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Research Article

Necessary and Sufficient Conditions of Oscillation in First Order Neutral Delay Differential Equations

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We are concerned with oscillation of the first order neutral delay differential equation $[x(t) - px(t-\tau)]' + qx(t-\sigma) = 0$ with constant coefficients, and we obtain some necessary and sufficient conditions of oscillation for all the solutions in respective cases $0 < p < 1$ and $p > 1$.

1. Introduction

Delay differential equations (DDEs) arose widely in many fields, like oscillation theory [1–9], stability theory [10–12], dynamical behavior of delayed network systems [13–15], and so on. Theoretical studies on oscillation of solutions for DDEs have fundamental significance (see [16, 17]). For this reason, DDEs have been attracting great interest of many mathematicians during the last few decades.

In this paper, we consider a class of neutral DDEs

$$[x(t) - px(t-\tau)]' + qx(t-\sigma) = 0, \quad t \geqslant t_0, \tag{1}$$

where t_0 is a positive number and p, q, τ, and σ are positive constants. Generally, a solution of (1) is called oscillatory if it is neither eventually positive nor eventually negative. Otherwise, it is nonoscillatory. It can be seen in the literature that the oscillation theory regarding solutions of (1) has been extensively developed in the recent years.

In [18], Zhang came to the following conclusion.

Theorem I. *Assume that $p \in (0,1)$ and $q\sigma e > 1 - p$; then all solutions of (1) are oscillatory.*

This result in Theorem I improves the corresponding result in [19]. Afterward, many authors have been devoted to studying this problem and have obtained many better results. For details, Gopalsamy and Zhang [20] obtained the improved result shown in Theorem II.

Theorem II. *If $p \in (0,1)$ and $q\sigma e > 1 - p[1 + q\tau/(1-p)]$, then all solutions of (1) are oscillatory.*

Further, Zhou and Yu [21] proved the following theorem.

Theorem III. *Suppose that $p \in (0,1)$ and $q\sigma e > 1 - p[1 + q\tau/(1-p) + (q\tau)^2/2(1-p)^2]$; then all solutions of (1) are oscillatory.*

Continuing to improve the research work, Xiao and Li [22] obtained the following.

Theorem IV. *Let $p \in (0,1)$ and $q\sigma e > 1 - pe^{q\tau/(1-p)}$; then all solutions of (1) are oscillatory.*

Finally, Lin [23] obtained the result shown in Theorem V.

Theorem V. *Assume that $p \in (0,1)$ and $q\sigma e > 1 - pe^{q\tau/(1-p-q\sigma)}$; then all solutions of (1) are oscillatory.*

However, all the conclusions mentioned above are limited to sufficient conditions in the case $0 < p < 1$. The aim of this paper is to establish systematically the necessary and sufficient conditions of oscillation for all solutions of (1) for the cases $0 < p < 1$ and $p > 1$.

2. Main Results

It is well known [24] that all solutions of (1) are oscillatory if and only if the characteristic equation of (1)

$$f(\lambda) \equiv \lambda - p\lambda e^{-\lambda\tau} + qe^{-\lambda\sigma} = 0 \quad (2)$$

has no real roots.

Theorem 1. *Assume that $p \in (0,1)$ and let*

$$\varphi(\mu) := q(\sigma\mu - 1) + p\tau\mu^2 e^{(\tau-\sigma)\mu}, \quad (3)$$

$$h(\mu) := qe^{\mu\sigma}\left[(\tau-\sigma)\mu + 1\right] - \tau\mu^2. \quad (4)$$

Then all solutions of (1) *are oscillatory if and only if*

$$h(\theta) = qe^{\theta\sigma}\left[(\tau-\sigma)\theta + 1\right] - \tau\theta^2 > 0, \quad (5)$$

where θ is a unique zero of $\varphi(\mu)$ in $(0, 1/\sigma)$.

Proof. It is easy to see that, for $\lambda \geqslant 0$, we have

$$f(\lambda) = \lambda\left(1 - pe^{-\lambda\tau}\right) + qe^{-\lambda\sigma} \geq qe^{-\lambda\sigma} > 0. \quad (6)$$

Thus any real root of (2) must be negative.

Next, let

$$g(\mu) = \frac{q}{\mu}e^{\mu\sigma} + pe^{\mu\tau} - 1 = 0. \quad (7)$$

We consider the monotonicity of the function $g(\mu) := f(-\mu)/\mu$. Differentiation yields

$$g'(\mu) = \frac{e^{\mu\sigma}\varphi(\mu)}{\mu^2}, \quad (8)$$

where $\varphi(\mu)$ satisfies the following properties:

(1) $\varphi(\mu) > 0$ for $\mu \in (1/\sigma, +\infty)$;

(2) $\varphi(\mu)$ is strictly increasing on $(0, 1/\sigma)$ since the function $\mu^2 e^{(\tau-\sigma)\mu}$ is strictly increasing on $(0, 1/\sigma)$.

In addition,

$$\varphi(0) = -q < 0, \qquad \varphi\left(\frac{1}{\sigma}\right) = p\tau\frac{1}{\sigma^2}e^{(\tau-\sigma)/\sigma} > 0. \quad (9)$$

Thus, we get that function $\varphi(\mu)$ has a unique zero θ in $(0, 1/\sigma)$. Hence $g'(\mu) < 0$ for $\mu \in (0, \theta)$ and $g'(\mu) > 0$ for $\mu \in (\theta, +\infty)$, which imply that $g(\mu)$ is decreasing on $(0, \theta)$ and increasing on $(\theta, +\infty)$. Therefore, $g(\mu) > 0$ for $\mu \in (0, +\infty)$ if and only if (7) has no real roots in $\mu \in (0, 1/\sigma)$. It is easy to see that $g(\theta)$ is the minimum value of $g(\mu)$ in $(0, 1/\sigma)$. Consequently, $g(\mu) = 0$ has no real roots in $(0, 1/\sigma)$ if and only if $g(\theta) > 0$. Since

$$g(\theta) = \frac{q}{\theta}e^{\theta\sigma} + pe^{\theta\tau} - 1 = \frac{h(\theta)}{\tau\theta^2}, \quad (10)$$

we obtain the result immediately. □

From Theorem 1, we obtain immediately the following.

Corollary 2. *If $p \in (0,1)$ and $\tau = \sigma$, then all solutions of* (1) *are oscillatory if and only if $qe^{\theta\sigma} > \sigma\theta^2$ holds, where $\theta = (\sqrt{q\sigma(q\sigma + 4p)} - q\sigma)/2p\sigma$.*

Theorem 3. *Suppose that $p \in (0,1)$; then all solutions of* (1) *are oscillatory if and only if one of the following conditions holds:*

(H_1) $q\sigma e \geqslant 1$;

(H_2) $\overline{\theta} > \theta$,

where θ and $\overline{\theta}$ are the unique zeros of $\varphi(\mu)$ and $h(\mu)$ (see (3) *and* (4)*) in $(0, 1/\sigma)$, respectively.*

Proof. Let $y(\mu) = h(\mu)/\mu^2 = qe^{\mu\sigma}((\tau-\sigma)/\mu + 1/\mu^2) - \tau$; then

$$y'(\mu) = \frac{qe^{\mu\sigma}z(\mu)}{\mu^3}, \quad (11)$$

where $z(\mu) = (\tau-\sigma)\sigma\mu^2 + (2\sigma - \tau)\mu - 2$, which satisfies

$$z(0) = -2 < 0, \qquad z\left(\frac{1}{\sigma}\right) = -1 < 0. \quad (12)$$

If $\tau \geqslant \sigma$, we get obviously that $z(\mu) < 0$ for all $\mu \in (0, 1/\sigma]$. If $\tau < \sigma$, we also get $z(\mu) < 0$ for all $\mu \in (0, 1/\sigma]$ since $z'(1/\sigma) = \tau > 0$. Thus, $z(\mu) < 0$ for all $\mu \in (0, 1/\sigma]$. From this and (11) we get that $y'(\mu) < 0$ for all $\mu \in (0, 1/\sigma]$. Consequently, $y(\mu)$ is strictly decreasing on $(0, 1/\sigma]$. Further,

$$\lim_{\mu\to 0^+} y(\mu) = +\infty, \qquad y\left(\frac{1}{\sigma}\right) = (qe\sigma - 1)\tau. \quad (13)$$

Therefore, if $q\sigma e \geqslant 1$, we have $y(\theta) > 0$. Hence $h(\theta) > 0$. If $q\sigma e < 1$, we have $y(1/\sigma) < 0$. Hence, it is easy to find that both functions $y(\mu)$ and $h(\mu)$ have an equal and unique zero $\overline{\theta} \in (0, 1/\sigma)$. Consequently, $h(\theta) > 0$ is equivalent to $\overline{\theta} > \theta$. □

From Theorem 1, all solutions of (1) are oscillatory if and only if one of (H_1) or (H_2) holds.

Theorem 4. *Assume that $p \in (0,1)$; then all solutions of* (1) *are oscillatory if one of the following conditions holds:*

(H_1) $q/\theta + q\sigma \geqslant 1 - p$;

(H_2) $q\sigma e \geqslant 1 - pe^{q\tau/(1-p-q\sigma)}$,

where θ is a unique zero of $\varphi(\mu)$ in $(0, 1/\sigma)$.

Proof. If $q/\theta + q\sigma \geqslant 1 - p$, we have that

$$\begin{aligned} g(\mu) &= \frac{q}{\mu}e^{\mu\sigma} + pe^{\mu\tau} - 1 > \frac{q}{\mu}(1 + \mu\sigma) + p - 1 \\ &= \frac{q}{\mu} + q\sigma + p - 1. \end{aligned} \quad (14)$$

From the proof of Theorem 1, all solutions of (1) are oscillatory. □

If $q\sigma e \geqslant 1 - pe^{q\tau/(1-p-q\sigma)}$, we suppose furthermore that $q/\theta + q\sigma < 1 - p$ (otherwise, all solutions of (1) are oscillatory by the above conclusion); that is, $\theta > q/(1-p-q\sigma)$. Since $q\sigma e$ is a minimum value of the function $(q/\mu)e^{\mu\sigma}$ at $\mu = 1/\sigma$, we have that

$$g(\theta) = \frac{q}{\theta}e^{\theta\sigma} + pe^{\theta\tau} - 1 > q\sigma e + pe^{q\tau/(1-p-q\sigma)} - 1 \geqslant 0, \tag{15}$$

and the result follows.

So far, for $p \in (0,1)$ we have discussed the necessary and sufficient conditions of oscillation for all solutions of (1). Our results have perfected the results in [23] (see Theorem 4). Next, we will discuss the behavior of oscillation of solutions of (1) in the case $p > 1$.

Lemma 5. *Let $p > 1$; then all solutions of* (1) *are oscillatory if and only if the equation*

$$g(\mu) = \frac{q}{\mu}e^{\mu\sigma} + pe^{\mu\tau} - 1 = 0 \tag{16}$$

has no real roots in $(-\ln p/\tau, 0)$.

Proof. By (14), we know that $g(\mu) > 0$ for $\mu \in (0,\infty)$. It is not difficult to see that $e^{\mu\sigma}/\mu$ is strictly decreasing on $(-\infty,0)$ while $e^{\mu\tau}$ is strictly increasing on $(-\infty,0)$. Notice that $pe^{\mu\tau} - 1 = 0$ at $\mu = -\ln p/\tau$; we find that

$$g(\mu) < 0 \quad \text{for } u \in \left(-\infty, \frac{-\ln p}{\tau}\right]. \tag{17}$$

Hence, $f(\lambda)$ has no real roots which is equivalent to $g(\mu)$ that has no real roots in $(-\ln p/\tau, 0)$. □

Theorem 6. *Suppose that $p > 1$ and $\tau = \sigma$; then all solutions of* (1) *are oscillatory if and only if*

$$qe^{\theta\sigma} < \sigma\theta^2, \tag{18}$$

where $\theta = (-\sqrt{q\sigma(q\sigma + 4p)} - q\sigma)/2p\sigma$.

Proof. It is similar to the proof of Theorem 1; $g(\theta)$ is the maximum value of $g(\mu)$ for $\mu \in (-\infty, 0)$. This and Lemma 5 imply the result. □

Theorem 7. *Assume that $p > 1$ and $\tau < \sigma$; then all solutions of* (1) *are oscillatory if and only if*

$$h(\theta) = qe^{\theta\sigma}\left[(\tau-\sigma)\theta + 1\right] - \tau\theta^2 < 0, \tag{19}$$

where θ is a unique zero of (3) *in* $(-\infty, 0)$.

Proof. Firstly, we prove that $\varphi(\mu)$ has a unique zero θ in $(-\infty,0)$. In fact,

$$\varphi'(\mu) = p\tau e^{(\tau-\sigma)\mu}\left[(\tau-\sigma)\mu^2 + 2\mu\right] + q\sigma. \tag{20}$$

It is easy to verify that $\varphi'(\mu)$ is strictly increasing on $(-\infty,0)$. In addition,

$$\varphi'(0) = q\sigma > 0, \qquad \varphi'(\mu) \to -\infty\,(\mu \to -\infty). \tag{21}$$

Therefore, $\varphi'(\mu)$ has a unique zero ω_0 in $(-\infty,0)$. Hence, $\varphi(\mu)$ is strictly decreasing on $(-\infty,\omega_0)$ and strictly increasing on $(\omega_0, 0)$, so that $\varphi(\mu)$ has a unique zero θ in $(-\infty,0)$ as $\varphi(0) = -q < 0$ and $\varphi(\mu) \to +\infty(\mu \to -\infty)$. □

Now, from (8), it follows that $g(\theta)$ is the maximum value of $g(\mu)$ in $(-\infty,0)$. By (10), we know that (19) is equivalent to $g(\mu) < 0$ for $\mu \in (-\infty,0)$.

From Theorem 7, we obtain the following corollary that extends Theorem 1 in [25] for $\tau < \sigma$.

Corollary 8. *If $p > 1$, $\tau < \sigma$, and $\tau q e^{-(\sigma/\tau)\ln p} \geqslant \tau \ln^2 p/(\sigma \ln p + \tau)$, then all solutions of* (1) *are oscillatory.*

Proof. The inequality $\tau q e^{-(\sigma/\tau)\ln p} \geqslant \tau\ln^2 p/(\sigma\ln p + \tau)$ is equivalent to $\varphi(-\ln p/\tau) \leqslant 0$. From the proof of Theorem 7, we get that $\theta \leqslant -\ln p/\tau$. This and (17) imply $g(\theta) < 0$; therefore, $h(\theta) < 0$. □

Theorem 9. *Suppose that $p > 1$ and $\tau > \sigma$; then all solutions of* (1) *are oscillatory if and only if one of the following conditions holds:*

(H_1) $q\sigma e^{2-\sqrt{2}} \geqslant (2\sqrt{2} - 2)p\tau/(\tau - \sigma)$;

(H_2) $\sigma(\tau-\sigma)\omega_1^2 + (2\sigma - \tau)\omega_1 \leqslant 2$;

(H_3) $h(\theta_2) = qe^{\theta_2\sigma}[(\tau-\sigma)\theta_2 + 1] - \tau\theta_2^2 < 0$,

where ω_1 is a unique zero of $\varphi'(\mu)$ in $(-2/(\tau-\sigma), (\sqrt{2}-2)/(\tau-\sigma))$ and θ_2 is the maximum negative zero of $\varphi(\mu)$.

Proof. By Lemma 5, all solutions of (1) are oscillatory if and only if

$$g(\mu) < 0, \quad \text{for } \mu \in (-\infty, 0). \tag{22}$$

From (20), we have that

$$\varphi'(\mu) > 0 \quad \text{for } \mu \in \left(-\infty, \frac{-2}{\tau - \sigma}\right], \tag{23}$$

and $\varphi'(\mu)$ is strictly decreasing on $(-2/(\tau-\sigma), (\sqrt{2}-2)/(\tau-\sigma))$ and strictly increasing on $((\sqrt{2}-2)/(\tau-\sigma), 0)$. Thus, $\varphi'((\sqrt{2}-2)/(\tau-\sigma))$ is the minimum value of $\varphi'(\mu)$ in $(-2/(\tau-\sigma), 0)$. □

(1) If $\varphi'((\sqrt{2}-2)/(\tau-\sigma)) \geqslant 0$, which is the case of (H_1), we have that

$$\varphi'(\mu) \geqslant 0, \quad \mu \in \left(\frac{-2}{\tau-\sigma}, 0\right). \tag{24}$$

Combining (23) and (24), we obtain that

$$\varphi(\mu) \leqslant \varphi(0) = -q < 0, \quad \mu \in (-\infty, 0). \tag{25}$$

This means that $g(\mu)$ is strictly decreasing on $(-\infty,0)$ and, consequently,

$$g(\mu) < \lim_{\mu\to-\infty} g(\mu) = -1. \tag{26}$$

(2) If $\varphi'((\sqrt{2}-2)/(\tau-\sigma)) < 0$, $\varphi'(\mu)$ has a unique zero ω_1 in $(-2/(\tau-\sigma), (\sqrt{2}-2)/(\tau-\sigma))$ and a unique zero ω_2 in

$((\sqrt{2}-2)/(\tau-\sigma),0)$ since $\varphi'(-2/(\tau-\sigma)) = q\sigma > 0$. Hence $\varphi(\mu)$ is strictly increasing on $(-\infty,\omega_1)$, strictly decreasing on (ω_1,ω_2), and strictly increasing on $(\omega_2,0)$. Consequently, $\varphi(\omega_1)$ is the maximum value of $\varphi(\mu)$ in $(-\infty,\omega_2)$. Now, it is easy to find that (22) holds if $\varphi(\omega_1) \leqslant 0$.

On the other hand, applying $\varphi'(\omega_1) = 0$, we can get

$$\varphi(\omega_1) = \frac{q\left[\sigma(\tau-\sigma)\omega_1^2 + (2\sigma-\tau)\omega_1 - 2\right]}{(\tau-\sigma)\omega_1 + 2}. \quad (27)$$

So $\varphi(\omega_1) \leqslant 0$ is equivalent to $\sigma(\tau-\sigma)\omega_1^2 + (2\sigma-\tau)\omega_1 \leqslant 2$. This is the case of (H_2).

If $\varphi(\omega_1) > 0$, we obtain that $\varphi(\mu)$ has a unique zero θ_1 in $(-\infty,\omega_1)$ and a unique zero θ_2 in (ω_1,ω_2). Therefore, $g(\mu)$ is strictly decreasing on $(-\infty,\theta_1)$, strictly increasing on (θ_1,θ_2), and strictly decreasing on $(\theta_2,0)$. Therefore, it is not difficult to find that (22) holds if and only if $g(\theta_2) < 0$ and it is the case of (H_3).

From Theorem 9, we obtain the following corollary immediately.

Corollary 10. *If $p > 1$, $\tau > \sigma$, and $q\sigma \geqslant p\tau/(\tau-\sigma)$, then all solutions of* (1) *are oscillatory.*

Example 11. Consider the following neutral delay differential equation:

$$[x(t) - 20x(t-12)]' + 10.5x(t-2) = 0. \quad (28)$$

It is not difficult to see that $p = 20$, $q = 10.5$, $\tau = 12$, and $\sigma = 2$. Consequently, $\tau > \sigma$, and

$$\begin{aligned} q\sigma e^{2-\sqrt{2}} - \frac{(2\sqrt{2}-2)p\tau}{\tau-\sigma} &> 21(3-\sqrt{2}) - 24(2\sqrt{2}-2) \\ &= 3(37-23\sqrt{2}) > 0, \end{aligned} \quad (29)$$

so that all the solutions of (28) are oscillatory from Theorem 9.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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