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Research Article

State-Feedback Stabilization for a Class of Stochastic Feedforward Nonlinear Time-Delay Systems

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We investigate the state-feedback stabilization problem for a class of stochastic feedforward nonlinear time-delay systems. By using the homogeneous domination approach and choosing an appropriate Lyapunov-Krasovskii functional, the delay-independent state-feedback controller is explicitly constructed such that the closed-loop system is globally asymptotically stable in probability. A simulation example is provided to demonstrate the effectiveness of the proposed design method.

1. Introduction

In recent years, the study on stochastic lower-triangular nonlinear systems has received considerable attention from both theoretical and practical point of views see, for instance, [1–19] and the references therein. This paper will further consider the following stochastic feedforward nonlinear time-delay systems described by

$$\begin{aligned}
dx_1 &= x_2 dt + f_1(\tilde{x}_3, \tilde{x}_3(t-d(t)))\,dt \\
&\quad + g_1^T(\tilde{x}_2, \tilde{x}_2(t-d(t)))\,d\omega, \\
&\;\vdots \\
dx_{n-2} &= x_{n-1} dt + f_{n-2}(\tilde{x}_n, \tilde{x}_n(t-d(t)))\,dt \\
&\quad + g_{n-2}^T(\tilde{x}_{n-1}, \tilde{x}_{n-1}(t-d(t)))\,d\omega, \\
dx_{n-1} &= x_n dt + g_{n-1}^T(\tilde{x}_n, \tilde{x}_n(t-d(t)))\,d\omega, \\
dx_n &= u\,dt,
\end{aligned} \tag{1}$$

where $x = (x_1, \ldots, x_n)^T \in R^n$ and $u \in R$ are the system state and input signal, respectively, $\tilde{x}_i = (x_i, \ldots, x_n)^T$, $\tilde{x}_i(t-d(t)) = (x_i(t-d(t)), \ldots, x_n(t-d(t)))^T$ is the time-delayed state vector, and $d(t) : R_+ \to [0, d]$ is the time-varying delay. ω is an m-dimensional standard Wiener process defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ with Ω being a sample space, $\mathcal{F}$ being a σ-field, $\{\mathcal{F}_t\}_{t\geq 0}$ being a filtration, and P being a probability measure. $f_i : R^{n-i-1} \times R^{n-i-1} \to R$ and $g_j : R^{n-j} \times R^{n-j} \to R^m$ are assumed to be locally Lipschitz with $f_i(0,0) = 0$ and $g_j(0,0) = 0$, $i = 1, \ldots, n-2$, $j = 1, \ldots, n-1$.

Feedforward (also called upper-triangular) system is another important class of nonlinear systems. Firstly, from a theoretical point of view, since they are not feedback linearizable and maybe not stabilized by applying the conventional backstepping method, the stabilization problem of these systems is more difficult than that of lower-triangular systems. Secondly, many physical devices, such as the cart-pendulum system in [20] and the ball-beam system with a friction term in [21], can be described by equations with the feedforward structure. In the recent papers, the stabilization problems for feedforward nonlinear (or time-delay) systems have achieved remarkable development; see, for example, [22–29] and the references therein.

However, all these above-mentioned results are limited to deterministic systems. There are fewer results on stochastic feedforward nonlinear systems until now, due to the special characteristics of this system. To the best of the authors' knowledge, [30] is the only paper to consider this kind of stochastic feedforward nonlinear systems, but the assumptions on the nonlinearities are restrictive.

The purpose of this paper is to further weaken the assumptions on the drift and diffusion terms of system (1) and solve the state-feedback stabilization problem. By using the homogeneous domination approach in [26] and choosing an appropriate Lyapunov-Krasovskii functional, a delay-independent state-feedback controller is explicitly constructed such that the closed-loop system is globally asymptotically stable in probability.

The paper is organized as follows. Section 2 provides some preliminary results. The design and analysis of state-feedback controller are given in Sections 3 and 4, respectively, following a simulation example in Section 5. Section 6 concludes this paper.

2. Preliminary Results

The following notations, definitions, and lemmas are to be used throughout the paper.

R_+ denotes the set of all nonnegative real numbers, and R^n denotes the real n-dimensional space. For a given vector or matrix X, X^T denotes its transpose, $\mathrm{Tr}\{X\}$ denotes its trace when X is square, and $|X|$ is the Euclidean norm of a vector X. $\mathscr{C}([-d,0];R^n)$ denotes the space of continuous R^n-value functions on $[-d,0]$ endowed with the norm $\|\cdot\|$ defined by $\|f\| = \sup_{x\in[-d,0]}|f(x)|$ for $f \in \mathscr{C}([-d,0];R^n)$; $\mathscr{C}^b_{\mathscr{F}_0}([-d,0];R^n)$ denotes the family of all $\mathscr{F}_0$-measurable bounded $\mathscr{C}([-d,0];R^n)$-valued random variables $\xi = \{\xi(\theta) : -d \le \theta \le 0\}$. $\mathscr{C}^i$ denotes the set of all functions with continuous ith partial derivatives; $\mathscr{C}^{2,1}(R^n \times [-d,\infty);R_+)$ denotes the family of all nonnegative functions $V(x,t)$ on $R^n \times [-d,\infty)$ which are $\mathscr{C}^2$ in x and $\mathscr{C}^1$ in t; $\mathscr{C}^{2,1}$ denotes the family of all functions which are $\mathscr{C}^2$ in the first argument and $\mathscr{C}^1$ in the second argument. $\mathscr{K}$ denotes the set of all functions $R_+ \to R_+$, which are continuous, strictly increasing, and vanishing at zero; $\mathscr{K}_\infty$ denotes the set of all functions which are of class $\mathscr{K}$ and unbounded; $\mathscr{KL}$ is the set of all functions $\beta(s,t)$: $R_+ \times R_+ \to R_+$, which are of $\mathscr{K}$ for each fixed t and decrease to zero as $t \to \infty$ for each fixed s.

Consider the following stochastic time-delay system:

$$dx(t) = f(x(t), x(t-d(t)), t)\,dt + g(x(t), x(t-d(t)), t)\,d\omega, \quad \forall t \ge 0, \tag{2}$$

with initial data $\{x(\theta) : -d \le \theta \le 0\} = \xi \in \mathscr{C}^b_{\mathscr{F}_0}([-d,0];R^n)$, where $d(t) : R_+ \to [0,d]$ is a Borel measurable function, ω is an m-dimensional standard Wiener process defined on the complete probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\ge0}, P)$, and $f : R^n \times R^n \times R_+ \to R^n$ and $g : R^n \times R^n \times R_+ \to R^{n\times m}$ are locally Lipschitz in $(x(t), x(t-d(t)))$ uniformly in t with $f(0,0,t) \equiv 0$ and $g(0,0,t) \equiv 0$.

Definition 1 (see [6]). For any given $V(x(t),t) \in \mathscr{C}^{2,1}$ associated with system (2), the differential operator $\mathscr{L}$ is defined as

$$\mathscr{L}V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2}\mathrm{Tr}\left\{g^T \frac{\partial^2 V}{\partial x^2} g\right\}. \tag{3}$$

Definition 2 (see [6]). The equilibrium $x(t) = 0$ of system (2) is said to be globally asymptotically stable (GAS) in probability if for any $\epsilon > 0$ there exists a function $\beta(\cdot,\cdot) \in \mathscr{KL}$ such that $P\{|x(t)| \le \beta(\|\xi\|, t)\} \ge 1-\epsilon$ for any $t \ge 0$, $\xi \in \mathscr{C}^b_{\mathscr{F}_0}([-d,0];R^n) \setminus \{0\}$, where $\|\xi\| = \sup_{\theta\in[-d,0]}|x(\theta)|$.

Definition 3 (see [26]). For fixed coordinates $(x_1,\ldots,x_n)^T \in R^n$ and real numbers $r_i > 0, i = 1,\ldots,n$, one has the following.

(i) The dilation $\Delta_\varepsilon(x)$ is defined by $\Delta_\varepsilon(x) = (\varepsilon^{r_1}x_1,\ldots,\varepsilon^{r_n}x_n)$ for any $\varepsilon > 0$; $r_1,\ldots,r_n$ are called as the weights of the coordinates. For simplicity, we define dilation weight $\Delta = (r_1,\ldots,r_n)$.

(ii) A function $V \in \mathscr{C}(R^n,R)$ is said to be homogeneous of degree τ if there is a real number $\tau \in R$ such that $V(\Delta_\varepsilon(x)) = \varepsilon^\tau V(x_1,\ldots,x_n)$ for any $x \in R^n\setminus\{0\}, \varepsilon > 0$.

(iii) A vector field $h \in \mathscr{C}(R^n,R^n)$ is said to be homogeneous of degree τ if there is a real number $\tau \in R$ such that $h_i(\Delta_\varepsilon(x)) = \varepsilon^{\tau+r_i}h_i(x)$ for any $x \in R^n \setminus \{0\}, \varepsilon > 0$, $i = 1,\ldots,n$.

(iv) A homogeneous p-norm is defined as $\|x\|_{\Delta,p} = (\sum_{i=1}^n |x_i|^{p/r_i})^{1/p}$ for any $x \in R^n$, where $p \ge 1$ is a constant. For simplicity, in this paper, one chooses $p = 2$ and writes $\|x\|_\Delta$ for $\|x\|_{\Delta,2}$.

Lemma 4 (see [6]). *For system* (2), *if there exist a function* $V(x(t),t) \in \mathscr{C}^{2,1}(R^n \times [-d,\infty);R_+)$, *two class* $\mathscr{K}_\infty$ *functions* α_1, α_2, *and a class* $\mathscr{K}$ *function* α_3 *such that*

$$\alpha_1(|x(t)|) \le V(x(t),t) \le \alpha_2\left(\sup_{-d\le s\le0}|x(t+s)|\right),$$
$$\mathscr{L}V(x(t),t) \le -\alpha_3(|x(t)|), \tag{4}$$

then there exists a unique solution on $[-d,\infty)$ *for* (2), *the equilibrium* $x(t) = 0$ *is GAS in probability, and* $P\{\lim_{t\to\infty}|x(t)| = 0\} = 1$.

Lemma 5 (see [26]). *Given a dilation weight* $\Delta = (r_1,\ldots,r_n)$, *suppose that* $V_1(x)$ *and* $V_2(x)$ *are homogeneous functions of degrees* τ_1 *and* τ_2, *respectively. Then* $V_1(x)V_2(x)$ *is also homogeneous with respect to the same dilation weight* Δ. *Moreover, the homogeneous degree of* $V_1 \cdot V_2$ *is* $\tau_1 + \tau_2$.

Lemma 6 (see [26]). *Suppose that* $V : R^n \to R$ *is a homogeneous function of degree* τ *with respect to the dilation weight* Δ; *then (i)* $\partial V/\partial x_i$ *is homogeneous of degree* $\tau - r_i$ *with* r_i *being the homogeneous weight of* x_i; *(ii) there is a constant* c *such that* $V(x) \le c\|x\|^\tau_\Delta$. *Moreover, if* $V(x)$ *is positive definite, then* $V(x) \ge \underline{c}\|x\|^\tau_\Delta$, *where* $\underline{c}$ *is a positive constant.*

Lemma 7 (see [5]). *Let c and d be positive constants. For any positive number $\overline{\gamma}$, then $|x|^c|y|^d \leq (c/(c+d))\overline{\gamma}|x|^{c+d} + (d/(c+d))\overline{\gamma}^{-c/d}|y|^{c+d}$.*

3. Design of State-Feedback Controller

The objective of this paper is to design a state-feedback controller for system (1) such that the equilibrium of the closed-loop system is globally asymptotically stable in probability.

3.1. Assumptions. For system (1), we need the following assumptions.

Assumption 8. For $i = 1, \ldots, n-1$, there exist positive constants a_1 and a_2 such that

$$\begin{aligned} &\left|f_i\left(\widetilde{x}_{i+2}, \widetilde{x}_{i+2}\left(t-d\left(t\right)\right)\right)\right| \\ &\quad \leq a_1\left(\sum_{j=i+2}^{n}\left|x_j\right| + \sum_{j=i+2}^{n}\left|x_j\left(t-d\left(t\right)\right)\right|\right), \\ &\left|g_i\left(\widetilde{x}_{i+1}, \widetilde{x}_{i+1}\left(t-d\left(t\right)\right)\right)\right| \\ &\quad \leq a_2\left(\sum_{j=i+1}^{n}\left|x_j\right| + \sum_{j=i+1}^{n}\left|x_j\left(t-d\left(t\right)\right)\right|\right), \end{aligned} \tag{5}$$

where $x_{n+1} = x_{n+1}(t-d(t)) = 0$.

Assumption 9. The time-varying delay $d(t)$ satisfies $\dot{d}(t) \leq \gamma < 1$ for a constant γ.

Remark 10. When $x_{i+1} = x_{i+1}(t-d(t)) = 0$ in diffusion term g_i $(i = 1, \ldots, n-1)$, Assumption 8 reduces to the same form as in [30], from which one can see that system (1) is more general than [30]. The significance and reasonability of Assumption 8 are illustrated in that paper.

Firstly, we introduce the following coordinate transformation:

$$\eta_i = \frac{x_i}{\kappa^{i-1}}, \quad \nu = \frac{u}{\kappa^n}, \quad i = 1, \ldots, n, \tag{6}$$

where $0 < \kappa < 1$ is a scalar to be designed. By (6), (1) can be expressed as

$$\begin{aligned} d\eta_1 &= \kappa\eta_2 dt + \overline{f}_1\left(\widetilde{\eta}_3, \widetilde{\eta}_3\left(t-d\left(t\right)\right)\right)dt \\ &\quad + \overline{g}_1^T\left(\widetilde{\eta}_2, \widetilde{\eta}_2\left(t-d\left(t\right)\right)\right)d\omega, \\ &\;\;\vdots \\ d\eta_{n-2} &= \kappa\eta_{n-1}dt + \overline{f}_{n-2}\left(\widetilde{\eta}_n, \widetilde{\eta}_n\left(t-d\left(t\right)\right)\right)dt \\ &\quad + \overline{g}_{n-2}^T\left(\widetilde{\eta}_{n-1}, \widetilde{\eta}_{n-1}\left(t-d\left(t\right)\right)\right)d\omega, \\ d\eta_{n-1} &= \kappa\eta_n dt + \overline{g}_{n-1}^T\left(\widetilde{\eta}_n, \widetilde{\eta}_n\left(t-d\left(t\right)\right)\right)d\omega, \\ d\eta_n &= \kappa\nu dt, \end{aligned} \tag{7}$$

where $\overline{f}_i = f_i/\kappa^{i-1}$, $\overline{g}_i = g_i/\kappa^{i-1}$, $i = 1, \ldots, n-1$, $\overline{f}_{n-1} = 0$.

3.2. State-Feedback Controller Design. We construct a state-feedback controller for system (7).

Step 1. Introducing $\xi_1 = \eta_1$ and choosing $V_1(\eta_1) = (1/4)\xi_1^4$, from (3) and (7), it follows that

$$\mathscr{L}V_1 = \kappa\xi_1^3\eta_2 + \frac{\partial V_1}{\partial\eta_1}\overline{f}_1 + \frac{1}{2}\operatorname{Tr}\left\{\overline{g}_1\frac{\partial^2 V_1}{\partial\eta_1^2}\overline{g}_1^T\right\}. \tag{8}$$

The first virtual controller

$$\eta_2^* = -c_{11}\xi_1 =: -\alpha_1\xi_1, \quad c_{11} > 0, \tag{9}$$

leads to $\mathscr{L}V_1 \leq -\kappa c_{11}\xi_1^4 + \kappa\xi_1^3(\eta_2 - \eta_2^*) + (\partial V_1/\partial\eta_1)\overline{f}_1 + (1/2)\operatorname{Tr}\{\overline{g}_1(\partial^2 V_1/\partial\eta_1^2)\overline{g}_1^T\}$.

Step i ($i = 2, \ldots, n$). In this step, we can get the following lemma.

Lemma 11. *Suppose that at step $i-1$, there is a set of virtual controllers $\eta_1^*, \ldots, \eta_i^*$ defined by*

$$\begin{aligned} \eta_1^* &= 0, \qquad \xi_1 = \eta_1 - \eta_1^* = \eta_1, \\ \eta_k^* &= -\alpha_{k-1}\xi_{k-1}, \quad \xi_k = \eta_k - \eta_k^*, \quad k = 2, \ldots, i, \end{aligned} \tag{10}$$

such that the $(i-1)$th Lyapunov function $V_{i-1}(\overline{\eta}_{i-1}) = (1/4)\sum_{j=1}^{i-1}\xi_j^4$ satisfies

$$\begin{aligned} \mathscr{L}V_{i-1} &\leq -\kappa\sum_{j=1}^{i-1}c_{i-1,j}\xi_j^4 + \kappa\xi_{i-1}^3\left(\eta_i - \eta_i^*\right) \\ &\quad + \sum_{j=1}^{i-1}\frac{\partial V_{i-1}}{\partial\eta_j}\overline{f}_j + \frac{1}{2}\sum_{p,q=1}^{i-1}\operatorname{Tr}\left\{\overline{g}_p\frac{\partial^2 V_{i-1}}{\partial\eta_p\partial\eta_q}\overline{g}_q^T\right\}, \end{aligned} \tag{11}$$

where α_j, $c_{i-1,j}$, $j = 1, \ldots, i-1$, are positive constants. Then there exists a virtual control law $\eta_{i+1}^* = -\alpha_i\xi_i$ such that

$$\begin{aligned} \mathscr{L}V_i &\leq -\kappa\sum_{j=1}^{i}c_{ij}\xi_j^4 + \kappa\xi_i^3\left(\eta_{i+1} - \eta_{i+1}^*\right) \\ &\quad + \sum_{j=1}^{i}\frac{\partial V_i}{\partial\eta_j}\overline{f}_j + \frac{1}{2}\sum_{p,q=1}^{i}\operatorname{Tr}\left\{\overline{g}_p\frac{\partial^2 V_i}{\partial\eta_p\partial\eta_q}\overline{g}_q^T\right\}, \end{aligned} \tag{12}$$

where $V_i(\overline{\eta}_i) = (1/4)\sum_{j=1}^{i}\xi_j^4 =: V_{i-1}(\overline{\eta}_{i-1}) + W_i(\overline{\eta}_i)$.

Proof. See the Appendix. □

At step n, choosing $V_n(\overline{\eta}_n) = (1/4)\sum_{i=1}^{n}\xi_i^4$ and

$$\begin{aligned} \nu &= \eta_{n+1}^* = -\alpha_n\xi_n \\ &= -\left(\overline{\alpha}_n\eta_n + \overline{\alpha}_{n-1}\eta_{n-1} + \cdots + \overline{\alpha}_1\eta_1\right), \end{aligned} \tag{13}$$

with the help of (3), (12), and (13), one obtains

$$\begin{aligned}\mathscr{L}V_n &\leq -\kappa\sum_{i=1}^{n} c_{ni}\xi_i^4 + \kappa\xi_n^3\left(v - \eta_{n+1}^*\right) \\ &\quad + \sum_{i=1}^{n}\frac{\partial V_n}{\partial \eta_i}\overline{f}_i + \frac{1}{2}\sum_{p,q=1}^{n}\operatorname{Tr}\left\{\overline{g}_p\frac{\partial^2 V_n}{\partial \eta_p \partial \eta_q}\overline{g}_q^T\right\} \\ &= -\kappa\sum_{i=1}^{n} c_{ni}\xi_i^4 + \frac{\partial V_n}{\partial \eta}F + \frac{1}{2}\operatorname{Tr}\left\{G\frac{\partial^2 V_n}{\partial \eta^2}G^T\right\},\end{aligned} \tag{14}$$

where $F = (\overline{f}_1, \ldots, \overline{f}_{n-2}, 0, 0)^T$, $G = (\overline{g}_1, \ldots, \overline{g}_{n-1}, 0)$, $\xi_n = \eta_n - \eta_n^*$, $\overline{\alpha}_i = \alpha_n \cdots \alpha_i$, c_{ni}, $i = 1, \ldots, n$, are positive constants. The system (7) and (13) can be written as

$$\begin{aligned} d\eta &= \kappa E(\eta)\,dt + F(\eta, \eta(t - d(t)))\,dt \\ &\quad + G^T(\eta, \eta(t - d(t)))\,d\omega,\end{aligned} \tag{15}$$

where $\eta = \overline{\eta}_n = (\eta_1, \ldots, \eta_n)^T$, $E(\eta) = (\eta_2, \ldots, \eta_n, v)^T$, and F and G are defined as in (14). Introducing the dilation weight $\Delta = (\underbrace{1, 1, \ldots, 1}_{\text{for } \eta_1, \ldots, \eta_n})$, by (10) and $V_n(\eta) = (1/4)\sum_{i=1}^{n}\xi_i^4$, one has

$$\begin{aligned} V_n(\Delta_\varepsilon(\eta)) &= \frac{1}{4}\sum_{i=1}^{n}\left(\varepsilon\eta_i + \alpha_{i-1}\varepsilon\eta_{i-1} + \cdots + \alpha_{i-1}\cdots\alpha_1\varepsilon\eta_1\right)^4 \\ &= \varepsilon^4 V_n(\eta),\end{aligned} \tag{16}$$

from which and Definition 3, we know that $V_n(\eta)$ is homogeneous of degree 4.

4. Stability Analysis

We state the main result in this paper.

Theorem 12. *If Assumptions 8 and 9 hold for the stochastic feedforward nonlinear time-delay system* (1)*, under the state-feedback controller $u = \kappa^n v$ and* (13)*, then*

(i) *the closed-loop system has a unique solution on $[-d, \infty)$;*

(ii) *the equilibrium at the origin of the closed-loop system is GAS in probability.*

Proof. We prove Theorem 12 by four steps.

Step 1. Since f_i, g_i, $i = 1, \ldots, n$, are assumed to be locally Lipschitz, so the system consisting of (13) and (15) satisfies the locally Lipschitz condition.

Step 2. We consider the following entire Lyapunov function for system (15):

$$V(\eta) = V_n(\eta) + \frac{(\overline{c}_{02} + \overline{c}_{03})\kappa^2}{1 - \gamma}\int_{t-d(t)}^{t}\|\eta(\sigma)\|_\Delta^4\,d\sigma, \tag{17}$$

where $\overline{c}_{02}$ and $\overline{c}_{03}$ are positive parameters to be determined. It is easy to verify that $V(\eta)$ is $\mathscr{C}^2$ on η. Since $V_n(\eta)$ is continuous, positive definite, and radially unbounded, by Lemma 4.3 in [31], there exist two class $\mathscr{K}_\infty$ functions β_1 and α_{21} such that

$$\beta_1(|\eta|) \leq V_n(\eta) \leq \alpha_{21}(|\eta|). \tag{18}$$

By Lemma 4.3 in [31] and Lemma 6, there exist positive constants $\underline{c}$ and $\overline{c}$, class $\mathscr{K}_\infty$ functions $\underline{\alpha}_{22}$ and $\overline{\alpha}_{22}$, and a positive definite function $U(\eta)$ whose homogeneous degree is 4 such that

$$\begin{gathered}\underline{c}\|\eta\|_\Delta^4 \leq U(\eta) \leq \overline{c}\|\eta\|_\Delta^4, \\ \underline{\alpha}_{22}(|\eta|) \leq U(\eta) \leq \overline{\alpha}_{22}(|\eta|).\end{gathered} \tag{19}$$

From $d(t) : R_+ \to [0, d]$ and (19), it follows that

$$\begin{aligned} &\frac{(\overline{c}_{02} + \overline{c}_{03})\kappa^2}{1 - \gamma}\int_{t-d(t)}^{t}\|\eta(\sigma)\|_\Delta^4\,d\sigma \\ &\quad\leq \tilde{c}\int_{t-d(t)}^{t}\overline{\alpha}_{22}(|\eta(\sigma)|)\,d\sigma \\ &\quad\overset{\sigma=s+t}{=} \tilde{c}\int_{-d(t)}^{0}\overline{\alpha}_{22}(|\eta(s+t)|)\,d(s+t) \\ &\quad\leq \tilde{c}\int_{-d}^{0}\overline{\alpha}_{22}(|\eta(s+t)|)\,d(s+t) \\ &\quad\leq c\sup_{-d\leq s\leq 0}\overline{\alpha}_{22}(|\eta(s+t)|) \\ &\quad\leq \alpha_{22}\left(\sup_{-d\leq s\leq 0}|\eta(s+t)|\right),\end{aligned} \tag{20}$$

where $\tilde{c}$, c are positive constants and α_{22} is a class $\mathscr{K}_\infty$ function. Since $|\eta| \leq \sup_{-d\leq s\leq 0}|\eta(s+t)|$, $\alpha_{21}(|\eta|) \leq \alpha_{21}(\sup_{-d\leq s\leq 0}|\eta(s+t)|)$. Defining $\beta_2 = \alpha_{21} + \alpha_{22}$, by (17), (18), and (20), one gets

$$\beta_1(|\eta|) \leq V(\eta) \leq \beta_2\left(\sup_{-d\leq s\leq 0}|\eta(s+t)|\right). \tag{21}$$

Step 3. By Lemma 6 and (14), there exists a positive constant c_{01} such that

$$\frac{\partial V_n}{\partial \eta}\kappa E(\eta) \leq -c_{01}\kappa\|\eta\|_\Delta^4. \tag{22}$$

By Assumption 8, (6), and $0 < \kappa < 1$, one has

$$\begin{aligned} &\left|\overline{f}_i\left(\tilde{\eta}_{i+2}, \tilde{\eta}_{i+2}(t - d(t))\right)\right| \\ &\quad\leq \frac{a_1\left(\sum_{j=i+2}^{n}\left|\kappa^{j-1}\eta_j\right| + \sum_{j=i+2}^{n}\left|\kappa^{j-1}\eta_j(t - d(t))\right|\right)}{\kappa^{i-1}} \\ &\quad\leq a_1\kappa^2\left(\sum_{j=i+2}^{n}\left|\eta_j\right| + \sum_{j=i+2}^{n}\left|\eta_j(t - d(t))\right|\right) \\ &\quad\leq \delta_1\kappa^2\left(\|\eta\|_\Delta + \|\eta(t - d(t))\|_\Delta\right),\end{aligned} \tag{23}$$

where δ_1 is a positive constant. Using Lemmas 5–7 and (23), one gets

$$\begin{aligned}\frac{\partial V_n}{\partial \eta}F(\eta,\eta(t-d(t))) &= \sum_{i=1}^{n-2}\frac{\partial V_n}{\partial \eta_i}\overline{f}_i(\tilde{\eta}_{i+2},\tilde{\eta}_{i+2}(t-d(t))) \\ &\leq \tilde{c}_{02}\kappa^2\sum_{i=1}^{n-2}\|\eta\|_\Delta^3(\|\eta\|_\Delta+\|\eta(t-d(t))\|_\Delta) \\ &\leq \kappa^2\left(c_{02}\|\eta\|_\Delta^4+\overline{c}_{02}\|\eta(t-d(t))\|_\Delta^4\right),\end{aligned} \tag{24}$$

where c_{02}, $\overline{c}_{02}$, and $\tilde{c}_{02}$ are positive constants. Similar to (23), there is a positive constant δ_2 such that

$$|\overline{g}_i(\tilde{\eta}_{i+1},\tilde{\eta}_{i+1}(t-d(t)))| \leq \delta_2\kappa(\|\eta\|_\Delta+\|\eta(t-d(t))\|_\Delta), \tag{25}$$

from which and Lemmas 5–7, one gets

$$\begin{aligned}&\frac{1}{2}\operatorname{Tr}\left\{G(\eta,\eta(t-d(t)))\frac{\partial^2 V_n}{\partial \eta^2}G^T(\eta,\eta(t-d(t)))\right\} \\ &\leq \frac{1}{2}m\sqrt{m}\sum_{i,j=1}^{n-1}\left|\frac{\partial^2 V_n}{\partial\eta_i\partial\eta_j}\right||\overline{g}_i(\tilde{\eta}_{i+1},\tilde{\eta}_{i+1}(t-d(t)))| \\ &\qquad\times\left|\overline{g}_j(\tilde{\eta}_{j+1},\tilde{\eta}_{j+1}(t-d(t)))\right| \\ &\leq \tilde{c}_{03}\kappa^2\sum_{i,j=1}^{n-1}\|\eta\|_\Delta^2(\|\eta\|_\Delta+\|\eta(t-d(t))\|_\Delta)^2 \\ &\leq \kappa^2\left(c_{03}\|\eta\|_\Delta^4+\overline{c}_{03}\|\eta(t-d(t))\|_\Delta^4\right),\end{aligned} \tag{26}$$

where c_{03}, $\overline{c}_{03}$, and $\tilde{c}_{03}$ are positive constants. With the help of (3), (15), (17), (22), (24), (26), and Assumption 9, one has

$$\begin{aligned}\mathscr{L}V &\leq \frac{\partial V_n}{\partial\eta}\kappa E(\eta)+\frac{\partial V_n}{\partial\eta}F(\eta,\eta(t-d(t))) \\ &\quad+\frac{1}{2}\operatorname{Tr}\left\{G(\eta,\eta(t-d(t)))\frac{\partial^2 V_n}{\partial\eta^2}G^T(\eta,\eta(t-d(t)))\right\} \\ &\quad+(\overline{c}_{02}+\overline{c}_{03})\kappa^2\left(\frac{1}{1-\gamma}\|\eta\|_\Delta^4-\|\eta(t-d(t))\|_\Delta^4\right) \\ &\leq -c_{01}\kappa\|\eta\|_\Delta^4+\left(c_{02}+c_{03}+\frac{\overline{c}_{02}+\overline{c}_{03}}{1-\gamma}\right)\kappa^2\|\eta\|_\Delta^4 \\ &= -\kappa\left(c_{01}-\left(c_{02}+c_{03}+\frac{\overline{c}_{02}+\overline{c}_{03}}{1-\gamma}\right)\kappa\right)\|\eta\|_\Delta^4.\end{aligned} \tag{27}$$

Since c_{01} is a constant independent of c_{02}, c_{03}, $\overline{c}_{02}$, $\overline{c}_{03}$, and γ, by choosing

$$0<\kappa<\kappa^* =: \min\left\{1,\frac{c_{01}}{c_{02}+c_{03}+((\overline{c}_{02}+\overline{c}_{03})/(1-\gamma))}\right\}. \tag{28}$$

Equation (27) becomes $\mathscr{L}V \leq -c_0\|\eta\|_\Delta^4$, where c_0 is a positive constant. By (19), one obtains

$$\mathscr{L}V \leq -\frac{c_0}{c}\underline{\alpha}_{22}(|\eta|). \tag{29}$$

By Steps 1–3 and Lemma 4, the system consisting of (13) and (15) has a unique solution on $[-d,\infty)$, $\eta=0$ is GAS in probability, and $P\{\lim_{t\to\infty}|\eta|=0\}=1$.

Step 4. Since (6) is an equivalent transformation, so the closed-loop system consisting of (1), $u=\kappa^n v$, and (13) has the same properties as the system (13) and (15). Theorem 12 holds. □

Remark 13. In this paper, the homogeneous domination idea is generalized to stochastic feedforward nonlinear time-delay systems (1). The underlying philosophy of this approach is that the state-feedback controller is first constructed for system (7) without considering the drift and diffusion terms, and then a low gain κ in (6) (whose the value range is (28)) is introduced to state-feedback controller to dominate the drift and diffusion terms.

Remark 14. Due to the special upper-triangular structure and the appearance of time-varying delay, there is no efficient method to solve the stabilization problem of system (1). By combining the homogeneous domination approach with stochastic nonlinear time-delay system criterion, the state-feedback stabilization of system (1) was perfectly solved in this paper.

Remark 15. One of the main obstacles in the stability analysis is how to deal with the effect of time-varying delay. In this paper, by constructing an appropriate Lyapunov-Krasovskii functional (17), this problem was effectively solved.

Remark 16. It is worth pointing out that the rigorous proof of Theorem 12 is not an easy job.

5. A Simulation Example

Consider the following stochastic nonlinear system:

$$\begin{aligned}dx_1 &= x_2dt+\frac{1}{10}(x_2+x_2(t-d(t))\cos x_2)\,d\omega, \\ dx_2 &= udt,\end{aligned} \tag{30}$$

where $d(t) = 1+(1/2)\sin t$. It is easy to verify that Assumptions 8 and 9 are satisfied with $a_1=0$, $a_2=1/10$, and $\dot{d}(t)=(1/2)\cos t<1$.

Design of Controller. Introducing the following coordinate transformation:

$$\eta_1 = x_1,\qquad \eta_2=\frac{x_2}{\kappa},\qquad \nu=\frac{u}{\kappa^2}, \tag{31}$$

system (30) becomes

$$\begin{aligned}d\eta_1 &= \kappa\eta_2dt+\overline{g}_1d\omega, \\ d\eta_2 &= \kappa v dt,\end{aligned} \tag{32}$$

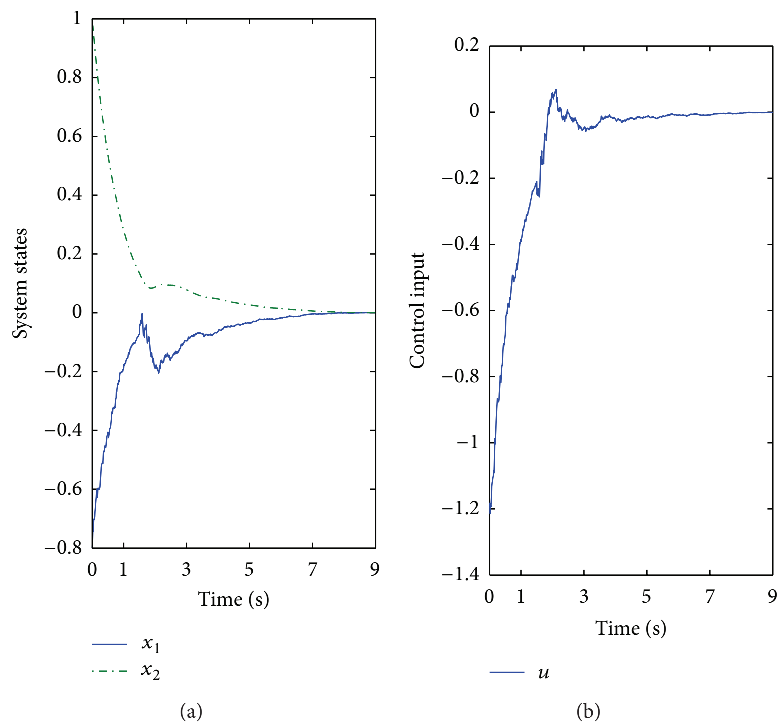

FIGURE 1: (a) The response of the closed-loop system (30) and (b) the response of the controller (37).

where $\overline{g}_1 = (1/10)(\kappa\eta_2 + \kappa\eta_2(t-d(t))\cos\kappa\eta_2)$. Choosing $\xi_1 = \eta_1$ and $V_1(\eta_1) = (1/4)\xi_1^4$, we obtain $\mathscr{L}V_1 \le -2\kappa\xi_1^4 + \kappa\xi_1^3(\eta_2 - \eta_2^*) + (1/2)(\partial^2 V_1/\partial\eta_1^2)\overline{g}_1^2$, where $\eta_2^* = -2\eta_1 =: -\alpha_1\xi_1$. By $\xi_2 = \eta_2 - \eta_2^*$ and $V_2(\overline{\eta}_2) = V_1(\eta_1) + (1/4)\xi_2^4$, a direct calculation leads to

$$\mathscr{L}V_2 \le -2\kappa\xi_1^4 + \kappa\xi_1^3\xi_2 + \kappa\xi_2^3 v + \kappa\alpha_1\xi_2^3\eta_2 + \frac{1}{2}\frac{\partial^2 V_2}{\partial\eta_1^2}\overline{g}_1^2. \quad (33)$$

By Lemma 7, one has

$$\xi_1^3\xi_2 \le 0.5\xi_1^4 + 0.8438\xi_2^4,$$
$$\alpha_1\xi_2^3\eta_2 \le 0.5\xi_1^4 + 5.7797\xi_2^4. \quad (34)$$

Choosing

$$v = -7.6235\xi_2 =: -\alpha_2\xi_2 \quad (35)$$

and substituting (34) into (33), one gets

$$\mathscr{L}V_2 \le -\kappa\left(\xi_1^4 + \xi_2^4\right) + \frac{1}{2}\frac{\partial^2 V_2}{\partial\eta_1^2}\overline{g}_1^2. \quad (36)$$

By (31) and (35), one obtains the actual controller

$$u = -\alpha_2\left(\kappa x_2 + \alpha_1\kappa^2 x_1\right). \quad (37)$$

The Choice of κ^*. Defining $\|\eta\|_\Delta = (\eta_1^2 + \eta_2^2)^{1/2}$ and choosing

$$V(\eta) = V_2(\eta) + \kappa^2\int_{t-d(t)}^{t}\|\eta(\sigma)\|_\Delta^4 d\sigma, \quad (38)$$

by (3), (36), and $d(t) = 1 + (1/2)\sin t$, one obtains

$$\begin{aligned}\mathscr{L}V &\le -\kappa\|\eta\|_\Delta^4 + \kappa^2\left(1.5\|\eta\|_\Delta^4 + 0.5\|\eta(t-d(t))\|_\Delta^4\right)\\ &\quad + \kappa^2\left(\|\eta\|_\Delta^4 - 0.5\|\eta(t-d(t))\|_\Delta^4\right)\\ &= -\kappa(1-2.5\kappa)\|\eta\|_\Delta^4,\end{aligned} \quad (39)$$

from which we get the critical value $\kappa^* = 0.4$; that is, $\kappa \in (0, 0.4)$.

In simulation, we choose the initial values $x_1(0) = -0.8$, $x_2(0) = 1$, and $\kappa = 0.3$. Figure 1 demonstrates the effectiveness of the state-feedback controller.

6. A Concluding Remark

By using the homogeneous domination approach, this paper further studied the state-feedback stabilization problem for a class of stochastic feedforward nonlinear time-delay systems (1). The delay-independent state-feedback controller is explicitly constructed such that the closed-loop system is globally asymptotically stable in probability.

There still exist some problems to be investigated. One is to consider the output-feedback control of switched stochastic system (1) by using average dwell time method in [32]. Another is to find a practical example (similar to [33–35]) for system (1). The last is to generalize the networked control systems (such as [36–41]) to stochastic feedforward networked systems.

Appendix

Proof of Lemma 11. According to (3), (7), (10), and (11), one has

$$
\begin{aligned}
\mathscr{L}V_i \leq & -\kappa\sum_{j=1}^{i-1} c_{i-1,j}\xi_j^4 + \kappa\xi_{i-1}^3\xi_i + \sum_{j=1}^{i-1}\frac{\partial V_{i-1}}{\partial \eta_j}\overline{f}_j \\
& + \frac{1}{2}\sum_{p,q=1}^{i-1}\operatorname{Tr}\left\{\overline{g}_p\frac{\partial^2 V_{i-1}}{\partial\eta_p\partial\eta_q}\overline{g}_q^T\right\} \\
& + \sum_{k=1}^{i}\frac{\partial W_i}{\partial \eta_k}\left(\kappa\eta_{k+1} + \overline{f}_k\right) + \frac{1}{2}\operatorname{Tr}\left\{\overline{g}_i\frac{\partial^2 W_i}{\partial\eta_i^2}\overline{g}_i^T\right\} \\
& + \sum_{j=1}^{i-1}\operatorname{Tr}\left\{\overline{g}_i\frac{\partial^2 W_i}{\partial\eta_i\partial\eta_j}\overline{g}_j^T\right\} + \frac{1}{2}\sum_{p,q=1}^{i-1}\operatorname{Tr}\left\{\overline{g}_p\frac{\partial^2 W_i}{\partial\eta_p\partial\eta_q}\overline{g}_q^T\right\} \\
= & -\kappa\sum_{j=1}^{i-1} c_{i-1,j}\xi_j^4 + \kappa\xi_{i-1}^3\xi_i + \frac{\partial W_i}{\partial\eta_i}\kappa\eta_{i+1} \\
& + \left(\sum_{j=1}^{i-1}\frac{\partial V_{i-1}}{\partial\eta_j}\overline{f}_j + \sum_{k=1}^{i}\frac{\partial W_i}{\partial\eta_k}\overline{f}_k\right) + \sum_{k=1}^{i-1}\frac{\partial W_i}{\partial\eta_k}\kappa\eta_{k+1} \\
& + \left(\frac{1}{2}\sum_{p,q=1}^{i-1}\operatorname{Tr}\left\{\overline{g}_p\frac{\partial^2 V_{i-1}}{\partial\eta_p\partial\eta_q}\overline{g}_q^T\right\} + \frac{1}{2}\operatorname{Tr}\left\{\overline{g}_i\frac{\partial^2 V_i}{\partial\eta_i^2}\overline{g}_i^T\right\}\right. \\
& \quad + \sum_{j=1}^{i-1}\operatorname{Tr}\left\{\overline{g}_i\frac{\partial^2 V_i}{\partial\eta_i\partial\eta_j}\overline{g}_j^T\right\} \\
& \quad \left. + \frac{1}{2}\sum_{p,q=1}^{i-1}\operatorname{Tr}\left\{\overline{g}_p\frac{\partial^2 W_i}{\partial\eta_p\partial\eta_q}\overline{g}_q^T\right\}\right) \\
= & -\kappa\sum_{j=1}^{i-1} c_{i-1,j}\xi_j^4 + \kappa\xi_i^3\eta_{i+1} + \sum_{j=1}^{i}\frac{\partial V_i}{\partial\eta_j}\overline{f}_j \\
& + \frac{1}{2}\sum_{p,q=1}^{i}\operatorname{Tr}\left\{\overline{g}_p\frac{\partial^2 V_i}{\partial\eta_p\partial\eta_q}\overline{g}_q^T\right\} \\
& + \kappa\xi_{i-1}^3\xi_i - \kappa\xi_i^3\sum_{k=1}^{i-1}\frac{\partial\eta_i^*}{\partial\eta_k}\eta_{k+1}.
\end{aligned} \tag{A.1}
$$

We concentrate on the last two terms on the right-hand side of (A.1).

Using (10) and Lemma 7, one obtains

$$
\begin{aligned}
\xi_{i-1}^3\xi_i &\leq l_{i,i-1,1}\xi_{i-1}^4 + \rho_{i1}\xi_i^4, \\
-\xi_i^3\sum_{k=1}^{i-1}\frac{\partial\eta_i^*}{\partial\eta_k}\eta_{k+1} & \\
&\leq \left|\xi_i\right|^3\left|\sum_{k=1}^{i-1}\alpha_{i-1}\cdots\alpha_k\left(\xi_{k+1}-\alpha_k\xi_k\right)\right| \\
&\leq \left|\xi_i\right|^3\left(\sum_{k=1}^{i-1}\left(\alpha_{i-1}\cdots\alpha_{k-1}+\alpha_{i-1}\cdots\alpha_{k+1}\alpha_k^2\right)\left|\xi_k\right|\right. \\
&\qquad \left. + \alpha_{i-1}\left|\xi_i\right|\right) \\
&\leq \sum_{k=1}^{i-1} l_{ik2}\xi_k^4 + \rho_{i2}\xi_i^4,
\end{aligned} \tag{A.2}
$$

where $l_{i,i-1,1}$, $l_{ik2}(k = 1,\ldots,i-1)$, ρ_{i1}, and ρ_{i2} are positive constants, $\alpha_0 = 0$.

Choosing

$$
\begin{aligned}
c_{ij} &= \begin{cases} c_{i-1,j} - l_{ij2} > 0, & j = 1,\ldots,i-2, \\ c_{i-1,i-1} - l_{i,i-1,1} - l_{i,i-1,2} > 0, & j = i-1, \end{cases} \\
\eta_{i+1}^* &= -\left(c_{ii} + \rho_{i1} + \rho_{i2}\right)\xi_i =: -\alpha_i\xi_i, \quad c_{ii} > 0,
\end{aligned} \tag{A.3}
$$

and substituting (A.2)-(A.3) into (A.1), one gets the desired result. □

Conflict of Interests

The authors declare that there is no conflict of interests.

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