Research Article

Some New Inclusion and Neighborhood Properties for Certain Multivalent Function Classes Associated with the Convolution Structure

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We use the familiar convolution structure of analytic functions to introduce two new subclasses of multivalently analytic functions of complex order, and prove several inclusion relationships associated with the (n, δ)-neighborhoods for these subclasses. Some interesting consequences of these results are also pointed out.

1. Introduction and preliminaries

Let $\mathcal{A}_p(n)$ denote the class of functions of the form

$$f(z) = z^p + \sum_{k=n}^{\infty} a_k z^k \quad (p < n; n, p \in \mathbb{N} = \{1, 2, \dots\}), \tag{1.1}$$

which are analytic and p-valent in the open unit disk

$$\mathbb{U} = \{z; z \in \mathbb{C} : |z| < 1\}. \tag{1.2}$$

If $f \in \mathcal{A}_p(n)$ is given by (1.1) and $g \in \mathcal{A}_p(n)$ is given by

$$g(z) = z^p + \sum_{k=n}^{\infty} b_k z^k, \tag{1.3}$$

then the Hadamard product (or convolution) $f*g$ of f and g is defined (as usual) by

$$(f*g)(z) := z^p + \sum_{k=n}^{\infty} a_k b_k z^k := (g*f)(z). \tag{1.4}$$

We denote by $\mathcal{T}_p(n)$ the subclass of $\mathcal{A}_p(n)$ consisting of functions of the form

$$f(z) = z^p - \sum_{k=n}^{\infty} a_k z^k \quad (p < n; a_k \geq 0(k \geq n); n, p \in \mathbb{N}), \tag{1.5}$$

which are p-valent in $\mathbb{U}$.

For a fixed function $g(z) \in \mathcal{A}_p(n)$ defined by

$$g(z) = z^p + \sum_{k=n}^{\infty} b_k z^k \quad (p < n; b_k \geq 0(k \geq n); n, p \in \mathbb{N}), \tag{1.6}$$

we introduce a new class $\mathcal{S}_p^\lambda(g; n, b, m)$ of functions belonging to the subclass of $\mathcal{T}_p(n)$, which consists of functions $f(z)$ of the form (1.5), satisfying the following inequality:

$$\left| \frac{1}{b} \left(\frac{z(f*g)^{(m+1)}(z) + \lambda z^2 (f*g)^{(m+2)}(z)}{\lambda z(f*g)^{(m+1)}(z) + (1-\lambda)(f*g)^{(m)}(z)} - (p-m) \right) \right| < 1 \tag{1.7}$$

$$(z \in \mathbb{U}; p \in \mathbb{N}; m \in \mathbb{N}_0; p > m; 0 \leq \lambda \leq 1; b \in \mathbb{C} \setminus \{0\}).$$

We note that there exist several interesting new (or known) subclasses of our function class $\mathcal{S}_p^\lambda(g; n, b, m)$. For example, if $\lambda = 0$ in (1.7), we obtain the class $\mathcal{S}_p(g; n, b, m)$ studied very recently by Prajapat et al. [1]. On the other hand, if the coefficients b_k in (1.6) are chosen as follows:

$$b_k = \left(\frac{k+\mu}{p+\mu} \right)^r \quad (\mu \geq 0; k \geq n; r, p, n \in \mathbb{N}), \tag{1.8}$$

and n is replaced by $n+p$ in (1.4) and (1.5), then we obtain the class $\mathcal{S}_{n,m}^p(\mu, r, \lambda, b)$ of p-valently analytic functions (involving the multiplier transformation operator $I_p(r, \mu)$ defined in [2]) which was studied recently by Srivastava et al. [3]. Also, if we set $\lambda = 0$ in (1.7) and if the arbitrary sequence b_k in (1.6) is selected as follows:

$$b_k = \binom{\mu + k - 1}{k - p} \quad (\mu > -p; k \geq n; p, n \in \mathbb{N}), \tag{1.9}$$

also if n is replaced by $n+p$ in (1.4) and (1.5), then we obtain the class $\mathcal{H}_{n,m}^p(\mu, b)$ of p-valently analytic functions (involving the familiar Ruscheweyh derivative operator) investigated by Raina and Srivastava [4]. Further, when

$$\lambda = 0, \qquad m = 0, \qquad b = p(1-\alpha) \quad (p \in \mathbb{N}; 0 \leq \alpha < 1), \tag{1.10}$$

in (1.7), then $\mathcal{S}_p^\lambda(g;n,b,m)$ reduces to the class studied recently by Ali et al. [5]. Moreover, when

$$g(z) = z^p + \sum_{k=n}^{\infty} \frac{(\alpha_1)_{k-p} \cdots (\alpha_q)_{k-p}}{(\beta_1)_{k-p} \cdots (\beta_s)_{k-p}(k-p)!} z^k,$$

(1.11)

$$(\alpha_j \in \mathbb{C} \ (j = 1,2,\ldots,q), \ \beta_j \in \mathbb{C} \setminus \{0,-1,-2,\ldots\} \ (j = 1,2,\ldots,s))$$

with the parameters

$$\alpha_1,\ldots,\alpha_q, \qquad \beta_1,\ldots,\beta_s$$

(1.12)

being so chosen that the coefficients b_k in (1.6) satisfy the following condition:

$$b_k = \frac{(\alpha_1)_{k-p} \cdots (\alpha_q)_{k-p}}{(\beta_1)_{k-p} \cdots (\beta_s)_{k-p}(k-p)!} \geq 0,$$

(1.13)

then the class $\mathcal{S}_p^\lambda(g;n,b,m)$ transforms into a (presumably) new class $\mathcal{S}_p^\lambda(n,b,m)$ defined by

$$\mathcal{S}_p^\lambda(n,b,m) = \left\{ f \in \mathcal{T}_p(n) : \left| \frac{1}{b} \left(\frac{z(H_s^q[\alpha_1]f)^{(m+1)}(z) + \lambda z^2(H_s^q[\alpha_1]f)^{(m+2)}}{\lambda z(H_s^q[\alpha_1]f)^{(m+1)}(z) + (1-\lambda)(H_s^q[\alpha_1]f)^{(m)}} - (p-m) \right) \right| < 1 \right\}$$

$$(z \in \mathbb{U}; \ q \leq s+1; \ m,q,s \in \mathbb{N}_0; \ 0 \leq \lambda \leq 1, \ p \in \mathbb{N}; \ b \in \mathbb{C} \setminus \{0\}).$$

(1.14)

The operator

$$(H_s^q[\alpha_1]f)(z) := H_s^q(\alpha_1,\ldots,\alpha_q; \ \beta_1,\ldots,\beta_s)f(z),$$

(1.15)

involved in (1.14), is the Dziok-Srivastava linear operator (see for details [6]; see also [7, 8]) which contains such well-known operators as the Hohlov linear operator, Saitoh generalized linear operator, Carlson-Shaffer linear operator, Ruscheweyh derivative operator as well as its generalized version, the Bernardi-Libera-Livingston operator, and the Srivastava-Owa fractional derivative operator. One may refer to [7] or [6] for further details and references for these operators. The Dziok-Srivastava linear operator defined in [6] has further been generalized by Dziok and Raina [7] (see also [8, 9]).

Following a recent investigation by Frasin and Darus [10], let $f(z) \in \mathcal{T}_p(n)$, $\delta \geq 0$, then a (q,δ)-neighborhood of the function $f(z)$ is defined by

$$\mathcal{N}_{n,\delta}^q(f) = \left\{ h : h \in \mathcal{T}_p(n) : h(z) = z^p - \sum_{k=n}^{\infty} c_k z^k, \ \sum_{k=n}^{\infty} k^{q+1} |a_k - c_k| \leq \delta \right\}.$$

(1.16)

It follows from the definition (1.16) that if

$$e(z) = z^p \quad (p \in \mathbb{N}),$$

(1.17)

then

$$\mathcal{N}^q_{n,\delta}(e) = \left\{ h : h \in \mathcal{T}_p(n) : h(z) = z^p - \sum_{k=n}^{\infty} c_k z^k, \ \sum_{k=n}^{\infty} k^{q+1}|c_k| \le \delta \right\}. \qquad (1.18)$$

We observe that

$$\mathcal{N}^0_{2,\delta}(f) = \mathcal{N}_\delta(f),$$
$$\mathcal{N}^1_{2,\delta}(f) = \mathcal{M}_\delta(f), \qquad (1.19)$$

where $\mathcal{N}_\delta(f)$ and $\mathcal{M}_\delta(f)$ denote, respectively, the δ-neighborhoods of the function

$$g(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (a_k \ge 0, \ z \in \mathbb{U}), \qquad (1.20)$$

defined by Ruscheweyh [11] and Silverman [12].

Finally, for a fixed function

$$g(z) = z^p + \sum_{k=n}^{\infty} b_k z^k \in \mathcal{A}_p(n) \quad (p < n; \ b_k > 0 \ (k \ge n); \ n, p \in \mathbb{N}), \qquad (1.21)$$

let $\mathcal{P}^\lambda_p(g; n, b, m)$ denote the subclass of $\mathcal{T}_p(n)$ consisting of functions $f(z)$ of the form (1.5) which satisfy the following inequality:

$$\left| \frac{1}{b} \left\{ [1 - \lambda(p - m - 1)](f*g)^{(m+1)}(z) + \lambda z (f*g)^{(m+2)}(z) - (p - m) \right\} \right| < p - m$$

$$(z \in \mathbb{U}, \ m \in \mathbb{N}_0; \ p \in \mathbb{N}; \ p > m; \ 0 \le \lambda \le 1, \ b \in \mathbb{C} \setminus \{0\}). \qquad (1.22)$$

The object of the present paper is to investigate the various properties and characteristics of functions belonging to the above-defined subclasses

$$\mathcal{S}^\lambda_p(g; n, b, m), \qquad \mathcal{P}^\lambda_p(g; n, b, m) \qquad (1.23)$$

of p-valently analytic functions in $\mathbb{U}$. Apart from deriving coefficient inequalities for each of these function classes, we establish several inclusion relationships involving the (n, δ)-neighborhoods of functions belonging to these subclasses.

2. Coefficient bound inequalities

We begin by proving a necessary and sufficient condition for the function $f(z) \in \mathcal{T}_p(n)$ to be in each of the classes

$$\mathcal{S}^\lambda_p(g; n, b, m), \qquad \mathcal{P}^\lambda_p(g; n, b, m). \qquad (2.1)$$

Theorem 2.1. *Let $f(z) \in \mathcal{T}_p(n)$ be given by (1.5). Then $f(z)$ is in the class $\mathcal{S}^\lambda_p(g; n, b, m)$ if and only if*

$$\sum_{k=n}^{\infty} a_k b_k [\lambda(k - m - 1) + 1](k - p + |b|) \binom{k}{m} \le |b|[\lambda(p - m - 1) + 1] \binom{p}{m}. \qquad (2.2)$$

Proof. Assume that $f(z) \in S_p^\lambda(g; n, b, m)$. Then, in view of (1.5)–(1.7), we get

$$\Re\left(\frac{\lambda z^2(f*g)^{(m+2)}(z) + z[1 - \lambda(p - m)](f*g)^{(m+1)}(z) - (1 - \lambda)(p - m)(f*g)^{(m)}(z)}{\lambda z(f*g)^{(m+1)}(z) + (1 - \lambda)(f*g)^{(m)}(z)}\right) > -|b|$$

(2.3)

which yields

$$\Re\left(\frac{\sum_{k=n}^{\infty} a_k b_k \binom{k}{m}(k - p)[\lambda(k - m - 1) + 1]z^{k-p}}{\binom{p}{m}[\lambda(p - m - 1) + 1]z^{p-m} - \sum_{k=n}^{\infty} a_k b_k \binom{k}{m}[\lambda(k - m - 1) + 1]z^{k-p}}\right) < |b| \quad (z \in \mathbb{U}).$$

(2.4)

Putting $z = r$ $(0 \leq r < 1)$ in (2.4), the denominator expression on the left-hand side of (2.4) remains positive for $r = 0$, and also for all $r \in (0, 1)$. Hence, by letting $r \to 1^-$, through real values, inequality (2.4) leads to the desired assertion (2.2) of Theorem 2.1.

Conversely, by applying the hypothesis (2.2) of Theorem 2.1, and letting $|z| = 1$, we find that

$$\left|\frac{z(f*g)^{(m+1)}(z) + \lambda z^2(f*g)^{(m+2)}(z)}{\lambda z(f*g)^{(m+1)}(z) + (1 - \lambda)(f*g)^{(m)}(z)} - (p - m)\right|$$

$$\leq \frac{|b|\left\{\binom{p}{m}[\lambda(p - m - 1) + 1] - \sum_{k=n}^{\infty} a_k b_k[\lambda(k - m - 1) + 1]\binom{k}{m}\right\}}{\binom{p}{m}[\lambda(p - m - 1) + 1] - \sum_{k=n}^{\infty} a_k b_k[\lambda(k - m - 1) + 1]\binom{k}{m}} = |b|.$$

(2.5)

Hence, by the *maximum modulus principle*, we infer that $f(z) \in S_p^\lambda(g; n, b, m)$, which completes the proof of Theorem 2.1. □

Remark 2.2. In the special case when

$$\text{(i)} \quad b_k = \left(\frac{k + \mu}{p + \mu}\right)^r \quad (\mu \geq 0; \; k \geq n; \; r, p, n \in \mathbb{N}; \; n \mapsto n + p).$$

(2.6)

Theorem 2.1 corresponds to a result given recently by Srivastava et al. [3, Theorem 1, page 3]:

$$\text{(ii)} \quad \lambda = 0; \qquad b_k = \binom{\mu + k - 1}{k - p} \quad (\mu > -p; \; k \geq n; \; n, p \in \mathbb{N}; \; n \mapsto n + p).$$

(2.7)

Theorem 2.1 yields the result given recently by Raina and Srivastava [4, Theorem 1, page 3]:

$$\text{(iii)} \quad m = 0; \qquad p = 1, \qquad b_k = k^\Omega \quad (\Omega \in \mathbb{N}_0; \; k \geq n; \; n \in \mathbb{N}; \; n \mapsto n + p).$$

(2.8)

Theorem 2.1 reduces to the result of Orhan and Kamali [13, Lemma 1, page 57]:

$$\text{(iv)} \quad \lambda = 0, \qquad m = 0, \qquad b = p(1 - \alpha) \quad (p \in N; \ 0 \leq \alpha < 1). \tag{2.9}$$

Theorem 2.1 gives a recently established result due to Ali et al. [5, Theorem 1, page 181].

The following results concerning the class of functions $p_p^\lambda(g; n, b, m)$ can be proved on similar lines as given above for Theorem 2.1.

Theorem 2.3. *Let $f(z) \in \mathcal{T}_p(n)$ be given by (1.5). Then $f(z)$ is in the class $p_p^\lambda(g; n, b, m)$ if and only if*

$$\sum_{k=n}^{\infty} \left[\lambda(k - p) + 1 \right] (k - m) \binom{k}{m} a_k b_k \leq (p - m) \left[\frac{|b| - 1}{m!} + \binom{p}{m} \right]. \tag{2.10}$$

Remark 2.4. Making use of the same substitutions as mentioned above in (2.6), Theorem 2.3 yields another known result due to Srivastava et al. [3, Theorem 2, page 4]. Also, using the same substitutions as mentioned above in (2.8), we get the result of Orhan and Kamali [13, Lemma 2, page 58].

3. Inclusion properties

We now obtain some inclusion relationships for the function classes

$$\mathcal{S}_p^\lambda(g; n, b, m), \qquad p_p^\lambda(g; n, b, m), \tag{3.1}$$

involving the (n, δ)-neighborhood defined by (1.18).

Theorem 3.1. *If $b_k \geq b_n$ $(k \geq n)$ and*

$$\delta := \frac{n \left[\lambda(p - m - 1) + 1 \right] |b| \binom{p}{m}}{(n - p + |b|) \left[\lambda(n - m - 1) + 1 \right] \binom{n}{m} b_n} \quad (p > |b|), \tag{3.2}$$

then

$$\mathcal{S}_p^\lambda(g; n, b, m) \subset \mathcal{N}_{n,\delta}^0(e). \tag{3.3}$$

Proof. Let $f(z) \in \mathcal{S}_p^\lambda(g; n, b, m)$. Then, in view of assertion (2.2) of Theorem 2.1, and the given condition $b_k \geq b_n$ $(k \geq n)$, we get

$$\left[\lambda(n - m - 1) + 1 \right] (n - p + |b|) \binom{n}{m} b_n \sum_{k=n}^{\infty} a_k$$

$$\leq \sum_{k=n}^{\infty} a_k b_k \left[\lambda(k - m - 1) + 1 \right] (k - p + |b|) \binom{k}{m} \leq |b| \left[\lambda(p - m - 1) + 1 \right] \binom{p}{m}, \tag{3.4}$$

which implies that

$$\sum_{k=n}^{\infty} a_k \leq \frac{|b|[\lambda(p-m-1)+1]\binom{p}{m}}{(n-p+|b|)[\lambda(n-m-1)+1]\binom{n}{m}b_n}. \tag{3.5}$$

Applying the assertion (2.2) of **Theorem 2.1** again (in conjunction with (3.5)), we obtain

$$\binom{n}{m}[\lambda(n-m-1)+1]b_n \sum_{k=n}^{\infty} ka_k$$

$$\leq |b|[\lambda(p-m-1)+1]\binom{p}{m} + (p-|b|)[\lambda(n-m-1)+1]\binom{n}{m}b_n \sum_{k=n}^{\infty} a_k$$

$$\leq |b|[\lambda(p-m-1)+1]\binom{p}{m} + (p-|b|)[\lambda(n-m-1)+1]\binom{n}{m}b_n \tag{3.6}$$

$$\cdot \frac{|b|[\lambda(p-m-1)+1]\binom{p}{m}}{(n-p+|b|)[\lambda(n-m-1)+1]\binom{n}{m}b_n}$$

$$= \frac{n|b|[\lambda(p-m-1)+1]\binom{p}{m}}{(n-p+|b|)}.$$

Hence,

$$\sum_{k=n}^{\infty} ka_k \leq \frac{n[\lambda(p-m-1)+1]|b|\binom{p}{m}}{(n-p+|b|)[\lambda(n-m-1)+1]\binom{n}{m}b_n} := \delta \quad (p > |b|), \tag{3.7}$$

which by virtue of (1.18) establishes the inclusion relation (3.3) of **Theorem 3.1**. □

In the analogous manner, by applying the assertion (2.10) of **Theorem 2.3** instead of the assertion (2.2) of **Theorem 2.1** to the functions in the class $\mathcal{P}_p^{\lambda}(g;n,b,m)$, we can prove the following inclusion relationship.

Theorem 3.2. *If $b_k \geq b_n$ $(k \geq n)$ and*

$$\delta := \frac{(p-m)[(|b|-1)/m! + \binom{p}{m}]}{[\lambda(n-p)+1]\binom{n-1}{m}b_n}, \tag{3.8}$$

then

$$\mathcal{P}_p^{\lambda}(g;n,b,m) \subset N_{n,\delta}^0(e). \tag{3.9}$$

Remark 3.3. Applying the parametric substitutions listed in (2.6), Theorems 3.1 and 3.2 would yield the known results due to Srivastava et al. [3, Theorem 3, page 4; Theorem 4, page 5]. Also, using substitutions (as mentioned above in (2.8)) in Theorems 3.1 and 3.2, we get the results due to Orhan and Kamali [13, Theorem 1, page 58; Theorem 2, page 59].

4. Neighborhood properties

This concluding section determines the neighborhood properties for each of the classes

$$\mathcal{S}_p^{(\lambda,\alpha)}(g;n,b,m), \qquad \mathcal{P}_p^{(\lambda,\alpha)}(g;n,b,m) \tag{4.1}$$

which are defined as follows.

A function $f(z) \in \mathcal{T}_p(n)$ is said to be in the class $\mathcal{S}_p^{(\lambda,\alpha)}(g;n,b,m)$ if there exists a function $h(z) \in \mathcal{S}_p^{\lambda}(g;n,b,m)$ such that

$$\left| \frac{f(z)}{h(z)} - 1 \right| < p - \alpha \quad (z \in \mathbb{U};\ 0 \le \alpha < p). \tag{4.2}$$

Analogously, a function $f(z) \in \mathcal{T}_p(n)$ is said to be in the class $\mathcal{P}_p^{(\lambda,\alpha)}(g;n,b,m)$ if there exists a function $h(z) \in \mathcal{P}_p^{\lambda}(g;n,b,m)$ such that inequality (4.2) holds true.

Theorem 4.1. *If $h(z) \in \mathcal{S}_p^{\lambda}(g;n,b,m)$ and*

$$\alpha = p - \frac{\delta}{n^{q+1}} \cdot \frac{(n-p+|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m}b_n}{\left[(n-p+|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m}b_n - |b|\left[\lambda(p-m-1)+1\right]\binom{p}{m}\right]}, \tag{4.3}$$

then

$$\mathcal{N}_{n,\delta}^q(h) \subset \mathcal{S}_p^{(\lambda,\alpha)}(g;n,b,m). \tag{4.4}$$

Proof. Suppose that $f(z) \in \mathcal{N}_{n,\delta}^q(h)$. We then find from (1.16) that

$$\sum_{k=n}^{\infty} k^{q+1} |a_k - c_k| \le \delta, \tag{4.5}$$

which readily implies that

$$\sum_{k=n}^{\infty} |a_k - c_k| \le \frac{\delta}{n^{q+1}} \quad (n \in \mathbb{N}). \tag{4.6}$$

Next, since $h(z) \in \mathcal{S}_p^{\lambda}(g;n,b,m)$, we have in view of (3.5) that

$$\sum_{k=n}^{\infty} c_k \le \frac{|b|\left[\lambda(p-m-1)+1\right]\binom{p}{m}}{(n-p+|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m}b_n}, \tag{4.7}$$

so that

$$\left| \frac{f(z)}{h(z)} - 1 \right| \le \frac{\sum_{k=n}^{\infty} |a_k - c_k|}{1 - \sum_{k=n}^{\infty} c_k}$$

$$\le \frac{\delta}{n^{q+1}} \frac{1}{1 - |b|\left[\lambda(p-m-1)+1\right]\binom{p}{m} / (n-p+|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m}b_n}$$

$$\le \frac{\delta}{n^{q+1}} \frac{(n-p+|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m}b_n}{\left[(n-p+|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m}b_n - |b|\left[\lambda(p-m-1)+1\right]\binom{p}{m}\right]} \tag{4.8}$$

$$= p - \alpha,$$

provided that α is given by (4.3). Thus, by the above definition, $f \in \mathcal{S}_p^{(\lambda,\alpha)}(g;n,b,m)$ where α is given by (4.3), which proves Theorem 4.1. $\qquad\square$

The proof of **Theorem 4.2** below is similar to that of **Theorem 4.1** above, and its proof details are, therefore, omitted here.

Theorem 4.2. *If* $h(z) \in \mathcal{P}_p^\lambda(g; n, b, m)$ *and*

$$\alpha = p - \frac{\delta}{n^{q+1}} \frac{\left[\lambda(n-p)+1\right](n-m)\binom{n}{m}b_n}{\left[\left[\lambda(n-p)+1\right](n-m)\binom{n}{m}b_n - (p-m)\left\{(|b|-1)/m! + \binom{p}{m}\right\}\right]}, \tag{4.9}$$

then

$$\mathcal{N}_{n,\delta}^q(h) \subset \mathcal{P}_p^{(\lambda,\alpha)}(g; n, b, m). \tag{4.10}$$

Remark 4.3. Applying the parametric substitutions listed in (2.6), Theorems 4.1 and 4.2 would yield the corresponding results of Srivastava et al. [3, Theorems 5 and 6, page 6]. Also using substitutions as mentioned above in (2.8), we get the results due to Orhan and Kamali [13, Theorem 3, page 60; Theorem 4, page 61].

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