Time variable gravity field recovery in local areas by means of Slepian functions

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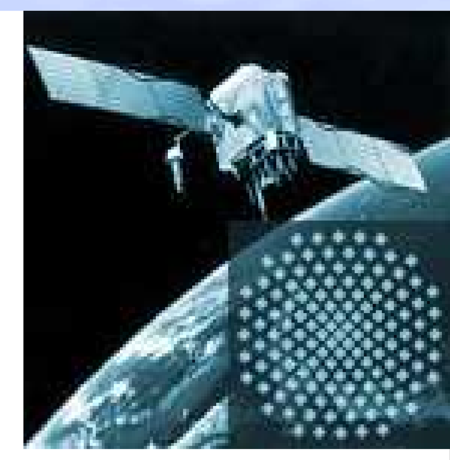

Introduction

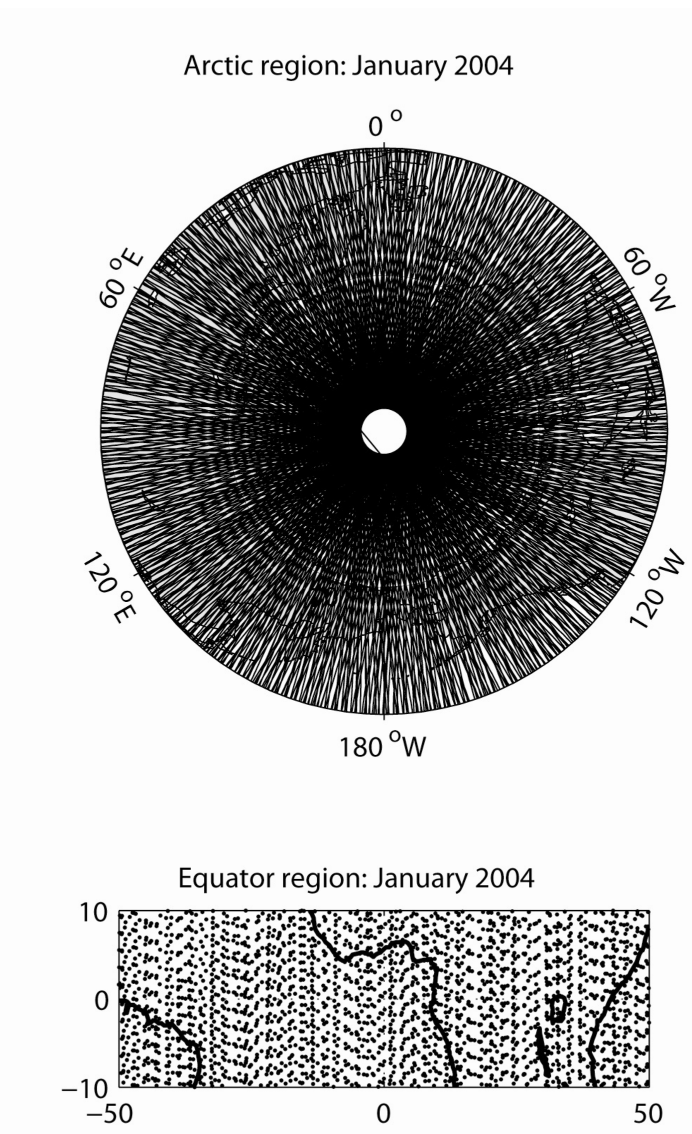

Figure 1: *Ground track for CHAMP*

It was shown by e.g. Han (2003) among others that an improvement can be reached by estimating local gravity field solutions, in high-latitude areas. Due to the almost polar orbit of the satellite missions CHAMP, GRACE and GOCE, the data density is much higher, cf. figure 1. One possible choice for a local analysis are the Slepian functions. This research aims at the demonstration of time-variable studies in the Slepian domain. For this, 45 GRACE monthly spherical harmonic solutions are transformed into the Slepian domain and the estimation of a trend and annual cycle is compared with studies from spherical harmonics. Due to the indication of their spatial concentration, Slepian coefficients can be identified which are improvable by local estimation.

Methodology

A function cannot be space limited and band limited at the same time. Simons et al. (2005) solved the problem of simultaneously concentrating a real-valued function in the spatial and the spectral domain which led to the so-called Slepian functions. The relation between Slepians and spherical harmonics is given as:

$$f = \sum_{i=1}^{N} \beta_i S_i = \sum_{i=1}^{N} \beta_i \sum_{l=0}^{L} \sum_{m=-l}^{l} g_{lm}^{i} Y_{lm} = \sum_{l=0}^{L} \sum_{m=-l}^{l} K_{lm} Y_{lm}$$

where β_i are the Slepian coefficients, K_{lm} the spherical harmonic coefficients and Y_{lm} the spherical surface functions. The solution of the spatial localization problem is essentially identical with the solution of an algebraic eigenvalue problem forming an orthogonal set of base functions S_i which are built using the eigenvectors g_{lm}^{i} of the system. From this equation we can derive the following relations between Slepian and spherical harmonic coefficients:

$$\beta_i = \sum_{l=0}^{L} \sum_{m=-l}^{l} K_{lm}\, g_{lm}^{i} \qquad K_{lm} = \sum_{i=1}^{N} \beta_i\, g_{lm}^{i}$$

These relations enable the forward and backward transformation of existing spherical harmonic coefficient sets into the Slepian domain. Note, that the Slepian coefficients and the number of well concentrated base functions within the area will depend on the choice of the area and the maximum degree *L*. The concentration is indicated by the eigenvalue, i.e., $\lambda \approx 1$ indicates good concentration, $\lambda \approx 0$ indicates poor concentration.

Time-frequency analysis of Slepians

The timely behavior of the Slepian coefficients can be investigated by a Fourier analysis. Figure 2 shows the analysis of the hydrological model GLDAS (Rodell et al., 2004) and the de-striped (Swenson and Wahr, 2006) GRACE Release 4 coefficients from the CSR, Texas. Both are determined for North America, expressed in equivalent water height and smoothed with a Gaussian filter with 400km halfwidth. The annual period can be identified in both pictures. In case of GRACE, some coefficients (red areas) are obviously more affected by noise than others.

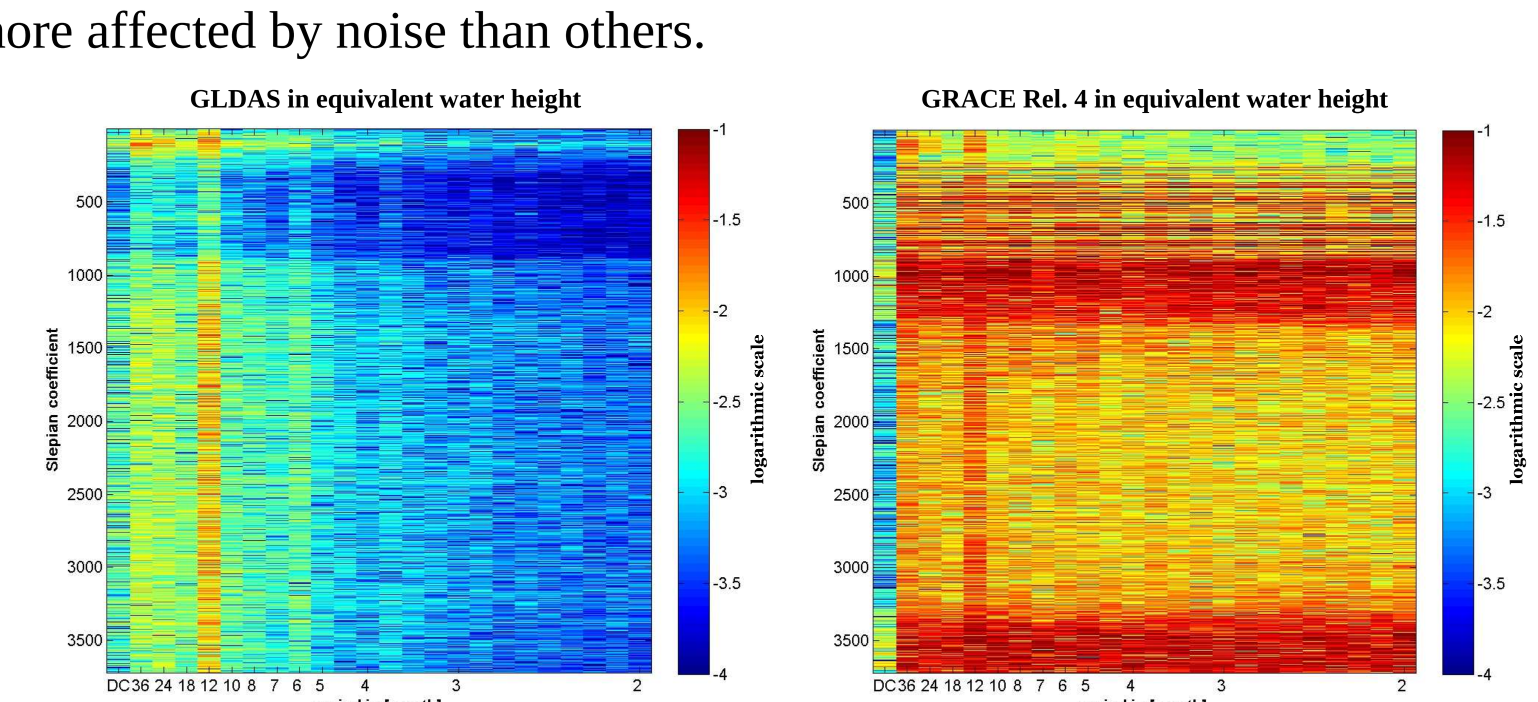

Figure 2: Fourier analysis of the Slepian coefficients

Estimation of trend and annual signal from GRACE

The time-variability of the Slepian coefficient can be modeled by a bias, a trend and an annual cycle with time t expressed in months, i.e.:

$$\beta_i(t) = \beta_i^0 + \beta_i^t (t - t_0) + \beta_i^c \cos\left(\frac{\pi}{6}(t - t_0)\right) + \beta_i^s \sin\left(\frac{\pi}{6}(t - t_0)\right)$$

Figure 3 shows on the left the trend over North America. The strongest part in Hudson Bay is mainly caused by post-glacial rebound. A decrease in the Western Cordillera and the Mississippi river basin is also visible and can be connected to hydrology. The right panel shows the difference to a least-squares fitting of the GRACE time series in the spatial domain (Van der Wal et al., 2007) and indicates that 97.5% of the signal has been recovered. Note, that out of 3721 possible Slepian coefficients (L = 60) only 427 were used.

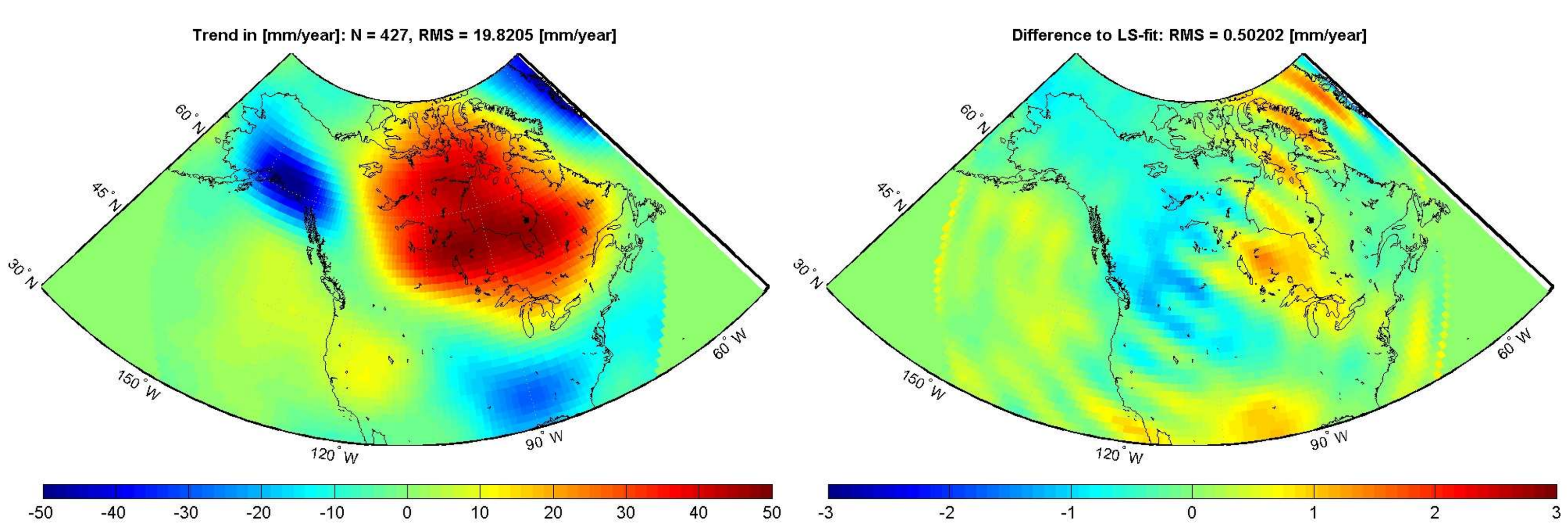

Figure 3: Trend from 45 months of GRACE data in equivalent water height

The magnitude of the annual cycle in figure 4 shows the characteristic patterns of the annual snow cycle in the Western Cordillera, the Mississippi river basin and the Quebec-Labrador region. The RMS indicates that 76.8% of the signal have been recovered using 466 Slepian coefficients. An improvement could not be achieved by using more coefficients.

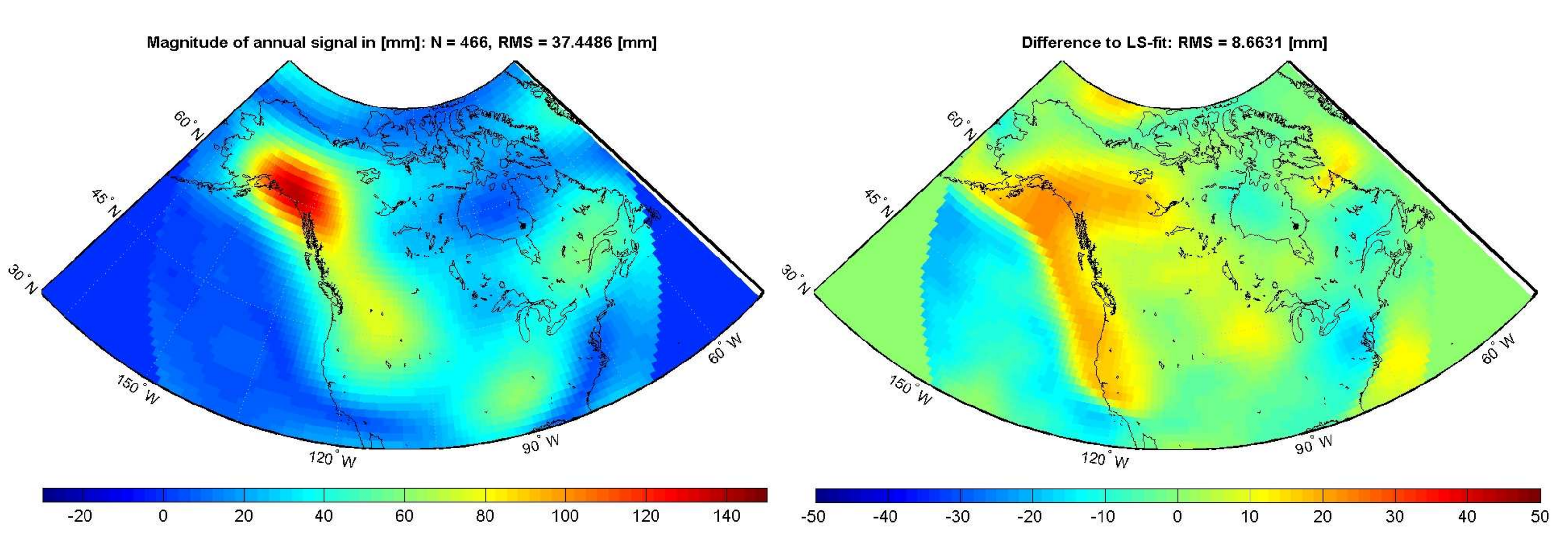

Figure 4: Magnitude of the annual cycle in equivalent water height

The difference in figure 4 can be explained with the relation between the coefficients. One Slepian coefficient is the linear combination of all spherical harmonic coefficients. Proper filtering is even more critical than in case of the spherical harmonic solution. The advantage of the Slepians will only be fully exploited by a least-squares estimation using local data.

Conclusions

It has been shown that a time-variable gravity signal can be recovered using Slepian analysis. The trend signal has been recovered up to 97.5%. The frequency analysis of the Slepian coefficients indicated the annual signal as the strongest time variable component but the recovery was deteriorated by the noise in the spherical harmonic coefficient. The reason is that each Slepian coefficient is derived from a linear combination of the spherical harmonic coefficients. Consequently and without proper filtering, the noise inside the spherical harmonics maps into each Slepian coefficient. The advantage of the Slepians will only be fully exploited when estimating Slepian coefficients from in-situ measurements.

Acknowledgement & References

We would like to acknowledge Dr. Simons.

- Han, S. (2003), Efficient Global Gravity Determination form Satellite-to-Satellite Tracking (SST). *PhD thesis*, Geodetic and GeoInformationScience, Ohio State University.
- Rodell, M. et al. (2004), The global land data assimilation system, *Bull. Am. Meteorol. Soc.* 85(3), 381-394
- Simons, F. et at. (2005), Spatiospectral concentration on a sphere, *Siam Reviews*
- Swenson, S. and Wahr, J.M. (2006), Post-processing removal of correlated errors in GRACE data, *Geophys. Res. Lett.*, vol 33.
- Van der Wal, W. et al. (2007), Comparison of GRACE and hydrology mass variations in North America studied by means of principal component analysis, *Poster presented at EGU 2007*