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Research Article

Robust Reliable H_∞ Control for Nonlinear Stochastic Markovian Jump Systems

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The robust reliable H_∞ control problem for a class of nonlinear stochastic Markovian jump systems (NSMJSs) is investigated. The system under consideration includes Itô-type stochastic disturbance, Markovian jumps, as well as sector-bounded nonlinearities and norm-bounded stochastic nonlinearities. Our aim is to design a controller such that, for possible actuator failures, the closed-loop stochastic Markovian jump system is exponential mean-square stable with convergence rate α and disturbance attenuation γ. Based on the Lyapunov stability theory and Itô differential rule, together with LMIs techniques, a sufficient condition for stochastic systems is first established in Lemma 3. Then, using the lemma, the sufficient conditions of the solvability of the robust reliable H_∞ controller for linear SMJSs and NSMJSs are given. Finally, a numerical example is exploited to show the usefulness of the derived results.

1. Introduction

In the past few decades, Markovian jump systems (MJSs) have been considerably studied since this kind of hybrid systems consists of a number of subsystems and a switch signal, which includes applications in safety-critical and high-integrity systems (e.g., aircraft, chemical plants, nuclear power station, robotic manipulator systems, large-scale flexible structures for space stations such as antenna, and solar arrays) typically systems, which may experience abrupt changes in their structure, see, for example, [1] and the references therein. And now, some results of stability and stabilization for Itô type stochastic Markovian jump systems are also available in many papers, see, for example, [2–4] and the references therein.

The analysis and synthesis problems of Markovian jump systems (MJSs) or stochastic Markovian jump systems (SMJSs) have attracted plenty of attention from many researchers. Many important and remarkable achievements reasonable have obtained. If the control systems possess integrity against actuator and sensor failures, we called reliable control systems or fault-tolerant control systems [5]. Recently, the robust reliable control and filtering problems for time-delay systems or Markovian jump systems (MJSs) have attracted considerable attention, and several approaches have been developed, see, for example, [6–11] and the references therein. Via linear matrix inequalities (LMIs), the authors designed the robust reliable H_∞ controller for uncertain nonlinear systems [6]. In [7], for admissible uncertainties as well as actuator failures occurring among a prespecified subset of actuators, Zhang et al. studied the reliable dissipative control of Markovian jump impulsive systems. The reliable H_∞ control problem for discrete-time piecewise linear systems with infinite distributed delays have been investigated in [8]. Recently, the study of stochastic H_∞ filtering for the systems governed by stochastic Itô-type equations has attracted a great deal of attention, and Zhang and Chen [9] firstly solved the nonlinear stochastic delay-free H_∞ filtering problem by means of a stochastic bounded real lemma derived in [10]. The reliable H_∞ filtering problems for discrete time-delay systems with randomly occurred nonlinearities [11] and discrete time-delay Markovian jump systems with partly unknown transition probabilities [12] also has been studied, respectively. The reliable control problem for a class of Markovian jump systems with interval time-varying delays and stochastic failure is studied in [13]. In recent years, the research begins to focusing on robust reliable control problems for stochastic systems or stochastic switched nonlinear systems, see, for example, [14–16] and the references therein.

However, all the aforementioned results are mainly focusing on the reliable control and filtering problems of discrete-time-delay systems and Markovian jump systems. Up to now, to the best of the authors' knowledge, the robust reliable H_∞ control problem for nonlinear stochastic Markovian jump systems (NSMJSs) has not been fully investigated, which is an open problem and gives the motivation of our present investigation. In this paper, our aim is to design a robust reliable H_∞ controller for NSMJSs, such that the NSMJSs are globally mean exponential stable with convergence rate α and disturbance attenuation γ.

1.1. Notations

Throughout this paper, for symmetric matrices X and Y, the notation $X \geq Y$ (resp., $X > Y$) means that the Matrix X-Y is positive semidefinite (respectively, positive definite). I is an identity matrix with appropriate dimensions; the subscript "T" represents the Transposition. $E(\cdot)$ denotes the expectation operator with respect to some probability measure P. $\mathcal{L}_2[0,\infty)$ is the space of square integrable vector functions over $[0,\infty)$; let $(\Omega, \digamma, P)$ be a complete probability space which is relative to an increasing family $(\digamma_t)_{t>0}$ of σ algebras $(\digamma_t)_{t>0} \subset \digamma$, where Ω is the samples space, $\digamma$ is σ algebra of subsets of the sample space, and P is the probability measure on $\digamma$.$\|\cdot\|_{E_2} = \|E(\cdot)\|_2$, while $\|\cdot\|_2$ stands for the usual $\mathcal{L}_2[0,\infty)$ norm, R^n and $R^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. In this paper, we provide all spaces $\mathbb{K}^k, k \geq 1$ with the usual inner product $\langle\cdot,\cdot\rangle$ and its corresponding 2-norm$\|\cdot\|$. Let $L^2(\Omega, \mathbb{K}^k)$ denote the space of square-integrable $\mathbb{K}^k$-valued functions on the probability space $(\Omega, \digamma, P)$. For any $0 < T < \infty$, we write $[0,T]$ for the closure of the open interval $(0,T)$ in R and denote by $L_2^n([0,T]; L^2(\Omega, \mathbb{K}^k))$ the space of the nonanticipative stochastic processes $y(\cdot) = (y(\cdot))_{t\in[0,T]}$ with respect to $(\digamma_t)_{t\in[0,T]}$ satisfying $\|y(\cdot)\|^2_{L_2^n} = E(\int_0^T \|y(t)\|^2 \mathrm{d}t) = \int_0^T E(\|y(t)\|^2)\mathrm{d}t < \infty$. $V(x(t), t, r(t) = i) = V(x(t), t, i)$, $A(r(t) = i) = A_i$ $B(r(t) = i) = B_i$, $A_0(r(t) = i) = A_{0i}$, $B_0(r(t) = i) = B_{0i}$, $C(r(t) = i) = C_i$, $D(r(t) = i) = D_i$.

2. Problem Formulation and Failure Model

In this paper, we mainly consider the following nonlinear stochastic Markovian jump systems (NSMJSs) with actuator failures:

$$\begin{aligned} \mathrm{d}x(t) &= \Big[A(r(t))x(t) + B(r(t))u^f(t,r(t)) + E(r(t))v(t) + f(r(t),x(t))\Big]\mathrm{d}t \\ &\quad + \Big[C(r(t))x(t) + D(r(t))u^f(t,r(t)) + H(r(t))v(t) + g(r(t),x(t))\Big]\mathrm{d}w(t), \\ z(t) &= J(r(t))x(t), \\ x(t_0) &= x_0, \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is the system state, $u^f(t) \in R^l$ is the control input of actuator fault, $v(t) \in R^q$ is the exogenous disturbance input of the systems which belong to $\mathcal{L}_2[0,\infty)$, $z(t) \in R^r$ is the system control output, $w(t)$ is a zero mean real scalar Weiner processes on a probability space $(\Omega, \mathcal{F}, P)$ relative to an increase family $(\mathcal{F}_t)_{t>0}$ of σ algebras $(\mathcal{F}_t)_{t>0} \subset \mathcal{F}$. $A_i, B_i, E_i, C_i, D_i, F_i, H_i, J_i$ are the known real constant matrices with appropriate dimensions. Morever, we assume that

$$E(\mathrm{d}w(t)) = 0, \qquad E\Big((\mathrm{d}w(t))^2\Big) = \mathrm{d}t. \tag{2.2}$$

Let $r(t),\ t \geq 0$, be a right-continuous Markovian chain on the probability space taking values in a finite state space $S = 1, 2, \ldots, N$ with generator $\Gamma = (\lambda_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta) = j \mid r(t) = i\} = \begin{cases} \lambda_{ij}\Delta + o(\Delta) & \text{if } i \neq j, \\ 1 + \lambda_{ii}\Delta + o(\Delta) & \text{if } i = j, \end{cases} \tag{2.3}$$

where $\Delta > 0$. Here $\lambda_{ij} \geq 0$ is the transition rate from manner i to manner j, if $i \neq j$ while $\lambda_{ii} = -\sum_{j\neq i}\lambda_{ij}$. We assume that the Markovian chain $r(\cdot)$ is independent of the Wienner process $w(\cdot)$. It is well known that almost every sample path of $r(t)$ is a right-continuous step function with a finite number of simple jump in any finite subinterval of $R_+(:= [0, +\infty))$.

$f(\cdot,\cdot) : S \times R^n \rightarrow R^n$ is a unknown nonlinear function which describes the system nonlinearity satisfying the following sector-bounded conditions:

$$\left(f_i(x(t)) - T_{1i}x\right)^T \left(f_i(x(t)) - T_{2i}x\right) \leq 0, \quad i \in S, \tag{2.4}$$

$g(\cdot,\cdot) : S \times R^n \rightarrow R^n$ also is a unknown nonlinear function which describes the stochastic nonlinearity satisfying the following:

$$g_i^T(x(t))g_i(x(t)) \leq x^T G_i^T G_i x, \quad i \in S, \tag{2.5}$$

where $T_{1i},\ T_{2i},\ G_i$ are known real constant matrices with approximate dimensions.

Remark 2.1. The nonlinearities $f_i(x(t))$ are bounded by sectors, which belong to $[L_{1i}, L_{2i}]$, and are very general that include the usual Lipschitz conditions as a special case which is considerable investigated and includes several other classes well studied nonlinear systems [17–19]. The nonlinearities $g_i(x(t))$ satisfy the norm-bounded conditions.

When the actuator experiences failure, we use $u^f(t, r(t))$ to describe the control signal form actuators. Consider the following actuator failure model with failure parameter F_i:

$$u_i^f(t) = F_i u_i(t), \tag{2.6}$$

where F_i is the actuator fault matrix with

$$F_i = \operatorname{diag}(f_{i1}, f_{i2}, \dots, f_{im}), \quad 0 \le \underline{f}_{ij} \le f_{ij} \le \overline{f}_{ij}, \ \overline{f}_{ij} \ge 1, \ j = 1, 2, \dots, m. \tag{2.7}$$

In which the variables f_{ij} quantify the failures of the actuators. $f_{ij} = 0$ means that jth actuator completely fails, and $f_{ij} = 1$ means that the jth actuator is normal.

Define the following:

$$F_{0i} = \operatorname{diag}(f_{0i1}, f_{0i2}, \dots, f_{0im}) = \frac{\overline{F}_i + \underline{F}_i}{2}, \qquad f_{0ij} = \frac{\underline{f}_{ij} + \overline{f}_{ij}}{2}, \tag{2.8}$$

$$\widetilde{F}_{0i} = \operatorname{diag}\left(\widetilde{f}_{0i1}, \widetilde{f}_{0i2}, \dots, \widetilde{f}_{0im}\right) = \frac{\overline{F}_i - \underline{F}_i}{2}, \qquad f_{0ij} = \frac{\overline{f}_{ij} - \underline{f}_{ij}}{2}, \tag{2.9}$$

and hence, the matrix F_i can be rewritten as

$$F_i = F_{0i} + \Delta_i = F_{0i} + \operatorname{diag}(\varphi_{i1}, \varphi_{i2}, \dots, \varphi_{im}), \qquad |\varphi_{ij}| \le \widetilde{f}_{ij}, \quad j = 1, 2, \dots, m. \tag{2.10}$$

In this paper, our aim is to design the controller $u_i(t) = K_i x(t)$, $i \in S$, such that the closed-loop systems satisfy the following conditions:

(i) without the exogenous disturbance input (i.e., $v(t) = 0$), the closed-loop control systems (2.1) are globally exponentially stable with convergence rate $\alpha > 0$;

(ii) with zero initial condition (i.e., $x(t_0) = 0$) and nonzero exogenous disturbance input (i.e., $v(t) \ne 0$), the following inequality holds:

$$\|z\|_{E_2} < \gamma \|v\|_2 \left(\text{i.e.,} \int_0^T z^T(t) z(t) \mathrm{d}t \le \gamma^2 \int_0^T v^T(t) v(t) \mathrm{d}t \right). \tag{2.11}$$

If the above two conditions hold, we also called the systems that are exponential mean-square stable with convergence rate α and disturbance attenuation γ.

3. Main Results

Lemma 3.1 (Schur complement lemma [20]). *For a given matrix* $S = \begin{pmatrix} S_1 & S_3 \\ * & S_2 \end{pmatrix}$ *with* $S_1^T = S_1$, $S_2^T = S_2$, *the following conditions are equivalent:*

(1) $S < 0$,

(2) $S_2 < 0, S_1 - S_3 S_2^{-1} S_3^T < 0$,

(3) $S_1 < 0, S_2 - S_3 S_1^{-1} S_3^T < 0$.

Lemma 3.2 (see [21]). *Let* $x \in \mathbb{R}^n$ *and* $y \in \mathbb{R}^n$. *Then, for any positive scalar* ε, *we have*

$$x^T y + y^T x \leq \varepsilon x^T x + \varepsilon^{-1} y^T y. \tag{3.1}$$

3.1. Robust Reliable H_∞ for LSMJSs

To obtain our main results, we first consider the following linear stochastic Markovian jump systems (LSMJSs) without control input:

$$\begin{aligned} \mathrm{d}x(t) &= [A_i x(t) + E_i v(t)]\mathrm{d}t + [C_i x(t) + H_i v(t)]\mathrm{d}w(t), \\ z(t) &= J_i x(t), \\ x(t_0) &= x_0. \end{aligned} \tag{3.2}$$

Lemma 3.3. *Suppose that* $P(t, r(t)) > 0$ *is continuously differentiable, then the systems* (3.2) *are exponential mean-square stable with convergence rate* α *and disturbance attenuation* γ *if and only if the following matrix functional inequalities hold:*

$$\Xi_i(t) = \begin{pmatrix} M_i(t) + J_i^T J_i & P_i E_i & C_i^T \\ * & -\gamma^2 I & H_i^T \\ * & * & -P_i^{-1}(t) \end{pmatrix} < 0, \quad i \in S, \tag{3.3}$$

where $M_i(t) = A_i^T P_i(t) + P_i(t) A_i + \dot{P}(t) + \sum_{j \in S} \lambda_{ij} P_j(t)$.

Proof. At first, let $v(t) = 0$, and defining the following Lyapunov function:

$$V(x(t), t, i) = V(x(t), t, r(t) = i) = x^T(t) P(t, r(t) = i) x(t) = x^T(t) P_i(t) x(t). \tag{3.4}$$

By Itô formula, we get the following:

$$\mathcal{L}V(x(t), t, i) = x^T(t)\Big(M_i(t) + C_i^T P_i(t) C_i\Big) x(t), \tag{3.5}$$

the matrix function inequalities (3.3) imply that $\mathcal{L}V(x(t), t, i) < 0$, and let $a_i = \lambda_{\max}(-\Xi_i(t))$, $a = \max_{i \in S}(a_i)$, where $\lambda_{\max}(\cdot)$ means the maximum eigenvalue of matrix $(\cdot)$, and we have

$$\mathcal{L}V(x(t), t, i) \leq -a x^T(t) x(t). \tag{3.6}$$

Hence

$$d\left[e^{\alpha t}V(x(t),t,i)\right] = \alpha e^{\alpha t}V(x(t),t,i) + e^{\alpha t}dV(x(t),t,i)$$
$$\leq (b\alpha - a)e^{\alpha t}\|x(t)\|^2 + e^{\alpha t}2x^T(t)P_i(t)C_i x(t)dw(t), \tag{3.7}$$

where $b_i = \sup_{t\geq t_0}\{\lambda_{\max}(P_i(t))\}$, and $b = \max_{i\in S}(b_i)$. Integrating the both sides of above inequality from t_0 to $T$and taking expectation, we obtain that

$$Ee^{\alpha T}[V(x(T),T,i) - V(x_0,t_0,i)] \leq (b\alpha - a)E\int_{t_0}^{T} e^{\alpha s}\|x(s)\|^2 \mathrm{d}s. \tag{3.8}$$

Set $\alpha = a/b$, and the following inequality is obtained:

$$e^{\alpha T}\min_{i\in S}\lambda_{\min}(P_i(T))E\|x(T)\|^2 \leq E\left[e^{\alpha T}V(x(T),T,i)\right] \leq EV(x_0,t_0,i), \tag{3.9}$$

which implies that

$$E\|x(T)\|^2 \leq EV(x_0,t_0,i)\frac{1}{\min_{i\in S}\lambda_{\min}(P_i(T))}e^{-\alpha T}. \tag{3.10}$$

That is to say that the stochastic systems are globally exponentially stable with convergence rate $\alpha > 0$.

Then, considering the stochastic H_∞ performance level for the resulting systems (3.2) with nonzero exogenous disturbance input $(v(t) \neq 0)$, for any $t > 0$, we define that

$$J(t) = E\left\{\int_0^t \left[z^T(s)z(s) - \gamma^2 v^T(s)v(s)\right]\mathrm{d}s\right\}. \tag{3.11}$$

By general Itô formula, we get he following:

$$J(t) = E\left\{\int_{t_0}^{t}\left[z^T(s)z(s) - \gamma^2 v^T(s)v(s) + \mathcal{L}V(x(s),s,i)\right]\mathrm{d}s\right\} - E(V(x(t),t,i))$$
$$\leq E\left\{\int_0^t\left[z^T(s)z(s) - \gamma^2 v^T(s)v(s) + \mathcal{L}V(x(s),s,i)\right]\mathrm{d}s\right\} \leq E\left\{\int_0^t \eta^T(s)\Omega_i(s)\eta(s)\mathrm{d}s\right\}, \tag{3.12}$$

where $\eta^T(t) = (x^T(t)v^T(t))$, $\Omega_i(t) = \begin{pmatrix} M_i(t)+J_i^T J_i & P_i(t)E_i \\ E_i^T P_i(t) & -\gamma^2 I\end{pmatrix} + \begin{pmatrix} C_i^T \\ H_i^T\end{pmatrix}P_i(t)\begin{pmatrix} C_i^T \\ H_i^T\end{pmatrix}^T$ From (3.3) we know that $\Omega(t) < 0$, which implies that

$$J(t) < 0. \tag{3.13}$$

Therefore, the inequality $\|z\|_{E_2} < \gamma\|v\|_2$ holds. The proof is completed. □

In the following time, we consider the following linear stochastic Markovian jump systems (LSMJSs) under the state feedback controller:

$$\begin{aligned}
\mathrm{d}x(t) &= [(A_i + B_iF_iK_i)x(t) + E_iv(t)]\mathrm{d}t + [(C_i + D_iF_iK_i)x(t) + H_iv(t)]\mathrm{d}w(t), \\
z(t) &= J_ix(t), \\
x(t_0) &= x_0.
\end{aligned} \tag{3.14}$$

Theorem 3.4. *If there exist the positive matrices $X_i > 0$, and the constant matrices Y_i with approximate dimensions, such that the following LMIs hold*

$$\Theta_i = \begin{pmatrix} \Theta_{i1} & E_i & \Theta_{i2} & \Theta_{i3} \\ * & -\gamma^2 I & H_i^T & 0 \\ * & * & -X_i & 0 \\ * & * & * & \Theta_{i4} \end{pmatrix} < 0, \quad i \in S, \tag{3.15}$$

where $\Theta_{i1} = X_iA_i^T + A_iX_i + B_iF_iY_i + Y_i^TF_i^TB_i^T + \lambda_{ii}X_i$, $\Theta_{i2} = X_iC_i^T + Y_i^TF_i^TD_i^T$,

$$\begin{aligned}
\Theta_{i3} &= \left(\sqrt{\lambda_{i1}}X_i \ \cdots \ \sqrt{\lambda_{i,i-1}}X_i \ \sqrt{\lambda_{i,i+1}}X_i \ \cdots \ \sqrt{\lambda_{iN}}X_i \ \ X_iJ_i^T\right), \\
\Theta_{i4} &= \mathrm{diag}(-X_1, \ldots, -X_{i-1}, -X_{i+1}, \ldots, -X_N, -I),
\end{aligned} \tag{3.16}$$

then the LSMJSs (3.14) *are exponential mean-square stable with convergence rate α and disturbance attenuation γ. In this case, the desired controllers are given as follows:*

$$K_i = Y_iX_i^{-1}. \tag{3.17}$$

Proof. Defining the following Lyapunov function:

$$V(x(t), t, i) = V(x(t), t, r(t) = i) = x^T(t)P_ix(t). \tag{3.18}$$

By Lemma 3.3, and similar to the proof of Lemma 3.3, we can get the following:

$$\mathcal{L}V(x(t), t, i) \le \eta^T(t)\Xi_i\eta(t), \tag{3.19}$$

where $\Xi_i = \begin{pmatrix} M_i & P_iE_i & C_i^T + K_i^TF_i^TD_i^T \\ * & -\gamma^2 I & H_i^T \\ * & * & -P_i^{-1} \end{pmatrix}$ $M_i = (A_i + B_iF_iK_i)^TP_i + P_i(A_i + B_iF_iK_i) + \sum_{j\in S}\lambda_{ij}P_j$.

Using Schur complement lemma together with contragredient transformation, we know that LMIs (3.15) imply that $\Xi_i < 0$. So we have

$$\begin{aligned} J(t) &= E\left\{\int_0^t \left[z^T(s)z(s) - \gamma^2 v^T(s)v(s)\right]\mathrm{d}s\right\} \\ &= E\left\{\int_{t_0}^t \left[z^T(s)z(s) - \gamma^2 v^T(s)v(s) + \mathcal{L}V(x(s),s,i)\right]\mathrm{d}s\right\} - E(V(x(t),t,i)). \\ &\leq E\left\{\int_0^t \left[z^T(s)z(s) - \gamma^2 v^T(s)v(s) + \mathcal{L}V(x(s),s,i)\right]\mathrm{d}s\right\} < 0. \end{aligned} \tag{3.20}$$

Therefore, the inequality $\|z\|_{E_2} < \gamma\|v\|_2$ holds. The proof is completed. □

Theorem 3.5. *If there exist the positive matrices $X_i > 0$, the positive diagonal matrices $R_i > 0$, and the constant matrices Y_i with approximate dimensions, such that the following LMIs hold:*

$$\widetilde{\Theta}_i = \begin{pmatrix} \widetilde{\Theta}_{i1} & E_i & \widetilde{\Theta}_{i2} & \Theta_{i3} & B_iR_i & Y_i^T \\ * & -\gamma^2 I & H_i^T & 0 & 0 & 0 \\ * & * & -X_i & 0 & D_iR_i & 0 \\ * & * & * & \Theta_{i4} & 0 & 0 \\ * & * & * & * & -R_i & 0 \\ * & * & * & * & * & -R_i\widetilde{F}_{i0}^{-2} \end{pmatrix} < 0, \quad i \in S, \tag{3.21}$$

where $\widetilde{\Theta}_{i1} = X_iA_i^T + A_iX_i + B_iF_{i0}Y_i + Y_i^T F_{i0}^T B_i^T + \lambda_{ii}X_i$, $\widetilde{\Theta}_{i2} = X_iC_i^T + Y_i^T F_{i0}^T D_i^T$, Then the LSMJSs (3.14) *are exponential mean-square stable with convergence rate α and disturbance attenuation γ. In this case, the desired controllers are given as follows:*

$$K_i = Y_iX_i^{-1}. \tag{3.22}$$

Proof. Noticing (2.10), we can see that Θ_i in (3.15) can be rewritten as

$$\Theta_i = \Theta_{i0} + \begin{bmatrix} B_i^T & 0 & D_i^T & 0 \end{bmatrix}^T \Delta_i \begin{bmatrix} Y_i & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} Y_i & 0 & 0 & 0 \end{bmatrix}^T \Delta_i \begin{bmatrix} B_i^T & 0 & D_i^T & 0 \end{bmatrix}, \tag{3.23}$$

where $\Theta_{i0} = \begin{pmatrix} \widetilde{\Theta}_{i1} & E_i & \widetilde{\Theta}_{i2} & \Theta_{i3} \\ * & -\gamma^2 I & H_i^T & 0 \\ * & * & -X_i & 0 \\ * & * & * & \Theta_{i4} \end{pmatrix}$.

By Lemma 3.2, we have

$$\Theta_i \leq \Theta_{i0} + \begin{bmatrix} B_i^T & 0 & D_i^T & 0 \end{bmatrix}^T R_i \begin{bmatrix} B_i^T & 0 & D_i^T & 0 \end{bmatrix} + \begin{bmatrix} Y_i & 0 & 0 & 0 \end{bmatrix}^T R_i^{-1} F_{0i}^2 \begin{bmatrix} Y_i & 0 & 0 & 0 \end{bmatrix}, \tag{3.24}$$

by Schur complement, we know that $\widetilde{\Theta}_i < 0$ imply that $\Theta_i < 0$. Therefore, we can know from Theorem 3.4 that the LSMJSs (3.14) are stabilizable with convergence rate α and disturbance attenuation γ. This completes the proof. □

3.2. Robust Reliable H_∞ for NSMJSs

In this section, we consider the following nonlinear stochastic Markovian jump systems (NSMJSs) under the state feedback controller:

$$\begin{aligned}
\mathrm{d}x(t) &= \left[(A_i + B_iF_iK_i)x(t) + E_iv(t) + f_i(x(t))\right]\mathrm{d}t \\
&\quad + \left[(C_i + D_iF_iK_i)x(t) + H_iv(t) + g_i(x(t))\right]\mathrm{d}w(t), \\
z(t) &= H_ix(t), \\
x(t_0) &= x_0.
\end{aligned} \tag{3.25}$$

Theorem 3.6. *If there exist the positive matrices $X_i > 0$, and the constant matrices Y_i with approximate dimensions, for the positive constant ε_i and the given scalar λ_i, such that the following LMIs hold:*

$$\overline{\Theta}_i = \begin{pmatrix}
\Theta_{i1} & E_i & I - \lambda_iX_i\widehat{T}_{i2} & \Theta_{i2}^T & \Theta_{i2}^T & \overline{\Theta}_{i3} \\
* & -\gamma^2 I & 0 & H_i^T & H_i^T & 0 \\
* & * & -\lambda_i I & 0 & 0 & 0 \\
* & * & * & -X_i & 0 & 0 \\
* & * & * & * & -\varepsilon_i I & 0 \\
* & * & * & * & * & \overline{\Theta}_{i4}
\end{pmatrix} < 0, \quad i \in S, \tag{3.26}$$

where $\overline{\Theta}_{i3} = (\varepsilon_iG_i \;\; \lambda_iX_i\widehat{T}_{i1} \;\; \Theta_{i3})$, $\overline{\Theta}_{i4} = \mathrm{diag}(-\varepsilon_iI, -\lambda_i\widehat{T}_{i1}, \Theta_{i4})$, $\widehat{T}_{i1} = (T_{i1}^TT_{i2} + T_{i2}^TT_{i1})/2$, $\widehat{T}_{i2} = -(T_{i1}^T + T_{i2}^T)/2$, then the NSMJSs (3.25) are exponential mean-square stable with convergence rate α and disturbance attenuation γ. In this case, the desired controllers are given as follows:

$$K_i = Y_iX_i^{-1}. \tag{3.27}$$

Proof. Defining the following Lyapunov function:

$$V(x(t), t, i) = V(x(t), t, r(t) = i) = x^T(t)P_ix(t), \tag{3.28}$$

by Itô formula, we get the following:

$$\begin{aligned}\mathcal{L}V(x(t),t,i) &= 2x^T(t)P_i\left[(A_i+B_iF_iK_i)x(t)+E_iv(t)+f_i(x(t))\right]+\sum_{j\in S}\lambda_{ij}x^T(t)P_jx(t)\\&\quad+\left[(C_i+DB_iF_iK_i)x(t)+H_iv(t)+g_i(x(t))\right]^T\\&\quad\times P_i\left[(C_i+DB_iF_iK_i)x(t)+H_iv(t)+g_i(x(t))\right]\\&\le \sigma^T(t)\Sigma_i\sigma(t)+x^T(t)G_i^TP_iG_ix(t)+2[(C_i+DB_iF_iK_i)x(t)+H_iv(t)]^TP_ig_i(x(t)),\end{aligned}\tag{3.29}$$

where $\sigma^T(t)=[x^T(t),v^T(t),f_i^T(x(t))]$, $\Sigma_i=\begin{pmatrix}M_i & P_iE_i & P_i\\ E_i^TP_i & 0 & 0\\ P_i^T & 0 & 0\end{pmatrix}+\begin{bmatrix}C_i^T+K_i^TF_i^TD_i^T\\ H_i^T\\ 0\end{bmatrix}P_i\begin{bmatrix}C_i^T+K_i^TF_i^TD_i^T\\ H_i^T\\ 0\end{bmatrix}^T$.

By Lemma 3.2, it follows that

$$\begin{aligned}&2[(C_i+DB_iF_iK_i)x(t)+H_iv(t)]^TP_ig_i(x(t))\\&\quad\le\sigma^T(t)\begin{bmatrix}C_i^T+K_i^TF_i^TD_i^T\\ H_i^T\\ 0\end{bmatrix}\varepsilon_i^{-1}I\begin{bmatrix}C_i^T+K_i^TF_i^TD_i^T\\ H_i^T\\ 0\end{bmatrix}^T\sigma(t)+x^T(t)(\varepsilon_iP_iG_i)^T\varepsilon_i^{-1}I(\varepsilon_iP_iG_i)x(t),\end{aligned}\tag{3.30}$$

from (2.4) $(f_i(x(t))-T_{1i}x)^T(f_i(x(t))-T_{2i}x)\le 0$, $i\in S$ which are equivalent to

$$\begin{bmatrix}x(t)\\ f(x(t))\end{bmatrix}^T\begin{bmatrix}\widehat{T}_{i1} & \widehat{T}_{i2}\\ T_{i2}^T & I\end{bmatrix}\begin{bmatrix}x(t)\\ f(x(t))\end{bmatrix}\le 0,\quad i\in S.\tag{3.31}$$

Considering the stochastic H_∞ performance level for the resulting systems (3.25) with nonzero exogenous disturbance input $(v(t)\neq 0)$, for any $t>0$, we define that

$$J(t)=E\left\{\int_0^t\left[z^T(s)z(s)-\gamma^2v^T(s)v(s)\right]ds\right\}.\tag{3.32}$$

By general Itô formula, for a given positive scalar λ, we get the following:

$$\begin{aligned}
&J(t)\\
&= E\left\{\int_{t_0}^{t}\left[z^T(s)z(s)-\gamma^2 v^T(s)v(s)+\mathcal{L}V(x(s),s,i)\right]\mathrm{d}s\right\}-E(V(x(t),t,i))\\
&\leq E\left\{\int_{0}^{t}\left[z^T(s)z(s)-\gamma^2 v^T(s)v(s)+\mathcal{L}V(x(s),s,i)-\lambda_i\left(f_i(x(t))-T_{1i}x(t)\right)^T\left(f_i(x(t))-T_{2i}x(t)\right)\right]\mathrm{d}s\right\}\\
&\leq E\left\{\int_{0}^{t}\sigma^T(s)\overline{\Omega}_i\sigma(s)\mathrm{d}s\right\},
\end{aligned} \tag{3.33}$$

where

$$\begin{aligned}
\overline{\Omega}_i = \Sigma_i &+ \begin{pmatrix} (\varepsilon_i P_i G_i)^T\varepsilon_i^{-1}I(\varepsilon_i P_i G_i)+J_i^T J_i & 0 & 0\\ 0 & -\gamma^2 I & 0\\ 0 & 0 & 0\end{pmatrix}\\
&+\begin{bmatrix} C_i^T+K_i^T F_i^T D_i^T\\ H_i^T\\ 0\end{bmatrix}\varepsilon_i^{-1}I\begin{bmatrix} C_i^T+K_i^T F_i^T D_i^T\\ H_i^T\\ 0\end{bmatrix}^T+\begin{pmatrix} -\lambda\widehat{T}_{i1} & 0 & -\lambda\widehat{T}_{i2}\\ 0 & 0 & 0\\ -\lambda\widehat{T}_{i2}^T & 0 & -\lambda I\end{pmatrix}.
\end{aligned} \tag{3.34}$$

By Schur complement lemma, we see that $\overline{\Omega}_i < 0$ is equivalent to the following matrix inequalities:

$$\begin{pmatrix}
M_i-\lambda_i\widehat{T}_{i1} & E_i & P_i-\lambda_i\widehat{T}_{i2} & X_i^{-1}\Theta_{i2}^T & X_i^{-1}\Theta_{i2}^T & \varepsilon_i P_i G_i & J_i^T\\
* & -\gamma^2 I & 0 & H_i^T & H_i^T & 0 & 0\\
* & * & -\lambda_i I & 0 & 0 & 0 & 0\\
* & * & * & -P_i^{-1} & 0 & 0 & 0\\
* & * & * & * & -\varepsilon_i I & 0 & 0\\
* & * & * & * & * & -\varepsilon_i I & 0\\
* & * & * & * & * & * & -I
\end{pmatrix} < 0, \quad i \in S, \tag{3.35}$$

which is implied in LIMs (3.26). Hence $J(t) < 0$.

Therefore, the inequality $\|z\|_{E_2} < \gamma\|v\|_2$ holds. The proof is completed. □

Similar to the proof of Theorem 3.5, we can get the following theorem without proof immediately.

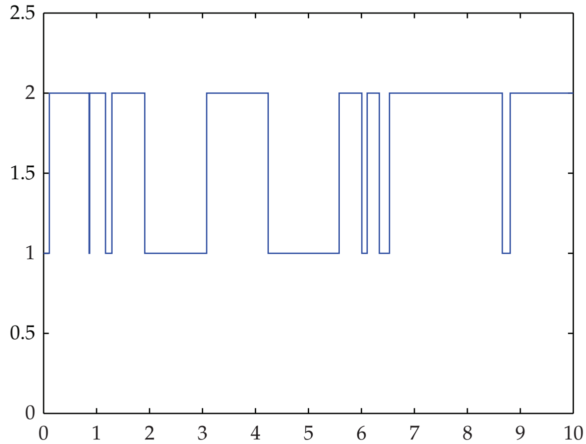

Figure 1: The Markovian chain $r(t)$.

Theorem 3.7. *If there exist the positive matrices $X_i > 0$, and the constant matrices Y_i with approximate dimensions, for the positive constant ε_i and the given scalar λ_i, such that the following LMIs hold*

$$\widehat{\Theta}_i = \begin{pmatrix} \widetilde{\Theta}_{i1} & E_i & I - \lambda_i X_i \widehat{T}_{i2} & \widetilde{\Theta}_{i2}^T & \widetilde{\Theta}_{i2}^T & B_i R_i & Y_i^T & \overline{\Theta}_{i3} \\ * & -\gamma^2 I & 0 & H_i^T & H_i^T & 0 & 0 & 0 \\ * & * & -\lambda_i I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -X_i & 0 & D_i R_i & 0 & 0 \\ * & * & * & * & -\varepsilon_i I & 0 & 0 & 0 \\ * & * & * & * & * & -R_i & 0 & 0 \\ * & * & * & * & * & * & -R_i \widetilde{F}_{i0}^{-2} & 0 \\ * & * & * & * & * & * & * & \overline{\Theta}_{i4} \end{pmatrix} < 0, \quad i \in S. \tag{3.36}$$

then the NSMJSs (3.27) *are exponential mean-square stable with convergence rate α and disturbance attenuation γ. In this case, the desired controllers are given as follows:*

$$K_i = Y_i X_i^{-1}. \tag{3.37}$$

4. Numerical Example with Simulation

In this section, we will give an example to show the usefulness of the derived results and the effectiveness of the proposed methods (Figure 1).

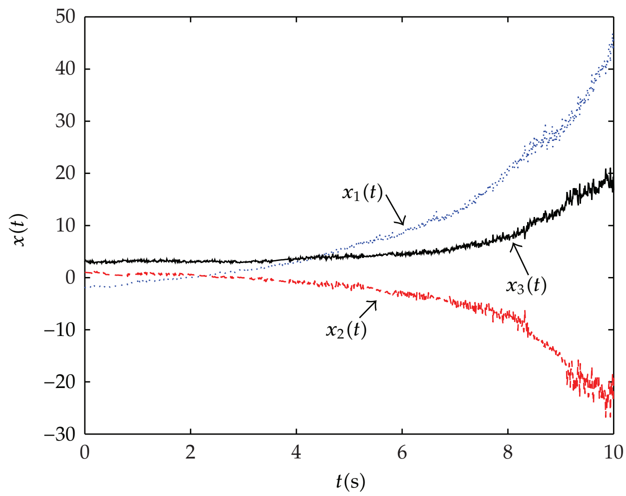

Figure 2: The state curve of uncontrolled LSMJSs (3.14).

Consider linear SMJSs (3.14) with $S = \{1, 2\}$, and the system parameters are given as follows:

$$
\begin{gathered}
A_1 = \begin{pmatrix} 0.3 & 0.3 & 0.5 \\ -0.2 & 0 & -0.3 \\ 0.1 & 0 & 0.3 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 0.5 & 0.2 & 0.2 \\ -0.2 & 0 & -0.4 \\ 0.2 & 0 & 0.2 \end{pmatrix}, \\
C_1 = \begin{pmatrix} 0.5 & 0.2 & 0.1 \\ 0 & 0.2 & -0.1 \\ 0.3 & -0.1 & -0.3 \end{pmatrix}, \qquad C_2 = \begin{pmatrix} 0.2 & 0.1 & 0.3 \\ 0.1 & -0.3 & 0.5 \\ 0 & 0.1 & -0.5 \end{pmatrix}, \\
B_1 = \operatorname{diag}(0.5, 0.4, 0.5), \qquad B_2 = \operatorname{diag}(0.5, 0.4, 0.5), \\
E_1 = E_2 = (0.3, 0.1, 0.5)^T, \qquad H_2 = H_1 = (0.2, 0.1, 0.3)^T, \\
D_2 = D_1 = \operatorname{diag}(0.2, 0.3, 0.4), \qquad J_1 = (0.3, 0.2, 0.6), \\
J_2 = (0.1, -0.1, 0.4), \qquad \gamma = 0.9.
\end{gathered} \tag{4.1}
$$

The actuator failure parameters are as follows:

$$
0.2 \leq f_{i1} \leq 0.4, \quad 0.1 \leq f_{i2} \leq 0.7, \quad 0.1 \leq f_{i3} \leq 0.9, \quad i \in S = \{1, 2\}. \tag{4.2}
$$

From (2.8) and (2.9), we have

$$
F_{10} = F_{20} = \operatorname{diag}(0.3, 0.4, 0.5), \qquad \widetilde{F}_{10} = \widetilde{F}_{20} = \operatorname{diag}(0.1, 0.3, 0.4). \tag{4.3}
$$

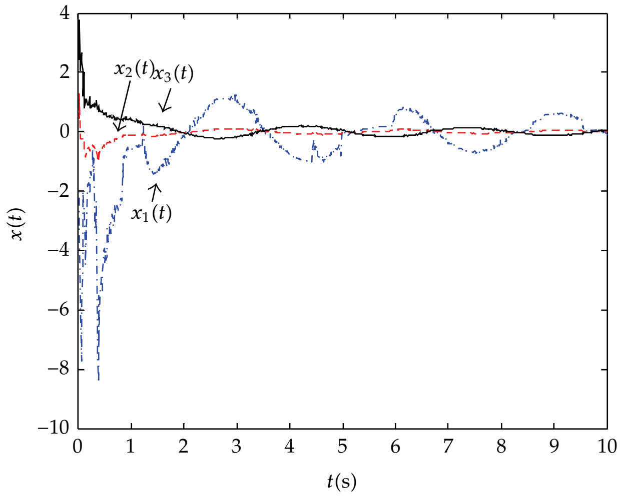

Figure 3: The state curve of closed-loop LSMJSs (3.14).

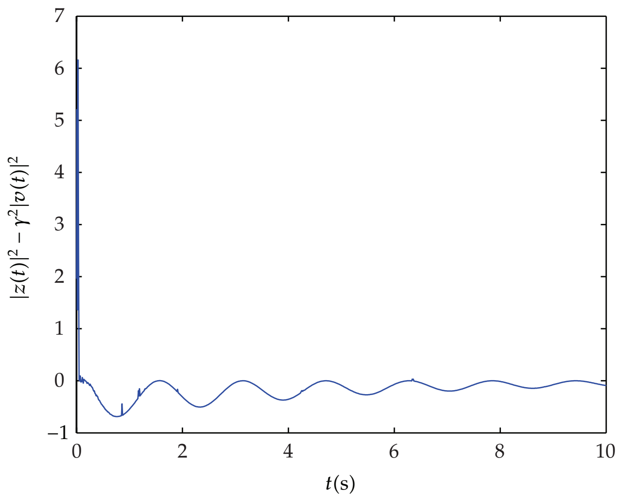

Figure 4: The curve of $|z(t)|^2 - \gamma^2|v(t)|$ for controlled LSMJSs (3.14).

From Figure 2, we can see that the uncontrolled LSMJSs are not stable, according to Theorem 3.5. By using the LMI toolbox, the controller parameters can be calculated as follows:

$$K_1 = \begin{pmatrix} -56.2264 & -6.3843 & -67.8069 \\ -1.1129 & -8.9588 & -3.6802 \\ -0.9754 & 0.1795 & -3.2600 \end{pmatrix}, \qquad K_2 = \begin{pmatrix} -41.7846 & 6.0578 & -200.8802 \\ -1.1365 & -7.5245 & -11.0209 \\ 0.1171 & -0.4055 & -0.7561 \end{pmatrix}. \tag{4.4}$$

Figures 3 and 4 give the simulation results of the response for the closed-loop LSMJSs, which confirm that the closed-loop LSMJSs are exponential mean-square stable with convergence rate α and disturbance attenuation γ.

5. Conclusions

In this paper, we have studied the robust reliable H_∞ control problems for a class of NSMJSs. The system under study contains Itô-type stochastic disturbance, Markovian jumps, sector-bounded nonlinearities, and norm-bounded stochastic nonlinearities. Based on the Lyapunov stability theory and Itô differential rule, sufficient condition which ensures exponential mean-square stable with convergence rate α and disturbance attenuation γ for SMJSs has been established in Lemma 3.3. By the lemma, together with the LMIs techniques, the sufficient conditions for the designation of the robust reliable H_∞ controller of linear SMJSs and NSMJSs have been obtained in terms of LMIs. Finally, a numerical example has been given to show the usefulness of the derived results and the effectiveness of the proposed methods. It is possible to extend our main results to the NSMJSs with time delay by using delay-dependent techniques, which is one of the future research topics.

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