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Research Article

Complex Formation Control of Large-Scale Intelligent Autonomous Vehicles

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A new formation framework of large-scale intelligent autonomous vehicles is developed, which can realize complex formations while reducing data exchange. Using the proposed hierarchy formation method and the automatic dividing algorithm, vehicles are automatically divided into leaders and followers by exchanging information via wireless network at initial time. Then, leaders form formation geometric shape by global formation information and followers track their own virtual leaders to form line formation by local information. The formation control laws of leaders and followers are designed based on consensus algorithms. Moreover, collision-avoiding problems are considered and solved using artificial potential functions. Finally, a simulation example that consists of 25 vehicles shows the effectiveness of theory.

1. Introduction

Distributed formation control of intelligent vehicles, including unmanned aerial vehicles, unmanned ground vehicles, and unmanned underwater submarines, has attracted a great attention, due to its broad applications, such as cooperative searching, surveillance, and combating using multiple unmanned air or ground vehicles. Roughly speaking, formation control strategy falls into six categories, namely, the behavior strategy [1–3], the potentials strategy [4–6], the virtual structure strategy [7–10], the distributed optimize strategy [11–14], and the consensus-based strategy [15–20].

The performances of the above strategies are almost excellent and effective when vehicles' number is small. However, when the number is large and formation is complex, formation problems become complicated. In [21], a formation control framework is developed through dividing information into two independent parts. It can greatly reduce data exchange and realize various kinds of static formations. But, framework is developed based on a first-order model and the information dividing criterion is not given. Another approach in controlling a large group of vehicles is flocking control strategies [22–26]. The essence difference between formation control and flocking is that flocking is concentrated on achieving three heuristic rules introduced by Reynolds [22]. The rules are cohesion, separation, and alignment in which formation requirements are not considered. In [23], a distributed flocking controller is proposed which only utilizes the neighbor information in both free and obstacle space. However, the flocking shape problems are studied in [24], a new piecewise smooth neighbor-based local controller is designed to regulate the flocking shape, three kinds of flocking shapes are obtained, such as, in a single line, vee shape or corner shape. Furthermore, flocking with formation within a constrained environment is studied in [25], a potential field-based method is proposed to achieve the specific formation, but each agent must know the relative positions of information. Also, the flocking shape problems are studied in the consensus framework [26] and in the Lie group setting [27].

Compared with the centralized formation, the communication and control structure of distributed formation is more flexible and is excluded from the restrictions of individual number. Consensus-based distributed formation control only needs local information acquired by wireless network and has received significant attention in the control community [28, 29]. Usually, intelligent vehicle acquires information through wireless network conveniently. However, communication time delays induced by network are inevitable. It is very important to study stability problems of time delay system. By using a small gain stability theorem and structured singular value in the frequency domain, a criterion to guarantee convergence is derived [30]. A leader-following consensus problem of autonomous agents with coupling delays is studied and system stability conditions are obtained by using Lyapunov functions [31]. Considering a finite set of bounded internal point and distributed delays, output-feedback stabilization is considered by linear dynamic controllers and an alternative stabilization technique on time delays in the controller is proposed in [32]. A multiestimation-based adaptive controller is proposed in [33] to improve the transparency property of the teleoperation scheme. The closed-loop system's stability is analyzed and stability capabilities of the overall scheme are carried out. With internal constant point delays, [34] researched the global asymptotic stability properties of switched systems under not very stringent conditions on the matrix functions of parameters. Several important and valued asymptotic stability conditions are obtained in this paper. In [35], by using algebraic graph theory, a method for decentralized stabilization of vehicle formations is proposed and a necessary and sufficient condition for an appropriate decentralized linear stabilizing feedback is obtained. In [36], the problems of packet dropouts, transmission delays, and quantization effects existing in the decentralized networked control systems are studied deeply. An efficient decentralized state feedback controller is designed and system stability conditions are acquired by using linear matrix inequalities.

When the number is large and formation is complex, it is impractical for each vehicle to obtain all the other vehicles' state information quickly. Moreover, there are many uncertainties and disturbances in environment, which may lead to information transmission failure. In fact, it is worth paying more attention to some key members in a formation. One simple example is the line formation with loose space constraint between vehicles. It is necessary and very important to design appropriate formation frameworks.

In this paper, we propose a new hierarchy decentralized formation framework. Our purpose is to realize large-scale vehicles' complex formations. In order to make the framework more close to reality, the second-order model and time delay are considered in formation control algorithm. The study is motivated by the work [19], the consensus-based formation control strategy [13–15], and the flocking shape problems [22–26]. The basic idea is given as follows.

(1) Formation is described using vertexes and edges, and the number of leaders and followers is ascertained, respectively.

(2) Automatically, based on initial position states and the formation information, vehicles are divided into two parts through wireless ad hoc network.

(3) Leaders receive global information from control center by tel-network and form desired geometric shape based on consensus control strategy, meanwhile, broadcast their states through local wireless ad hoc network.

(4) Followers receive leaders' information and track their own virtual leaders using tracking consensus control strategy separately.

(5) All vehicles adjust their states in a distributed manner.

The rest of the paper is organized as follows: in Section 2, some definitions are presented; in Section 3, formation control framework and control laws are discussed in detail; in Section 4, system stability conditions are obtained; in Section 5, numerical simulations are given to illustrate the effectiveness of the theoretical results; finally, concluding remarks are made in Section 6.

2. Problem Description and Preliminaries

Suppose the system under consideration consists of N autonomous vehicles, labeled $1 - N$. All these vehicles share a common state space and interact with each other via wireless network. Moreover, each vehicle updates its current states based upon the information received from its neighbors. The dynamics of the vehicles are all the same and described by double integrators, the ith vehicle has the dynamics as follows:

$$\dot{\mathbf{x}}_i(t) = \mathbf{v}_i(t), \quad \dot{\mathbf{v}}_i(t) = \mathbf{u}_i(t), \quad i = 1, 2, \ldots, N, \tag{2.1}$$

where $\mathbf{x}_i(t) \in \mathbb{R}^n$, $\mathbf{v}_i(t) \in \mathbb{R}^n$, $\mathbf{u}_i(t) \in \mathbb{R}^n$ denote the position state, velocity state, and control vector separately.

We use graphs to describe complex formations. Thus, a formation can be constructed by a sequence of vertexes and lines. Vehicles in the vertex positions are called leaders and determine the basic frame of a formation. Vehicles in the line positions are called followers and associate with two leaders. Some examples of formations are given in Figure 1.

Definition 2.1. In n dimension Euclidean space, each agent's position vector is $\mathbf{h}_i \in \mathbb{R}^n$, $i = 1, 2, \ldots, N$, and relative position vector is $\mathbf{h}_{ij} = \mathbf{h}_i - \mathbf{h}_j$, for all $i, j = 1, 2, \ldots, N$; thus, a formation vector $\mathbf{h} \in \mathbb{R}^{n \times N \times N}$ is defined as $\mathbf{h} = [\, h_{11}^1, h_{12}^1, \ldots, h_{1N}^1, \ldots, h_{N1}^n, \ldots, h_{NN}^n \,]^T$, where h_{ij}^k, $k = 1, 2, \ldots, n$ is the projection of $\mathbf{h}_{ij}$ in each coordinate axis.

Definition 2.2. When $t \to \infty$, if for all $i, j \in N, \mathbf{x}_i(t) - \mathbf{x}_j(t) = \mathbf{h}_{ij}, v_i(t) = \mathbf{v}_j(t)$ holds, then formation is formed and maintained.

Definition 2.3. Potential function V_{ij} is a differentiable, nonnegative function of the distance h_{ij} between vehicles i and j if $j \in N_i$ such that $V_{ij} \to \infty$ when $|h_{ij}| \to 0$ and V_{ij} attains its unique minimum when vehicles i and j are located at a desired distance.

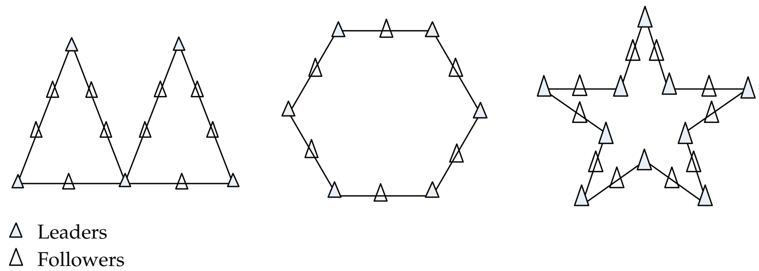


Figure 1: Leaders and followers in a formation.

Definition 2.4. The Kronecker of two matrixes $\mathbf{P} = [p_{ij}]$, $\mathbf{Q} = [q_{ij}]$ is defined as $\mathbf{P} \otimes \mathbf{Q} = [p_{ij}\mathbf{Q}]$.

It is natural to model the interaction among vehicles using directed graph. Let $G = (V, E, A)$ be a graph of order N, with a set of nodes $V = \{V_1, V_2, \ldots, V_n\}$, a set of edges $\varepsilon \subseteq V \times V$, and an adjacency matrix $A = [a_{ij}]$, for all $i, j, n \in I = \{1, 2, \ldots, n\}$. If there is information exchange between node V_i and V_j, $a_{ij} = 1$, else $a_{ij} = 0$. In a directed graph, if $(V_i, V_j) \in \varepsilon$, node V_j is V_j's out-degree neighbor, if $(V_j, V_i) \in \varepsilon$, node V_j is V_j's in-degree neighbor. A set of node V_i's out-degree neighbors is $N_{i(\text{out})}$. A set of node V_i's in-degree neighbors is $N_{i(\text{in})}$; similarly, a set of node V_i's neighbors is $N_i = N_{i(\text{out})} \bigcup N_{i(\text{in})}$. The degree matrix of graph G is a diagonal matrix $D = [d_{ij}]$, $d_{ij} = 0$, for all $i \neq j$ and $L = D - A$ is a Laplacian matrix of graph G. Furthermore, we can obtain the normalized Laplacian matrix $\overline{L} = D^{-1}L$.

3. Main Results

3.1. Overview of Formation Framework

All vehicles equip the same wireless communication devices. That is they can interact with each other through local ad hoc wireless network and interact with control center through tel-network on demanding. It is necessary to design appropriate ad hoc network architecture and communication protocol to improve system's performances. We adopt a cluster architecture ad hoc network and TDMA-type MAC protocol, where each vehicle is assigned to a specific time slot. We propose a distributed formation framework; see Figure 2.

Formation information is divided into global information and local information. Global information including number of leaders and followers, formation shape, velocity, and acceleration is transmitted by tel-network. Corresponding to global information, vehicle's position and velocity are defined as local information transmitted via a local ad hoc network.

Once a formation is given, global formation information is ascertained by a control center. Follow on, the control center sends global formation information to a leader by a tel-network and obtained by the other leaders via local Ad hoc network. It is worth to note that global information can be set, changed, and injected to leaders at any time by the control center. However, there is one exception that initial information must be known by each vehicle. Then, using the dividing method, vehicles are automatically divided into two groups according to initial position information. Based on consensus algorithm and artificial potentials' functions, leaders form a basic formation using global formation information set by the control center, and followers track their leaders to form the entire formation.

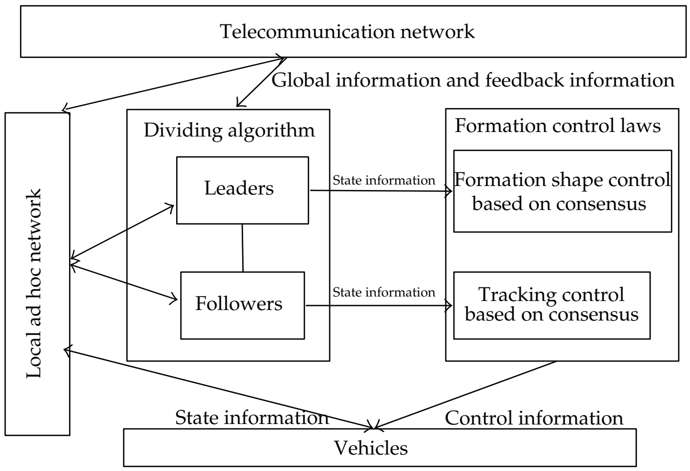


Figure 2: A distributed formation framework.

Assumption 3.1. Suppose that communication delays in the system are uniform and acquiring own state information has no delay, then, communication delay can be denoted by $\tau(t)$ and $\tau(t) = \tau$, $\tau > 0$.

3.2. Automatic Dividing Algorithm

When the number of vehicles is large and the formation is complex, it is very convenience to design an efficient dividing algorithm. Under the assumption that network is in the connectivity state at initial time, vehicles can elect leaders and followers automatically using the following algorithm.

Step 1. At the first time period, using flooding protocol, each vehicle acquires all the other's initial position information and the global formation information.

Step 2. Each vehicle calculates the center of initial positions $\overline{x} = (1/N)\sum_{i=1}^{N} x_i$ separately.

Step 3. Each vehicle calculates its distance to the center $d_i = \|x_i - \overline{x}\|$ and composites the results in descending order (small node ID has higher prior) separately. The first vehicle in the descending sequence is defined as a leader, written as $L1$.

Step 4. Each vehicle calculates the angle θ_{iL1} made by the center, local agent, and $L1$:

$$\theta_{iL1} = \arccos \frac{\|x_i - \overline{x}\|^2 + \|x_{L1} - \overline{x}\|^2 - \|x_{L1} - x_i\|^2}{2\|x_i - \overline{x}\|\|x_{L1} - \overline{x}\|}. \tag{3.1}$$

Step 5. Each vehicle calculates $k_{il1} = N_0\theta_{iL1}/2\pi$, and composites the results in descending order (small node id has higher prior) separately. Tick out the vehicle $k_{iL1} > 1$ and form a new sequence, then, the first vehicle in the descending sequence is defined as a leader, written as $L2$.

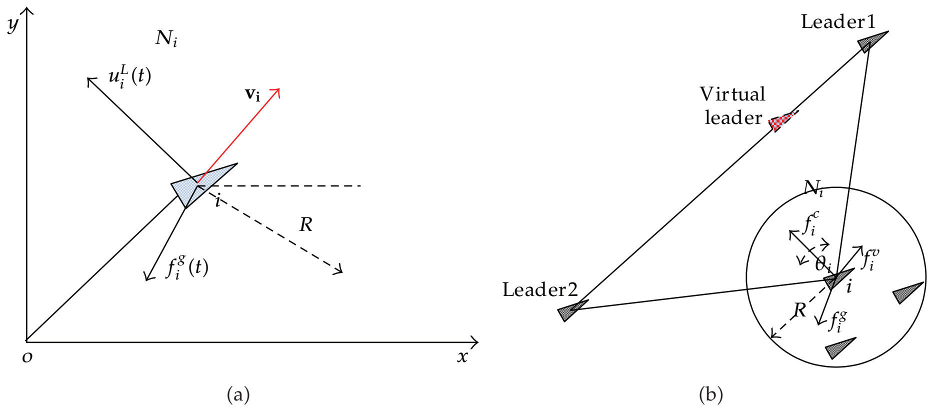


Figure 3: (a) Control of leaders. (b) Control of followers.

Step 6. Repeat Step 3 to Step 5 until find leader LN, then, other vehicles are followers.

Step 7. Ascertain a followers' leader. Compute the angle between leaders and followers θ_{iLj} in turn. According to the global formation information and the angle θ_{iLj}, followers associate with their two leaders which have max θ_{iLj}:

$$\theta_{iLj} = \arccos \frac{\left\|x_i - x_{Lj}\right\|^2 + \left\|x_i - x_{L(j+1)}\right\|^2 - \left\|x_{Lj} - x_{L(j+1)}\right\|^2}{2\left\|x_i - x_{Lj}\right\| \left\|x_i - x_{L(j+1)}\right\|}. \tag{3.2}$$

Step 8. Ascertain virtual leaders. The position and velocity of virtual leader is written as x_{ia}, v_{ia} and defined as follows:

$$x_{ia}(t) = (1 - p_i)x_{iL1}(t) + p_i x_{iL2}(t), \qquad v_{ia}(t) = 0.5(v_{iL1}(t) + v_{iL2}(t)), \tag{3.3}$$

where $p_i \in [0, 1]$ is a proportion factor correlative with expectation position.

A virtual leader can maintain that the virtual leader is at the expectation position of the line. The line is made by its leader each time.

3.3. *Formation Control Laws*

The design method of formation control laws are shown in Figure 3. Formation control law depends on the role of the vehicle if it is a leader, $\alpha_i = 1$, otherwise, $\alpha_i = 0$. Thus, the control law for an individual vehicle $u_i(t)$ is designed as follows:

$$u_i(t) = \alpha_i \mathbf{u}_i^L(t) + (1 - \alpha_i)\mathbf{u}_i^F(t) + \mathbf{f}_i{}^g(t), \tag{3.4}$$

where $\mathbf{u}_i^L(t)$, $\mathbf{u}_i^F(t)$, $\mathbf{f}_i{}^g(t)$ are leaders' formation control law, followers' formation control law, and collision avoidance control law separately.

The leaders' formation control law is designed based on consensus algorithm. The control purpose is used to form desired formation shape and movement. Considering communication delay, control law is designed as follows:

$$\mathbf{u}_i^L(t) = \alpha\dot{\mathbf{v}}_d - \beta(\mathbf{v}_i(t) - \mathbf{v}_d) - \frac{1}{N_i}\sum_{j=1}^{N} a_{ij}\left(v_i(t) - \mathbf{v}_j(t-\tau)\right) - \frac{1}{N_i}\sum_{j=1}^{N} a_{ij}\left(\left(\mathbf{x}_i(t) - \mathbf{x}_j(t-\tau) - \mathbf{h}_{ij}\right)\right). \tag{3.5}$$

Because of that,

$$\begin{aligned}
&\frac{1}{N_i}\left(-\sum_{j=1}^{N} a_{ij}\left(\mathbf{v}_i(t) - \mathbf{v}_j(t-\tau)\right) - \sum_{j=1}^{N} a_{ij}\left(\mathbf{x}_i(t) - \mathbf{x}_j(t-\tau) - \mathbf{h}_{ij}\right)\right) \\
&\quad = \frac{1}{N_i}\left(-\sum_{j=1}^{N} a_{ij}\mathbf{v}_i(t) + \sum_{j=1}^{N} a_{ij}\mathbf{v}_j(t-\tau) - \sum_{j=1}^{N} a_{ij}\mathbf{x}_i(t) + \sum_{j=1}^{N} a_{ij}\mathbf{x}_j(t-\tau) + \sum_{j=1}^{N} a_{ij}\mathbf{h}_{ij}\right) \\
&\quad = \frac{1}{N_i}\left(-\sum_{j=1}^{N} a_{ij}\mathbf{v}_i(t) - \sum_{j=1}^{N} a_{ij}\mathbf{x}_i(t) + \sum_{j=1}^{N} a_{ij}\mathbf{v}_j(t-\tau) + \sum_{j=1}^{N} a_{ij}\mathbf{x}_j(t-\tau) + \sum_{j=1}^{N} a_{ij}\mathbf{h}_{ij}\right) \\
&\quad = -\mathbf{v}_i(t) - \mathbf{x}_i(t) + \frac{1}{N_i}\sum_{j=1}^{N} a_{ij}\mathbf{v}_j(t-\tau) + \frac{1}{N_i}\sum_{j=1}^{N} a_{ij}\mathbf{x}_j(t-\tau) + \frac{1}{N_i}\sum_{j=1}^{N} a_{ij}\mathbf{h}_{ij} \\
&\quad = -\mathbf{v}_i(t) - \mathbf{x}_i(t) + \mathbf{v}_i(t-\tau) + \mathbf{x}_i(t-\tau) - \frac{1}{N_i}\sum_{j=1}^{N} a_{ij}\left(\mathbf{v}_i(t-\tau) - \mathbf{v}_j(t-\tau)\right) \\
&\qquad - \frac{1}{N_i}\sum_{j=1}^{N} a_{ij}\left(\mathbf{x}_i(t-\tau) - \mathbf{x}_j(t-\tau)\right) + \frac{1}{N_i}\sum_{j=1}^{N} a_{ij}\mathbf{h}_{ij}.
\end{aligned} \tag{3.6}$$

Then, we can rewrite $\mathbf{u}_i^L(t)$ as follows:

$$\begin{aligned}
\mathbf{u}_i^L(t) = {}& \alpha\dot{\mathbf{v}}_d - \beta(v_i(t) - \mathbf{v}_d) - \mathbf{v}_i(t) - \mathbf{x}_i(t) + v_i(t-\tau) + \mathbf{x}_i(t-\tau) + \frac{1}{N_i}\sum_{j=1}^{N} a_{ij}\mathbf{h}_{ij} \\
& - \frac{1}{N_i}\sum_{j=1}^{N} a_{ij}\left(\mathbf{v}_i(t-\tau) - \mathbf{v}_j(t-\tau)\right) - \frac{1}{N_i}\sum_{j=1}^{N} a_{ij}\left(\mathbf{x}_i(t-\tau) - \mathbf{x}_j(t-\tau)\right),
\end{aligned} \tag{3.7}$$

where $\mathbf{u}_i^L(t) \in \mathbb{R}^n$ is control vector; $\mathbf{v}_d, \dot{\mathbf{v}}_d \in \mathbb{R}^n$ are the expected velocity and acceleration; a_{ij} is the communication weight, if information flows from j to i, $a_{ij} = 1$, else $a_{ij} = 0$; $\alpha > 0$, $\beta > 0$ are control parameters; h_{ij} is desired relative distance vector and N_i is number of current neighbors.

According to the individual control law (3.7), we can get the leader system's control vector $\mathbf{u}_d(t)$

$$\begin{aligned}\mathbf{u}_d(t) &= \alpha(\mathbf{1}_N \otimes \dot{\mathbf{v}}_d) - \beta(\mathbf{v}(t) - \mathbf{1}_N \otimes \mathbf{v}_d) - \mathbf{v}(t) - \mathbf{x}(t) + \mathbf{v}(t-\tau) \\ &\quad + \mathbf{x}(t-\tau) \ - \left(\overline{\mathbf{L}} \otimes \mathbf{I}_n\right)v(t-\tau) - \left(\overline{\mathbf{L}} \otimes I_n\right)\mathbf{x}(t-\tau) + \mathbf{E} \\ &= \alpha(\mathbf{1}_N \otimes \dot{\mathbf{v}}_d) + \beta(\mathbf{1}_N \otimes \mathbf{v}_d) - (\beta+1)\mathbf{v}(t) - \mathbf{x}(t) + \mathbf{E} \\ &\quad + \left(\mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n\right)\mathbf{v}(t-\tau) + \left(\mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n\right)\mathbf{x}(t-\tau),\end{aligned} \tag{3.8}$$

where

$$\mathbf{E} = \begin{bmatrix} \mathbf{I}_n \otimes \overline{\mathbf{L}}_1 & & & \\ & \mathbf{I}_n \otimes \overline{\mathbf{L}}_2 & & \\ & & \cdots & \\ & & & \mathbf{I}_n \otimes \overline{\mathbf{L}}_N \end{bmatrix} \mathbf{h}. \tag{3.9}$$

Considering that,

$$\begin{aligned}&- \mathbf{x}(t) - (\beta+1)\mathbf{v}(t) + \left(\mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n\right)\mathbf{x}(t-\tau) + \left(\mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n\right)\mathbf{v}(t-\tau) \\ &\quad = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\mathbf{I}_{nN} & -(\beta+1)\mathbf{I}_{nN} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{v}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n & \mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n \end{bmatrix} \begin{bmatrix} \mathbf{x}(t-\tau) \\ \mathbf{v}(t-\tau) \end{bmatrix}.\end{aligned} \tag{3.10}$$

We can rewrite $\mathbf{u}_d(t)$ as follow:

$$\begin{aligned}\mathbf{u}_d(t) &= \alpha(\mathbf{1}_N \otimes \dot{\mathbf{v}}_d) + \beta(\mathbf{1}_N \otimes \mathbf{v}_d) + \mathbf{E} - \mathbf{x}(t) - (\beta+1)\mathbf{v}(t) \\ &\quad + \left(\mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n\right)\mathbf{x}(t-\tau) + \left(\mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n\right)\mathbf{v}(t-\tau) \\ &= \alpha(\mathbf{1}_N \otimes \dot{\mathbf{v}}_d) + \beta(\mathbf{1}_N \otimes \mathbf{v}_d) + \mathbf{E} \\ &\quad + \left[-\mathbf{I}_{nN} \ \ -(\beta+1)\mathbf{I}_{nN}\right] \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{v}(t) \end{bmatrix} + \left[\mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n \ \ \mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n\right] \begin{bmatrix} \mathbf{x}(t-\tau) \\ \mathbf{v}(t-\tau) \end{bmatrix}.\end{aligned} \tag{3.11}$$

Note that system model (2.1) can be rewritten as follows:

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{v}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{nN} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{v}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{u}_d(t) \end{bmatrix}. \tag{3.12}$$

Consequently, substituting equation (3.11) into system model (3.12), we have

$$\begin{aligned}\begin{bmatrix}\dot{\mathbf{x}}(t)\\ \dot{\mathbf{v}}(t)\end{bmatrix} &= \begin{bmatrix}\mathbf{0} & \mathbf{I}_{nN}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{v}(t)\end{bmatrix} + \begin{bmatrix}\mathbf{0}\\ \alpha(\mathbf{1}_N\otimes\dot{\mathbf{v}}_d)+\beta(\mathbf{1}_N\otimes\mathbf{v}_d)+\mathbf{E}\end{bmatrix}\\ &\quad+\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{I}_{nN} & -(\beta+1)\mathbf{I}_{nN}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{v}(t)\end{bmatrix} + \begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{I}_{nN}-\overline{\mathbf{L}}\otimes\mathbf{I}_n & \mathbf{I}_{nN}-\overline{\mathbf{L}}\otimes\mathbf{I}_n\end{bmatrix}\begin{bmatrix}\mathbf{x}(t-\tau)\\ \mathbf{v}(t-\tau)\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{0} & \mathbf{I}_{nN}\\ -\mathbf{I}_{nN} & -(\beta+1)\mathbf{I}_{nN}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{v}(t)\end{bmatrix} + \begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{I}_{nN}-\overline{\mathbf{L}}\otimes\mathbf{I}_n & \mathbf{I}_{nN}-\overline{\mathbf{L}}\otimes\mathbf{I}_n\end{bmatrix}\begin{bmatrix}\mathbf{x}(t-\tau)\\ \mathbf{v}(t-\tau)\end{bmatrix}\\ &\quad+\begin{bmatrix}\mathbf{0}\\ \alpha(\mathbf{1}_N\otimes\dot{\mathbf{v}}_d)+\beta(\mathbf{1}_N\otimes\mathbf{v}_d)+\mathbf{E}\end{bmatrix}.\end{aligned} \tag{3.13}$$

Furthermore, we define

$$\boldsymbol{\xi}(t) = \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{v}(t)\end{bmatrix}, \qquad \dot{\boldsymbol{\xi}}(t) = \begin{bmatrix}\dot{\mathbf{x}}(t)\\ \dot{\mathbf{v}}(t)\end{bmatrix}, \qquad \boldsymbol{\xi}(t-\tau(t)) = \begin{bmatrix}\mathbf{x}(t-\tau)\\ \mathbf{v}(t-\tau)\end{bmatrix}. \tag{3.14}$$

Thus, the close loop of the leader system state equation is derived:

$$\dot{\boldsymbol{\xi}}(t) = \mathbf{A}\boldsymbol{\xi}(t) + \mathbf{B}\boldsymbol{\xi}(t-\tau) + \mathbf{C}, \tag{3.15}$$

where

$$\begin{gathered}\mathbf{A} = \begin{bmatrix}\mathbf{0} & \mathbf{I}_{nN}\\ -\mathbf{I}_{nN} & -(\beta+1)\mathbf{I}_{nN}\end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{I}_{nN}-\overline{\mathbf{L}}\otimes\mathbf{I}_n & \mathbf{I}_{nN}-\overline{\mathbf{L}}\otimes\mathbf{I}_n\end{bmatrix},\\ \mathbf{C} = \begin{bmatrix}\mathbf{0}\\ \mathbf{E}+\beta(\mathbf{1}_N\otimes\mathbf{v}_d)+\alpha(\mathbf{1}_N\otimes\dot{\mathbf{v}}_d)\end{bmatrix}.\end{gathered} \tag{3.16}$$

The followers' formation control law is designed based on the consensus algorithm also. The control purpose is tracking its virtual leader, respectively. Considering communication delay, control law is designed as follows:

$$\begin{gathered}\mathbf{u}_i^F(t) = \mathbf{f}_i^{\,v}(t) + \mathbf{f}_i^{\,c}(t),\\ \mathbf{f}_i^v(t) = -\frac{c}{N_i}\sum_{j=1}^{N_i}\left(\mathbf{v}_i - \mathbf{v}_j(t-\tau)\right),\\ \mathbf{f}_i^c(t) = -(v_i(t) - \mathbf{v}_{ia}(t)) - \gamma(\mathbf{x}_i(t) - \mathbf{x}_{ia}(t)).\end{gathered} \tag{3.17}$$

The velocity feedback damp control term $\mathbf{f}_i^v(t)$ is used to regulate the vehicles' velocity and c is the damp control parameter. The consensus tracking control term $\mathbf{f}_i^c(t)$ is used to track its own virtual leader and γ is the consensus control parameter.

Let $\tilde{v}_i(t) = v_i(t) - \mathbf{v}_{ia}(t)$, $\tilde{\mathbf{x}}_i(t) = \mathbf{x}_i(t) - \mathbf{x}_{ia}(t)$, then we have

$$\mathbf{v}_i(t) - \mathbf{v}_j(t-\tau) = \tilde{v}_i(t) - \tilde{v}_j(t-\tau) + \mathbf{v}_{ia}(t) - \mathbf{v}_{ja}(t-\tau). \tag{3.18}$$

Since that,

$$\begin{aligned} -\frac{c}{N_i}\sum_{j=1}^{N} a_{ij}\left(\mathbf{v}_i(t) - \mathbf{v}_j(t-\tau)\right) &= -\frac{c}{N_i}\sum_{j=1}^{N} a_{ij}\left(\tilde{v}_i(t) - \tilde{v}_i(t-\tau) + \mathbf{v}_{ia}(t) - \mathbf{v}_{ja}(t-\tau)\right) \\ &= -\frac{c}{N_i}\sum_{j=1}^{N} a_{ij}\tilde{\mathbf{v}}_i(t) + \frac{c}{N_i}\sum_{j=1}^{N} a_{ij}\tilde{\mathbf{v}}_j(t-\tau) \\ &\quad -\frac{c}{N_i}\sum_{j=1}^{N} a_{ij}\left(\mathbf{v}_{ia}(t) - \mathbf{v}_{ja}(t-\tau)\right). \end{aligned} \tag{3.19}$$

The followers' formation control law is rewritten as follows:

$$\begin{aligned} \tilde{\mathbf{u}}_i^F(t) = &-\tilde{\mathbf{v}}_i(t) - \gamma\tilde{\mathbf{x}}_i(t) - c\tilde{\mathbf{v}}_i(t) + c\tilde{\mathbf{v}}_i(t-\tau) - c\mathbf{v}_{ia}(t) + c\mathbf{v}_{ia}(t-\tau) \\ &- \frac{c}{N_i}\sum_{j=1}^{N} a_{ij}\left(\tilde{\mathbf{v}}_i(t-\tau) - \tilde{\mathbf{v}}_j(t-\tau)\right) - \frac{c}{N_i}\sum_{j=1}^{N} a_{ij}\left(\mathbf{v}_{ia}(t-\tau) - \mathbf{v}_{ja}(t-\tau)\right). \end{aligned} \tag{3.20}$$

Furthermore, we can acquire the formation control law of the follower system:

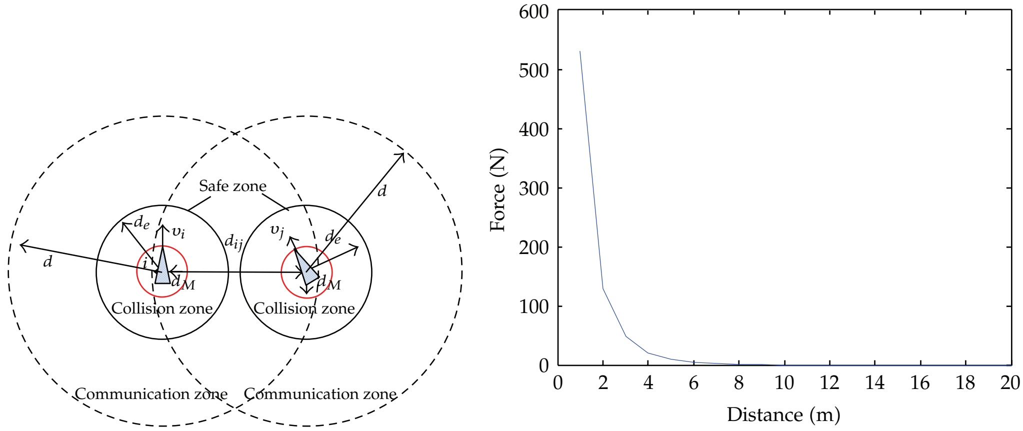
$$\begin{aligned} \tilde{\mathbf{u}}^F(t) &= -\tilde{v}(t) - \gamma\tilde{\mathbf{x}}(t) - c\tilde{v}(t) + c\tilde{v}(t-\tau) - c\left(\overline{\mathbf{L}} \otimes \mathbf{I}_n\right)\tilde{v}(t-\tau) - c\mathbf{v}_a(t) + c\left(\overline{\mathbf{L}} \otimes \mathbf{I}_n\right)\mathbf{v}_a(t-\tau) \\ &= -\gamma\tilde{\mathbf{x}}(t) - (1+c)\tilde{v}(t) + c\left(\mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n\right)\tilde{v}(t-\tau) - c\mathbf{v}_a(t) + c\mathbf{v}_a(t-\tau) \\ &\quad + c\left(\overline{\mathbf{L}} \otimes \mathbf{I}_n\right)\mathbf{v}_a(t-\tau). \end{aligned} \tag{3.21}$$

Consequently, the close loop of the follower system state equation is derived:

$$\dot{\tilde{\boldsymbol{\xi}}}(t) = \mathbf{D}\boldsymbol{\xi}(t) + \mathbf{E}\boldsymbol{\xi}(t-\tau) + \mathbf{F}(t), \tag{3.22}$$

where

$$\begin{aligned} \mathbf{D} &= \begin{bmatrix} \mathbf{0} & \mathbf{I}_{nN} \\ -\gamma\mathbf{I}_{nN} & -(1+c)\mathbf{I}_{nN} \end{bmatrix}, \qquad \mathbf{E} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & c\left(\mathbf{I}_{nN} - \overline{\mathbf{L}} \otimes \mathbf{I}_n\right) \end{bmatrix}, \\ \mathbf{F}(t) &= \begin{bmatrix} \mathbf{0} \\ -c\mathbf{v}_a(t) + c\mathbf{v}_a(t-\tau) + c\left(\overline{\mathbf{L}} \otimes \mathbf{I}_n\right)\mathbf{v}_a(t-\tau) \end{bmatrix}. \end{aligned} \tag{3.23}$$



(a) (b)

Figure 4: (a) Safe zone, communication zone, and collision zone. (b) Artificial potential function.

The collision avoidance control law is designed based on artificial potential functions and the design methods are showed in Figure 4.

Consequently, the collision avoidance control law is designed as follows:

$$\mathbf{f}_i^g(t) = \frac{1}{N_i} \sum_{j \in N_i(t)} x_{ij}(t) V_{ij}(\|x_{ij}(t)\|) \|x_{ij}(t)\|^{-1},$$
$$V_{ij}(\|x_{ij}(t)\|) = k\varphi(x_{ij}(t)) \|x_{ij}(t)\|^{-1}, \tag{3.24}$$
$$\varphi(x_{ij}(t)) = \begin{cases} \log^2(\|x_{ij}(t)\| d_e^{-1}) & \|x_{ij}(t)\| < d_e, \\ 0 & \|x_{ij}(t)\| \geq d_e. \end{cases}$$

The control term $\mathbf{f}_i^g(t)$ is designed on artificial potential functions which are used to avoid collision, $V_{ij}(\|x_{ij}(t)\|)$ is artificial potential function, d_e is the safe range, d is communication range, and k is positive control parameter.

4. Stability Analysis

In the following, we get two steps to study the system's stability problems.

Lemma 4.1 (Schur complement). *For a given symmetric matrix* $\mathbf{S}$ *with the form* $\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{12} & \mathbf{S}_{22} \end{bmatrix}$, *where* $\mathbf{S}_{11} \in \mathbb{R}^{r\times r}$, $\mathbf{S}_{22} \in \mathbb{R}^{(n-r)\times(n-r)}$, $\mathbf{S}_{12} \in \mathbb{R}^{r\times(n-r)}$, $\mathbf{S}_{21} \in \mathbb{R}^{(n-r)\times r}$, $\mathbf{S} < 0$ *if and only if* $\mathbf{S}_{22} < 0$ *and* $\mathbf{S}_{11} - \mathbf{S}_{12}\mathbf{S}_{22}^{-1}\mathbf{S}_{12}^T < 0$.

Step 1. The stability analysis of a leader system.

Theorem 4.2. *Given communication delay* $0 < \tau \leq d$, $0 < d < \infty$, *formation vector* h, *velocity* $\mathbf{v}_d$, *acceleration* $\dot{\mathbf{v}}_d$, *and control parameters* $\alpha > 0$, $\beta > 0$ *are bounded constants. Under the condition that* *directed network topology is connectivity, if there exit two symmetric matrixes* $\mathbf{P} > 0$, $\mathbf{Q} > 0$ *subject to* $\boldsymbol{\Gamma}_1 < 0$, *then for any time delay* τ, *the leader system is stable:*

$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} (\mathbf{A}+\mathbf{B})^T\mathbf{P} + \mathbf{P}(\mathbf{A}+\mathbf{B}) + 2\tau\left(\mathbf{A}^T\mathbf{Q}\mathbf{A} + \mathbf{B}^T\mathbf{Q}\mathbf{B}\right) & \mathbf{P}\mathbf{B} \\ \mathbf{B}^T\mathbf{P} & -\dfrac{\mathbf{Q}}{\tau} \end{bmatrix}. \tag{4.1}$$

Proof. When the formation vector $\mathbf{h}$ and desired velocity and acceleration $\mathbf{v}_d$, $\dot{\mathbf{v}}_d$ are known accurately and control parameters $\alpha > 0$ and $\beta > 0$ are bounded constant, the bias vector $\mathbf{C}$ is a bounded constant and plays no role in the system's convergence. So, the leader system's convergence (3.17) is the same as the following system:

$$\dot{\boldsymbol{\xi}}(t) = \mathbf{A}\boldsymbol{\xi}(t) + \mathbf{B}\boldsymbol{\xi}(t-\tau). \tag{4.2}$$

By the Newton-Leibniz formula, we have

$$\dot{\boldsymbol{\xi}} = (\mathbf{A}+\mathbf{B})\boldsymbol{\xi}(t) - \mathbf{B}\int_{t-\tau}^{t} \dot{\boldsymbol{\xi}}(s)ds. \tag{4.3}$$

Define a Lyapunov-Krasovskii function for system (4.3) as follows:

$$\begin{aligned} V(t) &= V_1(t) + V_2(t) + V_3(t), \\ V_1(t) &= \boldsymbol{\xi}^T(t)\mathbf{P}\boldsymbol{\xi}(t), V_2(t) = \int_0^{\tau}\int_{t-\beta}^{t} \dot{\boldsymbol{\xi}}^T(s)\mathbf{Q}\dot{\boldsymbol{\xi}}(s)ds\,d\beta, \\ V_3(t) &= 2\int_{t-\tau}^{t} \tau\boldsymbol{\xi}^T(s)\mathbf{B}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(s)ds. \end{aligned} \tag{4.4}$$

Differentiating $V(t)$ along the system (4.3), we can acquire

$$\begin{aligned} \dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t), \\ \dot{V}_1(t) &= \dot{\boldsymbol{\xi}}^T(t)\mathbf{P}\boldsymbol{\xi}(t) + \boldsymbol{\xi}^T(t)\mathbf{P}\dot{\boldsymbol{\xi}}(t) \\ &= \left((\mathbf{A}+\mathbf{B})\boldsymbol{\xi}(t) - \mathbf{B}\int_{t-\tau}^{t}\dot{\boldsymbol{\xi}}(s)ds\right)^T \mathbf{P}\boldsymbol{\xi}(t) + \boldsymbol{\xi}^T(t)\mathbf{P}\left((\mathbf{A}+\mathbf{B})\boldsymbol{\xi}(t) - \mathbf{B}\int_{t-\tau}^{t}\dot{\boldsymbol{\xi}}(s)ds\right) \\ &= \boldsymbol{\xi}^T(t)\left((\mathbf{A}+\mathbf{B})^T\mathbf{P} + \mathbf{P}(\mathbf{A}+\mathbf{B})\right)\boldsymbol{\xi}(t) - \int_{t-\tau}^{t}\left(\dot{\boldsymbol{\xi}}^T(s)\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t) + \boldsymbol{\xi}^T(t)\mathbf{P}\mathbf{B}\dot{\boldsymbol{\xi}}(s)\right)ds, \\ \dot{V}_2(t) &= \int_0^{\tau}\int_{t-\beta}^{t}\dot{\boldsymbol{\xi}}^T(s)\mathbf{Q}\dot{\boldsymbol{\xi}}(s)ds\,d\beta = \tau\dot{\boldsymbol{\xi}}^T(t)\mathbf{Q}\dot{\boldsymbol{\xi}}(t) - \int_{t-\tau}^{t}\dot{\boldsymbol{\xi}}^T(s)\mathbf{Q}\dot{\boldsymbol{\xi}}(s)ds, \\ \dot{V}_3(t) &= 2\tau\boldsymbol{\xi}^T(t)\mathbf{B}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t) - 2\tau\boldsymbol{\xi}^T(t-\tau)\mathbf{B}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t-\tau). \end{aligned} \tag{4.5}$$

Sum the integral terms in $\dot{V}_1(t), \dot{V}_2(t), \dot{V}_3(t)$, then, we have

$$
\begin{aligned}
&\int_{t-\tau}^{t}\left(-\dot{\boldsymbol{\xi}}^T(s)\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)-\boldsymbol{\xi}^T(t)\mathbf{P}\mathbf{B}\dot{\boldsymbol{\xi}}(s)-\dot{\boldsymbol{\xi}}^T(s)\mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)ds\\
&\quad=\int_{t-\tau}^{t}\left(-\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)+\mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)^T\mathbf{Q}^{-1}\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)+\mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)+\boldsymbol{\xi}^T(t)\mathbf{P}\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)\right)ds\\
&\quad=\int_{t-\tau}^{t}\left(-\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)+\mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)^T\mathbf{Q}^{-1}\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)+\mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)\right)ds+\int_{t-\tau}^{t}\left(\boldsymbol{\xi}^T(t)\mathbf{P}\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)\right)ds\\
&\quad=-\int_{t-\tau}^{t}\left(\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)+\mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)^T\mathbf{Q}^{-1}\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)+\mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)\right)ds+\tau\boldsymbol{\xi}^T(t)\mathbf{P}\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t).
\end{aligned}
\tag{4.6}
$$

Since that,

$$
\begin{aligned}
&\tau\dot{\boldsymbol{\xi}}^T(t)\mathbf{Q}\dot{\boldsymbol{\xi}}(t)+2\tau\boldsymbol{\xi}^T(t)\mathbf{B}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t)-2\tau\boldsymbol{\xi}^T(t-\tau)\mathbf{B}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t-\tau)\\
&\quad=\tau\left(\boldsymbol{\xi}^T(t)\mathbf{A}^T\mathbf{Q}\mathbf{A}\boldsymbol{\xi}(t)+\boldsymbol{\xi}^T(t)\mathbf{A}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t-\tau)+\boldsymbol{\xi}^T(t-\tau)\mathbf{B}^T\mathbf{A}\boldsymbol{\xi}(t)+\boldsymbol{\xi}^T(t-\tau)\mathbf{B}^T\mathbf{B}\boldsymbol{\xi}(t-\tau)\right)\\
&\qquad+2\tau\boldsymbol{\xi}^T(t)\mathbf{B}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t)-2\tau\boldsymbol{\xi}^T(t-\tau)\mathbf{B}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t-\tau),\\
&\boldsymbol{\xi}^T(t)\mathbf{A}^T\mathbf{Q}\mathbf{A}\boldsymbol{\xi}(t)+\boldsymbol{\xi}^T(t)\mathbf{A}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t-\tau)+\boldsymbol{\xi}^T(t-\tau)\mathbf{B}^T\mathbf{A}\boldsymbol{\xi}(t)+\boldsymbol{\xi}^T(t-\tau)\mathbf{B}^T\mathbf{B}\boldsymbol{\xi}(t-\tau)\\
&\qquad-2\boldsymbol{\xi}^T(t)\mathbf{A}^T\mathbf{Q}\mathbf{A}\boldsymbol{\xi}(t)-2\boldsymbol{\xi}^T(t-\tau)\mathbf{B}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t-\tau)+2\boldsymbol{\xi}^T(t)\mathbf{B}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t)+2\boldsymbol{\xi}^T(t)\mathbf{A}^T\mathbf{Q}\mathbf{A}\boldsymbol{\xi}(t)\\
&\quad=-(A\boldsymbol{\xi}(t)-\mathbf{B}\boldsymbol{\xi}(t-\tau))^T\mathbf{Q}(\mathbf{A}\boldsymbol{\xi}(t)-\mathbf{B}\boldsymbol{\xi}(t-\tau))+2\boldsymbol{\xi}^T(t)\mathbf{B}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t)+2\boldsymbol{\xi}^T(t)\mathbf{A}^T\mathbf{Q}\mathbf{A}\boldsymbol{\xi}(t).
\end{aligned}
\tag{4.7}
$$

According to (4.4) to (4.7), we have

$$
\begin{aligned}
\dot{V}(t)&=\boldsymbol{\xi}^T(t)\left((\mathbf{A}+\mathbf{B})^T\mathbf{P}+\mathbf{P}(\mathbf{A}+\mathbf{B})\right)\boldsymbol{\xi}(t)+\tau\boldsymbol{\xi}^T(t)\mathbf{P}\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)\\
&\quad-\tau(\mathbf{A}\boldsymbol{\xi}(t)-\mathbf{B}\boldsymbol{\xi}(t-\tau))^T\mathbf{Q}(\mathbf{A}\boldsymbol{\xi}(t)-\mathbf{B}\boldsymbol{\xi}(t-\tau))+2\tau\boldsymbol{\xi}^T(t)\mathbf{B}^T\mathbf{Q}\mathbf{B}\boldsymbol{\xi}(t)\\
&\quad+2\tau\boldsymbol{\xi}^T(t)\mathbf{A}^T\mathbf{Q}\mathbf{A}\boldsymbol{\xi}(t)-\int_{t-\tau}^{t}\left(\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(\mathbf{t})+\mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)^T\mathbf{Q}^{-1}\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)+\mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)\right)ds\\
&=\boldsymbol{\xi}^T(t)\left((\mathbf{A}+\mathbf{B})^T\mathbf{P}+\mathbf{P}(\mathbf{A}+\mathbf{B})+\tau\mathbf{P}\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}^T\mathbf{P}+2\tau\mathbf{A}^T\mathbf{Q}\mathbf{A}+2\tau\mathbf{B}^T\mathbf{Q}\mathbf{B}\right)\boldsymbol{\xi}(t)\\
&\quad-\tau(\mathbf{A}\boldsymbol{\xi}(t)-\mathbf{B}\boldsymbol{\xi}(t-\tau))^T\mathbf{Q}(\mathbf{A}\boldsymbol{\xi}(t)-\mathbf{B}\boldsymbol{\xi}(t-\tau))\\
&\quad-\int_{t-\tau}^{t}\left(\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)+\mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)^T\mathbf{Q}^{-1}\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t)+\mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)\right)ds.
\end{aligned}
\tag{4.8}
$$

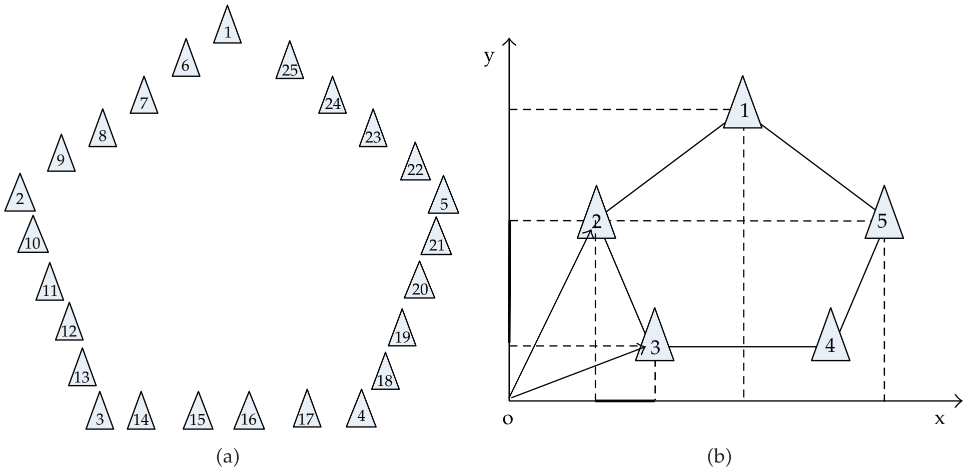


Figure 5: (a) The desired formation of all vehicles. (b) The desired formation shape of leaders.

According to (4.9), we get

$$\begin{aligned} &-(\mathbf{A}\boldsymbol{\xi}(t) - \mathbf{B}\boldsymbol{\xi}(t-\tau))^T \mathbf{Q}(\mathbf{A}\boldsymbol{\xi}(t) - \mathbf{B}\boldsymbol{\xi}(t-\tau)) < 0, \\ &-\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t) + \mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right)^T \mathbf{Q}^{-1}\left(\mathbf{B}^T\mathbf{P}\boldsymbol{\xi}(t) + \mathbf{Q}\dot{\boldsymbol{\xi}}(s)\right) < 0. \end{aligned} \tag{4.9}$$

Let

$$\boldsymbol{\Omega} = (\mathbf{A}+\mathbf{B})^T\mathbf{P} + \mathbf{P}(\mathbf{A}+\mathbf{B}) + \tau\mathbf{P}\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}^T\mathbf{P} + 2\tau\mathbf{A}^T\mathbf{Q}\mathbf{A} + 2\tau\mathbf{B}^T\mathbf{Q}\mathbf{B} < 0. \tag{4.10}$$

Then we have $\dot{V}(t) < 0$. By Lemma 4.1, we get $\boldsymbol{\Omega} < 0$ which is equivalent to $\boldsymbol{\Gamma}_1 < 0$:

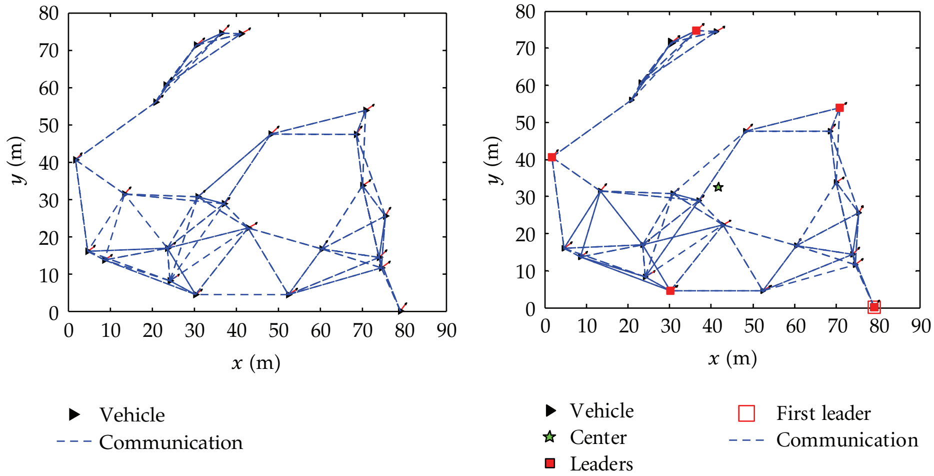
$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} (\mathbf{A}+\mathbf{B})^T\mathbf{P} + \mathbf{P}(\mathbf{A}+\mathbf{B}) + 2\tau\left(\mathbf{A}^T\mathbf{Q}\mathbf{A} + \mathbf{B}^T\mathbf{Q}\mathbf{B}\right) & \mathbf{P}\mathbf{B} \\ \mathbf{B}^T\mathbf{P} & -\dfrac{\mathbf{Q}}{\tau} \end{bmatrix}. \tag{4.11}$$

If $\boldsymbol{\Gamma}_1 < 0$, then we have $\dot{V}(t) < 0$. Therefore, the leader system is stable and a basic formation is formed and maintained. □

Step 2. The stability analysis of followers.

Theorem 4.3. *Under the condition of Theorem 4.2, if a leader system is stable and there exit two symmetric matrixes* $\mathbf{R} > 0$, $\mathbf{S} > 0$ *subject to* $\boldsymbol{\Gamma}_2 < 0$, *then for any delay* τ, *a follower system is stable:*

$$\boldsymbol{\Gamma}_2 = \begin{bmatrix} (\mathbf{D}+\mathbf{E})^T\mathbf{R} + \mathbf{R}(\mathbf{D}+\mathbf{E}) + 2\tau\left(\mathbf{D}^T\mathbf{S}\mathbf{D} + \mathbf{E}^T\mathbf{S}\mathbf{E}\right) & \mathbf{R}\mathbf{E} \\ \mathbf{E}^T\mathbf{R} & -\dfrac{\mathbf{Q}}{\tau} \end{bmatrix}. \tag{4.12}$$



(a) (b)

Figure 6: (a) Initial states of all vehicles. (b) States of all vehicles after grouping.

Proof. Similar to the proof of Theorem 4.2, we can derive the follower system's stability condition. It is that if there exit two symmetric matrixes $\mathbf{R} > 0$, $\mathbf{S} > 0$ subject to $\boldsymbol{\Gamma}_2 < 0$, then for any delay τ, the follower system is stable.

$$\boldsymbol{\Gamma}_2 = \begin{bmatrix} (\mathbf{D}+\mathbf{E})^T\mathbf{R} + \mathbf{R}(\mathbf{D}+\mathbf{E}) + 2\tau\left(\mathbf{D}^T\mathbf{S}\mathbf{D} + \mathbf{E}^T\mathbf{S}\mathbf{E}\right) & \mathbf{R}\mathbf{E} \\ \mathbf{E}^T\mathbf{R} & -\dfrac{\mathbf{Q}}{\tau} \end{bmatrix}. \tag{4.13}$$

According to the follower system's close-loop state equation, the equilibrium point of the dynamic system $\tilde{\boldsymbol{\xi}}_e = \mathbf{F}(t)$ is stable. If a leader system is stable, it means the condition $\mathbf{v}_i(t) = \mathbf{v}_j(t)$, for all $i, j \in N$, is true. Consequently, we can easily acquire the equation $\mathbf{v}_{ia}(t) = \mathbf{v}_{ja}(t) = \mathbf{v}_{ia}(t-\tau) = \mathbf{v}_{ja}(t-\tau)$. Thus, equations $c\mathbf{v}_a(t) = c\mathbf{v}_a(t-\tau)$ and $(\overline{\mathbf{L}} \otimes \mathbf{I}_n)\mathbf{v}_a(t-\tau) = 0$ are true which lead to $\mathbf{F}(t) = 0$. Furthermore, it is easy to see that $\tilde{v}_i(t) = v_i(t) - \mathbf{v}_{ia}(t) = 0$ and $\tilde{\mathbf{x}}_i(t) = \mathbf{x}_i(t) - \mathbf{x}_{ia}(t) = 0$ when $t \to \infty$. According to the virtual leader's definition, local formation can be formed and maintained. □

5. Simulations

In this section we give a simulation example of multiple vehicles formation in the plane to illustrate the effectiveness of the proposed algorithm in the paper. In the simulations, 25 vehicles are randomly distributed in $[0, 100]$ and $[0, 100]$; initial velocities are randomly distributed in $[0, 1]$ and $[0, 1]$. Expected velocity is $\mathbf{V}_d = [6, 6]^T$ and expected formation is

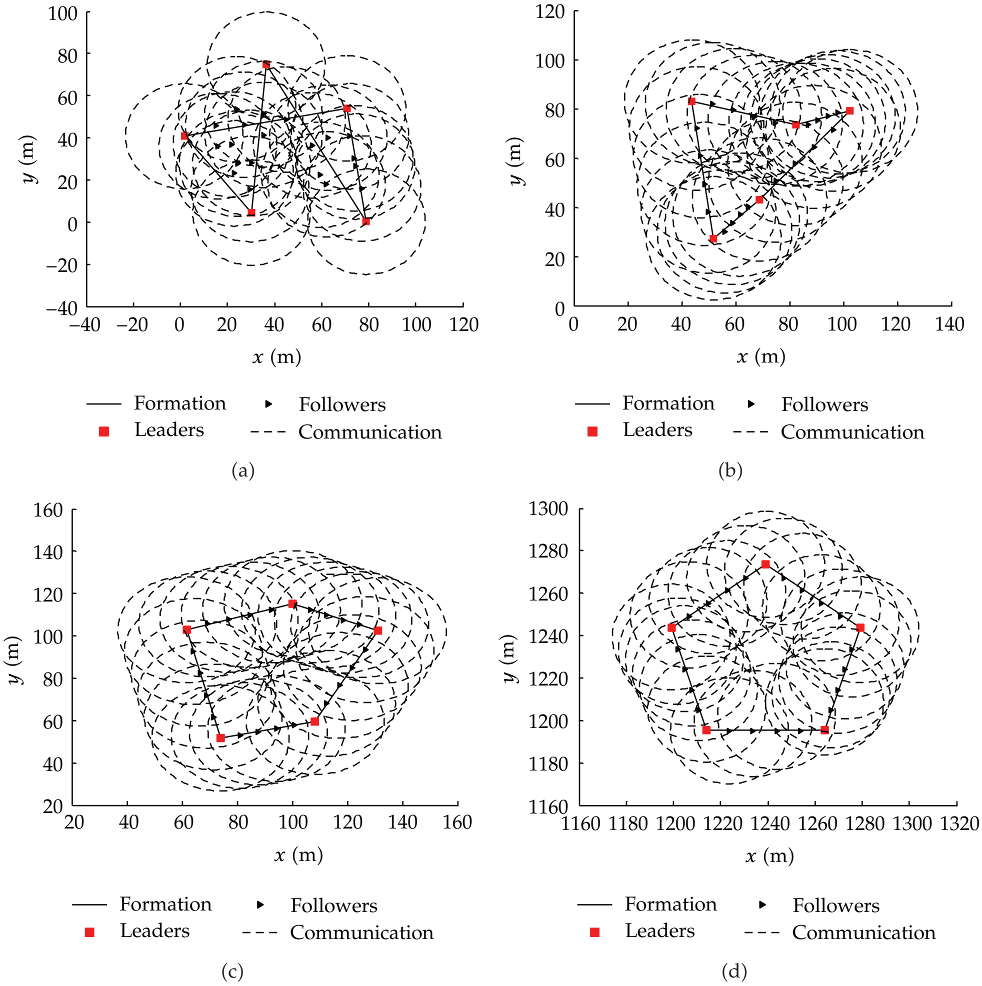


Figure 7: (a) States of all vehicles at time 5 seconds. (b) States of all vehicles at time 10 seconds. (c) States of all vehicles at time 15 seconds. (d) Final states of all vehicles.

shown as Figure 5. According to the proposed method, it is easily known that the number of leaders is 5 and the number of followers is 20. Leaders' formation vector is $\mathbf{h}^T$:

$$\mathbf{h} = [0, -30, -30, 30, 30, 0, 40, 100, 100, 40, 30, 0, 0, 60, 60, -40, 0, 60, 60, 0, 30, 0, 0, 60, 60, -100, \\ -60, 0, 0, -60, -30, -60, -60, 0, 0, -100, -60, 0, 0, -60, -30, -60, -60, 0, 0, -40, 0, 60, 60, 0]. \tag{5.1}$$

Control parameters are $\alpha = 1$, $\beta = 5$, $k = 10$, damp feedback coefficient $c = 1$, and velocity navigation coefficient $\gamma = 0.4$. Communication range is 20 and safe range is 6. Time delay is 0.2 second. Simulation step is not fixed and total time is 60 seconds.

The initial positions of all vehicles are randomly distributed and the initial network is connectivity as shown in Figure 6(a). In the designed formation framework, vehicles can elect

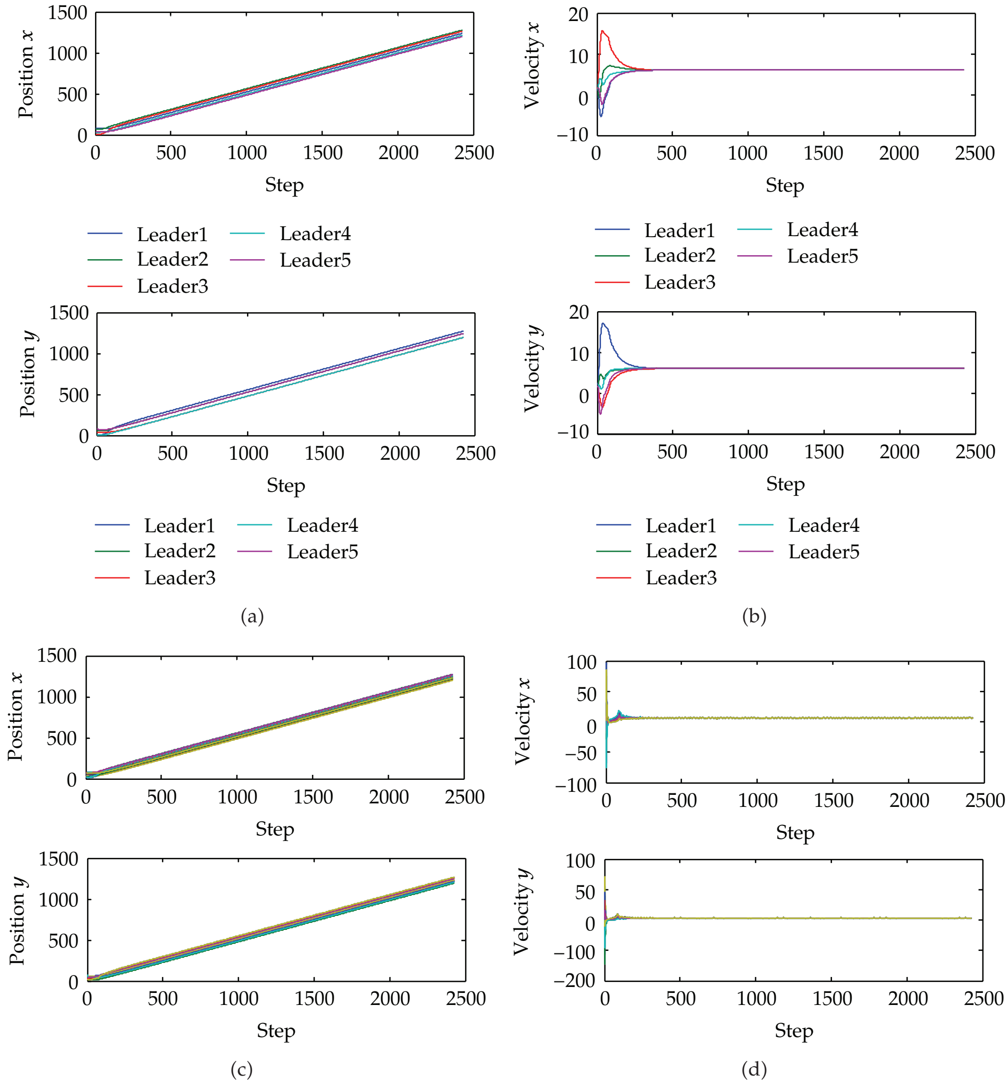


Figure 8: (a) Trajectories of leader vehicles. (b) Velocities of leader vehicles. (c) Trajectories of follower vehicles. (d) Velocities of follower vehicles.

leaders and followers automatically using the dividing algorithm. The states of all vehicles after grouping are shown in Figure 6(b).

Formation movement of 25 vehicles can be seen clearly in Figure 7. At time 5 seconds, followers are tracking their own virtual leader shown in Figure 7(a). At time 10 seconds, followers have tracked their own virtual leaders shown in Figure 7(b). At time 15 seconds, leaders are forming desired formation geometric shape using the global formation information shown in Figure 7(c). Final states of all vehicles are shown in Figure 7(d); the desired formation is formed and maintained.

To show the simulation results clearly, we choose position and velocity on x and y axes to make the results fill in Figure 8. Trajectories of leader vehicles are shown in Figure 8(a); it shows that the desired formation is formed and maintained. Velocities of leader vehicles are shown in Figure 8(b); it shows that desired velocities are achieved. Trajectories of follower vehicles are shown in Figure 8(c), it shows that local formation is formed and maintained. Velocities of follower vehicles are shown in Figure 8(d); it shows that desired formation velocities are achieved.

6. Conclusions

In this paper, we proposed a distributed formation framework of large-scale intelligent autonomous vehicles. Under the assumption that the initial network is connected, the vehicles are divided into leaders and followers automatically. Global formation information is only injected to leaders which can greatly reduce data exchange. Using the designed formation control law, the vehicles are able to realize various kinds of formation asymptotically while avoiding collision. It is illustrated that the proposed formation framework is effective by simulation results.

Nevertheless, there are still some other problems that need to be discussed in further detai, such as introducing more complex dynamic model and considering obstacle avoidance problems.

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