New CHAMP gravity results using the energy balance approach

5th Annual Scientific Conference of the GEOIDE Network

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In collaboration with IAPG, TU Munich

May 23rd, 2003

Outline

- **Energy integral (Jacobi integral) approach**
- **Data processing**
  - Orbits
  - Accelerometer calibration
- **Error analysis**
- **Results & Validation**
- **Conclusion**

Jacobi integral

Equation of motion in the rotating frame:

$$\ddot{\boldsymbol{x}} = \boldsymbol{f}+\boldsymbol{g}-\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{x})-2\boldsymbol{\omega}\times\dot{\boldsymbol{x}}-\dot{\boldsymbol{\omega}}\times\boldsymbol{x}$$

$$\int \dot{\boldsymbol{x}}\cdot\ddot{\boldsymbol{x}}dt = \int \dot{\boldsymbol{x}}\cdot(\boldsymbol{f}+\nabla V+\nabla Z-2\boldsymbol{\omega}\times\dot{\boldsymbol{x}})dt$$

$$\frac{1}{2}\dot{\boldsymbol{x}}\dot{\boldsymbol{x}} = \int\left(\dot{\boldsymbol{x}}\cdot\boldsymbol{f}-\frac{\partial V}{\partial t}\right)dt+V+Z+c$$

$$T+c = E_{kin}-U-Z-\int \boldsymbol{f}dx+V_t$$

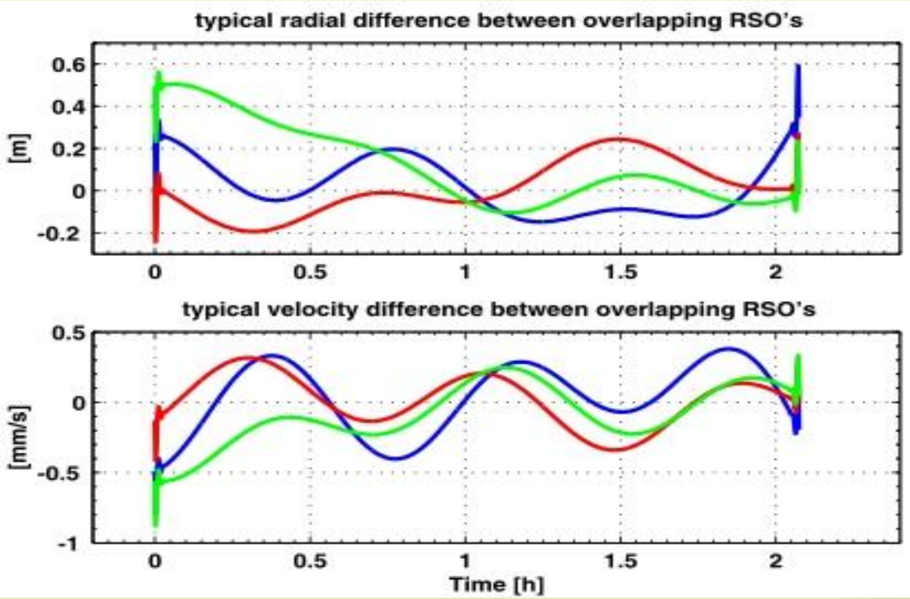
Data Processing: Overview

- preprocess RSO, RDO or kinematic data and ACC (synchronization, gaps, overlaps, . . .)
- calibrate accelerometer data
- calculate monthly solutions of **T** + *c*
- space-wise semi-analytical approach
- spherical harmonic analysis
- downward continuation in spectral domain
- iterate (calibration, continuation/gridding)

Orbit data (1/3)

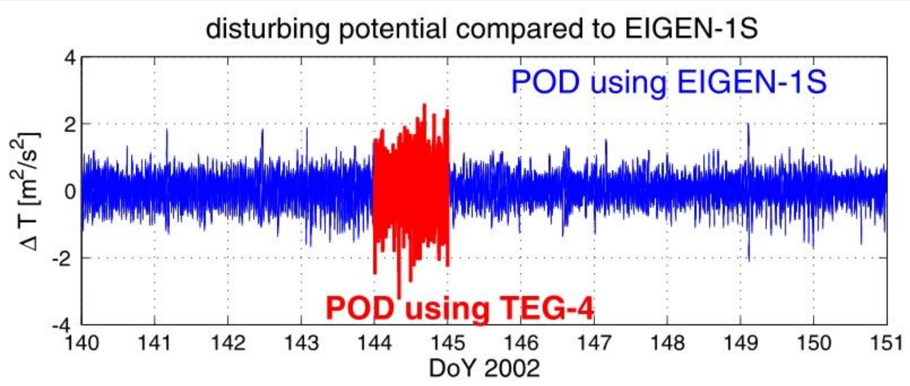
RSO: rapid science orbits

- **provided by the GFZ Potsdam**
- **Accuracy:**
  - Position: 2-3 dm
  - Velocity: ~0.3 mm/s



RDO: reduced dynamic orbits

- **provided by the IAPG, TU Munich**
- **Accuracy:**
  - Position: ~5 cm
  - Velocity: 0.1 mm/s

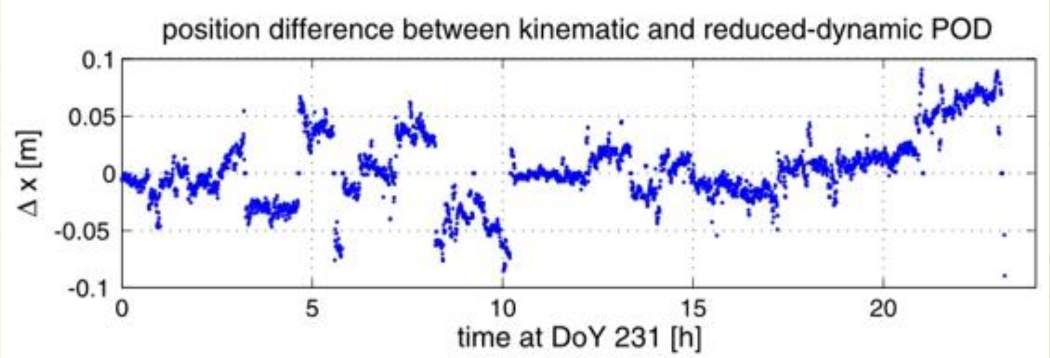


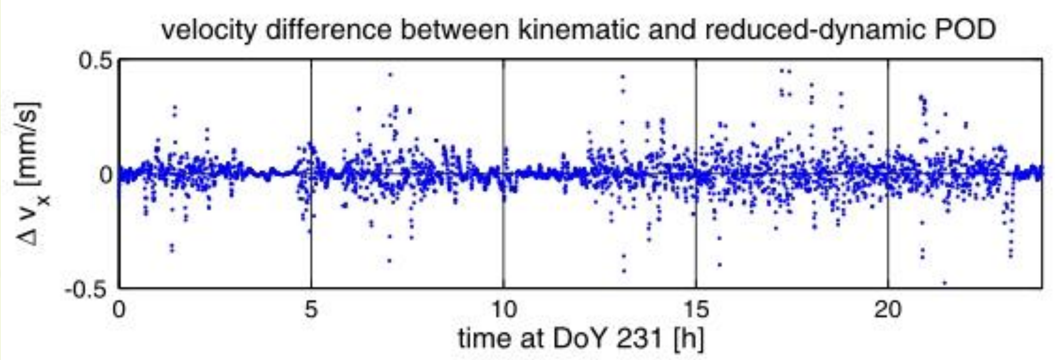
Both depend on a priori information

Orbit data (2/3)

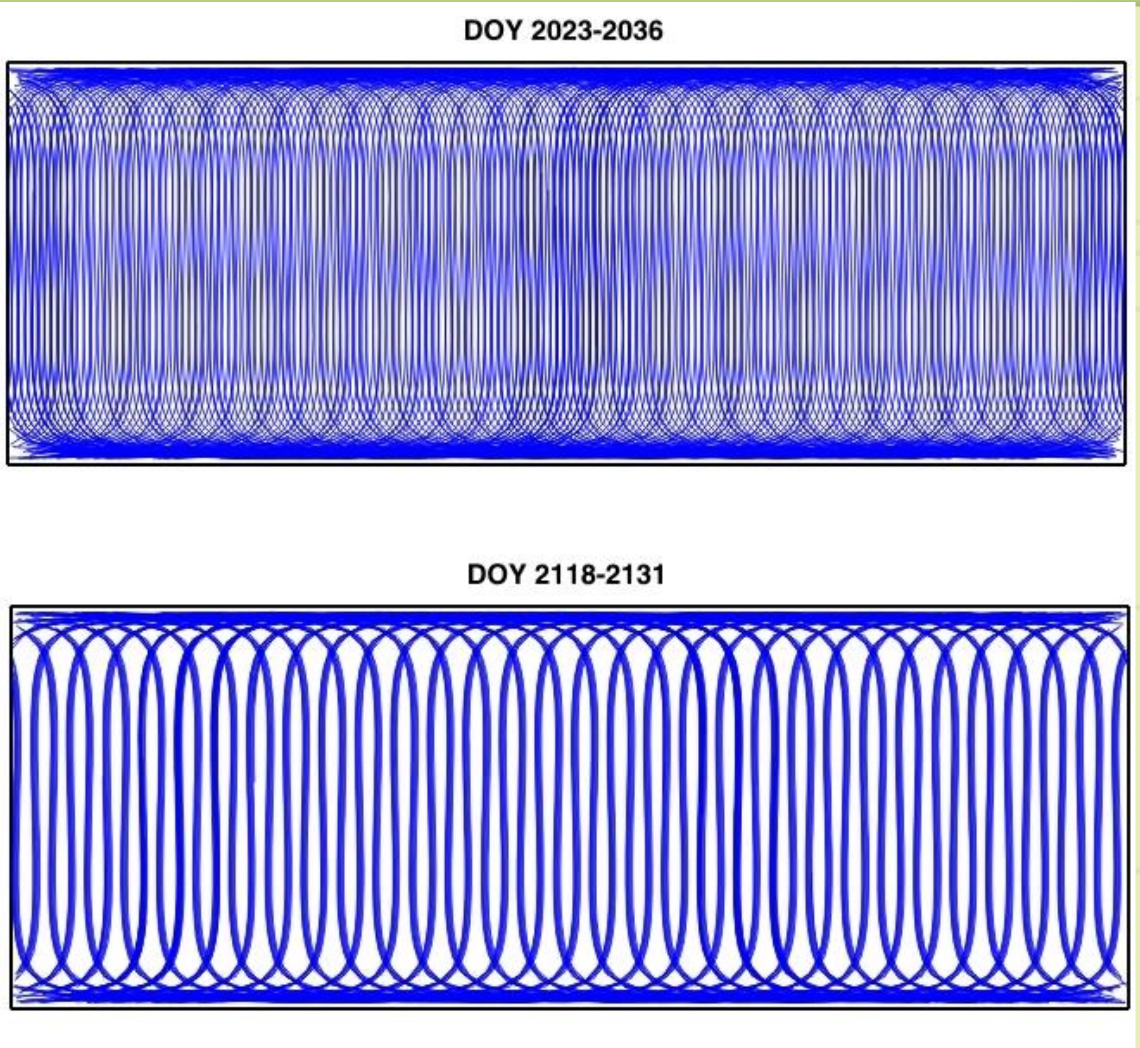
Kinematic Precise Orbit Determination:

- **provided by the IAPG, TU Munich**
- **pure geometric derivation of the position only**
- **velocity must be determined in an additional step**

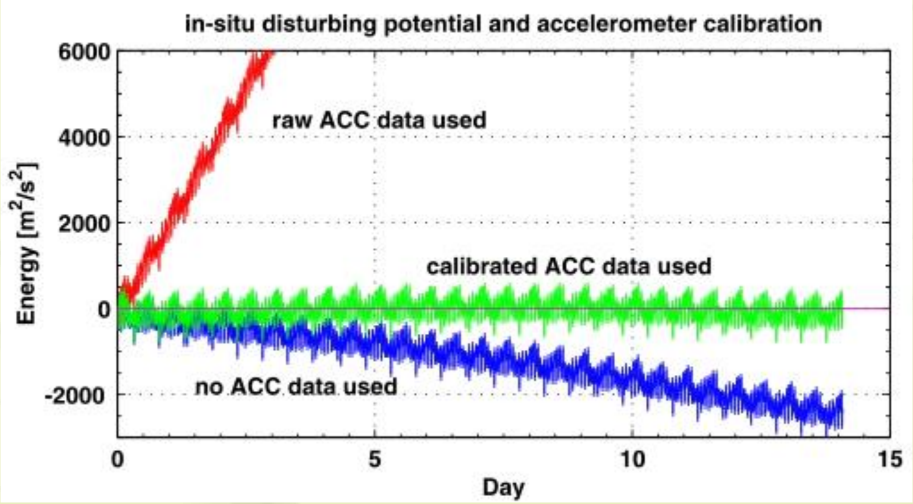


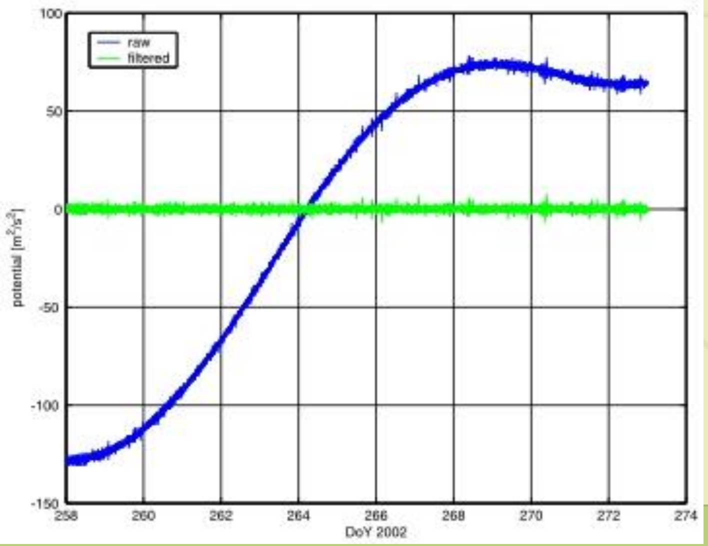


Orbit data (3/3)



Accelerometer Calibration (1/2)

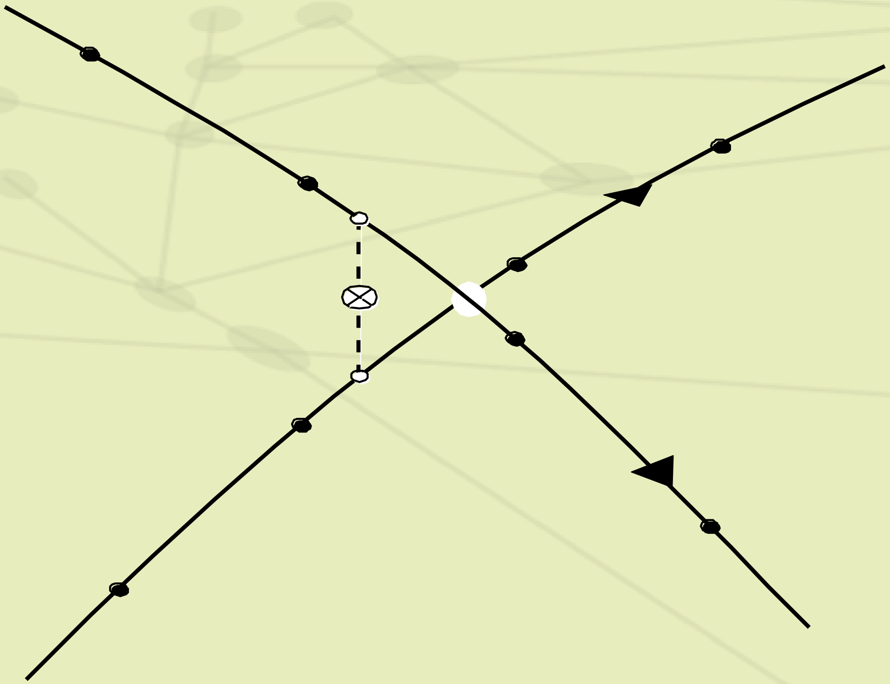




3 possibilities of calibration:

- **Trend analysis:**
  - Corresponding time series from a priori model
  - Solution forced toward the model
- **High pass Filtering:**
  - Reduction of a reference model
  - Solution similar to the reference model in the long wavelengths

Accelerometer Calibration (2/2)



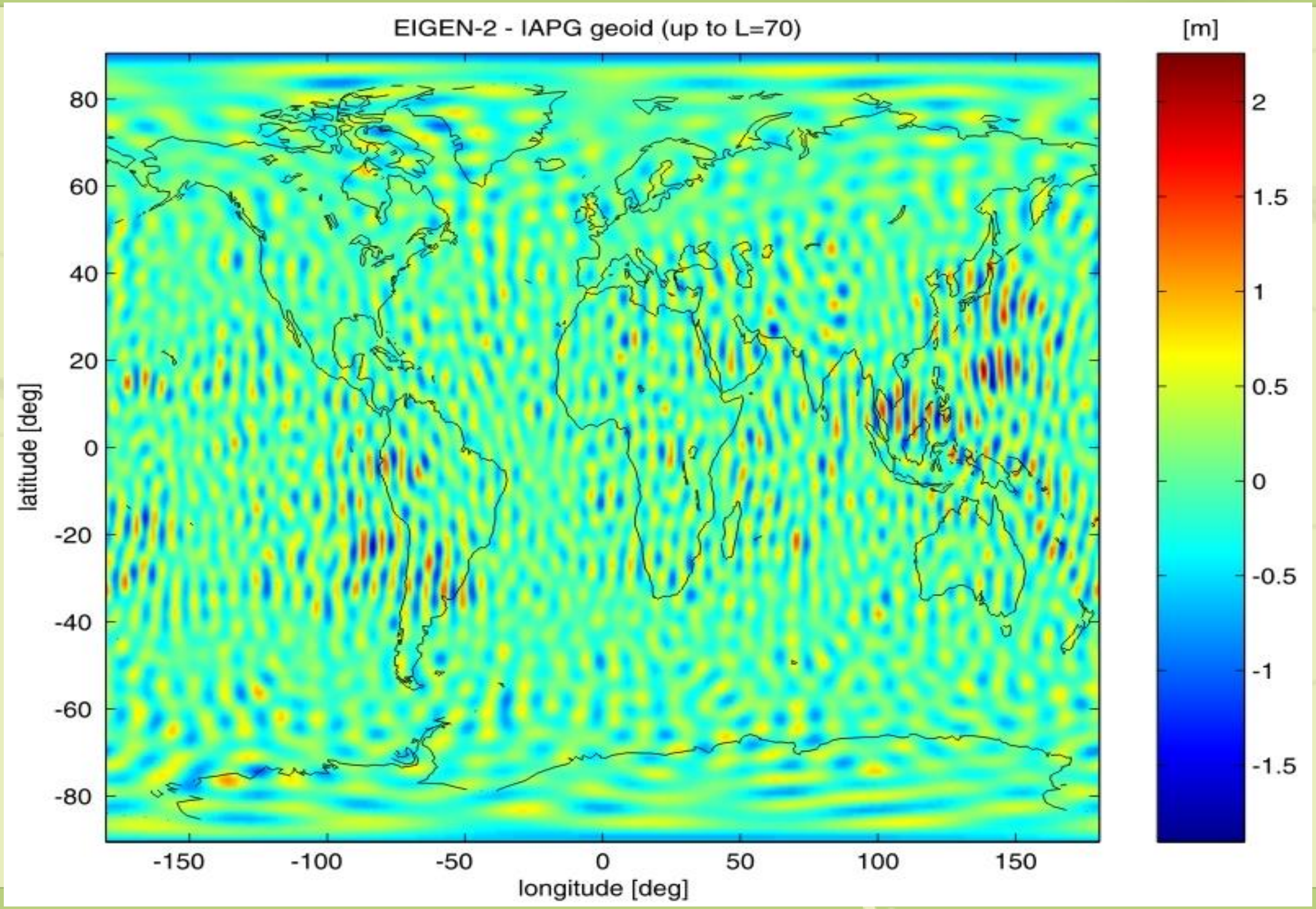
- **Cross over analysis:**
  - Energy level of descending and ascending arcs are the same
  - Interpolation necessary
  - Only weak dependency on a priori information
  - Ability to reveal unknown time dependent changes in the gravity field

Error analysis

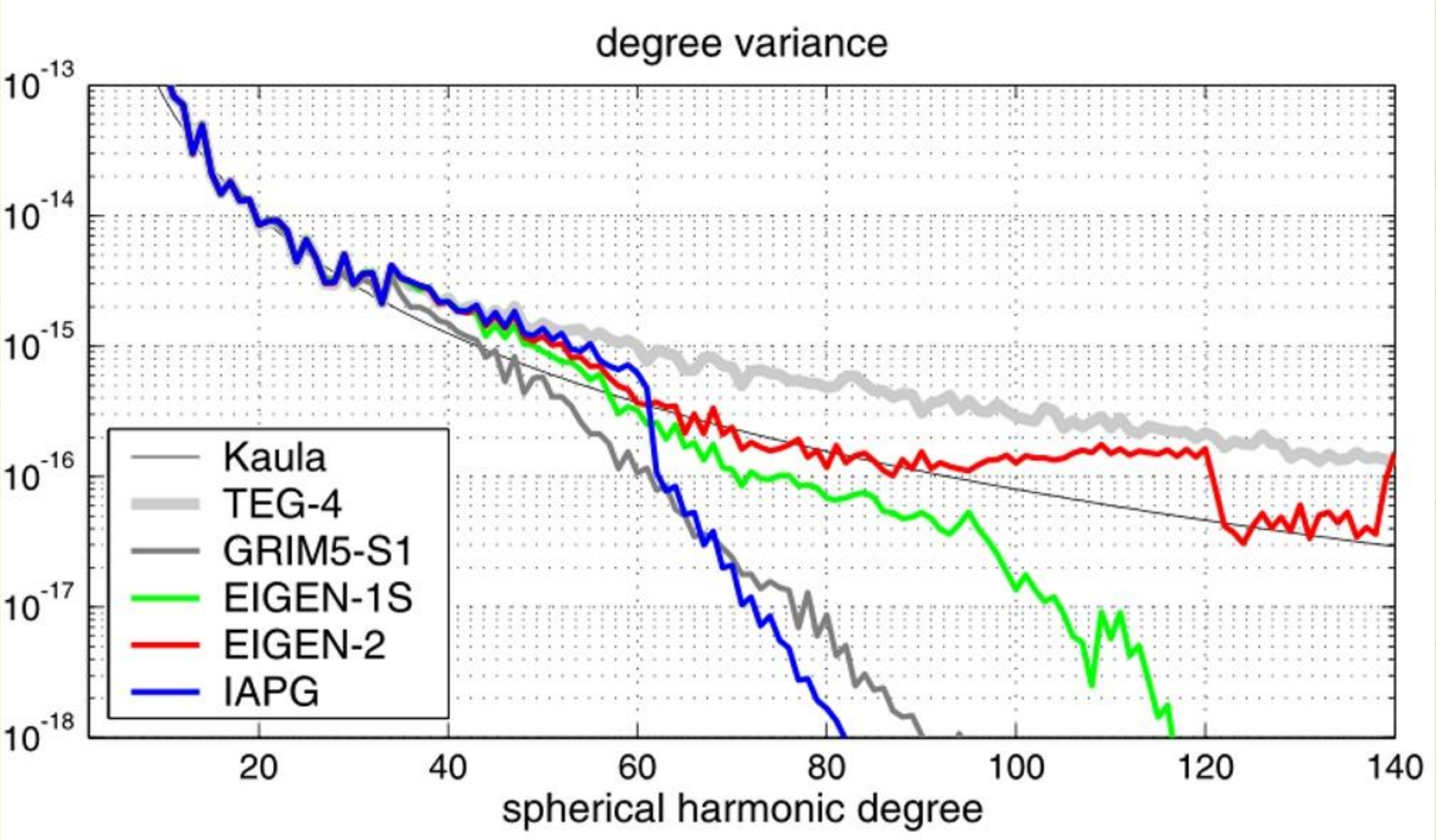
Suppose target: $\sigma_E = 1\ m^2/s^2$ → $\sigma_N = 10$ cm

- $\mathrm{d}E_{kin} = \dot{x} \cdot \mathrm{d}\dot{x}$ → **along-track velocity accuracy of 0.14 mm/s**
- **Determination of velocity from position:** $\sigma_{\dot{x}} = \sqrt{2}\sigma_x / \Delta t$
- $\mathrm{d}U = \gamma \cdot \mathrm{d}\mathbf{x}$ → **radial component better than 10 cm**
- $\mathrm{d}Z = \omega^2 (x\,\mathrm{d}x + y\,\mathrm{d}y)$ → **position requirement of several meter**
- $\mathrm{d}Z = (\boldsymbol{\omega} \times \mathbf{x}) \cdot (\mathrm{d}\boldsymbol{\omega} \times \mathbf{x})$ → **pole tide not significant**
- $\int f\,\mathrm{d}x$ → **controlled by calibration**
- V_t → **corrected for all known effects**

Results and Validation (1/4)



Results & Validation (2/5)



Results & Validation (3/5)

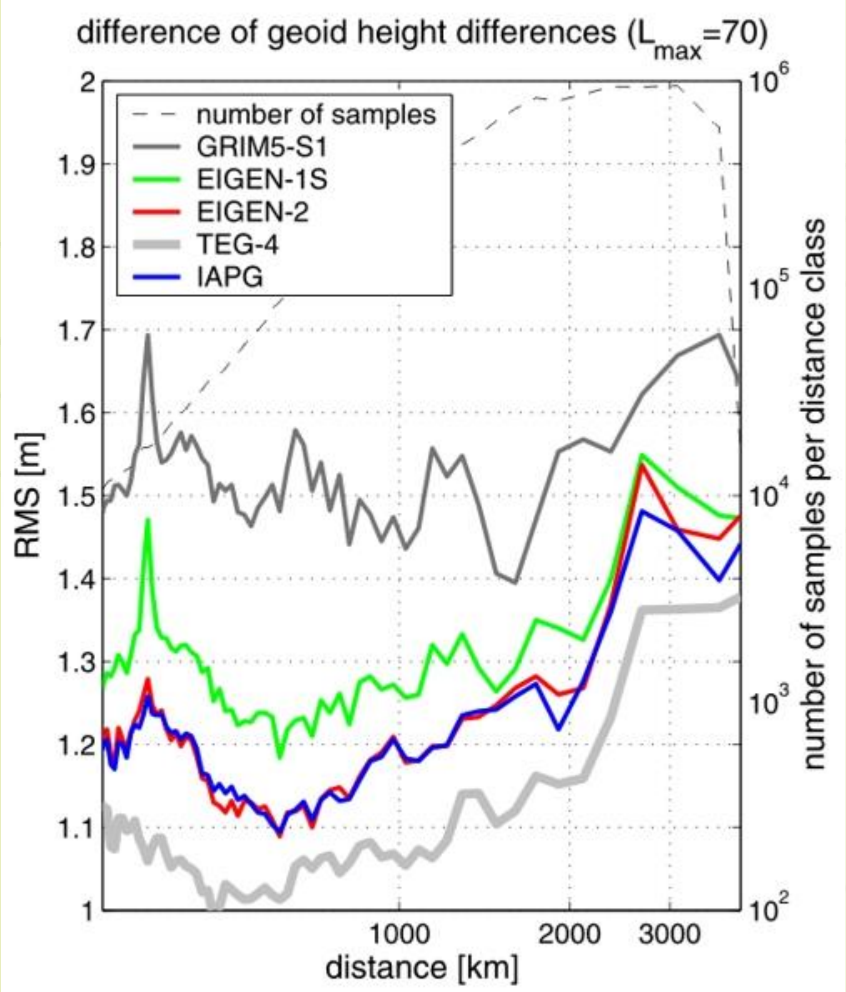


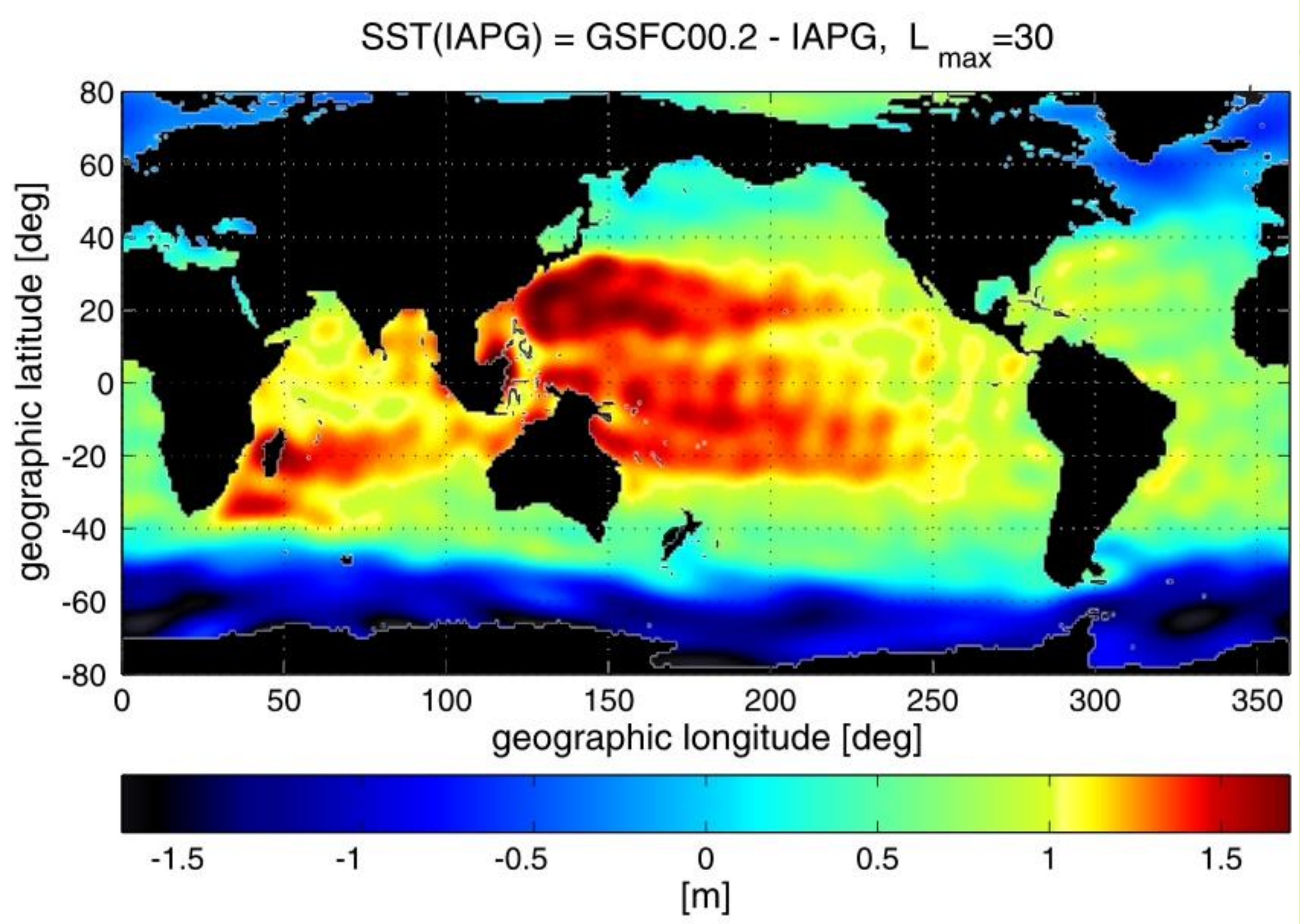
Table 4: *RMS of geoid height differences [m] in Canada (1587 points).*

L_{max}	GRIM5-S1	EIGEN-1S	EIGEN-2	TEG-4	IAPG
50	1.153	1.012	0.982	1.016	1.008
60	1.187	1.020	0.924	0.970	0.961
70	1.199	1.037	0.903	0.909	0.951

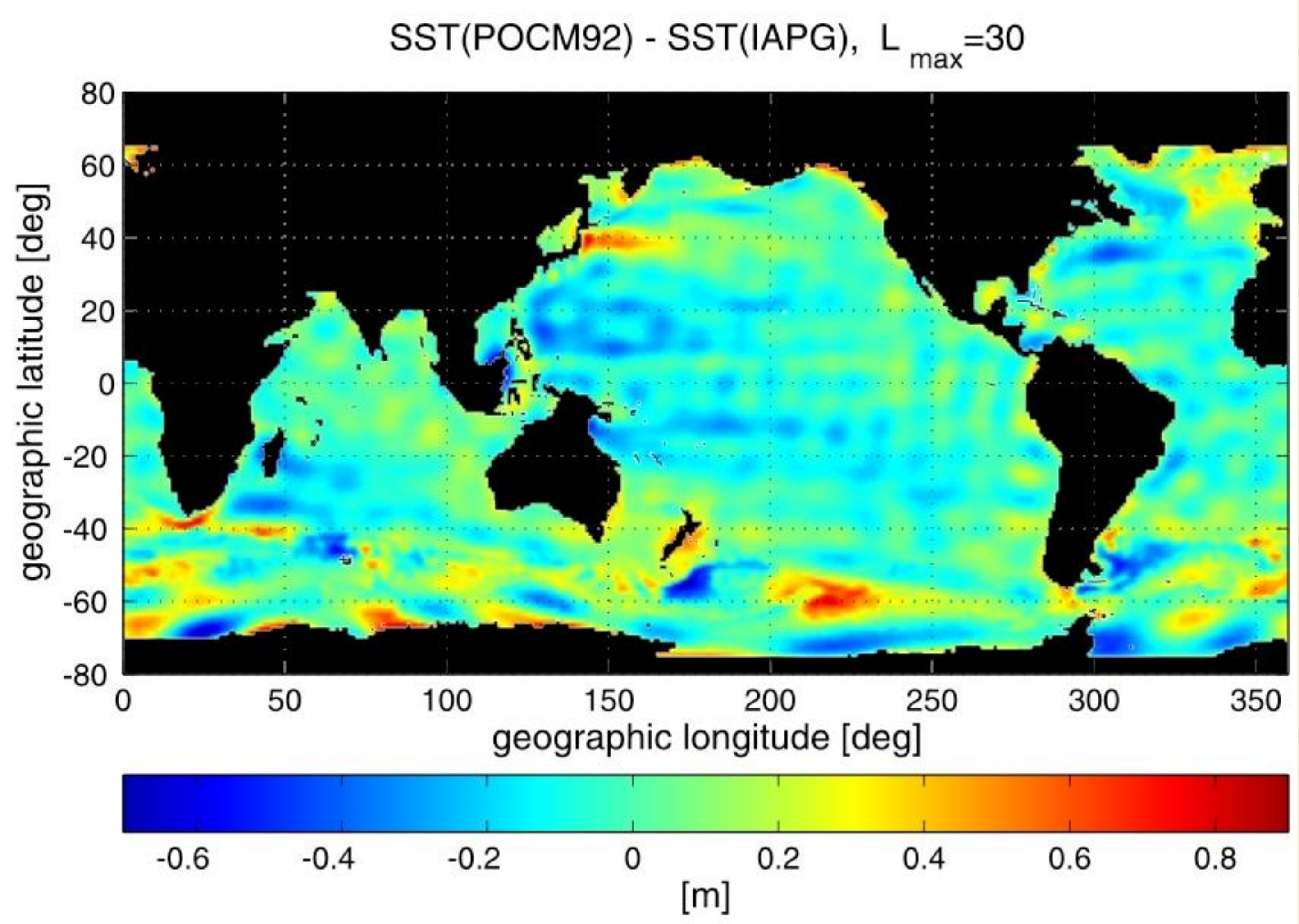
Table 3: *RMS of geoid height differences [m] in the USA (5168 points).*

L_{max}	GRIM5-S1	EIGEN-1S	EIGEN-2	TEG-4	IAPG
50	1.043	1.021	1.023	1.019	1.025
60	1.099	0.968	0.893	0.904	0.901
70	1.082	0.955	0.916	0.836	0.907

Results and Validation (4/5)



Results and Validation (5/5)



Conclusion

- **Gravity field recovery by energy integral approach is simple and efficient technique.**
- **Usage of kinematic precise orbit determination**
- **Cross over analysis is a powerful tool for accelerometer calibration**
- **New results based on purely kinematic orbit determination reasonable**

Acknowledgement

University of Calgary:

- **Nico Sneeuw**

IAPG, TU Munich:

- **Christian Gerlach**
- **Lorant Földvary**
- **Drăzen Švehla**
- **Thomas Gruber**
- **Björn Frommknecht**
- **Helmut Oberndorfer**
- **Thomas Peters**
- **Markus Rothacher**
- **Reiner Rummel**
- **Peter Steigenberger**
- **Martin Wermuth**

Thanks to the GEOIDE network and Dr. Sideris etc…

Questions ?

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