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Research Article

Quantum Computing in Decoherence-Free Subspace Constructed by Triangulation

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A formalism for quantum computing in decoherence-free subspaces is presented. The constructed subspaces are partial triangulated to an index related to environment. The quantum states in the subspaces are just projected states which are ruled by a subdynamic kinetic equation. These projected states can be used to perform ideal quantum logical operations without decoherence.

1. Introduction

Recent publications have formulated a remarkable theory for Decoherence-Free (DF) subspaces and subsystems in which quantum computing is performed in a DF subspace although the total space is still subject to decoherence [1–23]. Many proposals have considered the Born-Markov approximation or restrictions on the type of decoherence (e.g., symmetric and collective decoherence). However, there have been a number of reports on DF subspaces that do not invoke the Born-Markov approximation and do not place constraints on the type of decoherence [24]. These have significance for practical experimental implementation. So far, the most general framework may be called operator quantum error correction (OQEC), which encompasses active error correction and leads to improved threshold results in fault tolerant quantum computing [25–28], and has initiated the development of a structure theory for passive error correction [29, 30]. Experimentally, DF subspaces have recently been observed under some conditions, which shows that such DF subspaces do indeed exist, allowing logical qubits to be encoded without decoherence [22–34]. However, a practical procedure to construct DF subspace is still not a trivial problem, and hence, impedes building practical quantum logic gates. For this study, we present a different scheme to construct a DF subspace based on the Schrödinger type of subdynamic kinetic

equation (SSKE) which was inspired by theory of the subdynamics in Brussels-Austin school [35, 36].

2. Subdynamic Formalism

Consider a general quantum open system S interacting with an environment B, whose Hamiltonian is written as $H(t) = H_S(t) + H_B + \lambda H_{\text{int}}$ with coupling number λ. If one chooses the eigenprojectors and its orthogonal projectors of free Hamiltonian $H_0(t) = H_S(t) + H_B$ as P_n and Q_n, respectively (this is possible since the free Hamiltonian is assumed to be easily diagonalized), then by means of the subdynamics theory [37] one can introduce a creation (destruction) correlation operator (as a type of resolvent) as

$$C_n(t) = \frac{1}{E_n(t) - Q_n H(t) Q_n} Q_n H_1(t) P_n = D_n^{\dagger}(t), \tag{2.1}$$

where $E_n(t)$ is an eigenvalue of $H(t)$. This allows one to construct a Schrödinger type of kinetic equation for a projected state as

$$i\frac{\partial}{\partial t}\phi_{\text{proj}}(t) = \Theta(t)\phi_{\text{proj}}(t), \tag{2.2}$$

where the projected wave function ϕ_{proj} is defined as

$$\phi_{\text{proj}}(t) = \sum_n P_n \Pi_n(t) \varphi(t), \tag{2.3}$$

where the operator $\Pi_n(t)$ is expressed by

$$\Pi_n(t) = (P_n + C_n(t))(P_n + D_n(t)C_n(t))^{-1}(P_n + D_n(t)), \tag{2.4}$$

and the wave function $\varphi(t)$ satisfies the original Schrödinger equation, while the intermediate operator Θ is defined as

$$\Theta(t) = H_0(t) + \lambda \sum_n P_n H_1(t) C_n(t). \tag{2.5}$$

The creation operator $C_n(t)$ is independent of the representation with respect to the projectors P_n, Q_n and is not necessarily self-adjoint in the sense of extended functional space. This reveals that the eigenvalues of the intermediate operator may be complex and the corresponding evolution operator corresponding to two nonunitary semigroups evolutions, respectively. Using the physical boundary conditions, such as ϵ-rule [35], we can determine which semigroup is the correct one. Therefore, the evolution of the projected density operator for the open system can be time asymmetric and may exist in the generalized functional space beyond Hilbert space, such as rigged Hilbert space [38, 39]. This time asymmetric evolution is consistent with the second law of thermodynamics.

Moreover, by replacing the Hamiltonian $H(t)$ with the Liouvillian $L(t)$ and wave function φ with density operator ρ, as well as using the same approach as above, the SSKE can be transformed to the Liouvillian type of SKE (LSKE) for a quantum open system:

$$i\frac{\partial}{\partial t}\rho_{\text{proj}}(t) = \left[\Theta(t), \rho_{\text{proj}}(t)\right]. \tag{2.6}$$

The creation (destruction) correlation operator can be available by means of subdynamics theory. In fact, by introducing the eigen-projectors of the total Hamiltonian $\Pi_n(t)$, using the Heisenberg equation considering the eigenvalue problem of $\Pi_n(t)$ and the definition of the creation (destruction) operators, one can obtain the basic operator equation of the creation operator and its solutions can be expressed as the retarded or advanced integrals, corresponding to the two kinds of time evolution semigroups: $t \in [0, +\infty)$ or $t \in (-\infty, 0]$ and continuations of up and down half complex planes. In the same way, the basic equation for the destruction operator and relevant solution can be given.

This construction of SSKE or LSKE in subspace can be related to the original Schrödinger or Liouville equation. For instance, using the intertwining relations from subdynamics theory [35, 36, 40] $\Omega(t)\Theta(t)\Omega^{-1}(t) = H(t)$, one can arrive at the original Schrödinger equation

$$i\frac{\partial}{\partial t}\phi(t) = \Omega(t)\Theta(t)\Omega^{-1}(t)\phi(t) = H(t)\phi(t), \tag{2.7}$$

where the similarity operator is defined as

$$\Omega(t) = \sum_n (P_n + C_n(t)), \tag{2.8}$$

which is also not necessarily unitary. The eigenvectors of the time-independent total Hamiltonian H can be given by the eigenvectors of H_0, ϕ_n, as $|\varphi_n\rangle = (P_n + C_n)|\phi_n\rangle$ with the same structure of eigenvalues as Θ.

The second-order approximation for the LSKE also corresponds to the Master, Boltzmann, Pauli, and Fokker-Plank equations of kinetic theory and Brownian motion. For example, from the LSKE for a time-independent open system S, one can easily deduce the general Master equation by using Born-Markovian approximation. Indeed, if assuming that projector as

$$P = \frac{\exp(-\beta H_B)}{Tr_B \exp(-\beta H_B)} Tr_B \tag{2.9}$$

giving the reduced density operator for system S as $P\rho = \rho_S$, then a general Markovian equation can be obtained. This equation can also be reduced to a type of Lindblad equation or Boltzmann equation [41].

3. Decoherence-Free Partial Triangular Subspace

An interesting advantage using the above formalism is to construct a precise decoherence-free (DF) subspace. It is remarkable that the projected space on which subdynamics operates is a kind of DF subspace that occurs naturally by choosing a suitable basis to expand that subspace. In fact, if decoherence exists in a system arising from interactions with its environment, then the spectral decomposition of the Hamiltonian can be expressed using the subdynamic formalism as

$$H_0 = \Theta_0 = \sum_n E_n^0(t) P_n \Longrightarrow \sum_n \left(E_n^0(t) + E_n^1\right) P_n, \tag{3.1}$$

where it is not very difficult to see that the eigenprojector P_n for the free Hamiltonian $H_0(t)$ is invariant while the eigenvalue $E_n^0(t)$ is changed to $E_n^0(t) + E_n^1$. This produces a phase shift for the evolution state

$$\phi_{\mathrm{proj},n}(t) = \exp\left(-i\left(E_n^0(t) + E_n^1\right)t\right)\phi_{\mathrm{proj},n}(0), \tag{3.2}$$

which leads to a type of decoherence in the subspaces. For example, if the entangled states evolution in the subspace is $\sum_n \exp(-iE_n^0(t)t)\phi_n(0)$ before the interaction is exerted from the environment, then after the interaction the evolution of the state in the subspace becomes

$$\sum_n \exp\left(-iE_n^0(t)t\right)\phi_n(0) \Longrightarrow \sum_n \exp\left(-i\left(E_n^0(t) + E_n^1\right)t\right)\phi_n(0). \tag{3.3}$$

Hence for constructing an ideal DF subspace, one has to find a procedure to cancel the change of the eigenvalues. How can one realize it? *The key idea is to use the partial triangulation*.

For example, let us consider a typical two-qubit quantum computing system S, consisting of the spins $\mathbf{S}_1$ and $\mathbf{S}_2$, such as the two electrons of two ^{31}P confined in a germanium/silicon heterostructure of an electron spin-resonance transistor [42, 43] or the two electrons confined in two quantum dots [44]. Ignoring the influence of the environment, the Hamiltonian can be written using the Heisenberg model as $H_S(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2$, where $J(t)$ is the time-dependent exchange coupling parameter determined by the specific model considerations. In the case of spins of the two electrons (e.g., confined in two vertically, laterally, coupled quantum dots [45]), J is the difference in the energies of two-electrons ground state, a spin singlet at zero magnetic field, and the lowest spin-triplet state; J is also a function of the electric and magnetic field and the interdot distance. Using the relationship between $\mathbf{S}_1 \cdot \mathbf{S}_2$ and the square of the sum of $\mathbf{S}_1$ and $\mathbf{S}_2$, the eigenvalues and eigenvectors of $\mathbf{S}_1 \cdot \mathbf{S}_2$ can be found from $\mathbf{S}_1 \cdot \mathbf{S}_2 = (1/2)(\mathbf{S}^2 - (3/2))$ by

$$\begin{aligned} E_1 = \frac{1}{2} &\Longrightarrow \left\{ |\phi_1\rangle = |11\rangle,\ |\phi_2\rangle = |00\rangle,\ |\phi_3\rangle = \frac{1}{\sqrt{2}}|01\rangle + |10\rangle \right\}, \\ E_2 = -\frac{3}{4} &\Longrightarrow |\phi_4\rangle = \frac{1}{\sqrt{2}}|01\rangle - |10\rangle. \end{aligned} \tag{3.4}$$

A quantum Controlled-Not (CN) gate can be given by the sequence of operations

$$U_{\mathrm{CN}} = e^{i(\pi/2)S_1^z} e^{-i(\pi/2)S_2^z} U_{\mathrm{sw}}^{1/2} e^{i\pi S_1^z} U_{\mathrm{sw}}^{1/2}, \tag{3.5}$$

where U_{sw} is an ideal swap operator which can exchange the quantum states of qubits 1 and 2 and is determined generally by an evolution operator $U_s(J(\tau))$ by adjusting the coupling time between the two spins in the evolution of the system. For the particular spin-spin coupling duration τ_s, where $\int_0^{\tau_s} J(t)dt = \pi \pmod{2\pi}$, $U_{\mathrm{sw}} = U_s(\pi)$, and the swap operator which can exchange the quantum states of qubits 1 and 2 is given by [44]

$$U_{\mathrm{sw}} = e^{-i\int_0^{\tau_s}(H_S(\tau))d\tau} = \sum_{n=1}^{3} e^{-i(\pi/4)} \left|\phi_n\right\rangle\left\langle\phi_n\right| + e^{i(3\pi/4)} \left|\phi_4\right\rangle\left\langle\phi_4\right|. \tag{3.6}$$

In the presence of the environment, the nonideal action of the swap operation must be considered because it may introduce decoherence in the ideal swap operation. Here the environment is assumed to consist of a set of two-level particles randomly embedded in an environment B. This is a pure dephasing model whose Hamiltonian is given by $H_B = \sum_k \omega_k \sigma_k^z$, and the Hamiltonian coupling the two-qubit spin system is

$$\lambda H_{\mathrm{int}} = \sum_k \left(\sigma_1^z + \sigma_2^z\right)\left(g_k \sigma_k^+ + g_k^* \sigma_k^-\right), \tag{3.7}$$

where σ_k^+ (σ_k^-) is raising/lowering operator for the kth two-level particle, characterized by a generally complex coupling parameter g_k. Then, the Hamiltonian for the total system is $H(t) = H_S(t) + H_B + \lambda H_{\mathrm{int}}$, and the corresponding complete set of eigenvectors for $H_S(t) + H_B$ is denoted as $\{|\{k\} \otimes \phi_j\rangle, \langle\phi_j \otimes \{k\}|\}$.

To control the induced decoherence, we choose the time-independent eigen-projectors of $H_S(t) + H_B$ as

$$P_{nk} \equiv \left|\varphi_{nk}\right\rangle\left\langle\varphi_{nk}\right| = \left|\phi_n\right\rangle\left\langle\phi_n\right| \otimes |\{k\}\rangle\langle\{k\}| \tag{3.8}$$

and Q_{nk} as $Q_{nk} + P_{nk} = 1$, $n = 1, \ldots, 4$. Using the definition of the eigen-projectors P_{nk}, the spectral decomposition of the intermediate operator Θ is given by two cases:

$$\Theta(t) = \sum_{n=1}^{4} \left\langle\varphi_{nk}\right|\left(H_S(t) + H_B\right)\left|\varphi_{nk}\right\rangle P_{nk}, \quad \text{without the interaction,} \tag{3.9}$$

$$\begin{aligned}\Theta(t) = {} & \sum_{n=1}^{4} \left\langle\varphi_{nk}\right|\left(H_S(t) + H_B\right)\left|\varphi_{nk}\right\rangle P_{nk} \\ & + \left\langle\varphi_{1k}\right|\lambda H_{\mathrm{int}} Q_{1k} \frac{1}{E_{1k}(t) - Q_{1k}H(t)Q_{1k}} Q_{1k}\lambda H_{\mathrm{int}}\left|\varphi_{1k}\right\rangle P_{1k} \\ & + \left\langle\varphi_{2k}\right|\lambda H_{\mathrm{int}} Q_{2k} \frac{1}{E_{2k}(t) - Q_{2k}H(t)Q_{2k}} Q_{2k}\lambda H_{\mathrm{int}}\left|\varphi_{2k}\right\rangle P_{2k}, \quad \text{with the interaction,}\end{aligned} \tag{3.10}$$

where $E_{1k}(t)$ and $E_{2k}(t)$ are the eigenvalues of the total Hamiltonian, corresponding to the eigen-projectors of $\Theta(t)$, P_{1k}, and P_{2k}, respectively. This shows that the eigen-projectors P_{nk} of $\Theta(t)$ are invariant and independent of the interaction in the constructed subspace; however, for $n = 1, 2$, the eigenvalues are changed (index n is still diagonal to H_{int}, while index k is off-diagonal to H_{int}), which may introduce a phase shift in the evolution operator as a kind of decoherence.

For canceling this phase shift, we consider the triangular decomposition of the H_{int} as $H_{\text{int}}^{\text{utri}} + H_{\text{int}}^{\text{dtri}}$, where $H_{\text{int}}^{\text{utri}}$ is upper-triangular part of H_{int} and $H_{\text{int}}^{\text{dtri}}$ is lower-triangular part of H_{int}. Then it is easy to find that

$$\begin{aligned}
&\langle\varphi_{nk}|\left(H_{\text{int}}^{\text{utri}} + H_{\text{int}}^{\text{dtri}}\right)\frac{1}{E_{nk}(t) - Q_{nk}H(t)Q_{nk}}\left(H_{\text{int}}^{\text{utri}} + H_{\text{int}}^{\text{dtri}}\right)|\varphi_{nk}\rangle P_{nk} \\
&\quad = \langle\varphi_{nk}|H_{\text{int}}^{\text{utri}}\frac{1}{E_{nk}(t) - Q_{nk}H(t)Q_{nk}}H_{\text{int}}^{\text{dtri}}|\varphi_{nk}\rangle P_{nk} \\
&\quad\quad + \langle\varphi_{nk}|H_{\text{int}}^{\text{dtri}}\frac{1}{E_{nk}(t) - Q_{nk}H(t)Q_{nk}}H_{\text{int}}^{\text{utri}}|\varphi_{nk}\rangle P_{nk},
\end{aligned} \tag{3.11}$$

which shows that $H_{\text{int}}^{\text{utri}}$ part and $H_{\text{int}}^{\text{dtri}}$ part are to be moved out in the upper-triangular or lower-triangular subspace $\Phi_{\text{utri}}(\Phi_{\text{dtri}})$, respectively. Here the upper-triangular (or lower-triangular) subspace $\Phi_{\text{utri}}(\Phi_{\text{dtri}})$ can be defined by introducing upper-triangular (or lower-triangular) projector and upper-triangular (or lower-triangular) inner product given by $P_{kk'} = |k\rangle\langle k'|$, $k' \le k$ (or $k' \ge k$), and $\langle k|A|k'\rangle$, for any operator A with $k \le k'$ (or $k' \ge k$). For instance, any operator A defined in this upper-triangular subspace with respect to the index k can only be represented as $A = \sum_{k' \ge k}\langle k|A|k'\rangle|k\rangle\langle k'|$. Therefore we can construct the DF upper-triangular (or lower-triangular) subspace Φ_{utri} with respect to the environmental index k and, in the same time, enable the Hilbert space $\mathscr{H}_S$ to be invariant for the quantum computing system S, that is, $\mathscr{H}_S \otimes \Phi_{\text{utri}}$, in which one can allow the interaction terms in (3.10) to be zero

$$\langle\varphi_{jk}|\lambda H_{\text{int}}Q_{jk}\frac{1}{E_{jk}(t) - Q_{jk}H(t)Q_{jk}}Q_{jk}\lambda H_{\text{int}}|\varphi_{jk}\rangle P_{jk} = 0, \quad \text{for } j = 1, 2. \tag{3.12}$$

This demonstrates that the constructed intermediate operator $\Theta(t)$ on the upper-triangular subspace $\mathscr{H}_S \otimes \Phi_{\text{utri}}$ is independent upon the interaction part of the original Hamiltonian H_{int}; consequently the phase shift introduced by H_{int} is canceled in the subspaces although the total system experiences the decoherence introduced by H_{int}.

In the upper-triangular subspace $\mathscr{H}_S \otimes \Phi_{\text{utri}}$, the quantum Controlled-Not logic operation $e^{i(\pi/2)S_1^z}e^{-i(\pi/2)S_2^z}U_{\text{sw}}^{1/2}e^{i\pi S_1^z}U_{\text{sw}}^{1/2}$ can be executed by using a sequence of operations and its relevant swap operator U_{sw} remains invariant before and after interaction from the environment. This can be described by the following formula:

$$\begin{aligned}
&Tr_B e^{-i\int_0^{\tau_s}\Theta(\tau)d\tau} = Tr_B e^{-i\int_0^{\tau_s}(H_S(\tau)+H_B)d\tau} \\
&\quad = \left[\sum_{n=1}^{3} e^{-i(\pi/4)}|\phi_n\rangle\langle\phi_n| + e^{i(3\pi/4)}|\phi_4\rangle\langle\phi_4|\right]\sum_k\left[e^{-i(-a^2\omega_k+\omega_{k+1})/(-a^2+1)} + e^{-i(-a^2\omega_{k+1}+\omega_k)/(-a^2+1)}\right].
\end{aligned} \tag{3.13}$$

The above formalism can be extended to a general dephasing situation, for instance, a register of N two-level elements immersed in a quantized environment, whose Hamiltonian is described by

$$H = \sum_{n=0}^{N-1} \varepsilon \sigma_z^{(n)} + \sum_k \omega_k b_k^\dagger b_k + \sum_{n=0}^{N-1} \sum_k \sigma_z^{(n)} \left(\chi_{kn} b_k^\dagger + \chi_{kn}^* b_k \right). \tag{3.14}$$

From the preceding study, the procedure to construct the upper-triangular subspaces is simple, that is, if one performs the relevant quantum logical operations using the state $\phi(0)$, then one should perform the normal inner product with respect to the index n for the quantum computing system and the upper-triangular inner product with respect to the index k for the environment, and remember that the evolution of the states is described by $\exp(-i\Theta(t)t)\phi(0)$ in the upper-triangular subspace. The general considerations for the above model are given below.

4. General DF Subspace Constructed by Triangulation

Different models treat the interaction of the system with its environments quite differently; here, we propose a general procedure to construct a DF subspace by triangulation. Suppose that the states used in quantum computing system are the eigenvectors of the free Hamiltonian $H_0 = H_S(t) + H_B$, then the corresponding matrix of the Hamiltonian H_0 is diagonal. Thus the spectral decomposition for the intermediate operator Θ based on the above subdynamics formalism can be given by $\Theta = Tr_k(\sum_{n,k} E_n(t) P_{nk}) = H_S(t)$, where P_{nk} is an eigen-projector of H_0, and $E_n(t)$ is an eigenvalue of H_S.

Now, if the system is subject to decoherence induced from the environment by (a general) H_{int}, then (suppose) the matrix of Hamiltonian becomes off-diagonal to the index introduced from environment (and is diagonal to the index from the original system). Thus, one can construct the upper-triangular subspace $\mathscr{H}_s \otimes \Phi_{\text{utri}}$ by defining the rule of upper-triangular inner product in this space such that $\langle \varphi_{nk} | H_{\text{int}} | \varphi_{n'k'} \rangle \neq 0$, for $k \leq k'$; $\langle \varphi_{nk} | H_{\text{int}} | \varphi_{n'k'} \rangle = 0$, only for the index related to the environment $k > k'$. This leads to an upper-triangular matrix of H_{int}. Using this upper-triangular property, the interaction terms in the Θ operator should be zero $\sum_{k' \geq k} P_{nk} H_{\text{int}} P_{n'k'} C_{nk} P_{nk} = 0$; thus, the spectral decomposition of $\Theta = H_S$ maintains invariance, that is, $\Theta = Tr_k(\sum_{n,k} E_n(t) P_{nk} + 0) = H_S$, where P_{nk} is now chosen as an upper-triangular eigen-projector with respect to k.

It may be necessary to emphasize that the states required to perform quantum computing in the DF (triangular) subspace are just the projected states $\psi^{\text{proj}}(t)$ which can be spanned by $\{\varphi_{nk} = \phi_n \otimes \psi_k\}$. The projected states are measurable in this subspace since $\{\varphi_{nk} = \phi_n \otimes \psi_k\}$ is orthogonal (and triangular with k) and distinguishable, where the triangular inner product only is valid to the index which is related to the environment; here it means k, while for an other index relevant to the original system, such as n, the normal inner product is still used. Finally, one may ask whether the rule of the upper-triangular inner product in this space restricts or changes the original Controlled-Not logic operation and results in errors. The answer is no, as one can see there is no influence on decoherence to

the quantum Controlled-Not logic operation in (3.13). The term relative to the environment

$$\sum_{k}\left[e^{-i(-a^2\omega_k+\omega_{k+1})/(-a^2+1)}+e^{-i((-a^2\omega_{k+1}+\omega_k)/(-a^2+1))}\right] \tag{4.1}$$

now cannot influence the original quantum Controlled-Not logic operation

$$\left[\sum_{n=1}^{3}e^{-i(\pi/4)}\left|\phi_n\right\rangle\left\langle\phi_n\right|+e^{i(3\pi/4)}\left|\phi_4\right\rangle\left\langle\phi_4\right|\right]. \tag{4.2}$$

The entanglement between the system and environment is canceled in the constructed partial triangular subspace although it exists indeed in the original total space or even in the subspace with the normal inner product. The role of upper-triangular inner product with respect to the index from environment in the subspace is only to cancel the decoherence (phase shift) from the environment. Then, how can one realize the above procedure in the practical procedure for the quantum computing? We suppose to establish an additional measure or count system to read or calculate some dates, such as the eigenvalues and eigenvectors from the original system, and transfer the relevant dates to the expression in the frame of the partial triangular subspace based on (2.2) and the rule of the partial triangular product. Here the key is to establish a transformation system for the constructed partial triangular subspace, which allows the eigenvalues and eigenvectors from the original system to be expressed in the partial triangular subspace.

5. Conclusions and Remarks

A scheme for quantum computing in the DF triangular subspaces is presented. The DF subspaces are ruled by the subdynamic kinetic equation (SKE). The used quantum computing states in the DF subspaces are just the projected states $\phi_{\mathrm{proj}}(t)$ in the DF triangular subspaces. Moreover, this DF subspace is partial upper-triangular, in which an inner product is only upper-triangulated to index k corresponding to environment. That means, in a quantum computing process, that if one takes the upper-triangular inner product with respect to k and keeps the ordinary inner product to be invariance with respect to the index related to original quantum computing system, then the decoherence in the subdynamic spaces can be completely cancelled. The Markovian and non-Markovian decoherence and collective decoherence as studied in recent publications seem not to present any restrictions on the DF subspace considered here, and this proposal may be useful generally for the register of N two-level elements immersed in a quantized environment.

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