**THE EUROPEAN
PHYSICAL JOURNAL C**

Regular Article - Theoretical Physics

DGP cosmological model with generalized Ricci dark energy

Yeremy Aguilera[1,a], **Arturo Avelino**[2,b], **Norman Cruz**[3,c], **Samuel Lepe**[4,d], **Francisco Peña**[5,e]

[1] Departamento de Matemáticas y Ciencia de la Computación, Universidad de Santiago, Estación Central, Santiago, Chile
[2] Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA
[3] Departamento de Física, Facultad de Ciencia, Universidad de Santiago, Casilla 307, Santiago, Chile
[4] Facultad de Ciencias, Instituto de Física, Pontificia Universidad Católica de Valparaíso, Avenida Brasil 2950, Valparaiso, Chile
[5] Departamento de Ciencias Físicas, Facultad de Ingeniería y Ciencias, Universidad de La Frontera, Casilla 54-D, Temuco, Chile

Abstract The brane-world model proposed by Dvali, Gabadadze and Porrati (DGP) leads to an accelerated universe without cosmological constant or other form of dark energy for the positive branch ($\epsilon = +1$). For the negative branch ($\epsilon = -1$) we have investigated the behavior of a model with an holographic Ricci-like dark energy and dark matter, where the IR cutoff takes the form $\alpha H^2 + \beta \dot{H}$, H being the Hubble parameter and α, β positive constants of the model. We perform an analytical study of the model in the late-time dark energy dominated epoch, where we obtain a solution for $r_c H(z)$, where r_c is the leakage scale of gravity into the bulk, and conditions for the negative branch on the holographic parameters α and β, in order to hold the conditions of weak energy and accelerated universe. On the other hand, we compare the model versus the late-time cosmological data using the latest type Ia supernova sample of the *Joint Light-curve Analysis* (JLA), in order to constrain the holographic parameters in the negative branch, as well as $r_c H_0$ in the positive branch, where H_0 is the Hubble constant. We find that the model has a good fit to the data and that the most likely values for $(r_c H_0, \alpha, \beta)$ lie in the permitted region found from an analytical solution in a dark energy dominated universe. We give a justification to use a holographic cutoff in 4D for the dark energy in the 5-dimensional DGP model. Finally, using the Bayesian Information Criterion we find that this model is disfavored compared with the flat ΛCDM model.

[a] e-mail: yeremy.aguilera@usach.cl

[b] e-mail: aavelino@cfa.harvard.edu

[c] e-mail: norman.cruz@usach.cl

[d] e-mail: slepe@ucv.cl

[e] e-mail: francisco.pena@ufrontera.cl

1 Introduction

The acceleration in the expansion of the universe during recent cosmological times, first indicated by supernova observations [1–6] and also supported by the astrophysical data obtained from WMAP [7], indicates the existence of a fluid with negative pressure, which has been identified as dark energy due to its unknown nature. In order to explain the nature of this dark energy non-conventional approaches have advocated extra dimensions inspired by string and superstring theories. One of these models that have lead to an accelerated universe without cosmological constant or other form of dark energy is the brane-world model proposed by Dvali, Gabadadze, and Porrati (DGP) [8–10] (for reviews, see [11] and [12]). In a cosmological scenario, this approach leads to a late-time acceleration as a result of the gravitational leakage from a 3-dimensional surface (3-brane) to a fifth extra dimension on Hubble distances.

It is a well known fact that the DGP model has two branches of solutions: the self-accelerating branch and the normal one. The self-accelerating branch leads to an accelerating universe without invoking any exotic fluid, but it presents problems like ghosts [13]. Nevertheless, the normal branch requires a dark energy component to accommodate the current observations [14–16]. Extended models of gravity on the brane with f(R) terms have been investigated to obtain self-acceleration in the normal branch [17]. Solutions for a DGP brane-world cosmology with a k-essence field were found in [18] showing big rip scenarios and an asymptotically de Sitter phase in the future.

In the present work we explore a DGP cosmology in the framework of the holographic dark energy models [19–21], which are based on the holographic principle [22–24]. This principle has been advocated as a guideline to a complete theory of quantum gravity. A realization of this principle, based on the validity of the effective quantum field theory,

Springer

was formulated by Cohen et al. [19], by making the suggestion that the total energy in a region of size L should not exceed the mass of a black hole of the same size, which means $\rho_\Lambda \leq L^{-2}M_p^2$. The largest L is chosen by saturating this bound so that we obtain the holographic dark energy (HDE) density

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}, \tag{1}$$

where c is a free dimensionless $\mathcal{O}(1)$ parameter that can be determined by observations. Taking L as the Hubble radius $L = H_0^{-1}$ this ρ_Λ is comparable to the observed dark energy density, but it gives a wrong EoS for the dark energy [20].

For higher-dimensional space-times, the holographic principle in cosmological scenarios has been formulated considering the maximal uncompactified space of the model, i.e. in the bulk, leading to a crossing of phantom divide for the holographic dark energy, in 5D two-brane models [25]. Other investigations show that when the IR cutoff is the event horizon the vacuum energy would end up with a phantom phase with an inevitable Big Rip singularity [26].

Recently, a modified holographic dark energy model has been formulated using the mass of black holes in higher dimensions and the Hubble scale as the IR cutoff [27]. Using the future event horizon as the IR cutoff, it was found that the EoS of the holographic dark energy can cross the phantom divide [28]. The inclusion of a Gauss–Bonnet term in the bulk and an holographic energy density have been explored in [29], obtaining a late-time acceleration consistent with observations. In the same approach, but, using a Ricci-like dark energy, scenarios free from future singularities were found in [30].

Our aim in this work is to investigate a DGP model of a flat universe filled with an holographic Ricci-like dark energy [31,32] and dark matter. This holographic energy density takes the form [33]

$$\rho_h = (3/8\pi G)(\alpha H^2 + \beta \dot{H}), \tag{2}$$

where α and β are positive constants. This type of holographic dark energy works fairly well in fitting the observational data. Nevertheless, a global fitting on the parameters of this model using combined cosmic observations from type Ia supernovae, baryon acoustic oscillations, Cosmic Microwave Background and the observational Hubble data do not favor the holographic Ricci dark energy model over the ΛCDM model [34]. For the far future, the EoS behaves like a quintom model, crossing the phantom barrier [35–37]. The statefinder diagnostic of this model, in the framework of general relativity, indicates that interactions in the dark sector are favored [38]. It was found that without giving a priori some specific model for the interaction function, this can experience a change of sign during the cosmic evolution [39–45].

In the case of a DGP model, besides the holographic parameters, there exists the parameter $r_c H_0$, where r_c is the characteristic scale of the DGP model given by

$$r_c = \frac{1}{2} \frac{M_{(4)}^2}{M_{(5)}^3}, \tag{3}$$

which sets a length beyond which gravity starts to leak out into the bulk. $M_{(n)}$ is the n-dimensional Planck mass.

In this work we are interested in constraining the holographic parameters, α, β, and $r_c H_0$, and make a comparison with the ΛCDM model, using the Bayesian Information Criterion. This allows us to establish what model is most favored by cosmological observations and the suitability of an holographic Ricci-like dark energy in the DGP framework.

Our paper is organized as follows. In Sect. 2 we discuss the DGP model for a flat universe filled with a fluid obeying a barotropic EoS. Constraints for the parameter $r_c H$ are obtained. In Sect. 3 we study analytically the solution of the differential equation given by the model assuming a late-time evolution, where the density ρ_h dominates; the solution comes with restrictions from the weak energy condition WEC, the accelerated late-time expansion and the positivity of H. In Sect. 4 we work in the late-time phase universe and solve numerically a differential equation for $E = H/H_0$, where H is the Hubble parameter and show a table with the best estimates for the holographic parameters and figures of the confidence regions which was obtained for some variables for each branch. In Sect. 5 we explain the main calculation to use the SNIa data set and the Hubble parameter for different redshifts. In Sect. 6 we give arguments in order to justify the used holographic cutoff in 4D for the dark energy in the DGP model (5D). In Sect. 7 we discuss our results obtained for the different branches of the DGP model and compare it with the ΛCDM using the Bayesian Information Criterion.

2 DGP model

For an homogeneous and isotropic universe described by the FLRW metric the field equation is given by [9,10] (with $8\pi G = c = 1$)

$$\left(H^2 - \frac{\epsilon}{r_c}\sqrt{H^2 + \frac{k}{a^2}} \right) = \rho - \frac{3k}{a^2}, \tag{4}$$

where a is the cosmic scale factor, ρ is the total cosmic fluid energy density on the brane. The parameter $\epsilon = \pm 1$ represents the two branches of the DGP model. It is well known that the solution with $\epsilon = +1$ represents the self-accelerating branch, since even without dark energy the expansion of the universe accelerates. For late times the Hubble parameter approaches a constant, $H = 1/r_c$. In previous investiga-

tions, $\epsilon = -1$ has been named the normal branch, where the acceleration only appears if a dark energy component is included. By considering a flat universe as is suggested by the Planck results [46], Eq. (4) becomes

$$H^2 - \epsilon \frac{H}{r_c} = \rho, \tag{5}$$

and the weak energy condition (WEC) implies $r_c H \geq \epsilon$. If cosmic fluid satisfies a barotropic equation of state $p = \omega\rho$, the conservation equation is given by

$$\dot{\rho} + 3H(1+\omega)\rho = 0. \tag{6}$$

From Eqs. (5) and (6), we obtain an expression for the equation of state parameter, ω, in terms of the Hubble parameter, which is given by

$$1 + \omega = -\frac{1}{3} \left(\frac{2 - \epsilon(r_c H)^{-1}}{1 - \epsilon(r_c H)^{-1}} \right) \frac{\dot{H}}{H^2}. \tag{7}$$

According to Eq. (7), $1 + \omega < 0$ implies that $\dot{H} > 0$, since $(2 - \epsilon(r_c H)^{-1})/(1 - \epsilon(r_c H)^{-1}) > 0$ for both cases $(r_c H)^{-1} \lessgtr 1$. Besides, from this condition we see that WEC implies $r_c H > \epsilon$.

For $\epsilon = +1$ we notice that the leakage scale must be restricted to $r_c H_0 > 1$. For $\epsilon = -1$ we have $r_c H_0 > -1$, which does not imply any further constraint upon $r_c H_0$, since we are in the expanding phase with $H_0 > 0$.

3 Dark energy domination phase

In this section we consider the late-time behavior of our model in the normal branch, $\epsilon = -1$, where the holographic dark energy density ρ_h dominates and the matter density ρ_m can be neglected. In this case an analytical solution can be obtained solving Eq. (5) for ρ_h given by Eq. (2). We obtain the following expression for $r_c H(z)$:

$$r_c H(z) = \frac{1}{\alpha - 1} + \left[r_c H_0 - \frac{1}{\alpha - 1} \right] (1+z)^{\frac{\alpha-1}{\beta}}, \tag{8}$$

for $\alpha \neq 1$ and $\beta \neq 0$. We notice from this equation that $\alpha > 1$ is required, to ensure the positivity of $r_c H(z)$ for an expanding universe. The solution for the scale factor yields

$$a(t) = a_0[(\alpha - 1)r_c H_0(e^{\left(\frac{1}{\beta r_c}(t-t_0)\right)} - 1) + 1]^{\frac{\beta}{\alpha-1}}, \tag{9}$$

where the initial condition is $a(t = t_0) = a_0$, and t_0 is the present time. Notice that $a(t)$ is well behaved because the exponent $\beta/(\alpha - 1)$ is always positive. From Eqs. (2) and (5), and the expression for the acceleration $\frac{\ddot{a}}{a} = \dot{H} + H^2$ we have

$$\frac{\ddot{a}}{a} = \frac{H^2}{\beta} \left(1 - \alpha + \beta + \frac{1}{r_c H} \right), \tag{10}$$

we can obtain the conditions for the parameters α and β in order to have an accelerated late-time expansion. These conditions also ensure that WEC still holds. This late-time expansion behaves like a de Sitter phase.

4 The holographic dark energy and matter component

For a spatially flat FRW universe composed by the holographic dark energy as well as a matter component (dark and baryon matter), the Friedmann equation (5) in DGP cosmology has the form (with units)

$$H^2 - \epsilon \frac{H}{r_c} = \frac{8\pi G}{3}(\rho_h + \rho_m). \tag{11}$$

Conditions for dark energy domination phase		
Branch(ϵ)	α	β
-1	$\alpha > 1$	$\beta > -\left(1 - \alpha + \frac{1}{r_c H_0}\right)$

The pressureless matter scales in the usual way as $\rho_m = \rho_{m0}a^{-3}$, where ρ_{m0} is the present-day value of the matter density in the Universe. Inserting the expression (2) for ρ_h and $\rho_m = \rho_{m0}a^{-3}$ at Eq. (11), and reorganizing terms, we have

$$\beta \dot{H} - H^2(1-\alpha) + \epsilon \frac{H}{r_c} + \left(\frac{8\pi G}{3}\right) \frac{\rho_{m0}}{a^3} = 0. \tag{12}$$

We change the derivative of H with respect to time to the scale factor as $\dot{H} = (dH/da)\dot{a} = (dH/da)aH$; then Eq. (12) becomes

$$\beta a \frac{dH(a)}{da} - (1-\alpha)H(a) + \frac{\epsilon}{r_c} + \left(\frac{8\pi G}{3}\right) \frac{\rho_{m0}}{a^3 H(a)} = 0. \tag{13}$$

Dividing Eq. (13) by the Hubble constant H_0, defining the parameter density $\Omega_{m0} \equiv \rho_{m0}/\rho_{crit}^0$ where $\rho_{crit}^0 \equiv 3H_0^2/(8\pi G)$, changing of variable from the scale factor to the redshift, and defining the dimensionless Hubble parameter as $E \equiv H/H_0$, the differential equation (13) becomes

$$\beta(1+z)\frac{dE(z)}{dz} + (1-\alpha)E(z) - \frac{\epsilon}{r_c H_0} - \Omega_{m0}\frac{(1+z)^3}{E(z)} = 0. \tag{14}$$

We solve numerically this differential equation with the initial condition $E(z = 0) = 1$, and for both branches, $\epsilon = \pm 1$. The values of $(r_c H_0, \alpha, \beta, \Omega_{m0})$ are estimated and

constrained using the cosmological observations of type Ia supernovae as described in the next section.

5 Cosmological constraints

We test the viability of the model and constrain its free parameters ($r_c H_0$, α, β, Ω_{m0}) by using the *Joint Light-curve Analysis* (JLA) sample of type Ia supernovae (SNe Ia) of Betoule et al. [47], composed by 740 SNe that comes from nine different surveys. We compute the best-fit values and confidence intervals by sampling the parameter space using the Affine Invariant Markov Chain Monte Carlo (MCMC) of Goodman et al. [48], implemented in the *emcee* code [49].

The definition of the luminosity distance d_L in a flat FRW cosmology is given as

$$d_L(z, \vec{p}, H_0) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z', \vec{p})}, \tag{15}$$

where $E(z, \vec{p})$ is given by the numerical solution to the differential equation (14), "c" is the speed of light given in units of km/sec and $\vec{p}$ is the vector of parameters, i.e., $\vec{p} = (r_c H_0, \alpha, \beta, \Omega_{m0})$. The theoretical distance modulus is defined as

$$\mu^t(z_k, \vec{p}) = 5 \log_{10} \left[\frac{d_L(z, \vec{p})}{\text{Mpc}} \right] + 25, \tag{16}$$

where the superscript ' t' stands for the *theoretical* prediction of the distance modulus for a supernova at a redshift z_k.

On the other hand, the observed distance modulus for each supernova can be computed by modeling their intrinsic variability observed in their light curves [50], as

$$\mu = m_B^* - (M_B - \alpha_{lc} \times X_1 + \beta_{lc} \times C) \tag{17}$$

where M_B is the absolute magnitude of the SNe in the rest-frame B band. This parameter, together with (α_{lc}, β_{lc}), which characterizes the global properties of the light-curves of the SNe, are nuisance parameters that have to be computed and marginalized simultaneously with the cosmological parameters of interest.

On the other hand, m_B^*, X_1, C are the observed peak magnitude in the rest-frame B band, and the stretch and color parameters for each SN, respectively. They capture the intrinsic variability in the luminosity of the SNe. Their central values as well as the covariance matrices that account for all known sources of systematic uncertainties, as well as the statistical uncertainties, are publicly available at http://supernovae.in2p3.fr/sdss_snls_jla/ReadMe.html.

Following [47], the distance modulus for all the SNe are reorganized in a vector of $n = 740$ entries given as

$$\boldsymbol{\mu} = \mathbf{A}\boldsymbol{\eta} - \mathbf{M_B} \tag{18}$$

where the n-dimensional vector $\boldsymbol{\eta}$ and the $n \times n$ matrix $\mathbf{A}$ are given as

$$\boldsymbol{\eta} = ((m_{B,1}^*, X_{1,1}, C_1), \ldots, (m_{B,n}^*, X_{1,n}, C_n)) \tag{19}$$

$$\mathbf{A} = \mathbf{A}_0 + \alpha \mathbf{A}_1 - \beta \mathbf{A}_2, \quad \text{with } (\mathbf{A}_k)_{i,j} = \delta_{3i,i+k}. \tag{20}$$

With this, the χ^2 function to be minimized to compute the best-fit values and confidence intervals of the cosmological parameters has the form

$$\chi^2(\vec{p}, \vec{p}_{lc}) = (\boldsymbol{\mu}(\vec{p}_{lc}) - \boldsymbol{\mu}^t(z, \vec{p}))^T \mathbf{C}^{-1} (\boldsymbol{\mu}(\vec{p}_{lc}) - \boldsymbol{\mu}^t(z, \vec{p})) \tag{21}$$

where $\vec{p}_{lc} = (M_B, \alpha_{lc}, \beta_{lc})$ and $\mathbf{C}^{-1}$ is the inverse of the total covariance matrix reported in [47] that encapsulates all the known systematic and statistical errors. For a detailed discussion of $\mathbf{C}$ of the JLA sample see Betoule et al. [47]. We consider also the fixed fiducial value of $H_0 = 70$ km s^{-1} Mpc^{-1}.

5.1 Self-accelerated branch: $\epsilon = +1$

For the positive branch, $\epsilon = +1$, we consider the DGP brane filled with the baryon and dark matter component only, Ω_{m0}. We neglect the dark energy density because this branch is already accelerated. In this case, the only free parameter is the leakage scale $r_c H_0$, given that we set $\alpha = 0$, $\beta = 0$. In this case, the matter density Ω_{m0} is related to the leakage scale as

$$\Omega_{m0} = 1 - (1/r_c H_0). \tag{22}$$

This constraint comes directly from the differential equation (14) for this case.

The marginalized best-fit value for the leakage scale $r_c H_0$ is shown in Table 1, item (i), and Fig. 1 shows the joint credible regions for combinations of ($r_c H_0$, M_B, α_{lc}, β_{lc}) in pairs, as well as the marginalized probability density functions (PDFs) for each parameter. The relevant result for the leakage scale is that it is consistently having values in the physical region $r_c H_0 > 1$ (see Fig. 1). Marginalizing over the other parameter, the best-fit value is $r_c H_0 = 1.3^{+3.8}_{-0.3}$.

5.2 Normal branch: $\epsilon = -1$

For this branch we consider a universe filled with an holographic Ricci-like dark energy and dark matter. We solve

Table 1 Marginal best-fit values for the model parameters $(r_c H_0, \alpha, \beta, \Omega_{m0})$ as well as the light-curve parameters $(M_B, \alpha_{lc}, \beta_{lc})$ of type Ia supernovas of the JLA sample [47] and computed together with the cosmological parameters. The *first row* shows the best-fit values for the leakage scale $r_c H_0$ for the self-accelerated branch ($\epsilon = +1$) of a DGP brane model composed only of baryon and dark matter, Ω_{m0}, at late times. In this case, Ω_{m0} is found from the constraint equation $\Omega_{m0} = 1 - (1/r_c H_0)$ (see Sect. 5.1 for details). The *second row* shows the best-fits for $(r_c H_0, \alpha, \beta, \Omega_{m0})$ on the normal branch of a DGP brane model composed by Ω_{m0} and a Ricci-like holographic dark energy of the form $\rho_h = (3/8\pi G)(\alpha H^2 + \beta \dot{H})$, at late times. In (iii) are also

shown the best-fit values for the flat ΛCDM model in order to compare the results. We use the Bayesian Information Criterion (BIC) in order to assess the best model to fit the data, between the holographic DGP and the flat ΛCDM model. The model favored by the observations compared to the other corresponds to the one with the smallest value of BIC. In general, a difference of 2 in BIC between two models is considered as evidence against the model with the higher BIC, and a difference of 6 is strong evidence. We find that despite the good fit to the data by the holographic DGP model, it is disfavored compared with the ΛCDM model using the BIC criterion, as shown in the last column

Model	Best estimates								
	$r_c H_0$	α	β	Ω_{m0}	M_B	α_{lc}	β_{lc}	χ^2_{min}	BIC
(i) DGP ($\epsilon = +1$) only	$1.3^{+3.8}_{-0.3}$	–	–	$0.23^{+0.57}_{-0.23}$	$-18.9^{+0.14}_{-0.15}$	$0.13^{+0.06}_{-0.05}$	$3.3^{+0.84}_{-0.65}$	684.4	710.9
(ii) DGP ($\epsilon = -1$) + ρ_h	$1.2^{+3.5}_{-0.2}$	$2.1^{+3.4}_{-1.1}$	$2.45^{+5.5}_{-1.4}$	$0.46^{+0.54}_{-0.46}$	$-19.08^{+0.16}_{-0.15}$	$0.14^{+0.056}_{-0.054}$	$3.17^{+0.5}_{0.55}$	683.3	729.5
(iii) Flat ΛCDM	–	–	–	0.29 ± 0.03	-19.05 ± 0.02	0.14 ± 0.01	3.1 ± 0.01	683.0	709.4

numerically the differential equation (14) with the initial condition $E(z = 0) = 1$. In this case, the free parameters to be estimated are $(r_c H_0, \alpha, \beta, \Omega_{m0})$ together with the light-curve parameters $(M_B, \alpha_{lc}, \beta_{lc})$.

Figure 2 shows the joint credible regions in pairs of parameters as well as their marginalized PDFs. We find that the holographic best-fit parameters are $\alpha = 2.1^{+3.4}_{-1.1}$, $\beta = 2.45^{+5.5}_{-1.4}$, and $r_c H_0 = 1.2^{+3.5}_{-0.2}$ well compatible with the analytical constraints on the model described in Sect. 2.

For these parameters we notice that $r_c H_0 = 1.2$, $\alpha = 2.1 > 1$, and $\beta = 2.45 > -\left(1 - \alpha + \frac{1}{r_c H_0}\right) = 0.267$, which is in agreement with the constraint derived in Sect. 3 for an holographic dark energy domination phase. These values ensure WEC and accelerated expansion. For Ω_{m0} we find that the data does not impose tight constraints on it, so that any value in the range $0 < \Omega_{m0} < 1$ is equally likely according to the data.

6 Remarks for the holographic dark energy in higher-dimensional gravity

Let us discuss with some detail the results of the studies on the IR cutoff for holographic dark energy models in the framework of higher-dimensional gravity. In [27] was considered the mass of the Schwarzschild black hole in $N + 1$-dimensional space-time and then using consistently the formulation of Cohen et al. [19]. The direct use of the $N + 1$-dimensional solution for a spherically symmetric static matter source can be well justified taking into account that the Schwarzschild radius for the holographic considerations is H^{-1}. In this case the Vainshtein radius, $(r_c^2 H^{-1})^{1/3}$, which is the length scale at which gravity is modified, is of the same order of the Hubble radius and we expect that gravity

becomes 5-dimensional. Nevertheless, it has been pointed out by Viaggiu [51,52] that the condition (1) is derived considering the Schwarzschild solution, which represented an exact solution for symmetric perturbation of Minkowskian spacetime. But when we apply this condition to Friedmann universes filled with dark energy it leads to wrong results, due to the avoidance of black hole formation when a small but finite cosmological constant is present. In the case of DGP models, modifications introduced in the metric of a spherically symmetric, static matter source have been found in [53]. In this work the de Sitter background is considered. The main result lies in the absence of Birkhoff's theorem for DGP theory, which means that for even spherically symmetric sources the exact distribution of matter affects the gravitational force external to the source. The above results indicate that a fully consistent approach of the holographic dark energy in these models of modified gravity is currently under construction. Considering this situation and despite the improvements realized in [27], in order to consider higher dimensions in the holographic cutoff, we assume Eq. (1) valid as a first approximation.

Nevertheless, the above discussion brings forth the following question: how do both approaches differ for the holographic Ricci-like dark energy? In what follows we evaluate numerically these differences. From Eq. (2) and using the expression for the deceleration parameter $1 + q(z) = -\dfrac{\dot{H}}{H^2}$ we obtain

$$\rho(z) = 3\alpha \left[1 - \frac{\beta}{\alpha}(1 + q(z))\right] H^2(z) \tag{23}$$

$$= 3c^2(z) H^2(z) \tag{24}$$

where we have made the identification

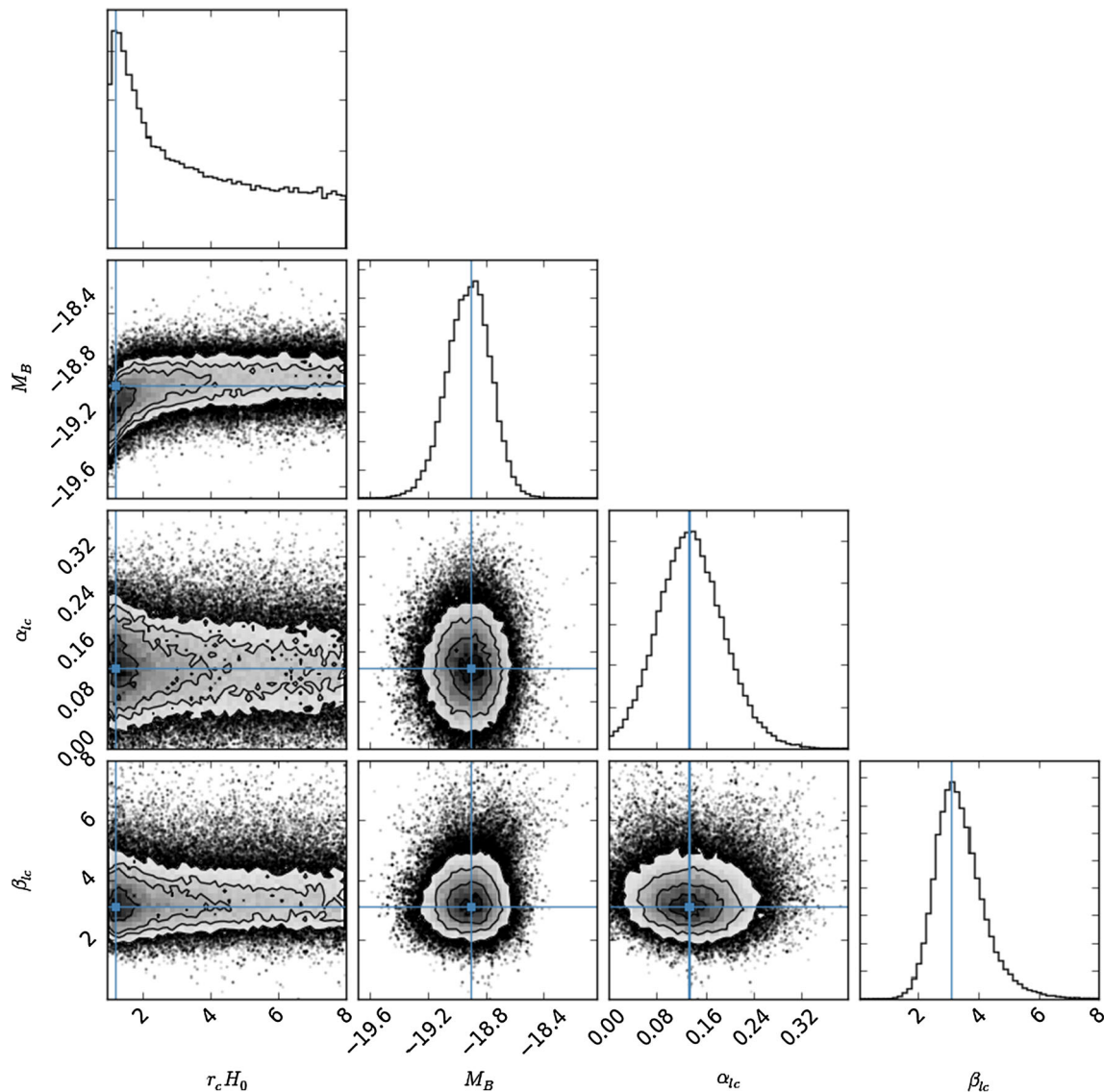$$c^2(z) = \alpha \left[1 - \frac{\beta}{\alpha}(1 + q(z))\right]. \tag{25}$$

Fig. 1 Joint and marginalized constraints on the leakage scale $r_c H_0$ of the self-accelerated branch ($\epsilon = +1$) of the DGP model in a spatially flat Universe. Also shown are the global light-curve parameters (M_B, α_{lc}, β_{lc}) of the JLA sample that were computed simultaneously with $r_c H_0$. The credible regions correspond to 1σ (68.3 %), 2σ (95.5 %), and 3σ (99.7 %) confidence level (CL). We assumed flat priors for all the parameters. We set the physical limit of $r_c H_0 > 1$. The individual best-fit values are shown in Table 1

This means that the Ricci cutoff can be seen as a generalization of the holographic dark energy density $\rho = 3c^2 H^2$, with $c = cte$. This condition has already been discussed in [54], where it is pointed out that if $c^2(z)$ grows with time the bound given by the holographic condition progressively saturates up to full saturation when, asymptotically, $c^{(}z)$ becomes constant.

If we rewrite Eq. (23) as

$$\rho(z) = 3\bar{c}^2(z) H(z), \tag{26}$$

with $\bar{c}^2(z) = c^2(z) H(z)$ the holographic Ricci bound it mimics the holographic bound in [27] for a 5-dimensional gravity

with H as the IR cutoff and a variable $\bar{c}^2(z)$ parameter. We will evaluate the variations of $\bar{c}^2(z)$ in the range of $0 \leq z \leq 1$. Using the expression for $q(z)$ in terms of H and $\dot{H}$ we can write

$$q(z) = \frac{(1+z)}{E(z)} \frac{\mathrm{d}E(z)}{\mathrm{d}z} - 1 \tag{27}$$

where $E(z)$ defined in Sect. 4 comes from the solution of the differential equation (14), using the information of this section and the parameters obtained with the data analysis we obtain Fig. 1.

From Fig. 3 we can see directly that the behavior of $\bar{c}^2(z)$ for future times is almost constant for the best-fit parameters
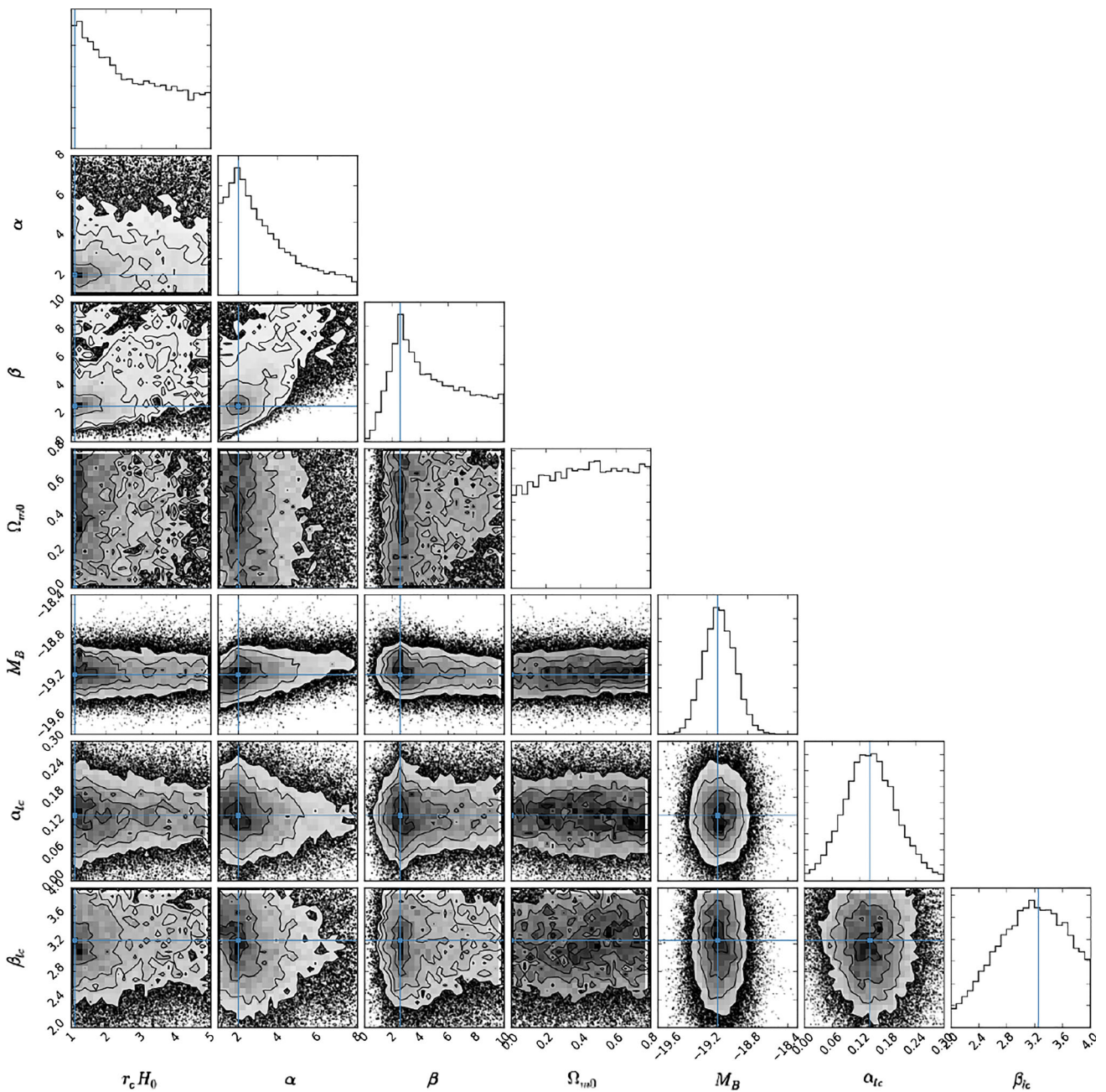
Fig. 2 Joint and marginalized constraints on the leakage scale $r_c H_0$ of the *non* self-accelerated branch ($\epsilon = -1$) of a DGP model in a spatially flat Universe, filled with a matter component (dark and baryonic matter), Ω_{m0}, and a Ricci-like holographic dark energy of the form $\rho_h = (3/8\pi G)(\alpha H^2 + \beta \dot{H})$. It is shown that the marginalized constraints on $(r_c H_0, \alpha, \beta, \Omega_{m0})$ as well as for the global light-curve parameters ($M_B, \alpha_{lc}, \beta_{lc}$) of the JLA sample that were computed simultaneously. The credible regions correspond to 1σ (68.3 %), 2σ (95.5 %), and 3σ (99.7 %) confidence level (CL). We assumed flat priors for all the parameters in the physical intervals: $0 < \Omega_m < 1$, $r_c H_0 > 1$, $\alpha > 1$, $\beta > 0$. The individual best-fit values are shown in Table 1

(red solid line). This evaluation indicates that our holographic energy density is proportional to the Hubble parameter for future times, which is similar to the behavior of the model discussed in [27].

7 Discussion and conclusions

We have investigated the viability of a cosmological model composed by a Ricci-like holographic dark energy in the
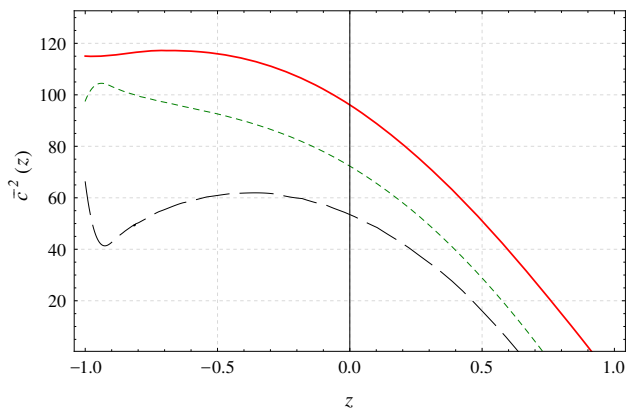
Fig. 3 Evolution of the function $\bar{c}^2(z)$ defined from Eq. (26), in the late time and future of the Universe. The *red solid line* corresponds to the best-fit values ($r_c H_0 = 1.3$, $\alpha = 2.1$, $\beta = 2.45$, $\Omega_{m0} = 0.46$), while the *green short dashed line* and *black long dashed line* correspond to the arbitrary values of (3, 1.1, 3, 0.3) and (6, 4, 8, 0.4) respectively, with the purpose to illustrate the behavior of $\bar{c}^2(z)$ for other values

non-self-accelerated branch of a DGP brane world, to explain the late-time accelerated expansion of the Universe.

We have discussed the holographic approach in higher-dimensional gravity, indicating that further research is required in order to have a fully consistent formulation. Nevertheless, an improvement in which the black hole solution in five dimensions is taken into account, was compared with our approach, showing that for future late times both holographic densities behave in a similar way.

For the dark energy domination phase in the normal branch $\epsilon = -1$, the model presents a de Sitter-like expansion, and the conditions for the parameter are $\alpha > 1$ and

$$\beta > -\left(1 - \alpha + \frac{1}{r_c H_0}\right).$$

When we computed in the positive branch the confidence interval for $r_c H_0$, we found that the leakage length scale is well compatible with positive values. Given that r_c is a length scale, the minimum requirement of positive values for this parameter is satisfied. For the case when we consider the negative branch $\epsilon = -1$ of the DGP brane world, we find that the parameters are constrained to be $1.2^{+3.5}_{-0.2}$, $\alpha = 2.1^{+3.4}_{-1.1}$, and $\beta = 2.45^{+5.5}_{-1.4}$ (see Table 1 (ii)), satisfying therefore the conditions for WEC and for the dark energy domination phase. So, the data indicates that the DGP model with an holographic Ricci-like dark energy might be a valid cosmological model.

On the other hand, in order to assess the viability of the present Ricci-like holographic DGP model to explain the late-time accelerated expansion of the Universe, we compare the flat ΛCDM model with our present model by using the Bayesian Information Criterion (BIC) [55] to determine which model is the most favored by the observations, taking into account the number of free parameters and the minimum value of χ^2.

In general, the value of BIC for the Gaussian errors of the data used is defined as

$$\text{BIC} = \chi^2_{\min} + \nu \ln N, \tag{28}$$

where ν and N are the number of free parameters of the model and the number of data used, respectively. The model favored by the observations compared to the others corresponds to that with the smallest value of BIC. In general, a difference of 2 in BIC between two models is considered as evidence against the model with the higher BIC, and a difference of 6 is a strong evidence.

In the last column of Table 1 are shown the BIC values for holographic DGP and flat ΛCDM models. We find that the flat ΛCDM model is the favored model by the data according to BIC, compared with the holographic DGP model investigated in the present work.

References

1. S. Perlmutter et al., Astrophys. J. **517**, 565 (1999)
2. P.M. Garnavich et al., Astrophys. J. **493**, L53 (1998)
3. A.G. Riess et al., Astron. J. **116**, 1009 (1998)
4. D.N. Spergel et al., Astrophys. J. Suppl. **148**, 175 (2003)
5. A.G. Riess, Astrophys. J. **607**, 665 (2004)
6. P.J. Peebles, B. Ratra, Rev. Mod. Phys. **75**, 559–606 (2003)
7. G.F. Hinshaw et al., Nine-year Wilkinson microwave anisotropy probe (WMAP) observations: cosmology results. ApJS **208**, 19 (2013)
8. G.R. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B **485**, 208 (2000)
9. C. Deffayet, Phys. Lett. B **502**, 199 (2001)
10. C. Deffayet, G.R. Dvali, G. Gabadadze, Phys. Rev. D **65**, 044023 (2002)
11. K. Koyama, Gen. Rel. Grav. **40**, 421 (2008)
12. M. Li, X. Li, S. Wang, Y. Wang, Commun. Theor. Phys. **56**, 525 (2011). arXiv:1103.5870
13. K. Koyama, Class. Quant. Grav. **24**, R231 (2007). arXiv:0709.2399 [hep-th]
14. A. Lue, G.D. Starkman, Phys. Rev. D **70**, 101501 (2004). astro-ph/0408246

15. V. Sahni, Y. Shtanov, JCAP **0311**, 014 (2003). astro-ph/0202346
16. R. Lazkoz, R. Maartens, E. Majerotto, Phys. Rev. D **74**, 083510 (2006). astro-ph/0605701
17. M. Bouhmadi-Lopez, JCAP **0911**, 011 (2009)
18. M. Bouhmadi-Lopez, L. Chimento, Phys. Rev. **D82**, 103506 (2010)
19. A.G. Cohen, D.B. Kaplan, A.E. Nelson, Phys. Rev. Lett. **82**, 4971 (1999). hep-th/9803132
20. S.D.H. Hsu, Phys. Lett. B **594**, 13 (2004). hep-th/0403052
21. M. Li, Phys. Lett. B **603**, 1 (2004). hep-th/0403127
22. P.F. González-Díaz, Phys. Rev. **D27**, 3042 (1983)
23. G. t Hooft, gr-qc/9310026
24. L. Susskind, J. Math. Phys. **36**, 6377 (1995). hep-th/9409089
25. E.N. Saridakis, JCAP **04**, 020 (2008)
26. X. Wu, R. Cai, Z. Zhu, Phys. Rev. **D77**, 043502 (2008). arXiv:0712.3604 [astro-ph]
27. Y. Gong, T. Li, Phys. Lett. B **683**, 241 (2010)
28. D. Liu, H. Wang, B. Yang, Phys. Lett. B **694**, 6–9 (2010). arXiv:1009.3776 [astro-ph.CO]
29. M. Bouhmadi-Lopez, A. Errahmani, T. Ouali, Phys. Rev. D **84**, 083508 (2011)
30. M. H. Belkacemi, M. Bouhmadi-Lopez, A. Errahmani, T. Ouali, arXiv:1112.5836 [gr-qc]
31. C. Gao, X. Chen, Y. Shen, F. Wu, Phys. Rev. D **79**, 043511 (2009)
32. K. Bamba, S. Capozziello, S. Nojiri, S.D. Odintsov, Astrophys Space Sci **342**, 155228 (2012)
33. L.N. Granda, A. Oliveros, Phys. Lett. **B669**, 275–277 (2008)
34. Y. Wang, L. Xu, Phys. Rev. **D81**, 083523 (2010)
35. B. Feng, X.L. Wang, X.M. Zhang, Phys. Lett. B **607**, 35 (2005)
36. B. Feng, M. Li, Y.S. Piao, X.M. Zhang, Phys. Lett. B **634**, 101 (2006)
37. S. Lepe, F. Peña, Eur. Phys. J. C **69**, 575–579 (2010)
38. F. Yu, J. Zhang, arXiv:1305.2792 [astro-ph.CO]
39. F. Arévalo. P. Cifuentes, S. Lepe, F. Peña, Astrophys. Space Sci. **352**, 899–907 (2014)
40. S. Wang, Y. Wang, X. Zhang, JCAP **0912**, 014 (2009). arXiv:0910.3855
41. M. Li, X. Li, S. Wang, X. Zhang, JCAP **0906**, 036 (2009). arXiv:0904.0928
42. X. Zhang, Phys. Lett. B **683**, 81–87 (2010)
43. L. Xu, JCAP **0909**, 016 (2009)
44. H. Wei, Nucl. Phys. B **819**, 210–224 (2009)
45. S.-F. Wu, P.-M. Zhang, G.-H. Yang, Class. Quant. Grav. **26**, 055020 (2009)
46. Planck Collaboration, P. A. R. Ade, et al., arXiv:1303.5076 (2013)
47. M. Betoule et al., arXiv:1401.4064 (2014)
48. J. Goodman, J. Weare, Commun. Appl. Math. Comput. Sci. **5**(1), 65–80 (2010). doi:10.2140/camcos.2010.5.65
49. D. Foreman-Mackey, D. W. Hogg, D. Lang, J. Goodman, Publications of the Astronomical Society of the Pacific, Vol. 125, No. 925 (pp. 306–312) (2013)
50. R. Tripp, Astron. Astrophys. **331**, 815–820 (1998)
51. S. Viaggiu, Mod. Phys. Lett. A **29** (2014)
52. S. Viaggiu, arXiv:1312.2889 (2013)
53. D. Dai, I. Maor, G.D. Starkman, Phys. Rev. **D77**, 064016 (2008)
54. D. Pavon, W. Zimdahl, Phys. Lett. **B628**, 206–210 (2005)
55. G. Schwarz, Ann. Stat. **6**, 461 (1978)
56. D. Foreman-Mackey, A. Price-Whelan, G. Ryan, Emily, M. Smith, K. Barbary, D.W. Hogg, B.J. Brewer, doi:10.5281/zenodo.10598