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Research Article

Multiswitching Synchronization of a Driven Hyperchaotic Circuit Using Active Backstepping

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An active backstepping technique is proposed for the realization of multiswitching synchronization of periodically forced hyperchaotic Van der Pol-Duffing oscillators. The active backstepping technique is a systematic design approach with recursive procedures that skillfully optimizes the choice of Lyapunov functions and active control technique. Using the active backstepping technique, the usual master-slave synchronization scheme is extended to study the synchronization of systems with different combinations of the slave states variables with master state variables. Our numerical results confirm the effectiveness of the proposed analytical technique.

1. Introduction

Nonlinear deterministic dynamical systems exhibiting sensitive dependence to initial conditions (chaos) have been found to exist and not unusual. Different methods used to describe their existence are found in the fields of sciences (physical and natural), medicine, and engineering [1–3]. Various attributes of nonlinear dynamical systems such as chaos, bifurcation, multistability, pattern formation, control, and synchronization have been found useful or having potential applications in many disciplines. One of the most important attributes of nonlinear dynamical systems is that of synchronization of chaotic systems which has become more fascinating and has generated interest from researchers in recent time due to its applications in information processing, secure communication, chemical reactions, and modelling brain activity [4–7]. Increasing interest in the study of synchronization of chaotic systems has led to the discovery of various types of synchronization which include complete synchronization [8], lag synchronization [9], phase synchronization [10], generalized synchronization [11–13], measure synchronization [14–17], projective synchronization [18–20], anticipated synchronization [21, 22], and reduced-order synchronization [23–31]. Recently multiswitching synchronization of chaotic systems was proposed by Uçar et al. [32]. The method was successfully implemented with Lorenz system using active control technique.

Several synchronization methods have evolved to achieve stable synchronization between two or more chaotic systems. These methods include adaptive control [33], active control [34], sliding mode control [35], impulsive control [36], linear feedback control [37], backstepping control [38], open plus close loop control [39], adaptive fuzzy feedback [40], and passive control [41]. Notable among this method is the backstepping control technique which has outstanding performance in the synchronization of identical and nonidentical chaotic systems [42, 43]. It has also been identified as a reliable design technique for stabilization and tracking [44]. In recent time, backstepping technique has been applied for controlling and

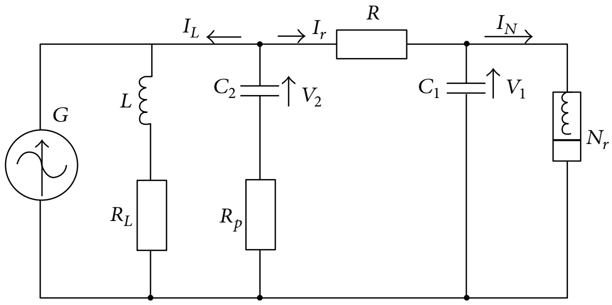

Figure 1: Unified and periodically forced Van der Pol-Duffing oscillator circuit.

tracking hyperchaotic systems [45]. In spite of the excellent performance of backstepping control technique, it has not been applied to multiswitching synchronization of chaotic system to the best of our knowledge. Motivated by the above discussion, we are reporting multiswitching synchronization of unified and periodically forced hyperchaotic Van der Pol-Duffing oscillator using active backstepping technique.

Van der Pol-Duffing oscillator is a popularly known and very significant classical model circuit that has been studied and even modified in some studies as reported by King et al. [46, 47], Fotsin et al. [48], and Fodjouong et al. [49]. The Van der Pol-Duffing oscillator in most reported cases is autonomous (unforced) whereas the periodically driven nonautonomous Van der Pol-Duffing oscillator considered in this paper displays more complex and richer dynamics as the amplitude and the frequency of the forcing signal are varied, thereby exhibiting chaos-hyperchaos transitions, and coexisting attractors as well as Hopf bifurcations in which quasiperiodic orbits are born [50]. Recently, there has been a renewed interest and several attempts to understand the dynamics of nonautonomous (forced) oscillators excited from equilibrium by different external forcing techniques. This is because deterministic influences often arise in practice as in cellular dynamics, blood circulation, and brain dynamics [51, 52].

In the present paper, we introduced the periodically forced Van der Pol-Duffing oscillator and study its synchronization. Specifically, we consider a more general form of synchronization, namely, *multiswitching synchronization*, in which a variable of a dynamical system synchronizes with another variable of the systems or another system. This type of synchronization was proposed by Uçar et al. [32]. Despite its relevance, it has received less attention. Here, we propose an active backstepping technique to realize the multiswitching synchronization. The rest of the paper is organized as follows: the basic dynamical properties of our model system are described in Section 2. In Section 3, the multiswitching synchronization of UVDP using active backstepping is discussed. Numerical simulations are provided in Section 4, while the paper is concluded in Section 5.

2. Model Description

2.1. Unified and Periodically Driven Van der Pol-Duffing Oscillator Circuit. The periodically driven Van der Pol-Duffing oscillator considered here can be modeled by the circuit shown in Figure 1, in which a dc series resistance is added to the C_2 branch of the circuit, while another dc resistance is placed parallel to it. In addition, a periodic signal generator, acting as periodic driving, is connected to the left end of the circuit as shown. By applying Kirchhoff's laws to the various branches of the circuit of Figure 1, and noting that the $i(v)$ characteristics of the nonlinear resistor (N) are approximated by the cubic polynomial $i(v) = v + av + bv^3$, $(a < 0, b > 0)$, we obtain the following set of equations:

$$\frac{dV_1}{dt} = -\frac{1}{C_1}\left(\frac{1}{R} + a\right)V_1 + \frac{V_2}{RC_1} - \frac{b}{C_1}V_1^3,$$
$$\frac{dV_2}{dt} = \frac{V_1}{RC_2} - \frac{\mu V_2}{RC_2} - \frac{i_L}{C_2} + \frac{i_G}{C_2}\sin\omega t, \quad (1)$$
$$\frac{di_L}{dt} = \frac{V_2}{L} - \frac{R_L}{L}i_L.$$

Making appropriate rescaling of (1) by setting $x = V_1\sqrt{bR}$, $y = V_2\sqrt{bR}$, $z = i_L\sqrt{bR^3}$, $\tau = t/RC_2$, $m = C_2/C_1$, $\alpha = -(1 + aR)$, $\beta = R^2C_2/L$, $\gamma = R_LRC_2/L$, $a_0 = R(\sqrt{bR})I_G$, $\Omega = \omega/RC_2$, and $\mu = (R + R_L)/R_L$, we obtain the following dimensionless equation:

$$\dot{x} = -m\left(x^3 - \alpha x - y\right),$$
$$\dot{y} = x - \mu y - z + a_0\sin\omega\tau, \quad (2)$$
$$\dot{z} = \beta y - \gamma z.$$

System (2) is a three-dimensional nonautonomous system. By letting $w = a_0\sin\Omega\tau$, the system of (2) can be rewritten as a system of four-dimensional autonomous dynamical system (including τ as a dynamical variable) as follows:

$$\dot{x} = -m\left(x^3 - \alpha x - y\right),$$
$$\dot{y} = x - \mu y - z + w,$$
$$\dot{z} = \beta y - \gamma z, \quad (3)$$
$$\dot{w} = \pm\omega\sqrt{a_0^2 - w^2},$$

where the new parameters ω and a_0 are the frequency and amplitude of the periodic driving force, respectively. Thus, our model system (2) or its equivalent system (3) has four Lyapunov exponents, allowing for hyperchaotic behaviour, that is, with the possibility of having two positive Lyapunov exponents along with one zero and one negative. Furthermore, under periodic driving, the unified Van der Pol-Duffing oscillator exhibits richer dynamical complexities, including the existence of hyperchaos-chaos transient and regime of periodic behaviour could be observed with the aid of bifurcations and Lyapunov exponents [50]. In Figure 2, we present an illustration of a typical dynamics of our model from [50]. The numerical results were performed using the standard fourth-order Runge-Kutta routine with step-size $h = 2\pi/(N\omega)$, where N is the number of iterations. The

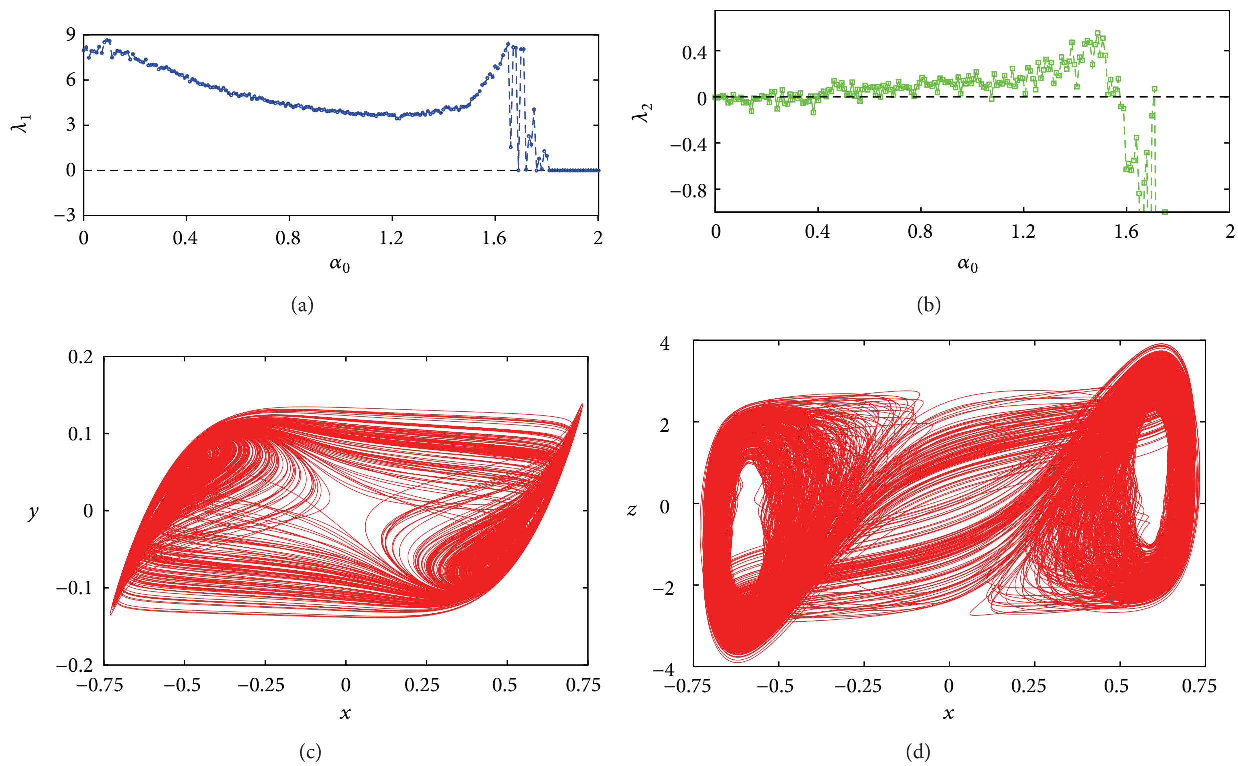

FIGURE 2: The first two Lyapunov exponents ((a) λ_1 and (b) λ_2) as functions of the driving amplitude, a_0. $\alpha = 0.35$, $\beta = 300$, $m = 100$, $\gamma = 0.2$, $\mu = 1.0$, and $\omega = 10$. (c) and (d) are phase portraits of the corresponding hyperchaotic attractors for the parameters $a_0 = 1.51$ and $\omega = 10$ for y versus x and z versus x.

complete Lyapunov exponent spectrum was computed using the Wolf et al.'s algorithm [53]. For clarity and brevity, we will present only the first two exponents, namely, λ_1 and λ_2, which determine exclusively the system's behaviours. Unless otherwise stated, the following parameters were fixed: $\alpha = 0.35$, $\beta = 300$, and $m = 100$, while the other parameters, namely, μ, γ, a_0, and ω, were varied for the different cases considered. In the absence of the forcing, we obtain the Fotsin and Woafo model when $\mu = 1.0$, and $\gamma = 0.2$ [48], whereas, when $\mu > 1.0$ and $\gamma = 0$, we have the Matouk and Agiza model [54]. In Figures 2(a) and 2(b), we see clearly that λ_1 and λ_2 are positive with increasing amplitude, a_0, confirming the existence of hyperchaos in this driving amplitude regime.

2.2. Stability and Equilibria. The changes discussed above maybe understood by examining the stability of system (3). First, we obtain the fixed points by solving the general equation $F(\dot{u}) = 0$, where u is the vector space containing x, y, and z at $a_0 = 0$. The equilibrium points are found by equating the right-hand sides of (2) to zero such that

$$\begin{aligned} \dot{x} &= -m\left(x^3 - \alpha x - y\right) = 0, \\ \dot{y} &= x - \mu y - z = 0, \\ \dot{z} &= \beta y - \gamma z = 0, \end{aligned} \tag{4}$$

from which the following fixed points were obtained:

$$\begin{aligned} E_+ &= +\left(\left(\alpha + \frac{\gamma}{\mu\gamma + \beta}\right)^{1/2}, +\gamma\chi, +\beta\chi\right), \quad \alpha > 0, \\ E_- &= -\left(\left(\alpha + \frac{\gamma}{\mu\gamma + \beta}\right)^{1/2}, -\gamma\chi, -\beta\chi\right), \quad \alpha < 0, \\ E_0 &= (0, 0, 0), \end{aligned} \tag{5}$$

where

$$\chi = \frac{\left(\gamma\left(\alpha\mu + 1\right) + \alpha\beta\right)^{1/2}}{\left(\mu\gamma + \beta\right)^{3/2}}. \tag{6}$$

The characteristic equation of the Jacobian matrix of system (5) about the equilibrium points $E = (\overline{x}, \overline{y}, \overline{z})$ is given as

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{7}$$

where

$$\begin{aligned} a_1 &= \mu + \gamma + m\left(3x^2 - \alpha\right), \\ a_2 &= m\left(3x^2 - \alpha\right)\left(\mu + \gamma\right) + \mu\gamma + \beta - m, \\ a_3 &= m\left(3x^2 - \alpha\right)\left(\mu\gamma + \beta\right) - m\gamma. \end{aligned} \tag{8}$$

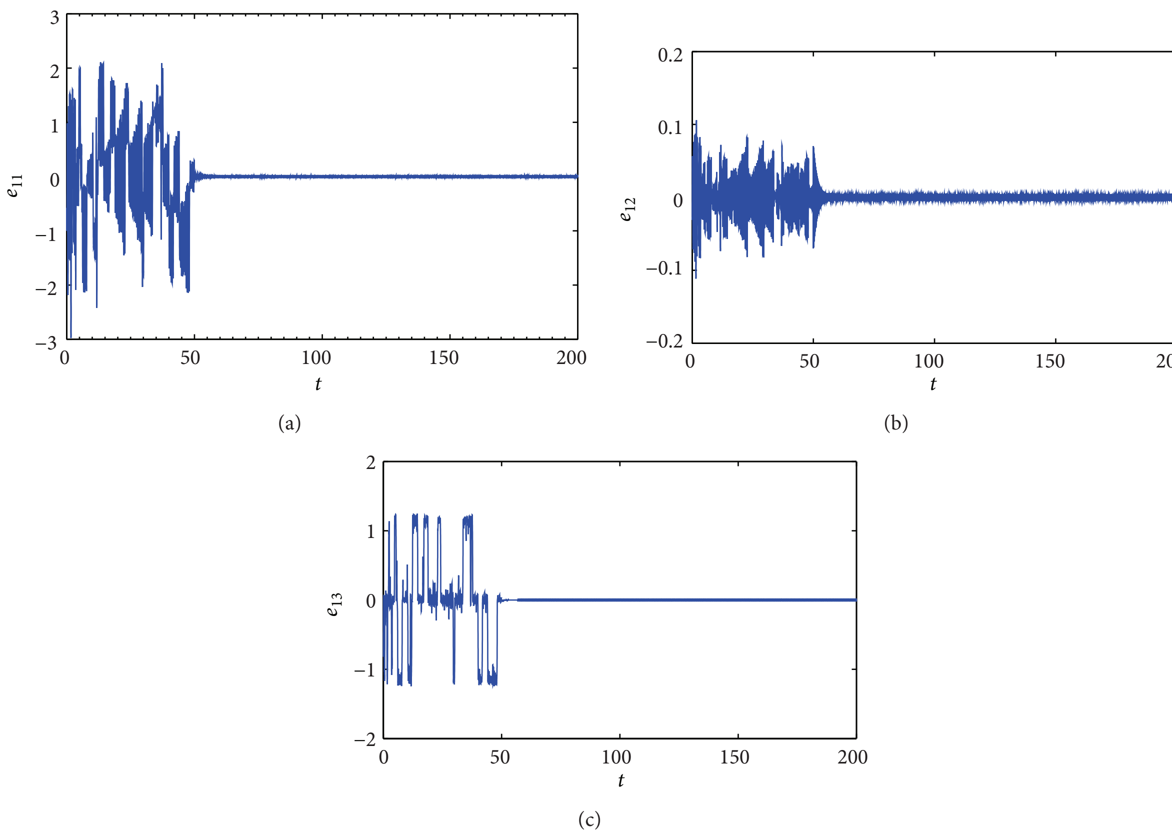

Figure 3: Error dynamics for the unified and periodically forced Van der Pol-Duffing oscillator of the system state for $N = 1$ with controller defined in (24) when the control was activated at $t = 50$ for (a) e_{11}, (b) e_{12}, and (c) e_{13}.

Before analyzing the stability properties of each points, it is important to consider the practical process of modeling system (4). It is clear that only α can take on negative or positive values. The other parameters are always positive. In order to study the stability conditions of the equilibrium point, E, we apply Routh-Hurwitz criterion [55], which states that all real eigenvalues and all real parts of complex conjugate eigenvalues are negative if and only if the following equation holds:

$$a_1 > 0, \quad a_3 > 0, \quad a_1 a_2 - a_3 > 0. \tag{9}$$

At values of $m = 100$, $\alpha = 0.35$, $\beta = 300$, $\gamma = 0.2$, and $\mu = 1.0$. For $E_0 = (0, 0, 0)$; $x = 0$, $y = 0$, $z = 0$. Values of a_1, a_2, and a_3 obtained are -33.8, 158.2, and -10527, respectively.

Remark 1. The equilibrium point E_0 of the system (5) has an unstable solution when α, m, β, μ, and γ are all positive real numbers because $a_1 < 0$, $a_2 > 0$, and $a_3 < 0$.

Stability Conditions of $E_\pm$. When $\alpha > 0$, the two equilibrium points E_+ and E_- appear. The two equilibrium points are symmetric. That is, $x = \pm 0.592170771$, $y = \pm 0.000394518$, $z = \pm 0.5917764$. $E_\pm = (\pm 0.5921707, \pm 0.0003945, \pm 0.5917764)$. Values of a_1, a_2, and a_3 obtained are 71.3998666, 284.4398399, and 21053.99995, respectively.

Remark 2 ($a_1 > 0, a_2 > 0, a_3 > 0$, and $a_1 a_2 > 0$). According to Routh-Hurwitz criteria, the equilibria E_+ and E_- are stable.

3. Multiswitching Synchronization via Backstepping

Here, we develop multi-switching synchronization procedure based on active-backstepping and applied to the periodically forced Van der Pol-Duffing oscillator due to its richer dynamical complexities and relevance in the area of chaotic systems that can be physically built and analyzed. It is convenient to transform the driven Van der Pol-Duffing oscillator system (2) by redefining the variables as follows: $x = z$, $y = y$, and $z = x$ such that we have the following master system:

$$\begin{aligned} \dot{x}_1 &= \beta y_1 - \gamma x_1, \\ \dot{y}_1 &= z_1 - \mu y_1 - x_1 + a_0 \sin \omega \tau, \\ \dot{z}_1 &= -m\left(z_1^3 - \alpha z_1 - y_1\right) \end{aligned} \tag{10}$$

and the corresponding slave system

$$\begin{aligned} \dot{x}_2 &= \beta y_2 - \gamma x_2 + U_{11}(t), \\ \dot{y}_2 &= z_2 - \mu y_2 - x_2 + a_0 \sin \omega \tau + U_{12}(t), \\ \dot{z}_2 &= -m\left(z_2^3 - \alpha z_2 - y_2\right) + U_{13}(t), \end{aligned} \tag{11}$$

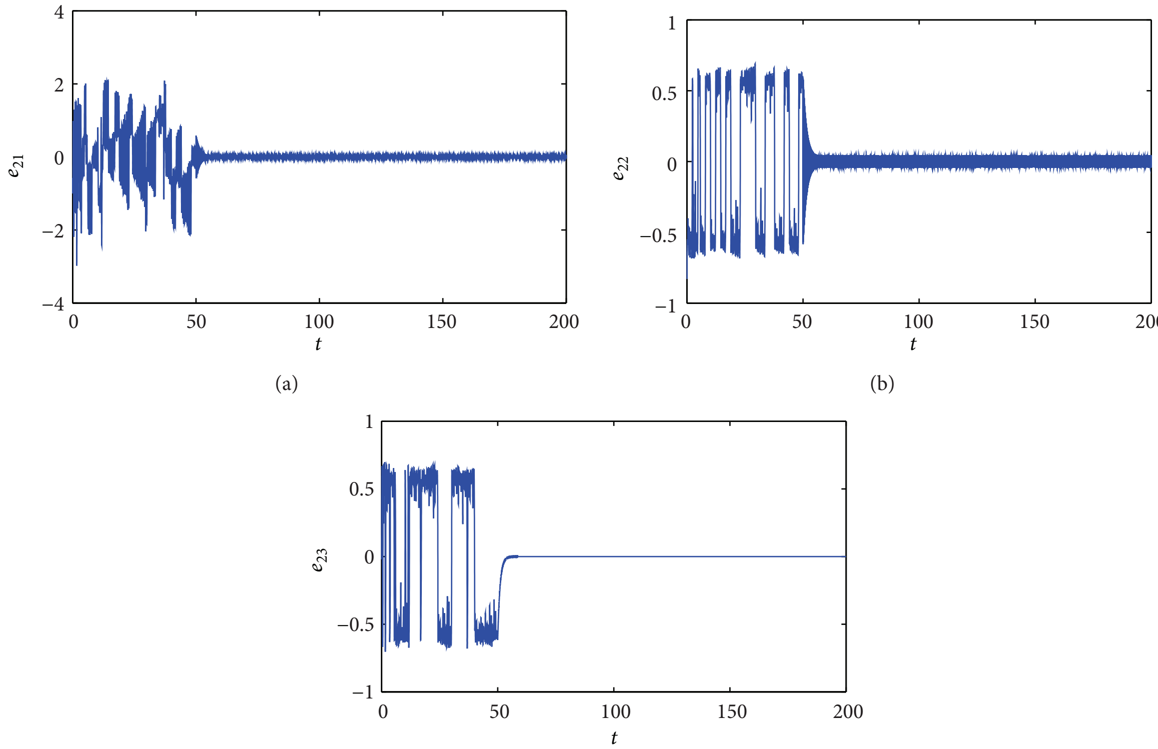

FIGURE 4: Error dynamics for the unified and periodically forced Van der Pol-Duffing oscillator of the system state for $N = 2$ with controller defined in (30) when the control was activated at $t = 50$ for (a) e_{21}, (b) e_{22}, and (c) e_{23}.

where $U_{11}(t)$, $U_{12}(t)$, and $U_{13}(t)$ are the set of nonlinear controller for the first switch ($N = 1$). We define the error signals for this switch as $e_{11} = x_2 - x_1$, $e_{12} = y_2 - y_1$, and $e_{13} = z_2 - z_1$. By considering the time derivative of the error signals together with (10) and (11), we determine the controller that would cause the three terms of error signals to asymptotically approach zero as $t \to \infty$. The controller was found from the following closed loop dynamics:

$$\begin{aligned} \dot{e}_{11} &= \beta e_{12} - \gamma e_{11} + U_{11}(t), \\ \dot{e}_{12} &= e_{13} - \mu e_{12} - e_{11} + U_{12}(t), \\ \dot{e}_{13} &= -m\left[z_2^3 - z_1^3\right] + m\alpha e_{13} + m e_{12} + U_{13}(t). \end{aligned} \tag{12}$$

With error dynamics represented by (12), the synchronization problem is equivalent to stabilizing the system at one of its equilibria. Since the equilibria E_+ and E_- are stable and symmetric, so if appropriate $U_{11}(t)$, $U_{12}(t)$, and $U_{13}(t)$ are chosen such that the equilibrium E_+ or E_- is stable and unchanged, then asymptotic stabilization would be realized leading to globally stable synchronization of systems (10) and (11). To start with, let $q_1 = e_{11}$, which implies that $\dot{q}_1 = \dot{e}_{11}$. Then, choose V_1 to be a Lyapunov function for the $\dot{q}_1$ subsystem given as

$$V_1 = \frac{1}{2} q_1^2, \tag{13}$$

and then

$$\dot{V}_1 = q_1 \dot{q}_1 = -\gamma q_1^2 + q_1\left(\beta e_{12} + U_{11}\right). \tag{14}$$

If $e_{12} = k_1$ is a virtual control and a function of e_{11}, making $U_{11} = 0$ and $k_1 = 0$, thus we obtain

$$\dot{V}_1 = -\gamma q_1^2, \tag{15}$$

which is negative definite, and this means that the subsystem, e_{11}, is fully stabilized. To stabilize the second subsystem in (12), let the error state between e_{12} and k_1 be q_2

$$q_2 = e_{12} - k_1 = e_{12}, \quad (k_1 = 0). \tag{16}$$

Introduce the second Lyapunov function V_2 given as

$$V_2 = V_1 + \frac{1}{2} q_2^2. \tag{17}$$

Taking time derivative of (16), we have

$$\begin{aligned} \dot{V}_2 = \dot{V}_1 + q_2 \dot{q}_2 &= -\gamma q_1^2 - \mu q_2^2 \\ &+ q_2\left(\beta q_1 - q_1 + e_{13} + U_{12}\right). \end{aligned} \tag{18}$$

Again, if $e_{13} = k_2 = 0$ is a virtual control and $U_{12} = q_1(1 - \beta)$, then

$$\dot{V}_2 = -\gamma q_1^2 - \mu q_2^2. \tag{19}$$

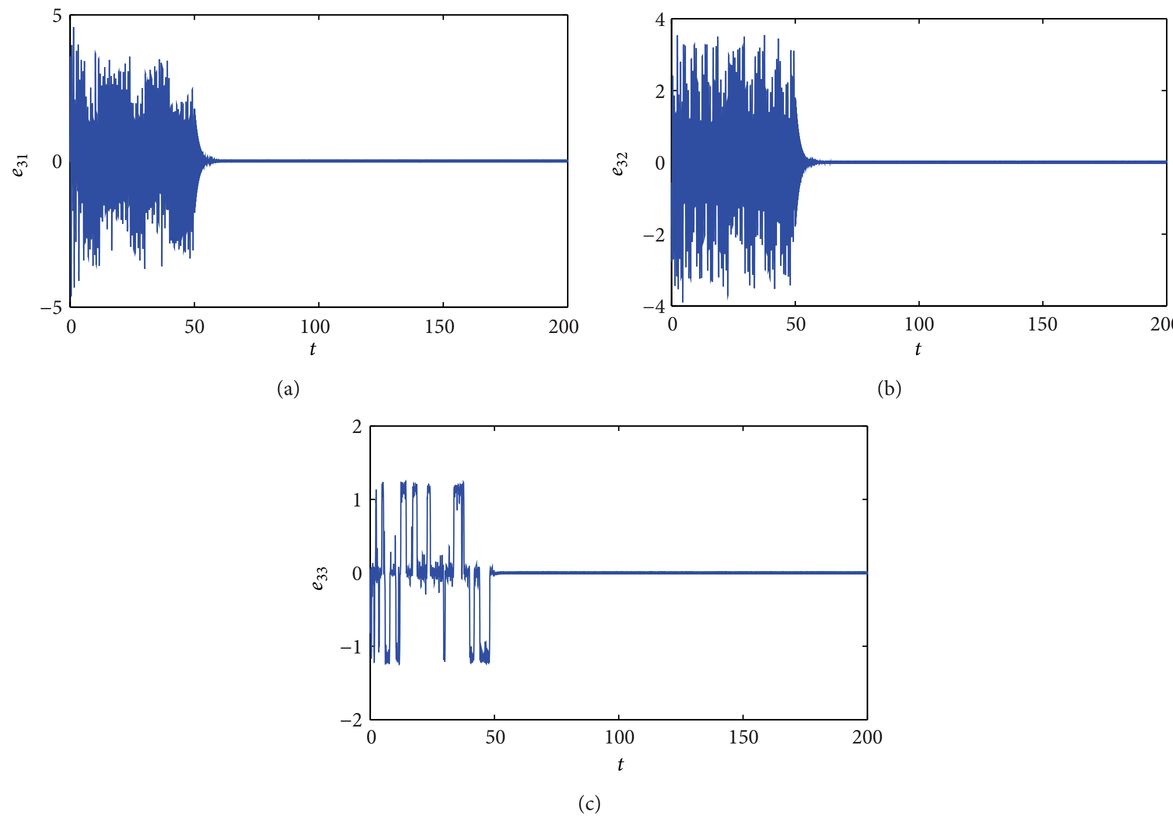

FIGURE 5: Error dynamics for the unified and periodically forced Van der Pol-Duffing oscillator of the system state for $N = 3$ with controller defined in (31) when the control was activated at $t = 50$ for (a) e_{31}, (b) e_{32}, and (c) e_{33}.

Equation (18) is negative definite, meaning that the subsystem, e_{12}, is also fully stabilized. To stabilize the third subsystem in (12), let the error state between e_{13} and k_2 be q_3

$$q_3 = e_{13} - k_2 = e_{13}, \quad (k_2 = 0). \tag{20}$$

We choose the third Lyapunov function and its time derivative expressed as

$$\begin{gathered} V_3 = V_2 + \frac{1}{2} q_3^2, \\ \dot{V}_3 = \dot{V}_2 + q_3 \dot{q}_3, \\ \dot{V}_3 = -\gamma q_1^2 - \mu q_2^2 \\ + q_3 \left(q_2 - m \left(z_2^3 - z_1^3 \right) + m\alpha q_3 + m q_2 + U_{13}(t) \right). \end{gathered} \tag{21}$$

If the controller $U_{13}(t)$ is chosen such that

$$U_{13}(t) = m \left(z_2^3 - z_1^3 \right) - q_2 (1 + m) - q_3 (1 + m\alpha), \tag{22}$$

then

$$\dot{V}_3 = -\gamma q_1^2 - \mu q_2^2 - q_3^2 \tag{23}$$

is negative definite, so that the two systems (10) and (11) are globally asymptotically synchronized. Summarily, the virtual controllers for the system state $N = 1$ are stated as follows:

$$\begin{gathered} U_{11} = 0, \\ U_{12} = q_1 (1 - \beta), \\ U_{13}(t) = m \left(z_2^3 - z_1^3 \right) - q_2 (1 + m) - q_3 (1 + m\alpha). \end{gathered} \tag{24}$$

More switching states that were investigated will be presented with the parameter symbol $N = 2, 3, 4, 5$, and 6. The error signals for these various switching states can be defined as follows:

$$N = 2; \quad e_{21} = x_2 - x_1; \quad e_{22} = y_2 - z_1; \quad e_{23} = z_2 - y_1, \tag{25}$$

$$N = 3; \quad e_{31} = x_2 - y_1; \quad e_{32} = y_2 - x_1; \quad e_{33} = z_2 - z_1, \tag{26}$$

$$N = 4; \quad e_{41} = x_2 - y_1; \quad e_{42} = y_2 - z_1; \quad e_{43} = z_2 - x_1, \tag{27}$$

$$N = 5; \quad e_{51} = x_2 - z_1; \quad e_{52} = y_2 - x_1; \quad e_{53} = z_2 - y_1, \tag{28}$$

$$N = 6; \quad e_{61} = x_2 - z_1; \quad e_{62} = y_2 - y_1; \quad e_{63} = z_2 - x_1. \tag{29}$$

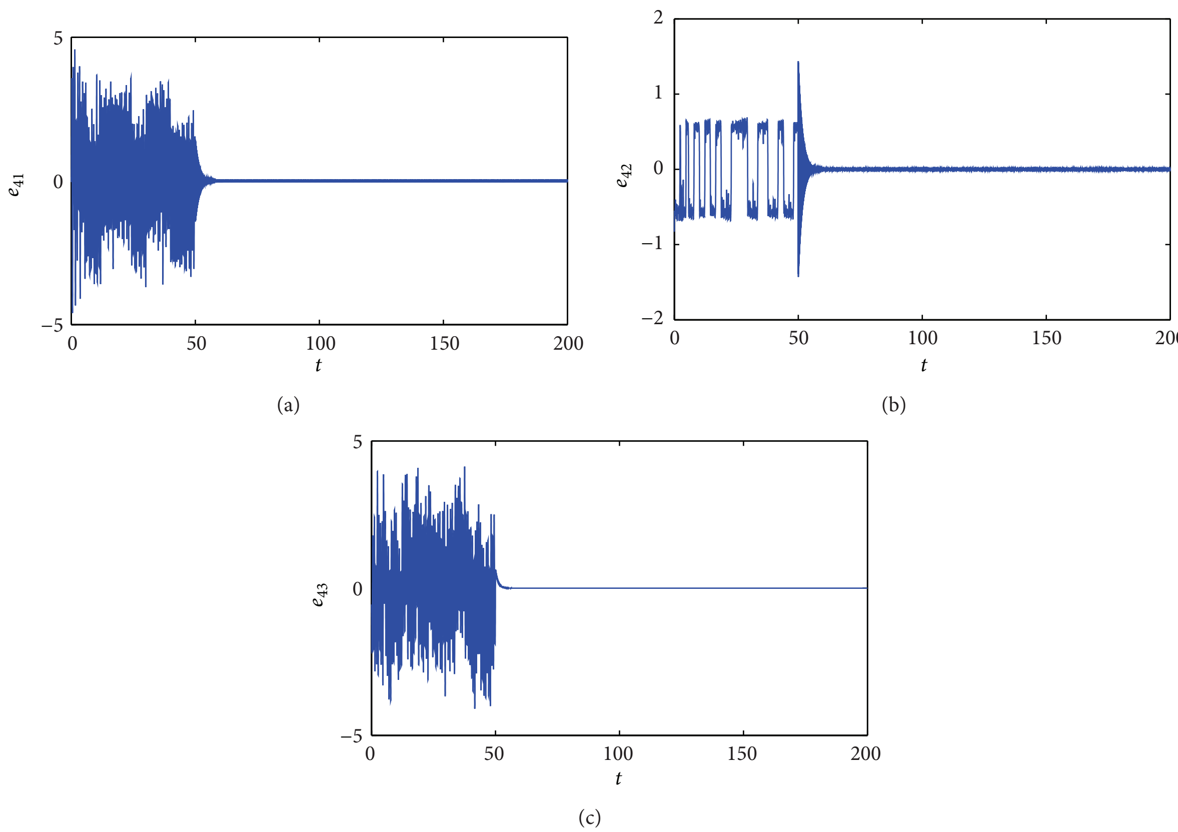

FIGURE 6: Error dynamics for the unified and periodically forced Van der Pol-Duffing oscillator of the system state for $N = 4$ with controller defined in (32) when the control was activated at $t = 50$ for (a) e_{41}, (b) e_{42}, and (c) e_{43}.

Following the same procedure that is itemized from (13) to (24), the controller signals $U_{N1}(t)$, $U_{N2}(t)$, and $U_{N3}(t)$ for $N = 2, 3, 4, 5$, and 6 switching, designed to cause the respective error dynamics to approach zero as time approaches infinity, are as follows:

$$N = 2 \Longrightarrow \begin{cases} U_{21}(t) = \beta(y_1 - z_1), \\ U_{22}(t) = q_1(1-\beta) - y_1(1-m) - mz_1^3 \\ \qquad + z_1(m\alpha + \mu) + x_1 - a_0 \sin \omega t, \\ U_{23}(t) = -q_2(1+m) - q_3(1+m\alpha) \\ \qquad + mz_2^3 - y_1(m\alpha + \mu) - z_1(m-1) \\ \qquad - x_1 + a_0 \sin \omega t, \end{cases} \tag{30}$$

$$N = 3 \Longrightarrow \begin{cases} U_{31}(t) = -x_1(\beta+1) - y_1(\mu - \gamma) + z_1 \\ \qquad + a_0 \sin \omega t, \\ U_{32}(t) = q_1(1-\beta) - x_1(\gamma - \mu) \\ \qquad + y_1(1+\beta) - z_1 - a_0 \sin \omega t, \\ U_{33}(t) = -q_3(1+m\alpha) - q_2(1+m) \\ \qquad + m(z_2^3 - z_1^3) - m(x_1 - y_1), \end{cases} \tag{31}$$

$$N = 4 \Longrightarrow \begin{cases} U_{41}(t) = -z_1(\beta-1) - y_1(\mu-\gamma) - x_1 \\ \qquad + a_0 \sin \omega t, \\ U_{42}(t) = q_1(1-\beta) - mz_1^3 + z_1(m\alpha + \mu) \\ \qquad + y_1(m+1) - x_1 - a_0 \sin \omega t, \\ N_{43}(t) = -q_3(1+m\alpha) - q_2(1+m) + mz_2^3 \\ \qquad - mz_1 + \beta y_1 - x_1(m\alpha + \gamma), \end{cases} \tag{32}$$

$$N = 5 \Longrightarrow \begin{cases} U_{51}(t) = -mz_1^3 + z_1(m\alpha + \gamma) + my_1 - \beta x_1, \\ U_{52}(t) = q_1(1-\beta) + z_1 - y_1(1-\beta) \\ \qquad - x_1(\gamma - \mu) - a_0 \sin \omega t, \\ U_{53}(t) = -q_3(1+m\alpha) - q_2(1+m) + mz_2^3 \\ \qquad + z_1 - y_1(m\alpha - \mu) - x_1(m-1) \\ \qquad - a_0 \sin \omega t, \end{cases} \tag{33}$$

$$N = 6 \Longrightarrow \begin{cases} U_{61}(t) = -mz_1^3 + z_1(m\alpha + \gamma) - y_1(\beta - m), \\ U_{62}(t) = q_1(1-\beta) + 2(z_1 - x_1), \\ U_{63}(t) = -q_3(1+m\alpha) - q_2(1+m) \\ \qquad + mz_2^3 - y_1(m-\beta) - x_1(\gamma + m\alpha). \end{cases} \tag{34}$$

4. Numerical Simulations

Numerical solutions are now presented to verify the effectiveness of controllers (24) and (30)–(34). In all six cases presented, the periodically driven oscillator parameters selected remain constant at $m = 100$, $\beta = 300$, $\alpha = 0.35$, $\gamma = 0.2$, $\mu = 1.0$, $a_0 = 1.51$, and $\omega = 10$. This step is very important such that the hyperchaotic state obtained in [50] and shown in Figure 2 is retained and also to disable the multistability property of the system. The synchronization of the slave (13) with the master system (12) for different cases represented by $N = 1, 2, 3, 4, 5$, and 6 is presented in Figures 3, 4, 5, 6, 7, and 8. Here, we find full/complete synchronization taking place

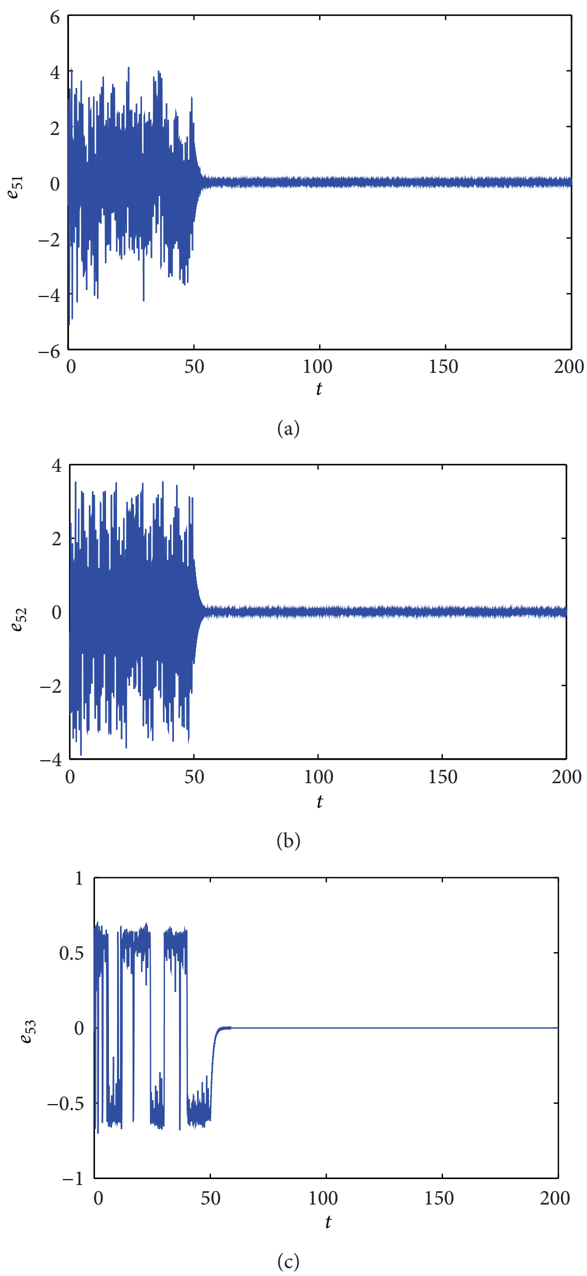

FIGURE 7: Error dynamics for the unified and periodically forced Van der Pol-Duffing oscillator of the system state for $N = 5$ with controller defined in (33) when the control was activated at $t = 50$ for (a) e_{51}, (b) e_{52}, and (c) e_{53}.

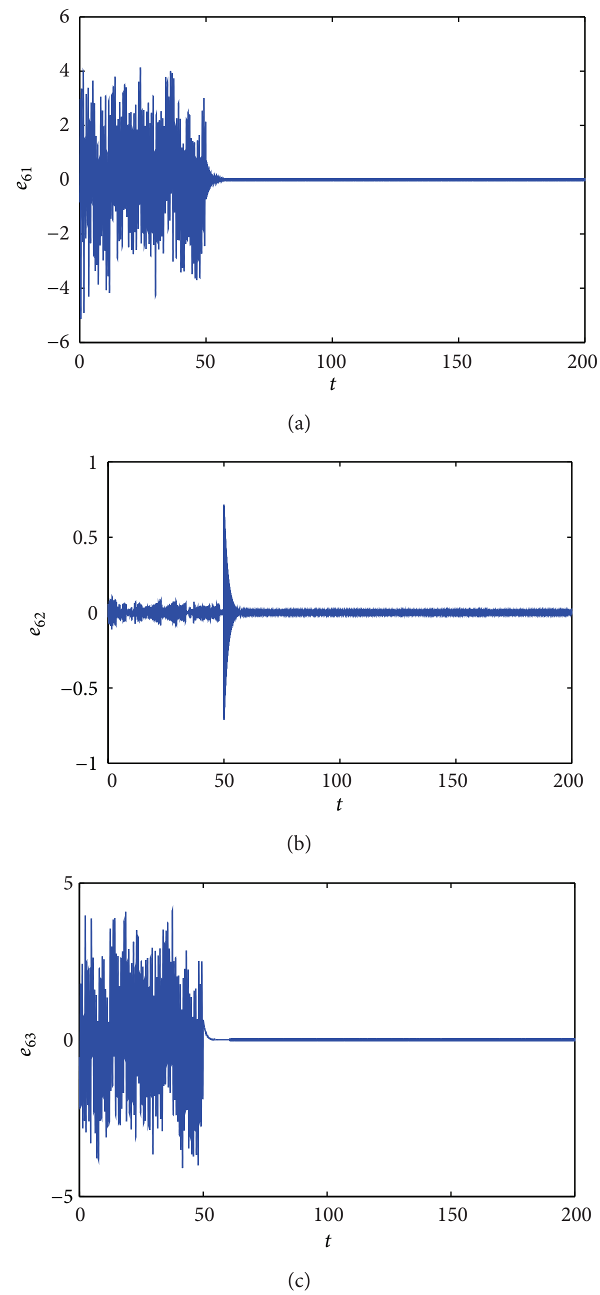

FIGURE 8: Error dynamics for the unified and periodically forced Van der Pol-Duffing oscillator of the system state for $N = 6$ with controller defined in (34) when the control was activated at $t = 50$ for (a) e_{61}, (b) e_{62}, and (c) e_{63}.

when each of the controllers was activated at $t \geq 50$, where the unified and periodically driven Van der Pol-Duffing oscillator was found to converge to zero as time tends to infinity which signifies that the multiswitching synchronization between systems (12) and (13) has been achieved. We remark, however, that quasi-synchronized (partial synchronization) state may also be achieved. Here, the oscillators suffer from achieving full synchronization but settle for a rather realistic form of complete synchronization wherein the state in which the defining limit of synchronization is bounded within a definite small region around zero. That is, $\lim_{t \to \infty} \|e_{ij}\| = \epsilon \neq 0$. We find this phenomenon when for N_1, the switching is between y_1 and y_2 as well as, for N_3, when, the switching is between y_2 and z_2, denoted as e_{12} and e_{22} in Figures 3(b) and 4(b), respectively. This kind of synchronization phenomena are intriguing in nature as well as in practical situations and have been previously reported [56–59].

5. Conclusion

In this paper, we have presented in brief the dynamics of a unified and periodically driven Van der Pol-Duffing oscillator circuit and have shown that the model admits hyperchaotic behaviour when the amplitude of the driving force is increased with a good choice of forcing frequency. Using our model circuit, an approach for realizing multiswitching synchronization was proposed based on active backstepping technique. The numerical results obtained confirmed the effectiveness of the proposed analytical method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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