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Research Article

Research on Mechanical Behavior of Viscoelastic Food Material in the Mode of Compressed Chewing

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The degeneration mechanism of viscoelastic food material in specific processing mode affects the formulation of food material processing technology. On the other hand, it determines the taste of food in the chewing process. The viscoelastic food material was taken as the research object, and experimental data were obtained through stress relaxation experiments and strain relaxation experiments of texture analyzer material. Based on Maxwell model and Kelvin model, describing small deformation of the nonlinear viscoelastic constitutive model, building a composite model was proposed. By making analysis and comparison between constructed composite model and Maxwell model and Kelvin model, it was verified that the constructed composite model can be better described as the mechanical behavior of viscoelastic food material under the mode of compressed chewing, which is also providing a more precise theoretical model for the processing and development of viscoelastic food material.

1. Introduction

Viscoelastic material is widely used, and the majority of food materials and industrial materials, such as toothpaste and paint, particularly those in the chewing mouth, mostly behaved as viscoelastic [1, 2]. In the process of preparation, even the final consumption, the mechanical behavior of viscoelastic food material not only affects the processing technology [3, 4] but also determines the perception of chewing [5].

In order to reflect the influence of various processing and testing parameters on viscoelastic material and accurately describe the stress-strain relation of material [6–8], many researchers had done a lot of research and proposed various calculation models [9–11]. The basic constitutive model of the viscoelastic material mainly included Maxwell model and Kelvin model. Both Maxwell model and Kelvin model supposed that the viscoelastic material was formed by solid mixed with liquid, showing the common characteristics of solid and liquid [12–14]. That is to say, the viscoelastic material showed characteristics of both spring and damping.

Maxwell model and Kelvin model could basically explain the mechanical behavior of most of viscoelastic material in a variety of loading modes, namely, the relationship between strain and stress. But to a certain particularized specific material, in specific loading mode (compression, tension, shear, and so on), these models could not accurately explain the mechanical behavior, which was required to construct a more precise model of stress and strain of material [15, 16].

Taking a particular food material as the experimental object, a composite model was constructed based on Maxwell model and Kelvin model. It verified that the model could better explain the mechanical behavior of material in the mode of compression compared to the Maxwell model and Kelvin model by stress-strain relaxation experiments of the viscoelastic material. In a word, the accuracy of stress-strain model of the material should be more high. The method had guiding significance on constructing stress-strain model of other viscoelastic materials in compressed mode.

2. Experimental Material and Experimental Method

Experimental material is 10 copies of French roll of the same original material, shape, and size produced by Fujian Dali

Food Group Company. Experimental instrument is TMS-PRO professional grade food texture analyzer produced by the FTC of America [17, 18]. The experiments were carried out at room temperature of 25°C.

The strain relaxation experiments and stress relaxation experiments were done under the situation of fixed deformation and fixed stress on experimental material. Open the Texture Lab Pro software, call, respectively, the fixed deformation and fixed stress program, and install the probe rightly; choosing the TMS 38.1 mm Perspex, the starting force was 0.01 N, the pretest velocity and the testing velocity were 10 mm/s, the fixed stress was set to 1 N, the deformation was set to 10%, the sample size is 20 mm × 20 mm × 25 mm, and the test time is 10 minutes.

3. Constructed Model of Viscoelastic Material

3.1. Constitutive Model of Viscoelastic Material. Maxwell model and Kelvin model are the main models to describe the typical mechanical behavior of viscoelastic material. Maxwell model supposed that the viscoelastic material was equivalent to a spring and a damping element in series, as was shown in Figure 1; as the two elements were connected in series, the subjected stress was equal, and the following could be obtained:

$$G\varepsilon(t) = \mu \frac{d\varepsilon(t)}{dt} = \sigma(t). \tag{1}$$

While the total deformations were the sum of the two strains, which could obtain the constitutive equation of Maxwell model,

$$\frac{1}{G}\frac{d\sigma(t)}{dt} + \frac{\sigma(t)}{\mu} = \frac{d\varepsilon(t)}{dt}. \tag{2}$$

In formula (2), $\varepsilon(t)$ is stress of viscoelastic material; $\sigma(t)$ is strain of viscoelastic material; G is elastic modulus of spring; and μ is viscosity of damping element.

Similarly, the constitutive equation of Kelvin model can be obtained:

$$G\varepsilon(t) + \mu \frac{d\varepsilon(t)}{dt} = \sigma(t). \tag{3}$$

In formula (3), the meaning of $\varepsilon(t)$, $\sigma(t)$, G, and μ was the same as the above marked.

3.1.1. Analysis on Stress Relaxation and Creepage of Maxwell Model and Kelvin Model

Analysis on Maxwell Model. In the case of creepage, $\sigma(t) = \sigma_0$, formula (2) turned to

$$\frac{d\varepsilon(t)}{dt} = \frac{\sigma_0}{\mu}. \tag{4}$$

In the case of stress relaxation, $d\varepsilon(t)/dt = 0$, formula (2) turned to

$$\frac{1}{G}\frac{d\sigma(t)}{dt} + \frac{\sigma(t)}{\mu} = 0. \tag{5}$$

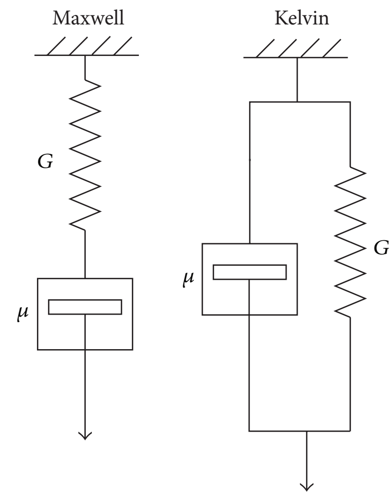

Figure 1: Maxwell model and Kelvin model.

Obtaining the equation of stress changed with time after integration,

$$\sigma(t) = ae^{-(1/b)t} + c. \tag{6}$$

Among it, a, b, and c are constants, and $b = \mu/G$, $c = 0$.

As was shown in formula (4), strain changed uniformly with time in Maxwell model in the case of creepage, which did not match with the actual situation of the viscoelastic material. Formula (6) suggested that the stress decayed exponentially with time, but it did not meet the general situation of viscoelastic material, as the creepage behavior of the material was very complicated. Furthermore, the actual stress relaxation behavior of viscoelastic material could not decay to zero for a long time. In a word, a simple exponential decayed item could not represent the real situation of creepage behavior.

Analysis on Kelvin Model. In the case of stress relaxation, formula (3) turned to

$$G\varepsilon(t) = \sigma(t). \tag{7}$$

It only showed the mechanical behavior of spring element, fully met with Hooke's law, which did not accord with the actual situation of viscoelastic material, which also suggested that the situation of stress relaxation was not suitable for this formula.

In the case of creepage, $\sigma(t) = \sigma_0$, formula (3) turned to

$$\frac{d\varepsilon(t)}{dt} = \frac{\sigma_0}{\mu} - \frac{G}{\mu}\varepsilon(t). \tag{8}$$

After integration of both sides of the above formula, obtain the equation in the following form:

$$\varepsilon(t) = de^{-1/f^t} + g. \tag{9}$$

Among it, $d = g = \sigma_0/G$.

As the initial deformation is zero, it could be obtained:

$$\varepsilon(t) = d\left(1 - e^{-1/f^t}\right). \tag{10}$$

Among it, the meaning of the parameters was the same as the above.

As was shown in formulas (7) and (8), Kelvin model could better describe the creepage property of viscoelastic material but could not exactly describe the relaxation property of viscoelastic materials.

3.1.2. Constructing a Custom Composite Model. Composite model referred to be connected by multiple Maxwell models, Kelvin model, and spring in parallel, the study was showed, composite model integrated the advantages of the above two models, and in accordance with the following constitutive relation formula [15],

$$\sigma(t) + \sum_{m=1}^{M} a_m \frac{d^m \sigma(t)}{dt^m} = G\varepsilon(t) + \sum_{n=1}^{N} b_n \frac{d^n \varepsilon(t)}{dt^n}. \quad (11)$$

At $M = 2$, $N = 2$ (two Maxwell models and a spring connected in parallel), the above formula could be expressed as

$$\sigma(t) + a_1 \frac{d\sigma(t)}{dt} + a_2 \frac{d^2\sigma(t)}{dt^2} = G\varepsilon(t) + b_1 \frac{d\varepsilon(t)}{dt} + b_2 \frac{d^2\varepsilon(t)}{dt^2}. \quad (12)$$

As could be seen, along with the number of Maxwell models and springs in parallel, more unknown quantities were introduced and required more groups of data; solving was also becoming very difficult. So this model was constructed by selection of formula (12), as was shown in Figure 2.

In the case of creepage ($d\varepsilon(t)/dt = 0$, $\varepsilon = \varepsilon_0$), it could be concluded that

$$\sigma(t) + a_1 \frac{d\sigma(t)}{dt} + a_2 \frac{d^2\sigma(t)}{dt^2} = G\varepsilon_0. \quad (13)$$

In the case of stress relaxation ($\sigma(t) = \sigma_0$), it could be concluded that

$$\sigma_0 = G\varepsilon(t) + b_1 \frac{d\varepsilon(t)}{dt} + b_2 \frac{d^2\varepsilon(t)}{dt^2}. \quad (14)$$

3.2. Model Validation. According to Maxwell model, fitting only needs to obtain the data of fixed strain, the fitting results were as shown in Figure 3, and the formula of function fitting relationship was as follows:

$$\sigma(t) = ae^{(1/-b)t} + c = 0.45418e^{(1/-22.66264)t}. \quad (15)$$

Substituting $\sigma(t)$ into formula (4), the relationship formula of $\varepsilon(t)$ was obtained. The model ratio could only obtain the ratio of μ and G, which was 22.66264, and it could not solve out the specific numerical values of μ and G.

According to the Kelvin model, fitting only used the data obtained from fixed stress, the fitting result was as shown in Figure 4, and the formula of function relationship obtained by origin fitting was

$$\varepsilon(t) = ae^{(1/-b)t} + c = -2.79136e^{(1/-4.52342)t} - 27.0611. \quad (16)$$

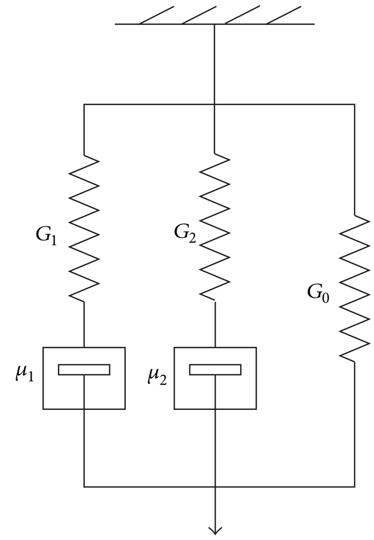

Figure 2: Constructed composite model.

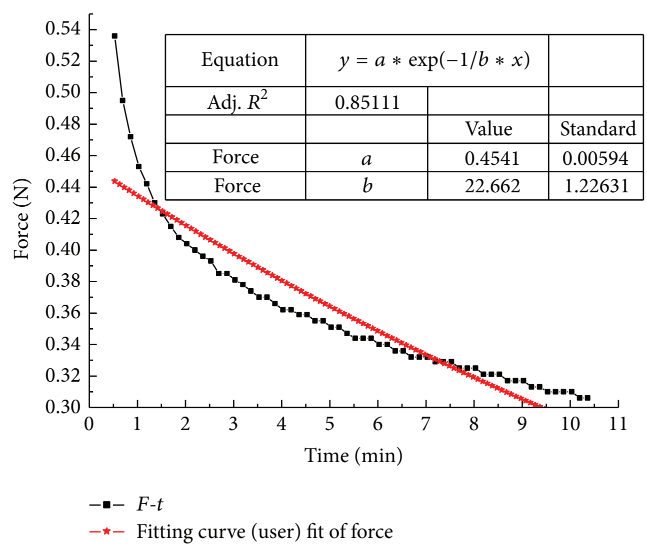

Figure 3: *F-t* fitting curve of Maxwell model.

After solving,

$$G = 0.01667, \qquad \mu = 0.07539. \quad (17)$$

The fitting curves of the constructed model were shown as in Figures 5 and 6, obtaining the following formula:

$$\sigma(t) = 0.32375e^{(1/-0.44747)t} + 0.17208e^{(1/-6.07104)t} - 0.27647,$$
$$\varepsilon(t) = -1.55703e^{(1/-0.44747)t} - 2.72363e^{(1/-6.07104)t} - 26.81475. \quad (18)$$

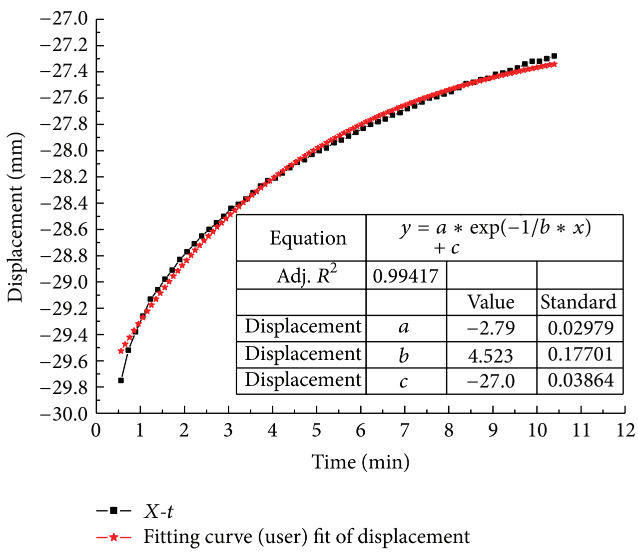

FIGURE 4: X-t fitting curve of Kelvin model.

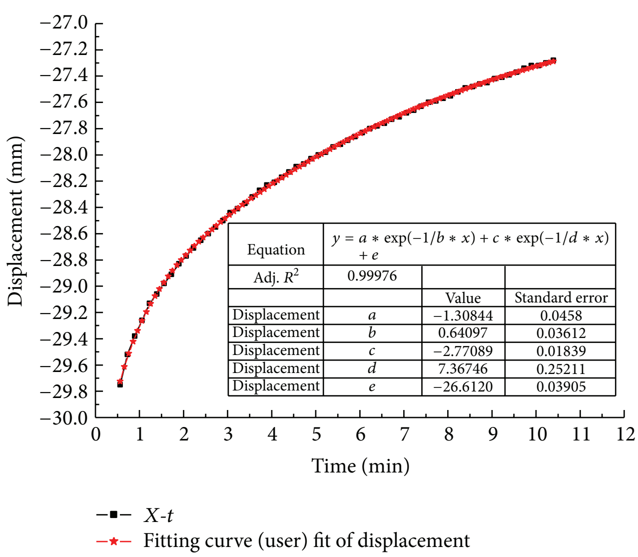

FIGURE 5: X-t fitting curve of constructed composite model.

After solving,

$$a_1 = 0.84793, \qquad a_2 = 7.5463 \times 10^{-4},$$
$$b_1 = 0.10864, \qquad b_2 = 7.54614 \times 10^{-4}, \qquad (19)$$
$$G = 0.01031.$$

The value of R represented the fitting effect; through the comparison of R values, it was calculated, relative to the one-sided description of Maxwell model and Kelvin model of viscoelastic food material; the fitting result of constructed model was the best, while the Maxwell model was the worst; the next was Kelvin model.

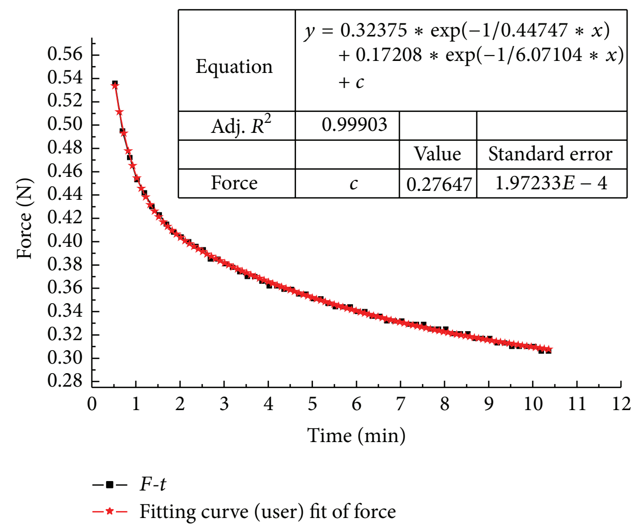

FIGURE 6: F-t fitting curve of constructed model.

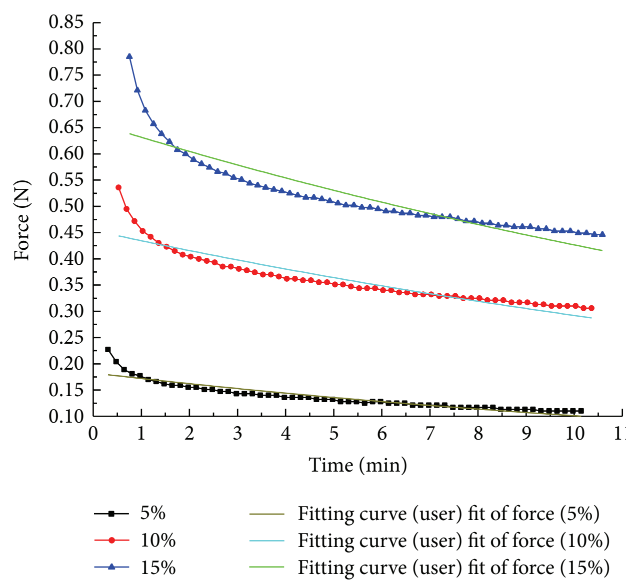

FIGURE 7: Fitting curves of Maxwell model in different compression ratio.

In the different compression ratio (5%, 10%, and 15%), fitting cases of Maxwell model and constructed model were further studied, as shown in Figures 7 and 8; the fitting correlation coefficients in the compression ratio of 5%, 10%, and 15% of Maxwell model were 0.85813, 0.85111, and 0.81914; the fitting effect was getting more and more worse with the increasing deformation; the fitting correlation coefficients in the compression ratio of 5%, 10%, and 15% of constructed model were 0.99714, 0.99896, and 0.99895; in the case of increasing deformation, the value of R was close to 1 and the fitting effect was better.

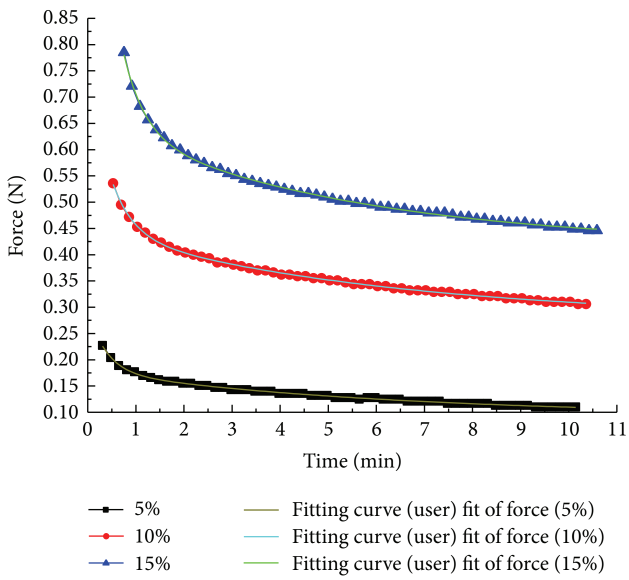

Figure 8: Fitting curves of constructed model in different compression ratio.

4. Conclusion

In this paper, through the stress-strain relaxation experiments of French roll produced by Fujian Dali Food Group Company, separately by Maxwell model, Kelvin model, and custom composite model, we studied mechanical behavior in the mode of compressed chewing, and established the stress-strain model in the mode of compressed chewing.

(1) In explaining the mechanical behavior of the viscoelastic food material under large deformation, the accuracy of Maxwell model and Kelvin model was not high.

(2) The constructed custom model through the stress relaxation experiments and strain relaxation experiments, compared to Maxwell model and Kelvin model, had a higher accuracy, which was better able to explain the mechanical behavior of this bread in the mode of chewing with large deformation.

(3) The method of constructed composite model was also applicable to construct the stress-strain model of other viscoelastic materials under different modes of masticatory, providing a more accurate theoretical model for the formulating of processing technology and selection of processing parameters of viscoelastic material.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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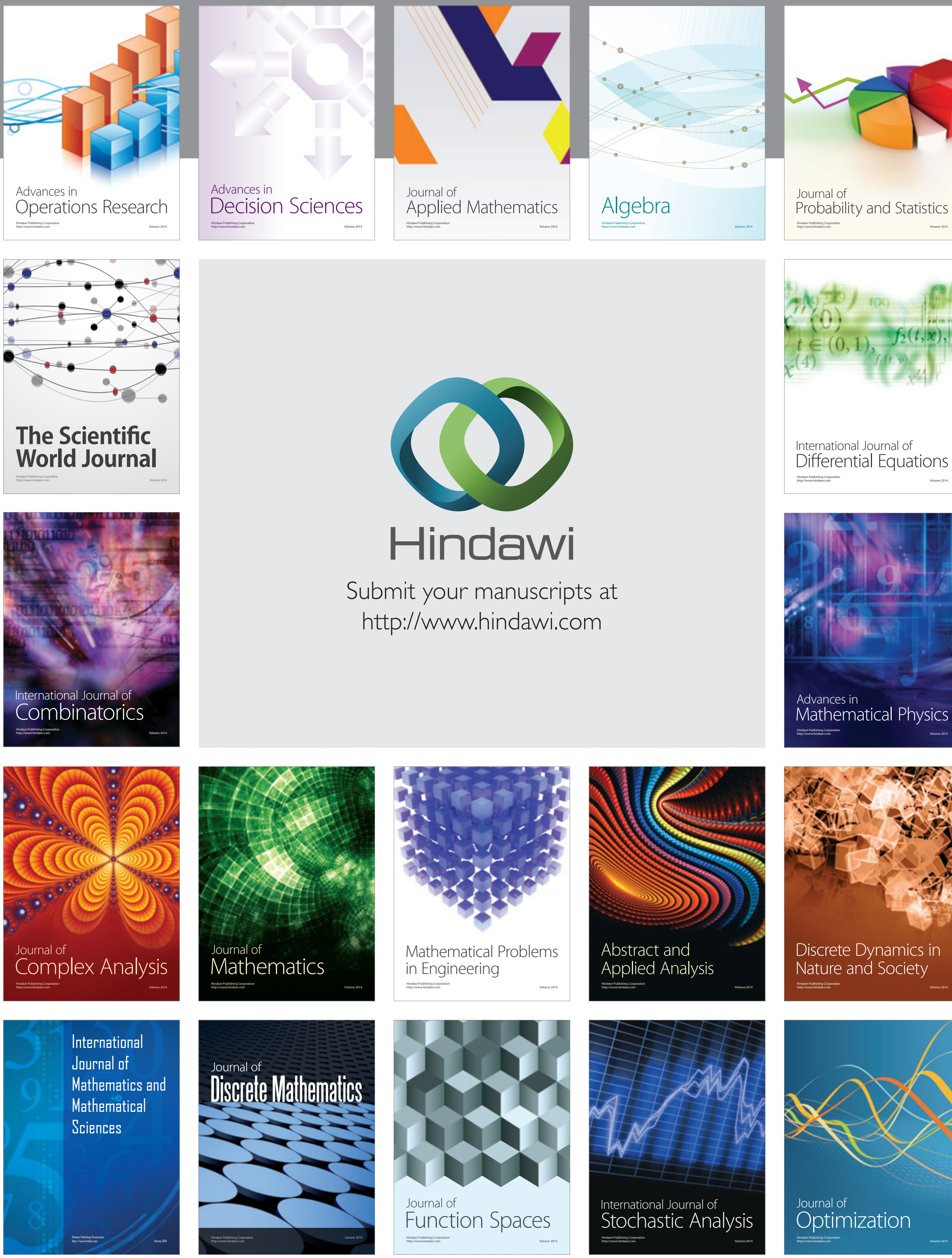
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