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Research Article

On Generalized Fractional Differentiator Signals

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By employing the generalized fractional differential operator, we introduce a system of fractional order derivative for a uniformly sampled polynomial signal. The calculation of the bring in signal depends on the additive combination of the weighted bring-in of N cascaded digital differentiators. The weights are imposed in a closed formula containing the Stirling numbers of the first kind. The approach taken in this work is to consider that signal function in terms of Newton series. The convergence of the system to a fractional time differentiator is discussed.

1. Introduction

Nowadays, fractional calculus (integral and differential operators) arises in signal processing and image possessing. The fractional calculation is able to enhance the quality of images, with interesting possibilities in edge detection and image restoration, to reveal faint objects in astronomical images and devoted to astronomical images analysis [1, 2]. Furthermore, fractional calculus is employed in image retrieval, design problems of variables and image denoising, digital fractional order for different filters [3–9]. In addition, the fractional calculus (differential operators) is used to reduce the error rate of handwritten signature verification system. All results based on the fractional calculus operators (differential and integral) show that this method is not only effective, but also good immunity. Therefore, the fractional calculus in the field of image processing and signal prosecuting that has broad application prospect.

The digital differentiator is a very helpful tool to compute and approximate the time derivatives of a given signal; such as, in radar and sonar applications, the velocity and acceleration are calculated from position measurements using differentiators. Digital fractional order differentiators are discrete-time digital systems fractional order differentiation. In view of signal processing, the generalization from integer to fractional orders that is an important concept for its possibility to enhance flexibility in designing digital differentiator has been well treated in the existing signal processing. There are comparatively published results respecting the fractional digital differentiators [10, 11].

In this work, by using the generalized Srivastava-Owa fractional differential operator [12] (involving two parameters α, μ), we introduce a system of generalized fractional order derivative for a uniformly sampled polynomial signal. The weights are obtained in a form containing the Pochhammer number. The convergence of the system to a fractional time differentiator is discussed. The output of the signal is determined by using the generalized hypergeometric function called the Fox-Wright function.

2. Design Technique

Ibrahim [13], has derived a formula for the generalized fractional integral. The n-fold integral for $n \in \mathbb{N} = \{1, 2, \ldots\}$ and real μ, is defined by

$$I_z^{\alpha,\mu} f(z) = \int_0^z \zeta_1^\mu d\zeta_1 \int_0^{\zeta_1} \zeta_2^\mu d\zeta_2 \cdots \int_0^{\zeta_{n-1}} \zeta_n^\mu f(\zeta_n)\, d\zeta_n. \quad (1)$$

Employing the Cauchy formula for iterated integrals yields

$$\int_0^z \zeta_1^\mu d\zeta_1 \int_0^{\zeta_1} \zeta^\mu f(\zeta)\, d\zeta = \int_0^z \zeta^\mu f(\zeta)\, d\zeta \int_\zeta^z \zeta_1^\mu d\zeta_1 = \frac{1}{\mu+1}\int_0^z \left(z^{\mu+1}-\zeta^{\mu+1}\right)\zeta^\mu f(\zeta)\, d\zeta. \tag{2}$$

Repeating the previous step $n-1$ times, we have

$$\int_0^z \zeta_1^\mu d\zeta_1 \int_0^{\zeta_1} \zeta_2^\mu d\zeta_2 \cdots \int_0^{\zeta_{n-1}} \zeta_n^\mu f(\zeta_n)\, d\zeta_n = \frac{(\mu+1)^{1-n}}{(n-1)!}\int_0^z \left(z^{\mu+1}-\zeta^{\mu+1}\right)^{n-1}\zeta^\mu f(\zeta)\, d\zeta, \tag{3}$$

which implies the fractional operator type

$$I_z^{\alpha,\mu} f(z) = \frac{(\mu+1)^{1-\alpha}}{\Gamma(\alpha)}\int_0^z \left(z^{\mu+1}-\zeta^{\mu+1}\right)^{\alpha-1}\zeta^\mu f(\zeta)\, d\zeta, \tag{4}$$

where α and $\mu \neq -1$ are real numbers, the function $f(z)$ is analytic in simply connected region of the complex z-plane $\mathbb{C}$ containing the origin, and the multiplicity of $(z^{\mu+1}-\zeta^{\mu+1})^{-\alpha}$ is removed by requiring $\log(z^{\mu+1}-\zeta^{\mu+1})$ to be real when $(z^{\mu+1}-\zeta^{\mu+1}) > 0$. When $\mu = 0$, we arrive at the standard Srivastava-Owa fractional integral operator, which is used to define the Srivastava-Owa fractional derivatives.

Corresponding to the generalized fractional integrals (4), we define the generalized differential operator of order α by

$$D_z^{\alpha,\mu} f(z) := \frac{(\mu+1)^{\alpha}}{\Gamma(1-\alpha)}\frac{d}{dz}\int_0^z \frac{\zeta^\mu f(\zeta)}{\left(z^{\mu+1}-\zeta^{\mu+1}\right)^{\alpha}} d\zeta; \quad 0 < \alpha \leq 1, \tag{5}$$

where the function $f(z)$ is analytic in simply connected region of the complex z-plane $\mathbb{C}$ containing the origin and the multiplicity of $(z^{\mu+1}-\zeta^{\mu+1})^{-\alpha}$ is removed by requiring $\log(z^{\mu+1}-\zeta^{\mu+1})$ to be real when $(z^{\mu+1}-\zeta^{\mu+1}) > 0$.

Example 1. We find the generalized derivative of the function $f(z) = z^\nu$, $\nu \in \mathbb{R}$. Let $\eta := (\zeta/z)^{\mu+1}$ then we have

$$\begin{aligned} D_z^{\alpha,\mu} z^\nu &= \frac{(\mu+1)^{\alpha}}{\Gamma(1-\alpha)}\frac{d}{dz}\int_0^z \frac{\zeta^{\mu+\nu}}{\left(z^{\mu+1}-\zeta^{\mu+1}\right)^{\alpha}} d\zeta \\ &= \frac{(\mu+1)^{\alpha-1}}{\Gamma(1-\alpha)}\frac{d}{dz} z^{(1-\alpha)(\mu+1)+\nu} \times \int_0^1 \eta^{(\nu+\mu+1)/(\mu+1)-1}(1-\eta)^{(1-\alpha)-1} d\eta \\ &= \frac{(\mu+1)^{\alpha-1}\Gamma(\nu/(\mu+1)+1)}{\Gamma(\nu/(\mu+1)+1-\alpha)} z^{(1-\alpha)(\mu+1)+\nu-1}. \end{aligned} \tag{6}$$

3. Fractional Digital Signal

In this section we will use the fractional differential operator (5) in order to compute the fractional signal. Assume the analytic signal $f(z)$, which can be represented as a Newton series around z_0

$$f(z) = \sum_{n=0}^{\infty} \frac{((z-z_0)/T)_n}{n!}\nabla_T^n f(z_0), \tag{7}$$

where $(a)_n$ is the Pochhammer symbol defined by

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = \begin{cases} 1, & n = 0 \\ a(a+1)\cdots(a+n-1), & n = \{1,2,\ldots\} \end{cases} = \sum_{k=0}^{\infty} \begin{bmatrix} n \\ k \end{bmatrix} a^k \tag{8}$$

such that $\left[\begin{smallmatrix} n \\ k \end{smallmatrix}\right]$ denotes the Stirling numbers of the first kind $S_1(n,k)$, and $\nabla_T^n f(z_0)$ is the backward difference operator defined by

$$\begin{aligned} \nabla_T^0 f(z_0) &= f(z_0) \\ \nabla_T^1 f(z_0) &= f(z_0) - f(z_0 - T) \\ &\vdots \\ \nabla_T^n f(z_0) &= \nabla_T^1\left[\nabla_T^{n-1} f(z_0)\right] \\ &= \nabla_T^{n-1} f(z_0) - \nabla_T^{n-1} f(z_0 - T). \end{aligned} \tag{9}$$

Now we assume that $z_0 = mT$ (T is the sampling period), $s(m) = f(mT)$ then we obtain

$$f(z) = \sum_{n=0}^{\infty} \frac{((z-mT)/T)_n}{n!}\nabla_1^n s(m), \tag{10}$$

where

$$\begin{aligned} \nabla_1^0 s(m) &= s(m) \\ \nabla_1^1 s(m) &= s(m) - s(m-1) \\ &\vdots \\ \nabla_1^n s(m) &= \nabla_1^{n-1} s(m) - \nabla_1^{n-1} s(m-1). \end{aligned} \tag{11}$$

By truncating the Newton series expansion at the Nth term ($n = N$), we assume the polynomial signal

$$p(z) = \sum_{n=0}^{N} c_n z^n, \quad (z \text{ is real}) \tag{12}$$

such that for all the differences of order $n \geq N + 1$ vanished.

The fractional differential digital signal is a discrete time system whose output $y(m)$ is the uniformly sampled version of the αth order derivative of $f(z)$. Therefore, we assume $\alpha > 0$. Specifically, we write

$$y(m) = D_z^{\alpha,\mu} f(z)\big|_{z=mT}. \tag{13}$$

The input $s(m) = f(mT)$ is supposed to be a polynomial of degree N. In view of Example 1, we have

$$D_z^{\alpha,\mu} f(z) = \sum_{n=0}^{N} \frac{\nabla_1^n s(m)}{n!} D_z^{\alpha,\mu}\left(\frac{z-mT}{T}\right)_n. \tag{14}$$

Next we proceed to evaluate the fractional order of the rising factorial power term at $mT = z$. We expand $(z - mT)^k$ using the binomial theorem, and we obtain

$$\begin{aligned}(z-mT)^k &= \binom{k}{0} z^k (mT)^0 \\ &\quad - \binom{k}{1} z^{k-1} (mT)^1 + \cdots + \binom{k}{k} (-mT)^k \\ &= \sum_{n=0}^{k} \binom{k}{n} z^n (-mT)^{k-n},\end{aligned} \tag{15}$$

where

$$\binom{k}{n} = \frac{k!}{n!\,(k-n)!} \tag{16}$$

consequently; by using Example 1, we have

$$\begin{aligned}&D_z^{\alpha,\mu}(z-mT)^k \\ &= \sum_{n=0}^{k} \binom{k}{n} \left[D_z^{\alpha,\mu} z^n\right] (-mT)^{k-n} \\ &= \sum_{n=0}^{k} \frac{k!}{n!\,(k-n)!} (-mT)^{k-n} \\ &\quad \times \frac{(\mu+1)^{\alpha-1}\Gamma(n/(\mu+1)+1)}{\Gamma(n/(\mu+1)+1-\alpha)} z^{(1-\alpha)(\mu+1)+n-1} \\ &= (\mu+1)^{\alpha-1}\Gamma(k+1)\, z^{(1-\alpha)(\mu+1)-1} (-mT)^k \\ &\quad \times \sum_{n=0}^{k} \frac{1}{\Gamma(k-n+1)} \frac{\Gamma(n/(\mu+1)+1)}{\Gamma(n/(\mu+1)+1-\alpha)} \frac{(z/mT)^n}{n!}\end{aligned} \tag{17}$$

as $k \to \infty$, $n < k$, and $\lim_{k\to\infty}\Gamma(k+1)/\Gamma(k-n+1) = 1$; thus we have

$$\begin{aligned}D_z^{\alpha,\mu}(z-mT)^k &= (\mu+1)^{\alpha-1} z^{(1-\alpha)(\mu+1)-1} (-mT)^k \\ &\quad \times {}_1\Phi_1\left[\begin{matrix}\left(1, \dfrac{1}{\mu+1}\right); \\ \left(1-\alpha, \dfrac{1}{\mu+1}\right);\end{matrix} \ \frac{z}{mT}\right],\end{aligned} \tag{18}$$

where ${}_q\Phi_p$ is the Fox-Wright function (the generalization of the hypergeometric function ${}_qF_p$) defined by

$$\begin{aligned}&{}_q\Phi_p\left[\begin{matrix}(\alpha_1, A_1), \ldots, (\alpha_q, A_q); \\ (\beta_1, B_1), \ldots, (\beta_p, B_p);\end{matrix} \ z\right] \\ &= {}_q\Phi_p\left[(\alpha_j, A_j)_{1,q}; (\beta_j, B_j)_{1,p}; z\right] \\ &:= \sum_{n=0}^{\infty} \frac{\Gamma(\alpha_1 + nA_1)\cdots\Gamma(\alpha_q + nA_q)}{\Gamma(\beta_1 + nB_1)\cdots\Gamma(\beta_p + nB_p)} \frac{z^n}{n!} \\ &= \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{q}\Gamma(\alpha_j + nA_j)}{\prod_{j=1}^{p}\Gamma(\beta_j + nB_j)} \frac{z^n}{n!},\end{aligned} \tag{19}$$

where $A_j > 0$ for all $j = 1, \ldots, q$, $B_j > 0$ for all $j = 1, \ldots, p$, and $1 + \sum_{j=1}^{p} B_j - \sum_{j=1}^{q} A_j \geq 0$ for suitable values $|z| < 1$. Hence

$$\begin{aligned}&D_z^{\alpha,\mu}(z-mT)^k\big|_{z=mT} \\ &\quad = (-1)^k (mT)^{k+(1-\alpha)(\mu+1)-1} (\mu+1)^{\alpha-1}\, {}_1\Phi_1[1].\end{aligned} \tag{20}$$

Substituting (20) into (14) we obtain (13)

$$\begin{aligned}&D_z^{\alpha,\mu} f(z) \\ &\quad = \sum_{n=0}^{N} \frac{\nabla_1^n s(m)}{n!} D_z^{\alpha,\mu}\left(\frac{z-mT}{T}\right)_n \\ &\quad = \sum_{n=0}^{N} \frac{\nabla_1^n s(m)}{n!} D_z^{\alpha,\mu}\left(\sum_{k=0}^{\infty} \begin{bmatrix} n \\ k \end{bmatrix} \left(\frac{z-mT}{T}\right)^k\right) \\ &\quad = \sum_{n=0}^{N} \frac{\nabla_1^n s(m)}{n!} \left(\sum_{k=0}^{\infty} \begin{bmatrix} n \\ k \end{bmatrix} D_z^{\alpha,\mu}\left(\frac{z-mT}{T}\right)^k\right)\end{aligned}$$

$$\begin{aligned}
&= \sum_{n=0}^{N} \frac{\nabla_1^n s(m)}{n!} \left(\sum_{k=0}^{\infty} \begin{bmatrix} n \\ k \end{bmatrix} \frac{1}{T^k} D_z^{\alpha,\mu} (z - mT)^k \right) \\
&= \sum_{n=0}^{N} \frac{\nabla_1^n s(m)}{n!} \left(\sum_{k=0}^{\infty} \begin{bmatrix} n \\ k \end{bmatrix} \frac{1}{T^k} \left[(-1)^k (mT)^{k+(1-\alpha)(\mu+1)-1} \right.\right. \\
&\qquad \left.\left. \times \left(\mu+1\right)^{\alpha-1} {}_1\Phi_1 [1] \right] \right) \\
&= T^{(1-\alpha)(\mu+1)-1} {}_1\Phi_1 [1] \left(\mu+1\right)^{\alpha-1} \sum_{n=0}^{N} \frac{\nabla_1^n s(m)}{n!} \\
&\quad \times \left(\sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix} \left[(-1)^k (m)^{k+(1-\alpha)(\mu+1)-1} \right] \right) \\
&:= y(m).
\end{aligned} \tag{21}$$

4. Experimental Results

In this section, we propose to apply the formula (21) on the signal $f(z) = \sqrt{z}$. We will assume $N = 5$. The Stirling numbers of the first kind $S_1(n,k)$ take the values

$$\begin{aligned}
&S_1(n,0) = \delta_{n,0} \\
&S_1(n,1) = (-1)^{n-1}(n-1)! \\
&\vdots \\
&S_1(n,n-1) = -\binom{n}{2} \quad \text{binomial coefficients},
\end{aligned} \tag{22}$$

where $\delta_{n,0}$ is the delta function

$$\delta_{n,0} = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0. \end{cases} \tag{23}$$

Now for sufficient small value of μ, the Fox-Wright function ${}_1\Phi_1[1]$ reduces to the hypergeometric function ${}_qF_p$

$$\begin{aligned}
&{}_q\Phi_p \left[\begin{matrix} (\alpha_1, 1), \ldots, (\alpha_q, 1); \\ (\beta_1, 1), \ldots, (\beta_p, 1); \end{matrix} \; z \right] \\
&\quad = \frac{\prod_{j=1}^{q} \Gamma(\alpha_j)}{\prod_{i=1}^{p} \Gamma(\beta_i)} {}_qF_p \left(\alpha_1, \ldots, \alpha_q, \beta_1, \ldots, \beta_p; z\right)
\end{aligned} \tag{24}$$

such that ${}_qF_p$ satisfies

$${}_2F_1(a,b;c;1) = \frac{\Gamma(c-a-b)\Gamma(c)}{\Gamma(c-a)\Gamma(c-b)}, \quad b \leq 0. \tag{25}$$

Employing the relations (24) and (25) yields

$$\begin{aligned}
{}_1\Phi_1[1] &\simeq \frac{\Gamma(\alpha_1)}{\Gamma(\beta_1)} {}_2F_1(1,0;1-\alpha;1) \\
&= \frac{1}{\Gamma(1-\alpha)} \frac{\Gamma(-\alpha)\Gamma(1-\alpha)}{\Gamma(-\alpha)\Gamma(1-\alpha)} \\
&= \frac{1}{\Gamma(1-\alpha)},
\end{aligned} \tag{26}$$

hence for $\alpha = 1/2 \Rightarrow \Gamma(1/2) = 1.772$, we impose ${}_1\Phi_1[1] \simeq 0.564$.

In virtue of (11), where $f(z) = f(mT) = s(m) = \sqrt{Tm}$ and $T = 1/25$, we pose

$$\begin{aligned}
\nabla_1^0 s(m) &= s(m) = \frac{\sqrt{m}}{5}, \\
\nabla_1^1 s(m) &= s(m) - s(m-1) = \frac{\sqrt{m} - \sqrt{m-1}}{5}, \\
\nabla_1^2 s(m) &= \frac{\sqrt{m} - 2\sqrt{m-1} + \sqrt{m-2}}{5}, \\
\nabla_1^3 s(m) &= \frac{\sqrt{m} - 3\sqrt{m-1} + 3\sqrt{m-2} - \sqrt{m-3}}{5}, \\
\nabla_1^4 s(m) &= \frac{\sqrt{m} - 4\sqrt{m-1} + 6\sqrt{m-2} - 4\sqrt{m-3} + \sqrt{m-4}}{5}, \\
\nabla_1^5 s(m) &= \left(\sqrt{m} - 5\sqrt{m-1} + 10\sqrt{m-2}\right. \\
&\quad \left. - 10\sqrt{m-3} + 5\sqrt{m-4} - \sqrt{m-5}\right) \\
&\quad \times (5)^{-1}.
\end{aligned} \tag{27}$$

For $\mu = 0$ and $\alpha = 0.5$, the 6th terms of $y(m)$ become

$$\begin{aligned}
&y(m) \\
&\quad = 2.82 \left\{ \frac{\nabla_1^0 s(m)}{\sqrt{m}} \right. \\
&\qquad - \sqrt{m} \left(\frac{\nabla_1^1 s(m)}{1!} + \frac{\nabla_1^2 s(m)}{2!} \right. \\
&\qquad \left. + 2\frac{\nabla_1^3 s(m)}{3!} + 6\frac{\nabla_1^4 s(m)}{4!} + 24\frac{\nabla_1^5 s(m)}{5!} \right)
\end{aligned}$$

$$+\sqrt{m^3}\left(\frac{\nabla_1^2 s(m)}{2!}+3\frac{\nabla_1^3 s(m)}{3!}+11\frac{\nabla_1^4 s(m)}{4!}+50\frac{\nabla_1^5 s(m)}{5!}\right)-\sqrt{m^5}\left(\frac{\nabla_1^3 s(m)}{3!}+6\frac{\nabla_1^4 s(m)}{4!}+35\frac{\nabla_1^5 s(m)}{5!}\right)+\sqrt{m^7}\left(\frac{\nabla_1^4 s(m)}{4!}+10\frac{\nabla_1^5 s(m)}{5!}\right)-\sqrt{m^9}\left(\frac{\nabla_1^5 s(m)}{5!}\right)+\cdots\Bigg\}. \quad (28)$$

Furthermore, for $\alpha = 0.5$ and $\mu = 0.25$, we have

$$y(m) \simeq 1.475\left\{\frac{\nabla_1^0 s(m)}{\sqrt[3]{m}}-\sqrt[3]{m^2}\left(\frac{\nabla_1^1 s(m)}{1!}+\frac{\nabla_1^2 s(m)}{2!}+2\frac{\nabla_1^3 s(m)}{3!}+6\frac{\nabla_1^4 s(m)}{4!}+24\frac{\nabla_1^5 s(m)}{5!}\right)+\sqrt[3]{m^5}\left(\frac{\nabla_1^2 s(m)}{2!}+3\frac{\nabla_1^3 s(m)}{3!}+11\frac{\nabla_1^4 s(m)}{4!}+50\frac{\nabla_1^5 s(m)}{5!}\right)-\sqrt[3]{m^8}\left(\frac{\nabla_1^3 s(m)}{3!}+6\frac{\nabla_1^4 s(m)}{4!}+35\frac{\nabla_1^5 s(m)}{5!}\right)+\sqrt[3]{m^{11}}\left(\frac{\nabla_1^4 s(m)}{4!}+10\frac{\nabla_1^5 s(m)}{5!}\right)-\sqrt[3]{m^{14}}\left(\frac{\nabla_1^5 s(m)}{5!}\right)+\cdots\right\}. \quad (29)$$

Also, for $\alpha = 0.5$ and $\mu = 0.5$, we have

$$y(m) \simeq 1.5\left\{\frac{\nabla_1^0 s(m)}{\sqrt[4]{m}}-\sqrt[4]{m^3}\left(\frac{\nabla_1^1 s(m)}{1!}+\frac{\nabla_1^2 s(m)}{2!}+2\frac{\nabla_1^3 s(m)}{3!}+6\frac{\nabla_1^4 s(m)}{4!}+24\frac{\nabla_1^5 s(m)}{5!}\right)+\sqrt[4]{m^7}\left(\frac{\nabla_1^2 s(m)}{2!}+3\frac{\nabla_1^3 s(m)}{3!}+11\frac{\nabla_1^4 s(m)}{4!}+50\frac{\nabla_1^5 s(m)}{5!}\right)-\sqrt[4]{m^{11}}\left(\frac{\nabla_1^3 s(m)}{3!}+6\frac{\nabla_1^4 s(m)}{4!}+35\frac{\nabla_1^5 s(m)}{5!}\right)+\sqrt[4]{m^{15}}\left(\frac{\nabla_1^4 s(m)}{4!}+10\frac{\nabla_1^5 s(m)}{5!}\right)-\sqrt[4]{m^{19}}\left(\frac{\nabla_1^5 s(m)}{5!}\right)+\cdots\right\}. \quad (30)$$

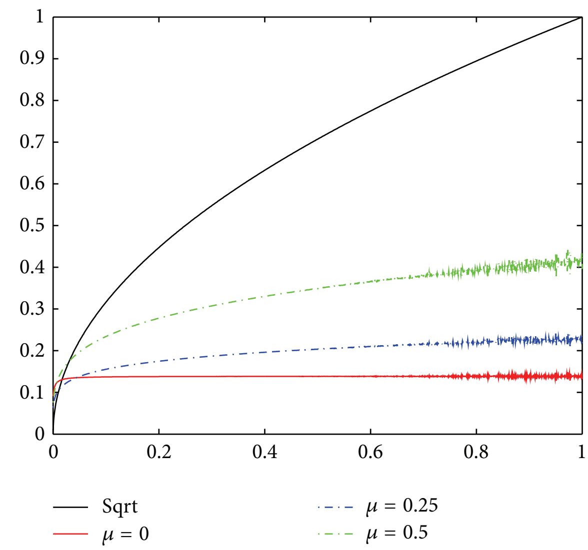

FIGURE 1: The outcome of the fractional signal when $\alpha = 0.5$, $T = 1/25$.

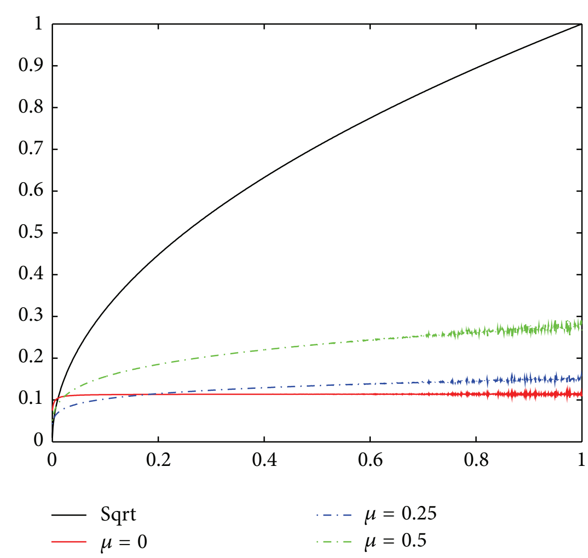

FIGURE 2: The outcome of the fractional signal when $\alpha = 0.75$, $T = 1/25$.

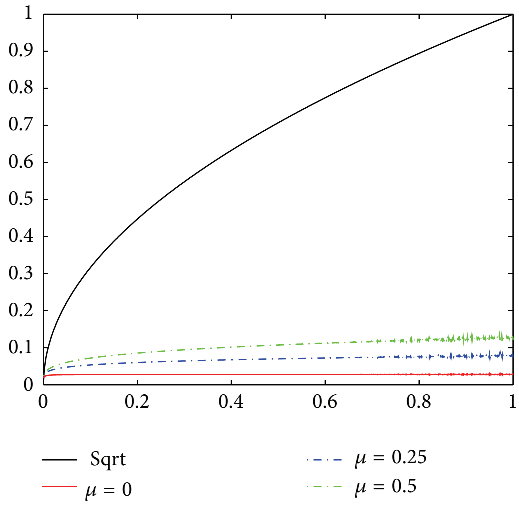

FIGURE 3: The outcome of the fractional signal when $\alpha = 0.5, T = 1$.

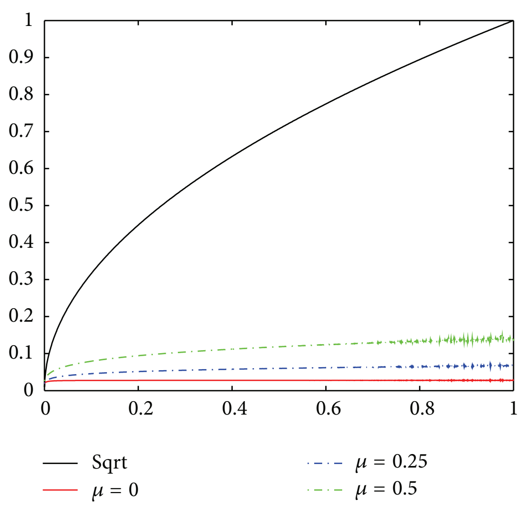

FIGURE 4: The outcome of the fractional signal when $\alpha = 0.75,\ T = 1$.

5. Discussion

For given values of N, fixed fractional number $\alpha = 0.5$, $\alpha = 0.75$, and different values of the second parameter $\mu = 0,\ 0.25,\ 0.5$, one can analyze the time-varying weights by plotting the time-varying impulse response of the system (Figures 1, 2, 3, and 4). For $\alpha = 1$ and any value of μ the system is a maximally linear differentiator (Figure 5). It follows from the relation (21) that the input/output characterizes the ideal digital differentiator, for fractional and integer values of α with the help of the fractional value of μ, of the polynomial signal. This relation shows for integer case of α that the weights are time-invariant. While the weights are varying time for fractional case.

Performance tests for the system proposed by this paper were implemented using MATLAB 2010a on Intel(R) Core i7 at 2.2 GHz, 4 GB DDR3 Memory, system type 64-bit, and Window 7.

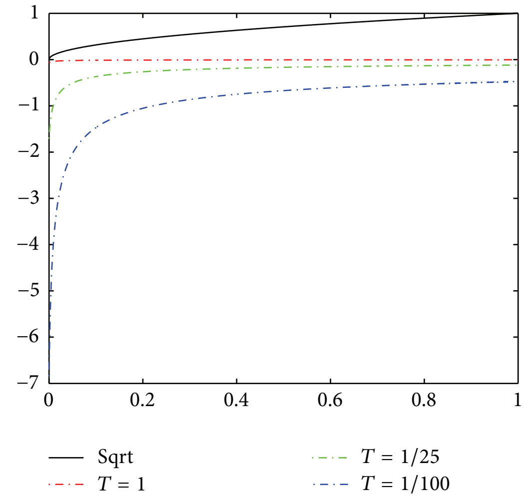

FIGURE 5: The outcome of the fractional signal when $\alpha = 1$.

6. Conclusion

The differential modeling of arbitrary order can be virtued as a signal processing method to develop numerical differential algorithms. There are various types of numerical fractional differential algorithms anticipated in the mathematics literature such as the Grünwald-Letnikov fractional differential operator and the Riemann-Liouville differential operator which based on one parameter α. In this paper, we modified the Newton series by using two-parameters (α, μ) fractional differential operator (generalized Srivastava-Owa operator). This approach implies zero error for the representation of the signal polynomial; thus it provides a means for the calculation of the fractional derivatives of $s(m)$. The system yields the arbitrary order derivative of the signal based on the current sample and N past samples of the signal. The value of N computed the truncation length in (14).

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References

[1] A. C. Sparavigna, "Using fractional differentiation in astronomy, Computer Vision and Pattern Recognition," 2010, http://arxiv .org/abs/0910. 2381.

[2] R. Marazzato and A. C. Sparavigna, "Astronomical image processing based on fractional calculus: the AstroFracTool, Instrumentation and Methods for Astrophysics," 2009, http://arxiv.org/abs/0910. 4637 .

[3] H. B. Kekre, S. D. Thepade, and A. Maloo, "Image retrieval using fractional coefficients of transformed image using DCT and Walsh transform," *International Journal of Engineering Science and Technology*, vol. 2, pp. 362–371, 2010.

[4] C. C. Tseng, "Design of variable and adaptive fractional order FIR differentiators," *Signal Processing*, vol. 86, no. 10, pp. 2554–2566, 2006.

[5] J. Hu, Y. Pu, and J. Zhou, "A novel image denoising algorithm based on riemann-liouville definition," *Journal of Computers*, vol. 6, no. 7, pp. 1332–1338, 2011.

[6] R. W. Schafer, "What is a savitzky-golay filter?" *IEEE Signal Processing Magazine*, vol. 28, no. 4, pp. 111–117, 2011.

[7] H. A. Jalab and R. W. Ibrahim, "Denoising algorithm based on generalized fractional integral operator with two parameters," *Discrete Dynamics in Nature and Society*, vol. 2012, Article ID 529849, 15 pages, 2012.

[8] H. A. Jalab and R. W. Ibrahim, "Texture enhancement for medical images based on fractional differential mask," *Discrete Dynamics in Nature and Society*, vol. 2013, Article ID 618536, 14 pages, 2013.

[9] H. A. Jalab and R. W. Ibrahim, "Texture enhancement based on the Savitzky-Golay fractional differential operator," *Mathematical Problems in Engineering*, vol. 2013, Article ID 149289, 8 pages, 2013.

[10] S. Samadi, M. O. Ahmad, and M. N. S. Swamy, "Exact fractional-order differentiators for polynomial signals," *IEEE Signal Processing Letters*, vol. 11, no. 6, pp. 529–532, 2004.

[11] S. C. Dutta Roy and B. Kumar, *Digital Differentiators*, North Holand, Amsterdam, The Netherlands, 1993.

[12] H. M. Srivastava and S. Owa, *Univalent Functions, Fractional Calculus, and Their Applications*, Halsted Press, John Wiley and Sons, New York, NY, USA, 1989.

[13] R. W. Ibrahim, "On generalized Srivastava-Owa fractional operators in the unit disk," *Advances in Difference Equations*, vol. 2011, article 55, 10 pages, 2011.