Refinement of the differential gravimetry approach for future inter-satellite observations

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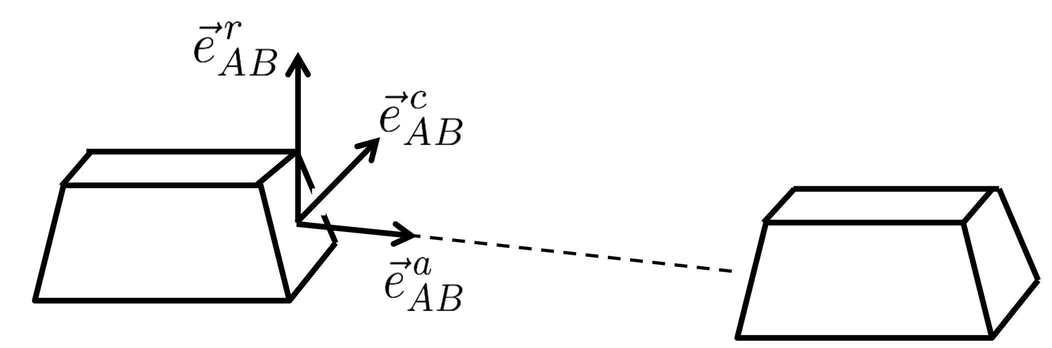
Introduction:

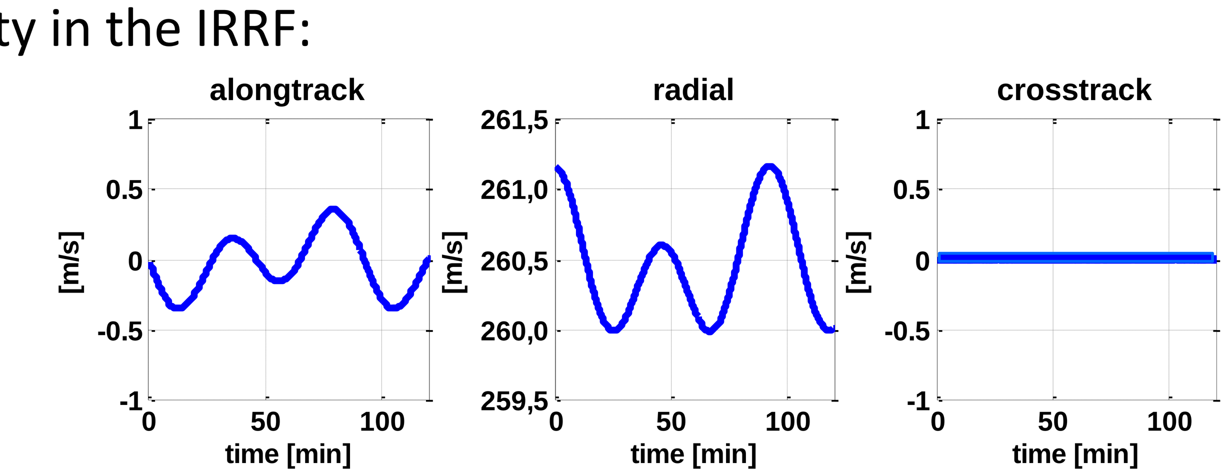
The differential gravimetry approach (aka. the relative acceleration approach) is one possible way of connecting GRACE-type inter-satellite observations to the gravity field. However, it depends on the combination with less accurate GPS-observations yielding a loss of accuracy in the solution. For the current GRACE mission this effect can be mitigated by reducing the observations to residual quantities. For the GRACE Follow-On mission this approach is not working anymore and thus a refinement of the solution strategy is necessary. It is based on the introduction of the variational equations for the critical GPS-related term only.

Instantaneous relative reference frame (IRRF):

- instantaneous alongtrack, radial and crosstrack direction
- best frame to understand the relative nature of the observables
- main part of radial relative velocity due to orbital motion
- no crosstrack component in this frame



Velocity in the IRRF:



Differential gravimetry approach:

Range observables:

$$\vec{X}_{AB} = \rho \cdot \vec{e}^{\,a}_{AB}$$

$$\dot{\vec{X}}_{AB} = \dot{\rho} \cdot \vec{e}^{\,a}_{AB} + \rho \cdot \dot{\vec{e}}^{\,a}_{AB}$$

$$\ddot{\vec{X}}_{AB} = \ddot{\rho} \cdot \vec{e}^{\,a}_{AB} + 2 \cdot \dot{\rho} \cdot \dot{\vec{e}}^{\,a}_{AB} + \rho \cdot \ddot{\vec{e}}^{\,a}_{AB}$$

Multiplication with unit vectors of the IRRF:

GRACE-type case

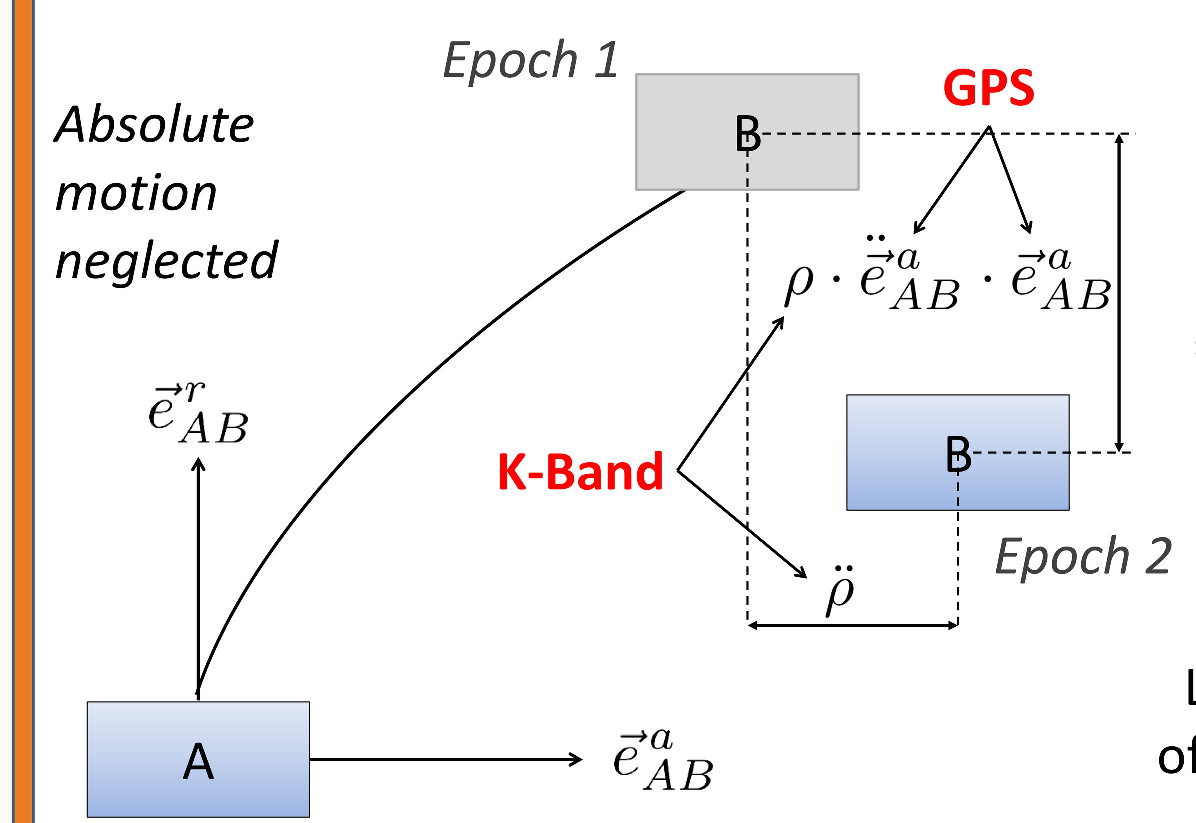
$$\ddot{\vec{X}}_{AB} \cdot \vec{e}^{\,a}_{AB} = \ddot{\rho} \quad + \quad 0 \quad + \quad \rho \cdot \ddot{\vec{e}}^{\,a}_{AB} \cdot \vec{e}^{\,a}_{AB}$$

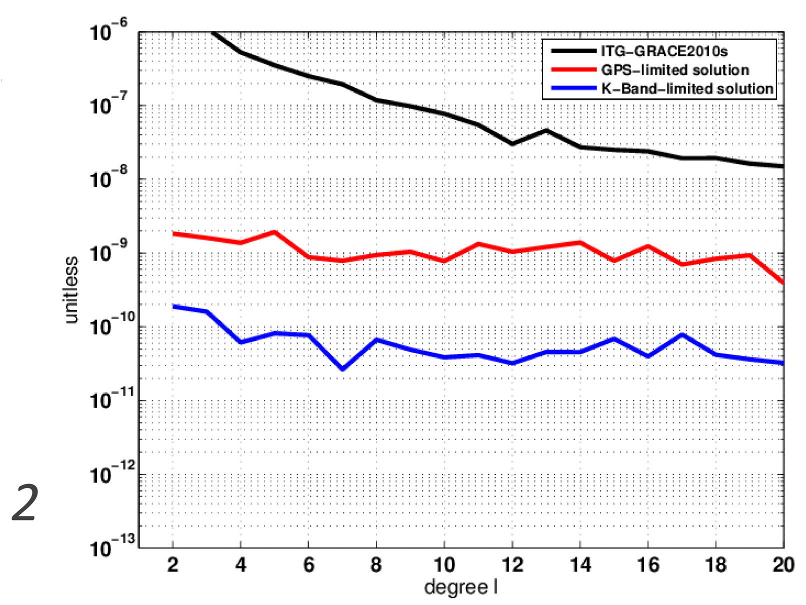
$$\ddot{\vec{X}}_{AB} \cdot \vec{e}^{\,r}_{AB} = 0 \quad + \quad 2 \cdot \dot{\rho} \cdot \|\dot{\vec{e}}^{\,a}_{AB}\| \quad + \quad \rho \cdot \ddot{\vec{e}}^{\,a}_{AB} \cdot \vec{e}^{\,r}_{AB}$$

$$\ddot{\vec{X}}_{AB} \cdot \vec{e}^{\,c}_{AB} = 0 \quad + \quad 0 \quad + \quad \rho \cdot \ddot{\vec{e}}^{\,a}_{AB} \cdot \vec{e}^{\,c}_{AB}$$

Relative motion between the satellites:

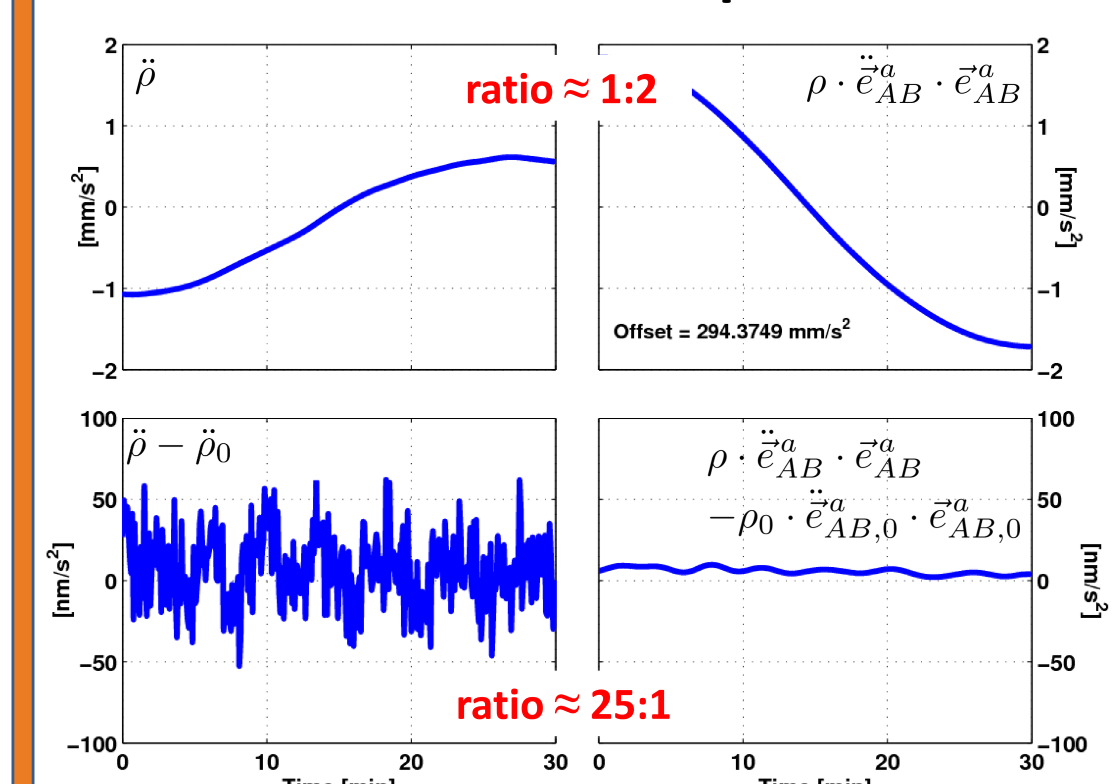
- GRACE-type observation equation with two components
- Range-acceleration $\ddot{\rho}$ derived directly from the K-Band observations
- Second terms needs augmentation with GPS (loss of accuracy)





Loss of accuracy of several order of magnitude due to combination with lower accurate GPS

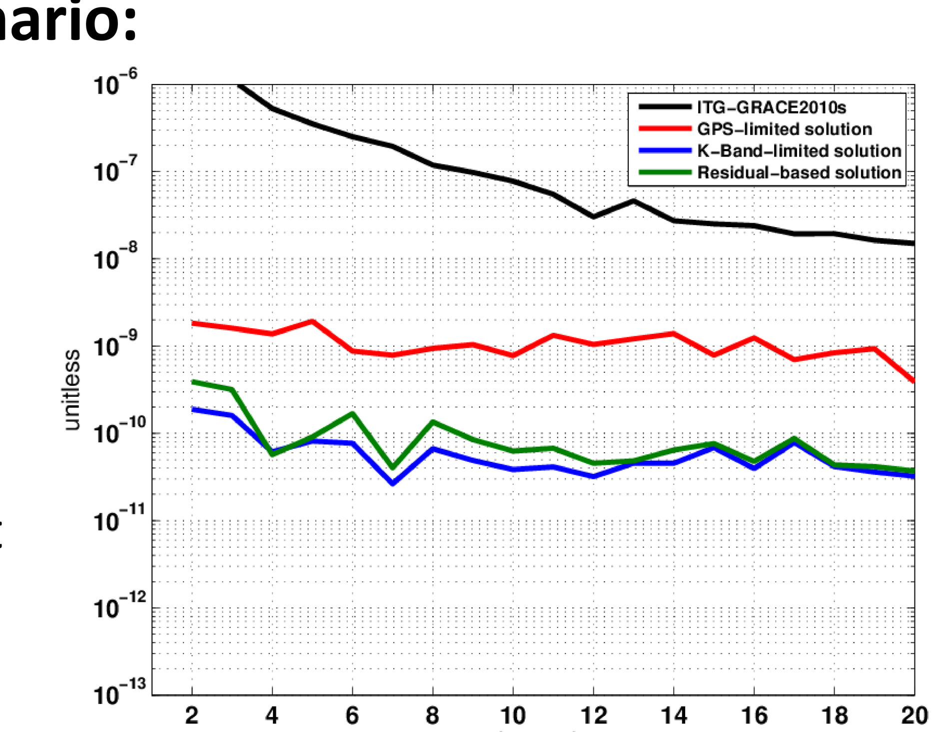
Reduction to residual quantities:



- Orbit fit using the homogeneous solution of the variational equation with a known a priori gravity field
- Avoiding the estimation of empirical parameters by using short arcs (e.g. one orbit)
- Strong reduction of the GPS-related term

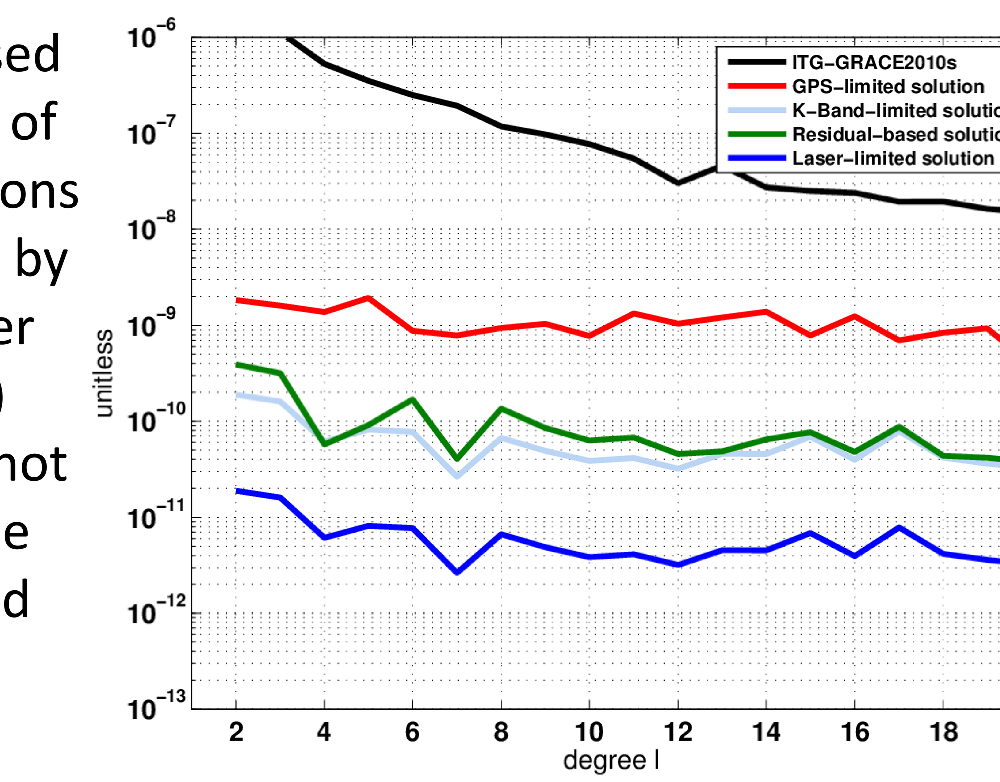
Solution for current GRACE scenario:

- Reduction of the GPS-related term -> neglect yields sufficient approximation
- Residual range-acceleration dominates the observation equations.
- Residual-based solution can exploit the potential of the K-band observation.



Applicability to GRACE – Follow On

- Laser interferometry used for the next generation of inter-satellite observations
- Expected improvement by approximately one order of magnitude (or more)
- Reduction to residuals not sufficient to mitigate the effect of the GPS-related term anymore



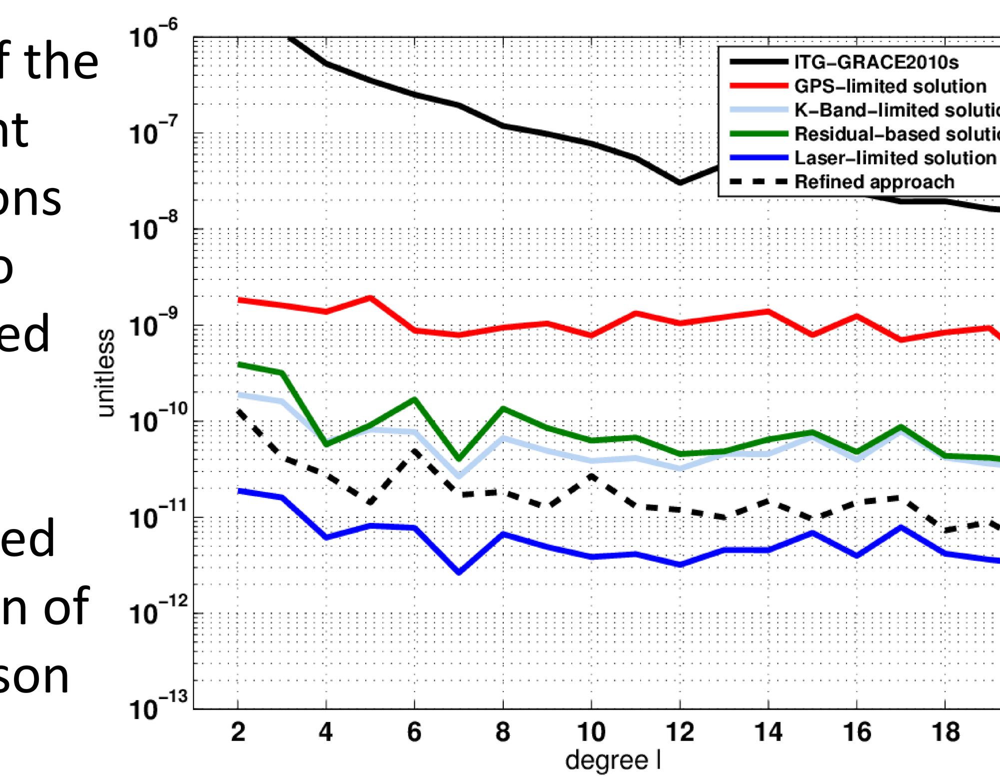
Refinement of the approach

- Modeling the GPS-related term velocity term by variational equations:

$$f = \rho \cdot \ddot{\vec{e}}^{\,a}_{AB} \cdot \vec{e}^{\,a}_{AB} = -\frac{1}{\rho}\left(\|\dot{\vec{X}}_{AB}\|^2 - \dot{\rho}^2\right)$$

$$\frac{\partial f}{\partial p_i} = \frac{\partial f}{\partial x_A}\frac{\partial x_A}{\partial p_i} + \frac{\partial f}{\partial x_B}\frac{\partial x_B}{\partial p_i} + \ldots\ldots\ldots + \frac{\partial f}{\partial \dot{z}_B}\frac{\partial \dot{z}_B}{\partial p_i}$$

- Applying the method of the variation of the constant
- Alternative: Hill equations
- Improved results due to refinement (black dashed curve)
- Full potential of laser observation not exploited
- Errors in initial condition of orbit fit as possible reason



Conclusions:

Previous solution strategies based on the differential gravimetry approach for GRACE-type observations are not directly applicable to laser interferometry based observations since the mitigation of the errors in the GPS-related term is not sufficient. Instead this term needs to be modeled by variational equations which, however, do not (yet) exploit the full potential of the new type of observation. Besides, the approach becomes questionable as it is practically not different to existing approaches connecting the range or range-rate observation to the gravity field on the basis of variational equations.