \theta_{f_{k1}}^{\sigma(t)}$ , a smooth function  $h_1(x_1)$  and an unknown positive constant  $l_1$  can be found:

$$\left| \sum_{k=1}^p \bar{f}_{k1}(x_1) \theta_{f_{k1}}^{\sigma(t)} \right| \leq l_1 h_1(x_1). \quad (5)$$

**Lemma 4** (see [10]). For any positive constant  $k_{b1}$ , any  $z_1$  satisfying  $|z_1| < k_{b1}$ , we have

$$\log \frac{k_{b1}^2}{k_{b1}^2 - z_1^2} < \frac{z_1}{k_{b1}^2 - z_1^2}. \quad (6)$$

In order to make the problem more analysable and tractable, practical assumption is given for the adaptive state controller.

**Assumption 5** (see [24]). We can find a smooth function  $\bar{f}_{k1}(x_1)$  such that

$$f_{k1}(x_1) = \bar{f}_{k1}(x_1) z_1. \quad (7)$$

## 3. Main Results

In this section, an adaptive controller is designed based on backstepping method by utilizing a BLF for systems (3).

*Step 1.* Define  $z_1$  for  $x_1$  as the tracking error which is  $z_1 = x_1 - y_d$ . Consider the first component of the system (3) and we have the first-order partial derivative as

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1 - \dot{y}_d \\ &= \sum_{k=1}^p f_{k1}(x_1) \theta_{f_{k1}}^{\sigma(t)} + \theta_b^{\sigma(t)} g_1(x_1) x_2 - \dot{y}_d,\end{aligned}\quad (8)$$

Choose the following BLF candidate:

$$V_1 = \frac{1}{2} \left( \log \frac{k_{b1}^2}{k_{b1}^2 - z_1^2} + \tilde{l}_1^2 \right), \quad (9)$$

where  $\tilde{l}_1 = l_1 - \hat{l}_1$ ,  $\hat{l}_1$  denotes the estimate of  $l_1$ , and  $\dot{\tilde{l}}_1 = -\dot{\hat{l}}_1$ . We design a stabilizing function and an adaptive law as

$$\begin{aligned}\alpha_1 &= \frac{1}{\bar{\theta}_b g_1} \left( -\hat{l}_1 h_1 z_1 - \lambda_1 z_1 + \dot{y}_d \right), \\ \dot{\hat{l}}_1 &= \frac{z_1^2 h_1}{k_{b1}^2 - z_1^2},\end{aligned}\quad (10)$$

where  $\lambda_1$  is a positive gain. Besides, let  $z_2 = x_2 - \alpha_1$ . On the basis of (10) and Assumption 1, we know that the time derivate of  $V_1$  satisfies

$$\begin{aligned}\dot{V}_1 &= \frac{z_1 \dot{z}_1}{k_{b1}^2 - z_1^2} + \tilde{l}_1 \dot{\tilde{l}}_1 = \frac{z_1 \left( \sum_{k=1}^p f_{k1}(x_1) \theta_{f_{k1}}^{\sigma(t)} + \theta_b^{\sigma(t)} g_1(z_2 + \alpha_1) \dot{y}_d \right)}{k_{b1}^2 - z_1^2} + \tilde{l}_1 \dot{\tilde{l}}_1 \\ &\leq \frac{|z_1|^2 l_1 h_1 + \theta_b^{\sigma(t)} g_1 z_1 \left( z_2 + (1/\bar{\theta}_b g_1) (-\hat{l}_1 h_1 z_1 - \lambda_1 z_1 + \dot{y}_d z_1) \right) - \dot{y}_d |z_1|^2}{k_{b1}^2 - z_1^2} + \tilde{l}_1 \left( -\frac{|z_1|^2 h_1}{k_{b1}^2 - z_1^2} \right) \leq -\frac{\lambda_1 z_1^2}{k_{b1}^2 - z_1^2},\end{aligned}\quad (11)$$

where  $\dot{y}_d$  can be found that  $\dot{y}_d = \dot{\tilde{y}}_d z_1$ .

*Step 2.* Consider the second component of system (3). Since  $\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1$ , we can find a  $\tilde{\alpha}_1$  satisfying  $\dot{\alpha}_1 = \tilde{\alpha}_1 z_2$  and choose the following BLF candidate:

$$V_2 = \sum_{i=1}^2 \frac{1}{2} \left( \log \frac{k_{bi}^2}{k_{bi}^2 - z_i^2} + \tilde{l}_i^2 \right). \quad (12)$$

The stabilizing function and the adaptive law at this step are designed as

$$\begin{aligned}\alpha_2 &= \frac{1}{\bar{\theta}_b g_2} \left( -\hat{l}_2 h_2 z_2 - \lambda_2 z_2 - \dot{\alpha}_1 - \frac{k_{b2}^2 - z_2^2}{k_{b1}^2 - z_1^2} \bar{\theta}_b g_1 z_1 \right), \\ \dot{\hat{l}}_2 &= \frac{z_2^2 h_2}{k_{b2}^2 - z_2^2}.\end{aligned}\quad (13)$$

Using (13) and Assumption 1, we know that the time derivate of  $V_2$  satisfies

$$\begin{aligned}\dot{V}_2 &= \frac{z_1 \dot{z}_1}{k_{b1}^2 - z_1^2} + \tilde{l}_1 \dot{\tilde{l}}_1 + \frac{z_2 \dot{z}_2}{k_{b2}^2 - z_2^2} + \tilde{l}_2 \dot{\tilde{l}}_2 \leq -\frac{\lambda_1 z_1^2}{k_{b1}^2 - z_1^2} \\ &\quad + \frac{z_1 z_2 \theta_b^{\sigma(t)} g_1}{k_{b1}^2 - z_1^2} + \frac{z_2 \dot{z}_2}{k_{b2}^2 - z_2^2} + \tilde{l}_2 \dot{\tilde{l}}_2 \leq -\frac{\lambda_1 z_1^2}{k_{b1}^2 - z_1^2} \\ &\quad + \frac{z_1 z_2 \theta_b^{\sigma(t)} g_1}{k_{b1}^2 - z_1^2} \\ &\quad + \frac{z_2 \left( \sum_{k=1}^p f_{k2}(x_2) \theta_{f_{k2}}^{\sigma(t)} + \theta_b^{\sigma(t)} g_2(x_2) (z_3 + \alpha_2) - \dot{\alpha}_1 \right)}{k_{b2}^2 - z_2^2} \\ &\quad + \tilde{l}_2 \dot{\tilde{l}}_2 \leq -\frac{\lambda_2 z_2^2}{k_{b2}^2 - z_2^2} - \frac{\lambda_1 z_1^2}{k_{b1}^2 - z_1^2} \leq 0.\end{aligned}\quad (14)$$

For the general case, we employ the BLF when  $n > 2$ :

$$V_i = \sum_{i=1}^j \frac{1}{2} \left( \log \frac{k_{bi}^2}{k_{bi}^2 - z_i^2} + \tilde{l}_i^2 \right), \quad 2 < j < n. \quad (15)$$

By the same token, we design stabilizing functions and adaptive laws:

$$\begin{aligned}\alpha_i &= \frac{1}{\bar{\theta}_b g_i} \left( -\hat{l}_i h_i z_i - \lambda_i z_i + \dot{\alpha}_{i-1} \right. \\ &\quad \left. - \frac{k_{bi}^2 - z_i^2}{k_{b(i-1)}^2 - z_{i-1}^2} \bar{\theta}_b g_{i-1} z_{i-1} \right), \\ \dot{\hat{l}}_i &= \frac{z_i^2 h_i}{k_{bi}^2 - z_i^2}.\end{aligned}\quad (16)$$

Finally, we obtain the stabilizing function and the adaptive law of step  $n$ :

$$\begin{aligned}\alpha_n &= \frac{1}{\bar{\theta}_b g_n} \left( -\hat{l}_n h_n z_n - \lambda_n z_n + \dot{\alpha}_{n-1} - \frac{\bar{\theta}_b g_{n-1} z_{n-1}}{k_{b(n-1)}^2 - z_{n-1}^2} \right),\end{aligned}\quad (17)$$

$$u = \alpha_n, \quad (18)$$

$$\dot{\hat{l}}_n = z_n^2 h_n. \quad (19)$$

**Theorem 6.** Considering system (3), the adaptive state feedback controller (18) and the adaptive law (19) can guarantee  $\lim_{t \rightarrow \infty} (x_1 - y_d) = 0$  and  $|x_i| \leq b$  under arbitrary switching signal and  $b$  is a positive constant.

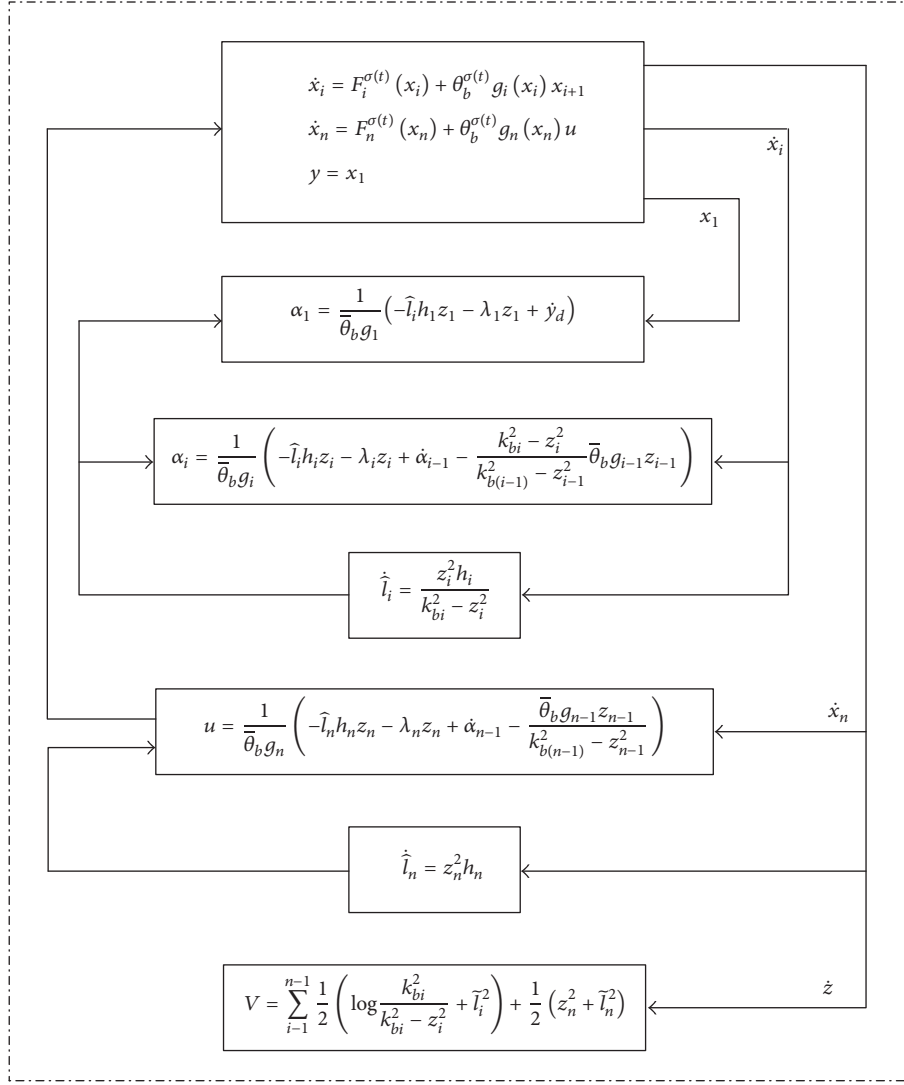


FIGURE 1: Block diagram of the adaptive control system.

*Proof.* Choose the BLF as follows:

$$V = \sum_{i=1}^{n-1} \frac{1}{2} \left( \log \frac{k_{bi}^2}{k_{bi}^2 - z_i^2} + \tilde{l}_i^2 \right) + \frac{1}{2} (z_n^2 + \tilde{l}_n^2). \quad (20)$$

According to the adaptive state feedback controller (18) and the adaptive law (19), we obtain the time derivative of the BLF:

$$\begin{aligned} \dot{V} &\leq -\sum_{i=1}^{n-1} \frac{\lambda_i z_i^2}{k_{bi}^2 - z_i^2} + \frac{z_{n-1} z_n \theta_b^{\sigma(t)} g_{n-1}}{k_{b(n-1)}^2 - z_{n-1}^2} + z_n \dot{z}_n + \tilde{l}_n \dot{\tilde{l}}_n \\ &\leq -\sum_{i=1}^n \frac{\lambda_i z_i^2}{k_{bi}^2 - z_i^2} \leq 0. \end{aligned} \quad (21)$$

It can be seen that all the signals are bounded. By Barbalat's Lemma,  $z_i$  will converge to zero. According to the nature of BLF, we can guarantee  $\lim_{t \rightarrow \infty} (x_1 - y_d) = 0$  and  $|x_i| \leq b$ , among them  $b$  being a positive constant. The output

of the system tracks the desired trajectory  $y_d$  asymptotically and certain states are constrained.

The design procedure of the proposed control scheme could be viewed from the block diagram in Figure 1.  $\square$

## 4. Simulations

In this section, we give examples and simulations to demonstrate the proposed result.

*Example 1.* Consider the following switched nonlinear system:

$$\begin{aligned} \dot{x}_1 &= [f_{11} \ f_{12} \ f_{13}] \theta_1^\sigma + \theta_b^\sigma g_1(x_1) x_2 \\ \dot{x}_2 &= [f_{21} \ f_{22} \ f_{23}] \theta_2^\sigma + \theta_b^\sigma g_2(x_2) u, \end{aligned} \quad (22)$$

$$\sigma(t) : [0, \infty) \longrightarrow \{1, 2\},$$

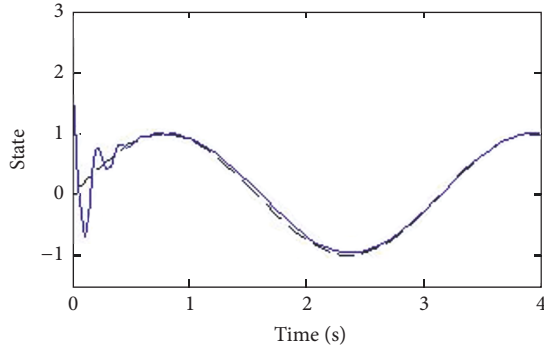


FIGURE 2: State trajectory  $x_1$  tracking converges the desired trajectory  $y_d = \sin x$ .

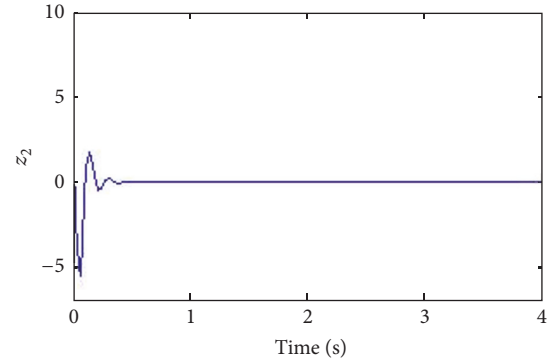


FIGURE 5: Error trajectory  $z_2$  converges to zero.

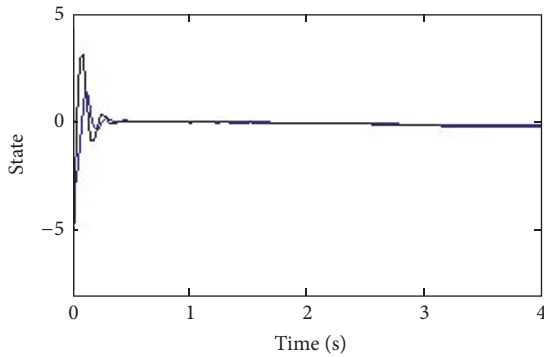


FIGURE 3: State trajectory  $x_2$  tracking converges the desired trajectory  $\alpha_1$ .

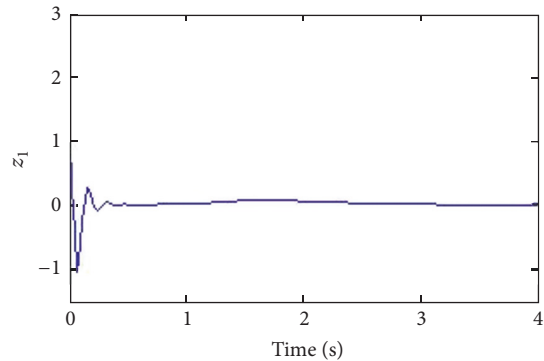


FIGURE 4: Error trajectory  $z_1$  converges to zero.

where  $[f_{11} \ f_{12} \ f_{13}] = [x_1^2 \ x_1 \ 1]$ ,  $[f_{21} \ f_{22} \ f_{23}] = [x_2^2 \ x_2 \ \cos(x_2)]$ ,  $g_1(x_1) = 2x_1^2 + 2$ ,  $g_2(x_2) = x_2 + 3$ . Parameters  $\theta_1^\sigma$ ,  $\theta_2^\sigma$ , and  $\theta_b^\sigma$  are unknown. According to previous discussions, we can find smooth functions and satisfy the conditions of Theorem 6 that

$$\begin{aligned} h_1(x_1) &= 2x_1^2 + 5, \\ h_2(x_2) &= x_2^2 + 2. \end{aligned} \quad (23)$$

The desired trajectory is  $y_d = \sin x$ , and the tracking error constraint is  $|z_1| < 2$ ,  $|z_2| < 10$ .

Figures 2 and 3 present the simulation results of the outputs of the system tracking the desired trajectory  $y_d$  asymptotically, and the tracking errors constraints are never violated. And it is evident from Figures 4 and 5 that tracking error trajectories converge to zero.

After that, we design a controller using QLF. Choose  $V_1 = (1/2)(z_1^2 + \bar{l}_1^2)$ , and we can show that  $\phi_1$  is a common virtual control for the first component of the system (22). Then, define  $z_2$  for  $x_2$  as the tracking error which  $z_2 = x_2 - \phi_1$ ,  $V_2 = \sum_{i=1}^2 (1/2)(z_i^2 + \bar{l}_i^2)$  is a common Lyapunov for the system (22). We will follow the same design procedure for the design of the controller based on the QLF method and compare the simulation results between BLF and QLF.

As it can be clearly seen from Figures 6 and 7, we know that the asymptotic tracking performance is achieved and  $x_1$  tracks the desired trajectory  $y_d$  asymptotically based on BLF method. However, when the QLF is utilized with the same parameters and design processes, the output is more volatile than the BLF one, and the convergence rate is slower than the BLF method.

**Example 2.** Considering the above system, the control objective is that the output of system  $x_1$  tracks the desired trajectory  $y_d = 0.3$ . Two different tracking error constraints are given that  $|z_1| < 0.2$ ,  $|z_1| < 0.15$ .

From Figure 8, we know that asymptotic tracking performance is achieved when the BLF is used with the initial value  $x(0) = (0.25, -1)^T$ . However, when the QLF is used with the same initial value, the state constraint is violated. The simulation results in Figures 9 and 10 show that we can know that tracking errors  $z_1$  satisfy constraints using the BLF. The errors  $z_1$  do not transgress boundary that  $|z_1| < 0.2$  and  $|z_1| < 0.15$ , and the objective is achieved.

## 5. Conclusions

In this paper, we have studied the constraint adaptive tracking control problem of switched nonlinear systems with uncertain parameters using the BLF and backstepping method. Asymptotic output tracking and states constraint have been ensured, and we guarantee that the state constraint is violated, which has been verified by the simulations. Next we will focus on studying constraint adaptive output feedback control and

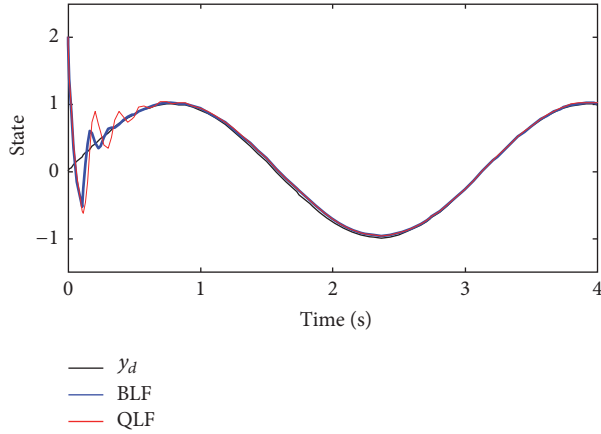


FIGURE 6: State trajectories  $x_1$  tracking converge the desired trajectory (BLF-QLF).

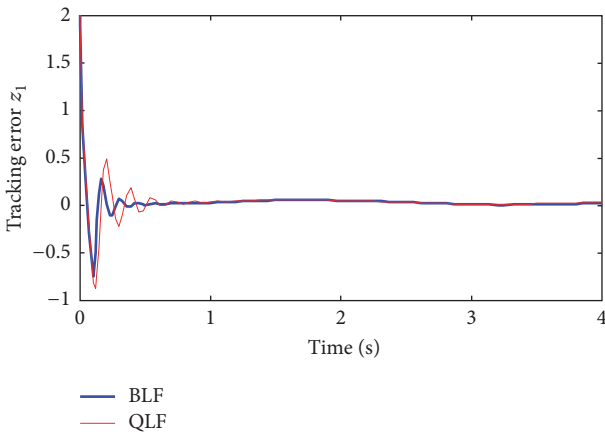


FIGURE 7: Error trajectories  $z_1$  converge to zero (BLF-QLF).

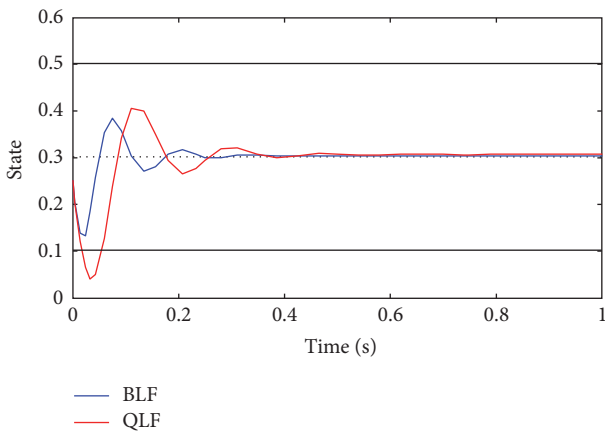


FIGURE 8: State trajectories  $x_1$  tracking converges the desired trajectory ( $|z_1| < 0.2$ ).

designing an adaptive controller and stabilization under arbitrary switching signals can be achieved by output feedback.

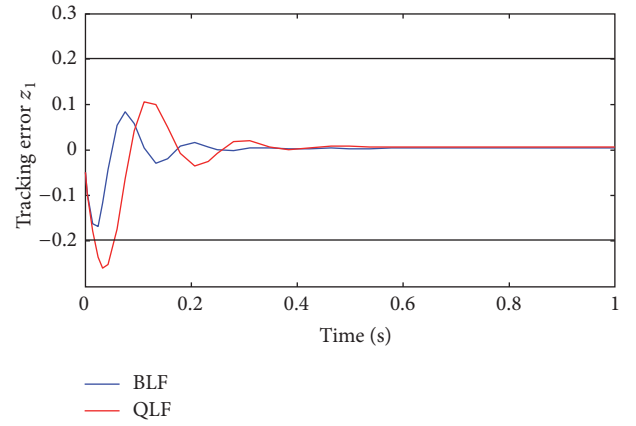


FIGURE 9: Tracking error trajectories  $z_1$  satisfies error constraint  $|z_1| < 0.2$ .

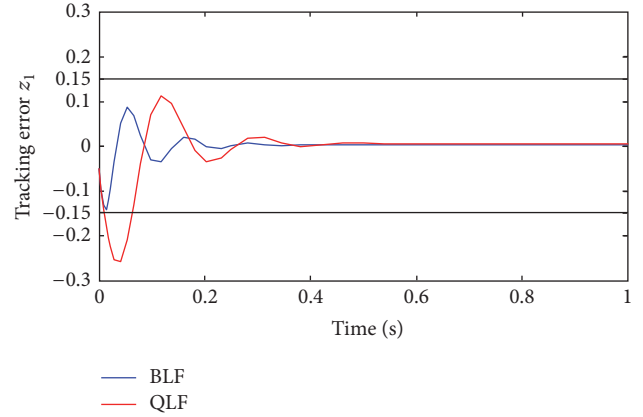


FIGURE 10: Tracking error trajectories  $z_1$  satisfies error constraint  $|z_1| < 0.15$ .

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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