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Quantization of rotating linear dilaton black holes

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Abstract In this paper, we focus on the quantization of four-dimensional rotating linear dilaton black hole (RLDBH) spacetime describing an action, which emerges in the Einstein–Maxwell-dilaton–axion (EMDA) theory. RLDBH spacetime has a non-asymptotically flat geometry. When the rotation parameter "a" vanishes, the spacetime reduces to its static form, the so-called linear dilaton black hole (LDBH) metric. Under scalar perturbations, we show that the radial equation reduces to a hypergeometric differential equation. Using the boundary conditions of the quasinormal modes (QNMs), we compute the associated complex frequencies of the QNMs. In a particular case, QNMs are applied in the rotational adiabatic invariant quantity, and we obtain the quantum entropy/area spectra of the RLDBH. Both spectra are found to be discrete and equidistant, and independent of the a-parameter despite the modulation of QNMs by this parameter.

1 Introduction

Quantization of black holes (BHs) has always interested physicists working on quantum gravity theory. The topic was introduced in the 1970s by Bekenstein, who proved that the entropy of a BH is proportional to the area of the BH horizon [1,2]. The equidistant area spectrum of a BH [3–5] is given by

$$A_n^{\mathrm{BH}} = 8\pi\xi\hbar = \epsilon n l_{\mathrm{p}}^2, \quad (n = 0, 1, 2\ldots), \tag{1}$$

where A_n^{BH} denotes the area spectrum of the BH horizon, n is the associated quantum number, ξ represents a number that is order of unity, ϵ is a dimensionless constant, and l_{p} is the Planck length. In his celebrated studies, Bekenstein considered the BH horizon area as an adiabatic invariant quantity, and he derived the discrete equally spaced area spectrum by Ehrenfest's principle. Equation (1) shows that when a test particle is swallowed by a BH, the horizon area increases by a minimum of $\Delta A_{\mathrm{min}}^{\mathrm{BH}} = \epsilon l_{\mathrm{p}}^2$. Throughout this paper, we normalize the fundamental units as $c = G = 1$ and $l_{\mathrm{p}}^2 = \hbar$. According to Bekenstein [3], a BH horizon consists of patches of equal area $\epsilon\hbar$, where $\epsilon = 8\pi$. However, although the derivation of the evenly spaced area spectra has received much attention, the value of ϵ has been somewhat controversial, because it depends on the method used to obtain the spectrum (for a topical review, the reader is referred to [6] and references therein).

QNMs, known as the characteristic ringing of BHs, are damped oscillations characterized by a discrete set of complex frequencies. They are determined by solving the wave equation under perturbations of the BH by an external field. A perturbed BH tends to equilibrate itself by emitting energy in the form of gravitational waves. For this reason, QNMs are also important for experiments such as LIGO [7], which aims to detect gravitational wave phenomena. Inspired by the above findings, Hod [8,9] hypothesized that ϵ can be computed from the QNMs of a vibrating BH. To this end, Hod adopted Bohr's correspondence principle [10], and conjectured that the real part of the asymptotic QNM frequency ($\mathrm{Re}\,\omega$) of a BH relates to the quantum transition energy between sequential quantum levels of the BH. Thus, this vibrational frequency induces a change $\Delta M = \hbar\,(\mathrm{Re}\,\omega)$ in the BH mass. For a Schwarzschild BH, Hod's calculations yield $\epsilon = 4\ln 3$ [8]. Later, Kunstatter [11] used the natural adiabatic invariant quantity I_{adb}, which is defined for a system with energy E by

$$I_{adb} = \int \frac{\mathrm{d}E}{\Delta\omega}, \tag{2}$$

where $\Delta\omega = \omega_{n-1} - \omega_n$ is the transition frequency. At large quantum numbers ($n \to \infty$), the Bohr–Sommerfeld quantization condition applies and I_{adb} behaves as a quantized quantity ($I_{adb} \simeq n\hbar$). By calculating $\Delta\omega$, using the real part

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Re ω, and replacing E with the mass M of the static BH in Eq. (2), Kunstatter retrieved Hod's result $\epsilon = 4\ln 3$ for a Schwarzschild BH. Maggiore [12] proposed that the proper physical frequency of the harmonic oscillator with a damping term takes the form $\omega = \sqrt{\mathrm{Re}\,\omega^2 + \mathrm{Im}\,\omega^2}$, where Im ω is the imaginary part of the QNM frequency. Actually, this form of the proper physical frequency was first proposed for QNM frequencies by Wang et al. [13]. Thus, Hod's result [8] is obtained whenever $\mathrm{Re}\,\omega \gg \mathrm{Im}\,\omega$.

On the other hand, as is well known, highly damped (excited) QNMs correspond to $\mathrm{Im}\,\omega \gg \mathrm{Re}\,\omega$. Consequently $\Delta\omega \approx \mathrm{Im}\,\omega_{n-1} - \mathrm{Im}\,\omega_n$, which is directly proportional to the Hawking temperature (T_{H}) of the BH [14]. This new interpretation recovers the original result of Bekenstein; i.e., $\epsilon = 8\pi$, for a Schwarzschild BH. Since its inception, Maggiore's method (MM) has been widely studied by other researchers investigating various BH backgrounds (see for example [15–24]).

Thereafter, Vagenas [25] and Medved [26] amalgamated the results of Kunstatter and Maggiore, and proposed the following rotational version of the adiabatic invariant:

$$I_{adb}^{\mathrm{rot}} = \int \frac{\mathrm{d}M - \Omega_{\mathrm{H}}\mathrm{d}J}{\Delta\omega} = n\hbar, \tag{3}$$

where Ω_{H} and J represent angular velocity and angular momentum of the rotating BH, respectively. Vagenas [25] and Medved [26] inserted the asymptotic form of the QNMs of the Kerr BH [27] into the above expression and obtained an equidistant area spectrum conditional on $M^2 \gg J$.

Following Refs. [28–33], one can use the first law of thermodynamics ($T_{\mathrm{H}}\mathrm{d}S^{\mathrm{BH}} = \mathrm{d}M - \Omega_{\mathrm{H}}\mathrm{d}J$) to safely derive the entropy spectra from the QNMs via

$$I_{adb}^{\mathrm{rot}} = \int \frac{T_{\mathrm{H}}\mathrm{d}S^{\mathrm{BH}}}{\Delta\omega} = n\hbar. \tag{4}$$

In the present study, we analyze the entropy/area spectra of the RLDBHs. The RLDBHs, developed by Clément et al. [34], are the non-asymptotically flat (NAF) solutions to the Einstein–Maxwell-dilaton–axion (EMDA) theory in four dimensions. These BHs have both dilaton and axion fields. Especially, the axion field may legitimate the existence of the cold dark matter [35,36] in the RLDBH spacetime. Although many studies have focused on non-rotating forms of these BHs (known as LDBHs) and their related topics such as Hawking radiation, entropic forces, higher dimensions, quantization, and the gravitational lensing effect on the Hawking temperature [37–47], studies of RLDBHs have remained very limited. To our knowledge, RLDBH studies have focused on branes, holography, greybody factors, Hawking radiation, and QNMs [48–50], while ignoring the quantization of the RLDBH. Our present study bridges this gap in the literature.

We compute the QNMs of the RLDBH by giving an analytical exact solution (as being an alternative solution to the solution presented by Li [50]) to the massless Klein–Gordon equation (KGE) in the RLDBH geometry. To this end, we use a particular transformation for the hypergeometric function, and impose the appropriate boundary conditions for the QNMs, which obey only ingoing wave at the event horizon and only outgoing wave at spatial infinity. Our results support the study of Li [50]. On the other hand, we remark that equation (45) of [50] provides only one of the two sets of the QNMs of the RLDBHs and is independent of the rotation parameter a. However, this parameter is required in a general form of the QNMs of the RLDBH. In fact, by inserting $\mathbf{b}_s = -n$ instead of $\mathbf{a}_s = -n$ (see equations (17) and (44) of [50]) in the QNMs evaluation, we derive a further set of QNMs involving the rotation term a, which reduces to the QNMs of the LDBHs as $a \to 0$. Somehow this possibility has been overlooked in [50]. In the present paper, we show that, in a particular case, the QNMs with rotation parameter a can be represented as a simple expression including the angular velocity and surface gravity terms, as previously obtained for the Kerr BH [27]. We then derive the equally spaced entropy/area spectra of the RLDBHs.

The remainder of this paper is arranged as follows. Section 2 introduces the RLDBH metric, and analyzes the KGE for a massless scalar field in this geometry. In Sect. 3, we show that the KGE reduces to a hypergeometric differential equation. We also read the QNMs of the RLDBHs and use one of their two sets to derive the entropy/area spectra of the RLDBHs. Conclusions are presented in Sect. 4.

2 RLDBH spacetime and separation of KGE

In this section, we provide a general overview of the RLDBH solution and present its basic thermodynamic features. We then investigate the KGE in the geometry of the RLDBH. We show that after separation by variables, the radial equation can be obtained.

The action of the EMDA theory, which comprises the dilaton field ϕ and the axion (pseudoscalar) $\varkappa$ coupled to an Abelian vector field $\mathcal{A}$ is given by

$$S = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{|g|}\left\{-R + 2\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}e^{4\phi}\partial_\mu\varkappa\partial^\mu\varkappa \right. \\ \left. - e^{-2\phi}F_{\mu\nu}F^{\mu\nu} - \varkappa F_{\mu\nu}\widetilde{F}^{\mu\nu}\right\}, \tag{5}$$

where $F_{\mu\upsilon}$ is the Maxwell two-form associated with a $U(1)$ subgroup of $E_8 \times E_8$ or Spin(32)/Z_2, and $\widetilde{F}^{\mu\nu}$ denotes the dual of $F_{\mu\upsilon}$. In the absence of a NUT charge, the RLDBH solution is given by [34]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + R(r)\left[d\theta^2 + \sin^2\theta\left(d\varphi - \frac{a}{R(r)}dt\right)^2\right], \tag{6}$$

where the metric functions are given by

$$R(r) = rr_0, \tag{7}$$

$$f(r) = \frac{\Lambda}{R(r)}. \tag{8}$$

In Eq. (8) $\Lambda = (r - r_2)(r - r_1)$, where r_1 and r_2 denote the inner and outer (event) horizons, respectively, under the condition $f(r) = 0$. These radii are given by

$$r_j = M + (-1)^j\sqrt{M^2 - a^2}, \quad (j = 1, 2), \tag{9}$$

where the integration constant M is related to the quasilocal mass (M_{QL}) of the RLDBH, and a is the above-mentioned rotation parameter determining the angular momentum of the RLDBH. The background electric charge Q is related to the constant parameter r_0 by $\sqrt{2}Q = r_0$. Furthermore, the background fields are given by

$$e^{-2\phi} = \frac{R(r)}{r^2 + a^2\cos^2\theta}, \tag{10}$$

$$\varkappa = -\frac{r_0 a\cos\theta}{r^2 + a^2\cos^2\theta}. \tag{11}$$

The Maxwell field ($F = d\mathcal{A}$) is derivable from the following electromagnetic four-vector potential:

$$\mathcal{A} = \frac{1}{\sqrt{2}}(e^{2\phi}dt + a\sin^2\theta d\varphi). \tag{12}$$

From Eq. (9), we infer that for a BH

$$M \geq a. \tag{13}$$

Meanwhile, because the RLDBH has a NAF geometry, M_{QL} can be defined by the formalism of Brown and York [52]. Thus, we obtain

$$M_{QL} = \frac{M}{2}. \tag{14}$$

The angular momentum J is computed as

$$J = \frac{ar_0}{2}. \tag{15}$$

According to the definition given by Wald [53], the surface gravity of the RLDBH is simply evaluated as follows:

$$\kappa = \left.\frac{f'(r)}{2}\right|_{r=r_2} = \frac{(r_2 - r_1)}{2r_2 r_0}, \tag{16}$$

where a prime on a function denotes differentiation with respect to its argument. Thus, the Hawking temperature [53,54] of the RLDBH is obtained as

$$T_H = \frac{\hbar\kappa}{2\pi}, = \frac{\hbar(r_2 - r_1)}{4\pi r_2 r_0}. \tag{17}$$

Clearly, the temperature vanishes in the extreme case ($M = a$) since $r_2 = r_1$. The Bekenstein–Hawking entropy of a BH is defined in Einstein's theory as [53]

$$S^{BH} = \frac{A^{BH}}{4\hbar} = \frac{\pi r_2 r_0}{\hbar}, \tag{18}$$

where A^{BH} is area of the surface of the event horizon. To satisfy the first law of BH mechanics, we also need the angular velocity which takes the following form for the RLDBH:

$$\Omega_H = \frac{a}{r_2 r_0}. \tag{19}$$

We can now validate the first law of thermodynamics in the RLDBH through the relation

$$dM_{QL} = T_H dS^{BH} + \Omega_H dJ. \tag{20}$$

It is worth mentioning that Eq. (20) is independent of electric charge Q. Here, Q is considered as a background charge of fixed value – a characteristic feature of linear dilaton backgrounds [34].

To obtain the RLDBH entropy spectrum via the MM, we first consider the massless KGE on that geometry. The general massless KGE in a curved spacetime is given by

$$\partial_j(\sqrt{-g}\partial^j\Phi) = 0, \quad j = 0, \ldots, 3. \tag{21}$$

We invoke the following ansatz for the scalar field Φ in the above equation:

$$\Phi = \frac{\Re(r)}{\sqrt{r}}e^{-i\omega t}Y_l^m(\theta, \varphi), \quad \mathrm{Re}(\omega) > 0, \tag{22}$$

where the $Y_l^m(\theta, \varphi)$ denote the well-known spheroidal harmonics with eigenvalues $\lambda = -l(l+1)$ [55]. Here, m and l denote the magnetic quantum number and orbital angular quantum number, respectively. Thus, we obtain the following radial equation:

$$\Lambda\Re''(r)+\left(\Lambda'-\frac{\Lambda}{r}\right)\Re'(r)$$
$$+\left[\frac{(\omega r r_0-ma)^2}{\Lambda}+\lambda+\frac{3\Lambda}{4r^2}-\frac{\Lambda'}{2r}\right]\Re(r)=0. \tag{23}$$

The tortoise coordinate r^* is defined as [51]

$$r^*=\int\frac{\mathrm{d}r}{f(r)}, \tag{24}$$

whereby

$$r^*=\frac{r_0}{r_2-r_1}\ln\left[\frac{\left(\frac{r}{r_2}-1\right)^{r_2}}{(r-r_1)^{r_1}}\right]. \tag{25}$$

It is easily checked that the asymptotic limits of r^* are

$$\lim_{r\to r_2} r^*=-\infty \quad\text{and}\quad \lim_{r\to\infty} r^*=\infty. \tag{26}$$

3 QNMs and entropy/area spectra of RLDBHs

In this section, we obtain an exact analytical solution to the radial equation (23). In computing the QNM frequencies, we impose the relevant boundary conditions: (1) the field is purely ingoing at the horizon; (2) the field is purely outgoing at spatial infinity. Finally, we represent how the quantum spectra of the RLDBH can be equidistant and fully agree with the Bekenstein conjecture (1).

In order to solve Eq. (23) analytically, one can redefine the r-coordinate with a new variable x:

$$x=\frac{r-r_2}{r_2-r_1}. \tag{27}$$

Thus, the radial equation (23) adopts the form

$$x(1+x)\Re''(x)+(1+2x)\Re'(x)+\Theta\Re(x)=0, \tag{28}$$

where

$$\Theta=\frac{\{r_0\omega\,[x\,(r_2-r_1)+r_2]-ma\}^2}{(r_2-r_1)^2\,x\,(1+x)}+\lambda. \tag{29}$$

The above equation can be rewritten as

$$\Theta=\frac{A^2}{x}+\frac{B^2}{1+x}-C, \tag{30}$$

where

$$A=\frac{\widehat{\omega}}{2\kappa}, \tag{31}$$

$$B=i\frac{\omega r_1-m\Omega_{\mathrm{H}}r_2}{2\kappa r_2}, \tag{32}$$

$$C=-\left(\lambda+\omega^2r_0^2\right), \tag{33}$$

and

$$\widehat{\omega}=\omega-m\Omega_{\mathrm{H}}. \tag{34}$$

The solution of the radial equation (28) reads as follows (see Appendix):

$$\Re(x)=C_1(x)^{iA}(1+x)^{-B}F\,(a_n,b_n;c_n;-x)$$
$$+C_2(x)^{-iA}(1+x)^{-B}F\left(\widetilde{a}_n,\widetilde{b}_n;\widetilde{c}_n;-x\right), \tag{35}$$

where $F\,(a_n,b_n;c_n;-x)$ is the hypergeometric function [57] and C_1, C_2 are constants. Here, the coefficients $\widetilde{a}_n$, $\widetilde{b}_n$, and $\widetilde{c}_n$ are given through the relations

$$\widetilde{a}_n=a_n-c_n+1,$$
$$\widetilde{b}_n=b_n-c_n+1,$$
$$\widetilde{c}_n=2-c_n, \tag{36}$$

where

$$a_n=\frac{1}{2}-B+iA+\frac{\sqrt{4C+1}}{2}, \tag{37}$$

$$b_n=\frac{1}{2}-B+iA-\frac{\sqrt{4C+1}}{2}, \tag{38}$$

$$c_n=1+2iA. \tag{39}$$

One can see that

$$-B+iA=i\omega r_0, \tag{40}$$

$$\frac{\sqrt{4C+1}}{2}=\eta=ir_0\sqrt{\omega^2-\tau^2}, \tag{41}$$

where

$$\tau=\frac{l+1/2}{r_0}. \tag{42}$$

In the neighborhood of the horizon ($x\to0$), the function $\Re(x)$ behaves as

$$\Re(x)=C_1(x)^{iA}+C_2(x)^{-iA}. \tag{43}$$

If we expand the metric function $f(r)$ as a Taylor series with respect to r around the event horizon r_2:

$$f(r)\simeq f'(r_2)(r-r_2)+\eth(r-r_2)^2,$$
$$\simeq2\kappa(r-r_2), \tag{44}$$

the tortoise coordinate (24) then transforms into

$$r^* \simeq \frac{1}{2\kappa}\ln x. \tag{45}$$

Therefore, the scalar field near the horizon can be written in the following way:

$$\Phi \sim C_1 e^{-i(\omega t-\widehat{\omega}r^*)} + C_2 e^{-i(\omega t+\widehat{\omega}r^*)}. \tag{46}$$

From the above expression, we see that the first term corresponds to an outgoing wave, while the second one represents an ingoing wave [50,56]. For computing the QNMs, we have to impose the requirement that there exist only ingoing waves at the horizon so that, in order to satisfy the condition, we set $C_1 = 0$. Then the physically acceptable radial solution (35) is given by

$$\Re(x) = C_2(x)^{-iA}(1+x)^{-B}F\left(\widetilde{a}_n, \widetilde{b}_n; \widetilde{c}_n; -x\right). \tag{47}$$

For matching the near-horizon and far regions, we are interested in the large r behavior ($r \simeq r^* \simeq x \simeq 1+x \to \infty$) of the solution (47). To achieve this aim, one uses $z \to \frac{1}{z}$ transformation law for the hypergeometric function [57,58]

$$\begin{aligned} F(\widetilde{a}, \widetilde{b}; \widetilde{c}; z) = &\frac{\Gamma(\widetilde{c})\Gamma(\widetilde{b}-\widetilde{a})}{\Gamma(\widetilde{b})\Gamma(\widetilde{c}-\widetilde{a})}(-z)^{-\widetilde{a}}F(\widetilde{a}, \widetilde{a}\\ &+1-\widetilde{c}; \widetilde{a}+1-\widetilde{b}; 1/z)\\ &+\frac{\Gamma(\widetilde{c})\Gamma(\widetilde{a}-\widetilde{b})}{\Gamma(\widetilde{a})\Gamma(\widetilde{c}-\widetilde{b})}(-z)^{-\widetilde{b}}F(\widetilde{b}, \widetilde{b}+1-\widetilde{c}; \widetilde{b}\\ &+1-\widetilde{a}; 1/z). \end{aligned} \tag{48}$$

Thus, we obtain the asymptotic behavior of the radial solution (47) as

$$\Re(r) = \frac{C_2}{\sqrt{r}}\left[\frac{\Gamma(\widetilde{c}_n)\Gamma(\widetilde{b}_n-\widetilde{a}_n)}{\Gamma(\widetilde{b}_n)\Gamma(\widetilde{c}_n-\widetilde{a}_n)}(r)^{-\eta} + \frac{\Gamma(\widetilde{c})\Gamma(\widetilde{a}-\widetilde{b})}{\Gamma(\widetilde{a})\Gamma(\widetilde{c}-\widetilde{b})}(r)^{\eta}\right]. \tag{49}$$

Since the QNMs impose the requirement that the ingoing waves spontaneously terminate at spatial infinity (meaning that only purely outgoing waves are allowed), the first term of Eq. (49) vanishes. This scenario is enabled by the poles of the Gamma function in the denominator of the second term; $\Gamma(\widetilde{c}_n-\widetilde{a}_n)$ or $\Gamma(\widetilde{b}_n)$. Since the Gamma function $\Gamma(\sigma)$ has the poles at $\sigma = -n$ for $n = 0, 1, 2, \ldots$, the wave function satisfies the considered boundary condition only upon the following restrictions:

$$\widetilde{c}_n - \widetilde{a}_n = 1 - a_n = -n, \tag{50}$$

or

$$\widetilde{b}_n = -n. \tag{51}$$

These conditions determine the QNMs. From Eqs. (37) and (50), we find

$$\begin{aligned} \omega_n &= -\frac{i}{(2n+1)r_0}\left[n(n+1) - l(l+1)\right],\\ &= -\frac{i}{r_0}\left[n+\frac{1}{2}-\frac{(2l+1)^2}{2(2n+1)}\right], \end{aligned} \tag{52}$$

which is nothing but the QNM solution represented in [50]. Obviously, it is independent from the rotation parameter a of the RLDBH. For the highly damped modes ($n \to \infty$), it reduces to

$$\omega_n = -i\left(n+\frac{1}{2}\right)\kappa_0, \quad (n \to \infty), \tag{53}$$

where $\kappa_0 = \frac{1}{2r_0}$ is the surface gravity of the LDBH (i.e., $a = 0$). Hence, its corresponding transition frequency becomes

$$\begin{aligned} \Delta\omega &\approx \mathrm{Im}\,\omega_{n-1} - \mathrm{Im}\,\omega_n,\\ &= \kappa_0. \end{aligned} \tag{54}$$

QNMs obtaining from Eq. (51) have a rather complicated form. However, if we consider the LDBH case, simply setting $a = 0$ in Eq. (51), one promptly reads the QNMs as represented in Eq. (52). On the other hand, for $a \neq 0$ case, we inferred from our detailed analysis that the quantization of the RLDBH, which is fully consistent with the Bekenstein conjecture (1) can be achieved when the case of large charge ($r_0 \gg$) and low orbital quantum number l is considered. Hence $\tau \ll$ [see Eq. (42)], and therefore $\eta \approx i\omega r_0$ in Eq. (41). In this case, Eq. (51) becomes

$$\widetilde{b}_n \approx \frac{1}{2} - i\frac{\widehat{\omega}}{2\kappa} = -n. \tag{55}$$

Thus, we compute the QNMs as follows:

$$\omega_n = m\Omega_{\mathrm{H}} - i\left(n+\frac{1}{2}\right)\kappa. \tag{56}$$

The structural form of the above result corresponds to the Hod QNM result obtained for the Kerr BH [27]. Now, the transition frequency between two highly damped neighboring states is

$$\begin{aligned} \Delta\omega &\approx \mathrm{Im}\,\omega_{n-1} - \mathrm{Im}\,\omega_n, \quad (n \to \infty)\\ &= \kappa,\\ &= 2\pi T_{\mathrm{H}}/\hbar, \end{aligned} \tag{57}$$

which reduces to Eq. (54) when setting $a = 0$ ($\kappa \to \kappa_0$). Thus, after substituting Eq. (57) into Eq. (4), the adiabatic invariant quantity reads

$$I_{adb}^{\mathrm{rot}} = \frac{\hbar S^{\mathrm{BH}}}{2\pi} = \frac{r_2 r_0}{2} = n\hbar. \tag{58}$$

From Eq. (58), the entropy spectrum is obtained as

$$S_n^{\mathrm{BH}} = 2\pi n. \tag{59}$$

Because $S^{\rm BH} = \frac{A^{\rm BH}}{4\hbar}$, the area spectrum is then obtained as

$$A_n^{\rm BH} = 8\pi n\hbar, \tag{60}$$

and the minimum area spacing becomes

$$\Delta A_{\rm min}^{\rm BH} = 8\pi\hbar. \tag{61}$$

The above results imply that the spectral-spacing coefficient becomes $\epsilon = 8\pi$, which is fully in agreement with Bekenstein's original result [3–5]. Furthermore, the spectra of the RLDBH are clearly independent of the rotation parameter a.

4 Conclusion

In this paper, we quantized the entropy and horizon area of the RLDBH. According to the MM, which considers the perturbed BH as a highly damped harmonic oscillator, the transition frequency in Eq. (4) (describing $I_{adb}^{\rm rot}$) is governed by $\operatorname{Im}\omega$ rather than by $\operatorname{Re}\omega$ of the QNMs. To compute $\operatorname{Im}\omega$, we solved the radial equation (23) as a hypergeometric differential equation. The parameters of the hypergeometric functions were rigorously found by straightforward calculation. We then computed the complex QNM frequencies of the RLDBH. In the case of $\tau \ll$, we showed that the obtained QNMs with the rotation term a are in the form of Hod's asymptotic QNM formula [27]. After obtaining the transition frequency of the highly damped QNMs, we derived the quantized entropy and area spectra of the RLDBH. Both spectra are equally spaced and independent of the rotation parameter a. Our results support the Kothawala et al.'s [59] conjecture, which states that the area spectrum is equally spaced under Einstein's gravity theory but unequally spaced under alternative (higher-derivative) gravity theories. On the other hand, the numerical coefficient was evaluated as $\epsilon = 8\pi$ or $\xi = 1$, which is consistent with previous results [3–5,12].

In future work, we will extend our analysis to the Dirac equation, which can be formulated in the Newman–Penrose picture [51,60,61]. In this way, we plan to analyze the quantization of stationary spacetimes using fermion QNMs.

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Appendix

By redefining the independent variable $x = -y$, Eq. (28) becomes

$$y(1-y)\Re''(y)+(1-2y)\Re'(y)+\left(\frac{A^2}{y}-\frac{B^2}{1-y}+C\right)\Re(y) = 0. \tag{62}$$

We can also redefine the function $\Re(y)$ as [62]

$$\Re(y) = y^{\alpha}(1-y)^{\beta}F(y). \tag{63}$$

Thus, the radial equation (62) becomes

$$y(1-y)F''(y) + [1+2\alpha-2y(\alpha+\beta+1)]\,F'(y) + \left(\frac{\widetilde{A}}{y}-\frac{\widetilde{B}}{1-y}+\widetilde{C}\right)F(y) = 0, \tag{64}$$

where

$$\widetilde{A} = A^2+\alpha^2, \tag{65}$$

$$\widetilde{B} = B^2-\beta^2, \tag{66}$$

$$\widetilde{C} = C-(\alpha+\beta)^2-(\alpha+\beta). \tag{67}$$

If we set

$$\alpha = iA, \quad \rightarrow \widetilde{A} = 0, \tag{68}$$

$$\beta = -B, \quad \rightarrow \widetilde{B} = 0, \tag{69}$$

then Eq. (64) is transformed to the following second order differential equation:

$$y(1-y)F''(y) + [1+2iA+2(B-1-iA)y]F'(y) + \left[(A+iB)^2+B+C-iA\right]F(y) = 0. \tag{70}$$

The above equation resembles the hypergeometric differential equation which is of the form [57]

$$y(1-y)F''(y)+[c_n-(1+a_n+b_n)y]F'(y)-a_nb_nF(y) = 0. \tag{71}$$

Equation (71) has two independent solutions, which can expressed around three regular singular points: $y=0$, $y=1$, and $y=\infty$ as [57]

$$F(y)=C_1F\,(a_n, b_n; c_n; y) + C_2y^{1-c_n}F\,(a_n-c_n+1, b_n-c_n+1; 2-c_n; y)\,. \tag{72}$$

By comparing the coefficients of Eqs. (70) and (71), one can obtain the following identities:

$$c_n = 1 + 2iA, \tag{73}$$
$$1 + a_n + b_n = -2(B - 1 - iA), \tag{74}$$
$$a_n b_n = -(A + iB)^2 - B - C + iA. \tag{75}$$

Solving system of Eqs. (74) and (75), we get

$$a_n = \frac{1}{2} - B + iA + \frac{\sqrt{4C+1}}{2}, \tag{76}$$
$$b_n = \frac{1}{2} - B + iA - \frac{\sqrt{4C+1}}{2}. \tag{77}$$

Using Eqs. (63), (72), (73), (76), and (77), one obtains the general solution of Eq. (28) as Eq. (35).

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