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Experimental and numerical investigations of free convection heat transfer in solar oven

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Abstract

The use of solar energy for baking, heating or drying represents a sustainable way of solar energy applications with negligible negative effects. Solar oven is an alternative to conventional oven that rely heavily on coal and wood or Electric oven that uses the power from the National grid of which the end users have little or no control. Since the Solar oven uses no fuel and it cost nothing to run, it use are widely promoted especially in situations where minimum fuel consumption or fire risks are considered highly important. As useful as the Solar Oven proved, it major setback in the area of applications has been its future sustainability. For the use of Solar Oven/Cookers to be sustained in the future, the design and development of solar oven must rely on sound analytical tools. Therefore, this work focused on the design and development of the solar oven. To test the performance of the Small Solar Oven a 5000cm³ beaker of water was put into the Oven and the temperature of the water was found to reach 81°C after about 3hrs under an average ambient temperature of 30°C. On no load test, the oven reached a maximum temperature of 112°C in 6hrs. In order to carry out the parametric studies and improve the performance of the Solar Oven, Mathematical models were developed and solved by using Characteristics-Based Split (CBS) Finite Element Method. The Model results were compared with the Experimental results and a good agreement ware found between the two results.

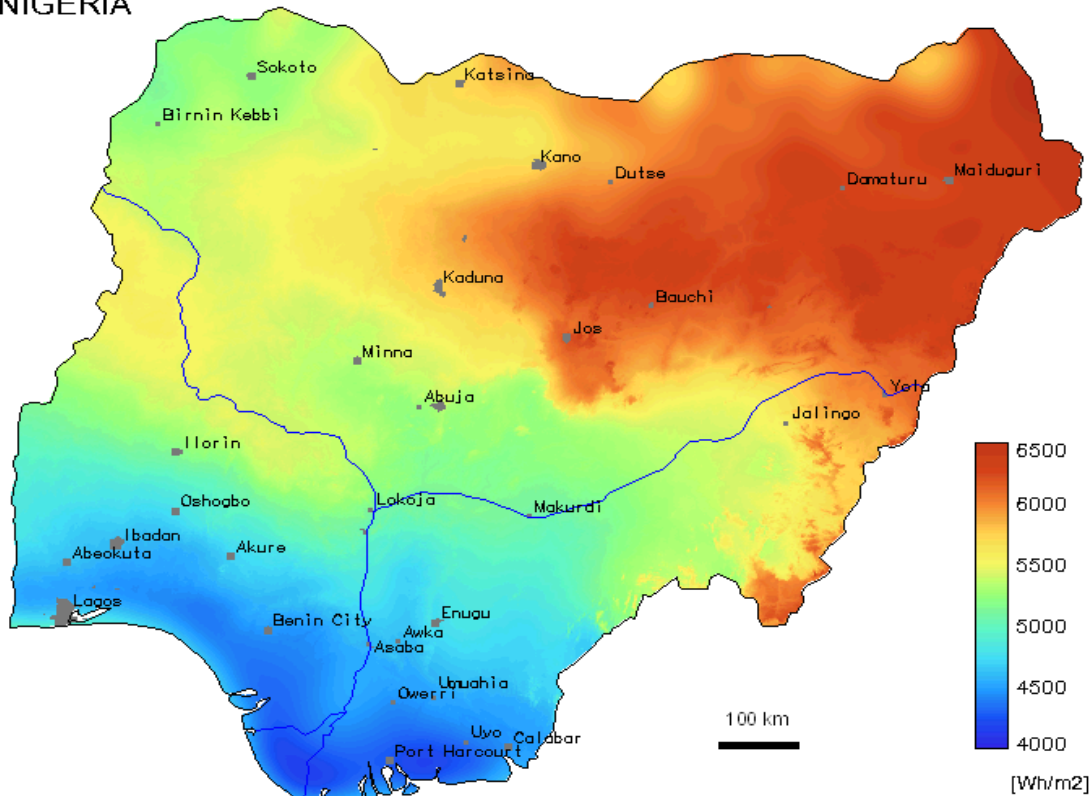
Key words: Solar Oven; Free Convection; Heat Transfer; CBS Finite Element Method

1. Introduction

The world rapidly growing population and exploding energy demands coupled with the climatic profile, socio-economic development, energy supply infrastructures and the threats on health and environment accompanying the utilization of fossil fuels strongly suggest the use of renewable energy powered systems. In fact, the strategies for attaining balance between energy demands and effects in various countries has provoked vast majority in considering the alternative sources. Furthermore, the convergence of rapidly rising costs, dwindling and threatened supply of fossil fuel are stimulating renewed efforts to find viable energy alternatives. Some of the proposed alternatives such as large scale use of bio-fuels may not be achievable or sustainable-or, eventually practical. Some other interesting possibilities such as geothermal energy may be viable but are local and limited. Solar energy that is virtually an inexhaustible natural source produces little or no greenhouse gases. It is known as a clean and environmental friendly energy source. Aside from the indisputable

advantages of solar energy mentioned above, the consideration of harnessing solar energy among all other alternative sources of energy sources to meet the needs especially in developing countries is based on well established visibility studies. Firstly, most of the countries called developing, like Nigeria, are in or adjacent to the tropics and have good solar radiation available as shown in Fig.1. Secondly, energy is a critical need of these countries but they do not have widely distributed readily available supplies of conventional energy resources. Thirdly, most of the developing countries are characterized by arid climates, dispersed and inaccessible populations and a lack of investment capital and are thus faced with practically insuperable obstacles to the provision of energy by conventional means, for example, by electrification. In contrast to this, solar energy is readily available and is already distributed to the potential users. Fourthly, because of the diffuse nature of solar energy the developments all over the world, the energy source has been in smaller units which fit well into the pattern of rural economics.

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Fig.1 Solar insolation in different States in Nigeria

The abundant solar radiation in the tropical and subtropical regions proved solar energy as the most attractive alternative energy source for baking, cooking, drying and heating. A number of government, non-government and private organizations along with a few research institutions and other individual researchers (Reddy et al, 1979; Mumba, 1995; Oosthuizen, 1995; Wisniewski and Pietruszko, 1997, Adelaja et al, 2009, Muhammad *et al*, 2011) have applied the knowledge of solar radiation on the earth surface for the fabrication, promotion and dissemination of different types of solar dryer (direct, indirect and mixed-mode types) in developing countries. In meeting the essential and basic need of human being, wood and fossil fuel as sources of energy for cooking food have played tremendous and invaluable roles. However, the direct combustion of wood and fossil fuel as the major sources of cooking has immensely contributed to global

warming and acid rains. Aside from these adverse effects, the inhalation of these gases irritates the lungs and the eyes and can cause diseases such as pneumonia. In order to obviate or eliminate this problem, a lot of researchers have developed different solar ovens/cookers over the years (Lof, 1965; Duffie *et al*, 1974; Daniel, 1990; Halacy *et al*, 1992, Wary, 1992, Warren, 1994, Barbara, 1994, Malik et al, 1996, Frederick, *et al*, 1997, Kulkarni *et al*, 1997, Adegoke, *et al*, 1998; Patel *et al*, 2000, Felix, *et al*, 2002, Amer, 2003; Ekechukwu *et al*, 2003; Folmer *et al*, 2003, Volker, 2003, 2004, Fronco et al, 2004, Negi *et al*, 2005; Hussein *et al*, 2008; Uhuegbu, 2011; Tuara and Musa, 2012). The range of temperature achieved in most of these ovens/cookers (80°C-100°C) is most suitable for cooking by boiling. Talking about the advantages, this application of solar energy has been assisting in preventing deforestation, saving of fossil fuel and reduction in



utility bills in most countries. Also, since the solar oven uses no fuel, it cost nothing to run and therefore, its use is widely promoted especially in situations where minimum fuel consumption or fire risks are considered highly important. However, as useful as the Solar Oven proved, its major setback in the area of applications has been its future sustainability. For the use of Solar Oven/Cookers to be sustained in the future, the design and development of solar oven must rely on sound analytical tools. Also, to understand the physical phenomena of solar oven and to optimize the performance of the forced convection solar dryer, theoretical investigation is essential. An adventure into the analytical methods in solving the resulting conservation equations for the theoretical investigations of the performance and optimisation of the Oven has proven abortive and over the years, recourse has been made to numerical methods such as Finite difference or Finite volume methods in analysing such class of problems involving conduction, convection and sometimes diffusion. The finite element method has been somehow stated to be limited to Computational Solid Mechanics or at its best in Thermo-Fluid

problems, conductive heat transfer. Actually, for most conduction equations in heat transfer problems, the finite element method especially Galerkin Method is straight forward. However, if similar Galerkin type approximation is applied in the solution of convection equations, the results will be marked with spurious oscillations in space if certain parameters exceed a critical value, element Peclet number. This method is not unique to finite elements as all other spatial discretization techniques have difficulties (Lewis et al, 2009). Direct application of the Characteristic Galerkin scheme to solve the momentum equation is difficult. Therefore, in this work, a Characteristic Based Split (CBS) scheme which satisfies the well-known Babuska-Brezzi condition (21, 22) as proposed by Lewis et al (20) is used to analyse and simulate the heat transfer in a photovoltaic-powered forced convection solar dryer. In order to optimize the performance of the solar dryer, the developed finite element models were used to investigate the effects of the blower speeds and the solar dryer chamber aspect ratios on the temperature distribution, velocity distribution and the pressure drop in the drying chamber.

2. The Working Principle of Solar Oven and Experimental Set up

When solar energy is radiated on the solar collector of the solar oven, some of the energy is reflected, some is absorbed; or some of it passes right through. But only the absorbed energy is converted to heat energy in the enclosure of the solar oven. When food is placed in this solar oven enclosure, moisture at the surface is evaporated and removed by the hot air and the low humidity of air in the oven. This establishes a moisture vapour pressure gradient, which causes movement of

moisture from the interior of the food to the surface. Depending on the nature of the food and the rate of heating when the extent of moisture loss exceeds the rate of movement from the interior, the zone of evaporation moves inside the food, the surface dries out, the temperature rises to the temperature of the hot air (119°-240°C) and a crust is formed (Halacy, 1978) but the internal temperature of the food does not exceed 100°C cause baking takes place at atmospheric pressure and moisture escapes freely from the food.



Fig. 2 Physical model of the solar oven

Since the food does not reach too high temperature in the oven, it can safely be left in the cooker all day without burning. Depending on the latitude and The two test carried above was conducted on solar oven. The results obtained are shown in the Fig.13 and Fig .14. The test at load which involved placing of 500cm³ beaker of water.

3. Model development for the solar oven

The numerical investigations of the solar oven involve setting up mathematical models. In order to achieve this, a set of conservation equations were set up for the numerical analysis

x-momentum equation.

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \quad 2$$

y-momentum equation.

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] + \beta g(T - T_o) \quad 3$$

Energy Equation.

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \quad 4$$

The initial conditions used in this model are:

weather, food can be cooked or baked either early or later in the day. The cooker can also be used to warm food, drinks and pasteurize water or milk.

3.1 Conservation Equations for the Analysis of Solar Oven

The following set of conservation equations were used for the finite element analysis.

Continuity Equation.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad 1$$

1



$$t = 0, \quad u = 0, \quad v = 0, \quad T = T_{\infty}, \quad p = p_{\infty} \quad \text{for } 0 < x < L, 0 < y < L$$

The boundary conditions used in this model are:

$$t > 0, \quad y = 0, \quad u = v = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial p}{\partial y} = 0 \quad \text{for } 0 < x < L$$

$$t > 0, \quad y = L, \quad u = v = 0, \quad T = T_w, \quad p = p_m \quad \text{for } 0 < x < L$$

$$t > 0, \quad x = 0 \quad u = v = 0, \quad \frac{\partial T}{\partial x} = 0, \quad \frac{\partial p}{\partial x} = 0 \quad \text{for } 0 < y < L$$

$$t > 0, \quad x = L \quad u = v = 0, \quad \frac{\partial T}{\partial x} = 0, \quad \frac{\partial p}{\partial x} = 0 \quad \text{for } 0 < y < L$$

5a

To obtain the non-dimensional form of the equation, the following non-dimensional scales are used.

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \tau = \frac{tu_{\infty}}{L}, \quad U = \frac{u}{u_{\infty}}, \quad V = \frac{v}{u_{\infty}}, \quad P = \frac{p}{\rho U_{\infty}^2}, \quad \theta = \frac{T - T_{\infty}}{T_w - T_{\infty}}$$

5b

Where L is the characteristics dimension, T_w is the wall temperature (reference temperature) and U_{∞} is the free stream velocity.

On substituting the above scales into the equations(1-4), we have:

Continuity equation.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{6}$$

X-momentum equation.

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \tag{7}$$

Y-momentum equation.

$$\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Gr}{Re^2} \theta \tag{8}$$

Energy Equation.

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re Pr} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \tag{9}$$

or



$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pe} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)$$

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The initial and Boundary conditions in dimensionless form are:

$$\begin{aligned} \tau > 0, \quad Y = 0, \quad U = V = 0, \quad \frac{\partial \theta}{\partial Y} = 0, \quad \frac{\partial P}{\partial Y} = 0 \quad \text{for } 0 < X < 1 \\ \tau > 0, \quad Y = 1, \quad U = V = 0, \quad T = 1, \quad P = p_m / \rho U^2 \quad \text{for } 0 < X < 1 \\ \tau > 0, \quad X = 0 \quad U = V = 0, \quad \frac{\partial \theta}{\partial X} = 0, \quad \frac{\partial P}{\partial X} = 0 \quad \text{for } 0 < y < 1 \\ \tau > 0, \quad X = 1 \quad U = V = 0, \quad \frac{\partial \theta}{\partial X} = 0, \quad \frac{\partial P}{\partial X} = 0 \quad \text{for } 0 < y < 1 \end{aligned}$$

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3.2 Characteristics-Based Split (CBS) Finite Element Analysis of the Conservation Equations

For the continuity equation

$$\frac{U_{i+1,j}^n - U_{i,j}^n}{\Delta X} + \frac{V_{i+1,j}^n - V_{i,j}^n}{\Delta Y} = 0 \quad \text{or} \quad \frac{\partial U_{ij}^n}{\partial X} + \frac{\partial V_{ij}^n}{\partial Y}$$

12

For the x momentum equation, the semi-discrete form is:

$$\begin{aligned} \frac{U_{ij}^{n+1} - U_{ij}^n}{\Delta \tau} = U_{ij}^n \frac{\partial U_{ij}^n}{\partial X} - V_{ij}^n \frac{\partial U_{ij}^n}{\partial Y} + \frac{1}{Re} \left(\frac{\partial^2 U_{ij}}{\partial X^2} + \frac{\partial^2 V_{ij}}{\partial Y^2} \right)^n - \frac{\partial P_{ij}}{\partial X} + U_{ij} \frac{\Delta \tau}{2} \frac{\partial}{\partial X} \left[U_{ij}^n \frac{\partial U_{ij}^n}{\partial X} + \right. \\ \left. V_{ij}^n \frac{\partial U_{ij}^n}{\partial Y} + \frac{\partial P_{ij}^n}{\partial X} \right] + V_{ij} \frac{\Delta \tau}{2} \frac{\partial}{\partial Y} \left[U_{ij}^n \frac{\partial U_{ij}^n}{\partial X} + V_{ij}^n \frac{\partial U_{ij}^n}{\partial Y} + \frac{\partial P_{ij}^n}{\partial X} \right] \end{aligned}$$

13a

For y momentum equation, the semi-discrete form is:

$$\begin{aligned} \frac{V_{ij}^{n+1} - V_{ij}^n}{\Delta \tau} = U_{ij}^n \frac{\partial V_{ij}^n}{\partial X} - V_{ij}^n \frac{\partial V_{ij}^n}{\partial Y} + \frac{1}{Re} \left(\frac{\partial^2 V_{ij}}{\partial X^2} + \frac{\partial^2 V_{ij}}{\partial Y^2} \right)^n - \frac{\partial P_{ij}}{\partial X} + U_{ij} \frac{\Delta \tau}{2} \frac{\partial}{\partial X} \left[U_{ij}^n \frac{\partial U_{ij}^n}{\partial X} + V_{ij}^n \frac{\partial}{\partial Y} \right. \\ \left. \frac{\partial P_{ij}^n}{\partial X} \right] + V_{ij} \frac{\Delta \tau}{2} \frac{\partial}{\partial Y} \left[U_{ij}^n \frac{\partial V_{ij}^n}{\partial X} + V_{ij}^n \frac{\partial V_{ij}^n}{\partial Y} + \frac{\partial P_{ij}^n}{\partial X} \right] + \frac{Gr}{Re^2} \theta_{ij}^n \end{aligned}$$

13b

On removing the pressure from equations 13a and 13b, we arrived equation 14-15

$$\begin{aligned} \frac{U_{ij} - U_{ij}^n}{\Delta \tau} = U_{ij}^n \frac{\partial U_{ij}^n}{\partial X} - V_{ij}^n \frac{\partial U_{ij}^n}{\partial Y} + \frac{1}{Re} \left(\frac{\partial^2 U_{ij}}{\partial X^2} + \frac{\partial^2 U_{ij}}{\partial Y^2} \right)^n + U_{ij} \frac{\Delta \tau}{2} \frac{\partial}{\partial X} \left(U_{ij}^n \frac{\partial U_{ij}^n}{\partial X} + V_{ij}^n \frac{\partial U_{ij}^n}{\partial Y} \right) \\ + V_{ij} \frac{\Delta \tau}{2} \frac{\partial}{\partial Y} \left(U_{ij}^n \frac{\partial U_{ij}^n}{\partial X} + V_{ij}^n \frac{\partial U_{ij}^n}{\partial Y} \right) \end{aligned}$$

14



$$\frac{v_{ij} - v_{ij}^n}{\Delta\tau} = U_{ij}^n \frac{\partial v_{ij}^n}{\partial X} - V_{ij}^n \frac{\partial v_{ij}^n}{\partial Y} + \frac{1}{Re} \left(\frac{\partial^2 v_{ij}}{\partial X^2} + \frac{\partial^2 v_{ij}}{\partial Y^2} \right)^n + U_{ij}^n \frac{\Delta\tau}{2} \frac{\partial}{\partial X} \left(U_{ij}^n \frac{\partial U_{ij}^n}{\partial X} + V_{ij}^n \frac{\partial U_{ij}^n}{\partial Y} \right) + V_{ij}^n \frac{\Delta\tau}{2} \frac{\partial}{\partial Y} \left(U_{ij}^n \frac{\partial v_{ij}^n}{\partial X} + V_{ij}^n \frac{\partial v_{ij}^n}{\partial Y} \right) + \frac{Gr}{Re^2} \theta_{ij}^n$$

15

On subtracting equation 14 and 15 from equations 13a and 13b respectively, we obtained the real velocity field equations(velocity or momentum correction equations) as:

$$\frac{u_{ij}^{n+1} - \tilde{u}_{ij}}{\Delta\tau} = -\frac{\partial P^n}{\partial X} + U \frac{\Delta\tau}{2} \frac{\partial}{\partial X} \left(\frac{\partial P}{\partial X} \right)^n + V \frac{\Delta\tau}{2} \frac{\partial}{\partial X} \left(\frac{\partial P}{\partial X} \right)^n$$

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$$\frac{v_{ij}^{n+1} - \tilde{v}_{ij}}{\Delta\tau} = -\frac{\partial P^n}{\partial Y} + U \frac{\Delta\tau}{2} \frac{\partial}{\partial X} \left(\frac{\partial P}{\partial X} \right)^n + V \frac{\Delta\tau}{2} \frac{\partial}{\partial Y} \left(\frac{\partial P}{\partial Y} \right)^n$$

17

On differentiating equation 17 with respect to X and also with respect to Y and adding the resulting equations together neglecting third-order terms gives.

$$\frac{\partial u_{ij}^{n+1}}{\partial X} + \frac{\partial v_{ij}^{n+1}}{\partial Y} - \frac{\partial \tilde{u}_{ij}}{\partial X} - \frac{\partial \tilde{v}_{ij}}{\partial Y} = -\Delta\tau \left(\frac{\partial^2 P_{ij}}{\partial X^2} + \frac{\partial^2 P_{ij}}{\partial Y^2} \right)^n$$

18

Which gives the pressure calculation

$$\left(\frac{\partial^2 P_{ij}}{\partial X^2} + \frac{\partial^2 P_{ij}}{\partial Y^2} \right)^n = \frac{1}{\Delta\tau} \left(\frac{\partial \tilde{u}_{ij}}{\partial X} + \frac{\partial \tilde{v}_{ij}}{\partial Y} \right)^n$$

19

Since (continuity equation);

$$\frac{\partial u_{ij}^{n+1}}{\partial X} + \frac{\partial v_{ij}^{n+1}}{\partial Y} = 0$$

20

Temperature Calculation

On applying the characteristic Galerkin procedure to the temperature equation (11), it gives:



$$\frac{\theta_{ij}^{n+1} - \theta_{ij}^n}{\Delta t} = U_{ij}^n \frac{\partial \theta^n}{\partial X} - V_{ij}^n \frac{\partial \theta_{ij}^n}{\partial Y} + \frac{1}{RePr} \left(\frac{\partial^2 \theta_{ij}}{\partial X^2} + \frac{\partial^2 \theta_{ij}}{\partial Y^2} \right)^n + U \frac{\Delta \tau}{2} \frac{\partial}{\partial X} \left(U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right) + V \frac{\Delta \tau}{2} \frac{\partial}{\partial Y} \left(U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right)^n$$

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In summary, we have the intermediate X-momentum equation is:

$$\frac{U_{ij} - U_{ij}^n}{\Delta t} = U \frac{\partial U_{ij}^n}{\partial X} - V \frac{\partial U_{ij}^n}{\partial Y} + \frac{1}{Re} \left(\frac{\partial^2 U_{ij}}{\partial X^2} + \frac{\partial^2 U_{ij}}{\partial Y^2} \right)^n + U_{ij}^n \frac{\Delta \tau}{2} \frac{\partial}{\partial X} \left(U_{ij}^n \frac{\partial v_{ij}}{\partial X} + V \frac{\partial v_{ij}}{\partial Y} \right)^n + V_{ij}^n \frac{\Delta \tau}{2} \frac{\partial}{\partial Y} \left(U_{ij}^n \frac{\partial U_{ij}}{\partial X} + V \frac{\partial U_{ij}}{\partial Y} \right)$$

22

And the intermediate Y-momentum equation is:

$$\frac{V_{ij} - V_{ij}^n}{\Delta t} = U_{ij} \frac{\partial v_{ij}^n}{\partial X} - V \frac{\partial v_{ij}^n}{\partial Y} + \frac{1}{Re} \left(\frac{\partial^2 v_{ij}}{\partial X^2} + \frac{\partial^2 v_{ij}}{\partial Y^2} \right)^n + U_{ij}^n \frac{\Delta \tau}{2} \frac{\partial}{\partial X} \left(U_{ij}^n \frac{\partial v_{ij}}{\partial X} + V \frac{\partial v_{ij}}{\partial Y} \right)^n + V_{ij}^n \frac{\Delta \tau}{2} \frac{\partial}{\partial Y} \left(U \frac{\partial U_{ij}}{\partial X} + V \frac{\partial U_{ij}}{\partial Y} \right)$$

23

Pressure Calculation

$$\left(\frac{\partial^2 P_{ij}}{\partial X^2} + \frac{\partial^2 P_{ij}}{\partial Y^2} \right)^n = \frac{1}{\Delta \tau} \left(\frac{\partial U_i}{\partial X} + \frac{\partial V}{\partial Y} \right)$$

24

Temperature equation

$$\frac{\theta_{ij}^{n+1} - \theta_{ij}^n}{\Delta t} = U_{ij}^n \frac{\partial \theta^n}{\partial X} - V_{ij}^n \frac{\partial \theta_{ij}^n}{\partial Y} + \frac{1}{RePr} \left(\frac{\partial^2 \theta_{ij}}{\partial X^2} + \frac{\partial^2 \theta_{ij}}{\partial Y^2} \right)^n + U \frac{\Delta \tau}{2} \frac{\partial}{\partial X} \left(U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right) + V \frac{\Delta \tau}{2} \frac{\partial}{\partial Y} \left(U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right)^n$$

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Spatial Discretization

Applying the standard Galerkin approximation for solving equations 20-25, assuming linear interpolation functions for all variables, the spatial variation for a linear triangular element may be written as :

$$U = N_i U_i + N_j U_j + N_k U_k = [N][U]$$

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$$V = N_i V_i + N_j V_j + N_k V_k = [N][V] \quad 27$$

$$P = N_i P_i + N_j P_j + N_k P_k = [N][P] \quad 28$$

$$\theta = N_i \theta_i + N_j \theta_j + N_k \theta_k = [N][\theta]$$

29

After performing spatial discretization, the Element Mass Matrix from the Characteristics-Based Split scheme is

$$[M_e] = \frac{A}{12} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

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And the Element Convection Matrix

$$[C_e] = \frac{1}{24} \begin{bmatrix} (wsw + u_i)b_i & (wsw + u_i)b_j & (wsw + u_i)b_k \\ (wsw + u_j)b_i & (wsw + u_j)b_j & (wsw + u_j)b_k \\ (wsw + u_k)b_i & (wsw + u_k)b_j & (wsw + u_k)b_k \end{bmatrix} + \frac{1}{24} \begin{bmatrix} (wrw + u_i)c_i & (wrw + u_i)c_j & (wrw + u_i)c_k \\ (wrw + u_j)c_i & (wrw + u_j)c_j & (wrw + u_j)c_k \\ (wrw + u_k)c_i & (wrw + u_k)c_j & (wrw + u_k)c_k \end{bmatrix}$$

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Where

$$wsw = u_i + u_j + u_k$$

$$wrw = v_i + v_j + v_k$$

$$[K_e] = \frac{Re}{4A} \begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_j b_i & b_j^2 & b_j b_k \\ b_k b_i & b_k b_j & b_k^2 \end{bmatrix} + \frac{Re}{4A} \begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_j c_i & c_j^2 & c_j c_k \\ c_k c_i & c_k c_j & c_k^2 \end{bmatrix}$$

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For the momentum diffusion, we have;

$$[K_{te}] = \frac{Pr}{4A} \begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_j b_i & b_j^2 & b_j b_k \\ b_k b_i & b_k b_j & b_k^2 \end{bmatrix} + \frac{Pr}{4A} \begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_j c_i & c_j^2 & c_j c_k \\ c_k c_i & c_k c_j & c_k^2 \end{bmatrix}$$

33

And for the Heat diffusion and Stabilization Matrix



$$[K_{se}] = \frac{U_{av}}{12} \begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_j b_i & b_j^2 & b_j b_k \\ b_k b_i & b_k b_j & b_k^2 \end{bmatrix} + \frac{U_{av}}{12} \begin{bmatrix} b_i c_i & b_i c_j & b_i c_k \\ b_j c_i & b_j c_j & b_j c_k \\ b_k c_i & b_k c_j & b_k c_k \end{bmatrix} + \frac{V_{av}}{12A} \begin{bmatrix} c_i b_i & c_i b_j & c_i b_k \\ c_j b_i & c_j b_j & c_j b_k \\ c_k b_i & c_k b_j & c_k b_k \end{bmatrix} + \frac{V_{av}}{12A} \begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_j c_i & c_j^2 & c_j c_k \\ c_k c_i & c_k c_j & c_k^2 \end{bmatrix}$$

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Where U_{av} and V_{av} are the average values of U and V over the element. The discretization of the CBS steps requires three more matrices and four forcing vectors to complete the process. The matrix from the discretized second-order terms for pressure calculations is:

$$K = \frac{1}{4A} \begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_j b_i & b_j^2 & b_j b_k \\ b_k b_i & b_k b_j & b_k^2 \end{bmatrix} + \frac{1}{4A} \begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_j c_i & c_j^2 & c_j c_k \\ c_k c_i & c_k c_j & c_k^2 \end{bmatrix}$$

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The first gradient matrix in the X-direction is:

$$[G_u] = \frac{1}{6} \begin{bmatrix} b_i & b_j & b_k \\ b_i & b_j & b_k \\ b_i & b_j & b_k \end{bmatrix}$$

36

And the second-gradient matrix in the Y-direction is

$$[G_v] = \frac{1}{6} \begin{bmatrix} c_i & c_j & c_k \\ c_i & c_j & c_k \\ c_i & c_j & c_k \end{bmatrix}$$

37

The forcing vectors of the X-component of the momentum equation is:



$$[f_1] = \frac{\Gamma}{4A} Re \begin{bmatrix} b_i u_i & b_j u_j & b_k u_k \\ b_i u_i & b_j u_j & b_k u_k \\ 0 & 0 & 0 \end{bmatrix}^n n_u + \frac{\Gamma}{4A} Re \begin{bmatrix} c_i u_i & c_j u_j & c_k u_k \\ c_i u_i & c_j u_j & c_k u_k \\ 0 & 0 & 0 \end{bmatrix}^n n_v$$

38

Where i, j is the boundary edge of an element.

The forcing vector of the Y- component momentum equation is:

$$[f_2] = \frac{\Gamma}{4A} Re \begin{bmatrix} b_i v_i & b_j v_j & b_k v_k \\ b_i v_i & b_j v_j & b_k v_k \\ 0 & 0 & 0 \end{bmatrix}^n n_u + \frac{\Gamma}{4A} Re \begin{bmatrix} c_i v_i & c_j v_j & c_k v_k \\ c_i v_i & c_j v_j & c_k v_k \\ 0 & 0 & 0 \end{bmatrix}^n n_v \quad 39$$

The forcing vector from the discretion of the second-order pressure terms in pressure calculations is:

$$[f_3] = \frac{\Gamma}{4A} \begin{bmatrix} b_i P_i & b_j P_j & b_k P_k \\ b_i P_i & b_j P_j & b_k P_k \\ 0 & 0 & 0 \end{bmatrix}^n n_u + \frac{\Gamma}{4A} \begin{bmatrix} c_i P_i & c_j P_j & c_k P_k \\ c_i P_i & c_j P_j & c_k P_k \\ 0 & 0 & 0 \end{bmatrix}^n n_v \quad 40$$

Finally, the forcing term due to the discretization of the second-order terms in the equation is:

$$[f_4] = \frac{\Gamma}{4A} Pr \begin{bmatrix} b_i \theta_i & b_j \theta_j & b_k \theta_k \\ b_i \theta_i & b_j \theta_j & b_k \theta_k \\ 0 & 0 & 0 \end{bmatrix}^n n_u + \frac{\Gamma}{4A} Pr \begin{bmatrix} c_i \theta_i & c_j \theta_j & c_k \theta_k \\ c_i \theta_i & c_j \theta_j & c_k \theta_k \\ 0 & 0 & 0 \end{bmatrix}^n n_v \quad 41$$

The four steps of the characteristic based split scheme may now be written in matrix form.

Step 1: Intermediate Velocity Calculation X-Component

$$\left[M \frac{\Delta[\bar{v}]}{\Delta t} \right] = -[C][U]^n - [K_m][U]^n - [K_b][U]^n + [f_1] \quad 42$$

Intermediate Velocity Calculation Y-Component



$$[M] \frac{\Delta[\tilde{U}]}{\Delta t} = -[C][V]^n - [K_m][V]^n - [K_s][V]^n + [f_2]$$

43

Step 2: Pressure Calculation

$$[K][P]^n = -\frac{1}{\Delta t} [G_u][\tilde{U}] + [G_u][\tilde{V}] + [f_3]$$

44

Step 3: Velocity Correction

$$[M][U]^{n+1} = [M][\tilde{U}] - \Delta\tau[G_v][P]^n$$

45a

$$[M][V]^{n+1} = [M][\tilde{V}] - \Delta\tau[G_v][P]^n$$

45b

Step 4: Temperature Computation

$$[M] \frac{\Delta[\theta]}{\Delta t} = -[C][\theta]^n - [K_f][\theta]^n - [K_s][\theta]^n + [f_4]$$

46

4. Results and Discussion

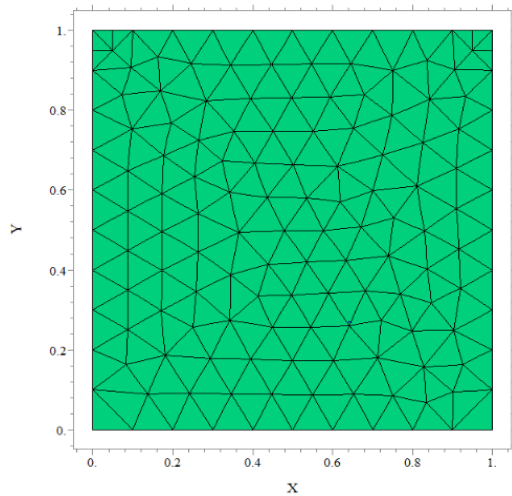


Fig.3 The Finite Element discretization of the solar oven enclosure 30mins

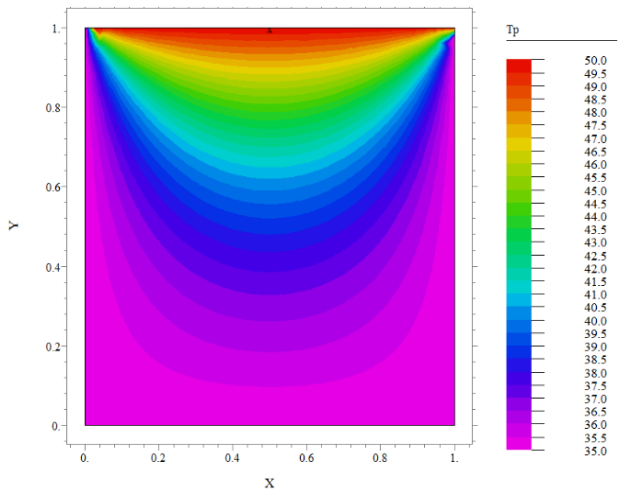


Fig.4 Temperature distribution in the oven after 30mins

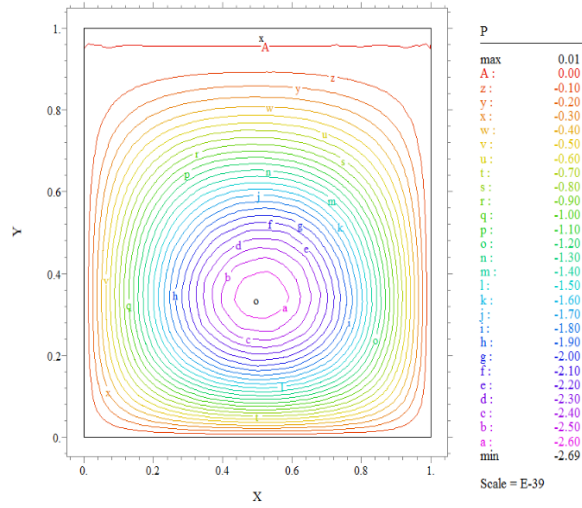


Fig.5 Pressure distribution in the oven after 30mins

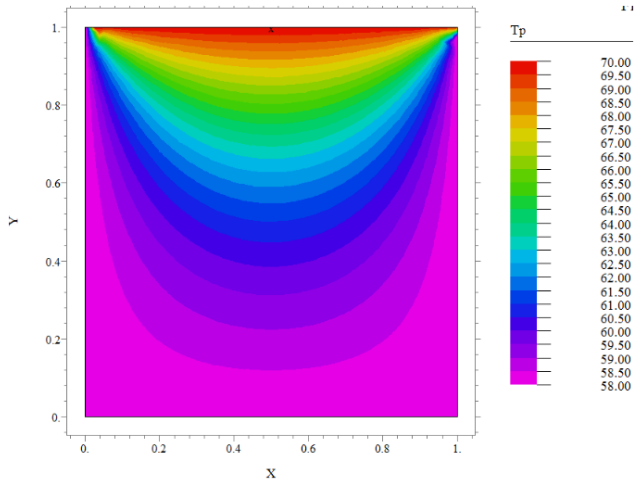


Fig.6 Temperature distribution in the oven after 2hrs

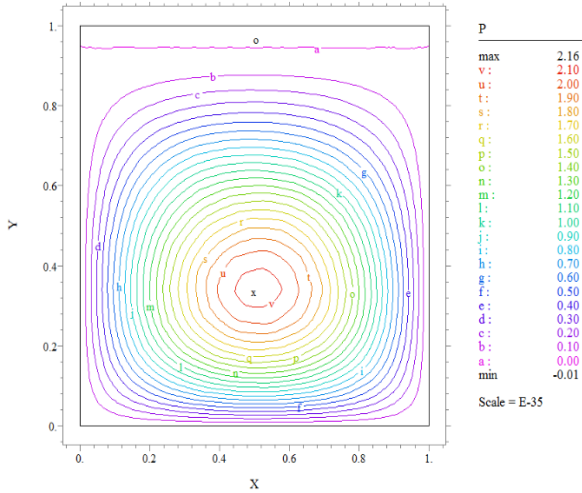


Fig.7 Pressure distribution in the oven after 2hrs 30mins

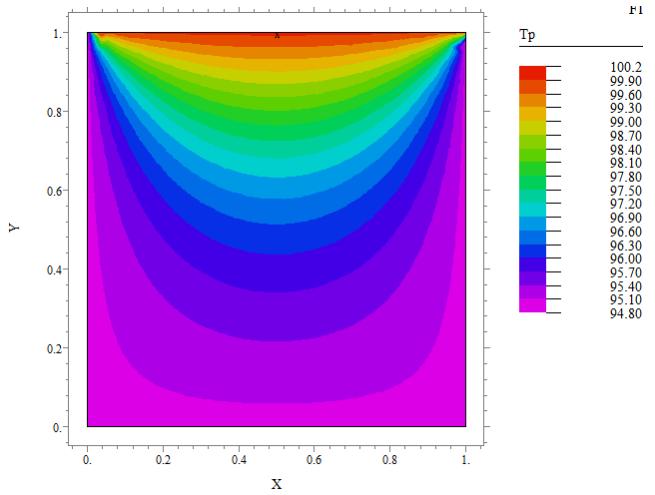


Fig.8 Temperature distribution in the oven after 4hrs

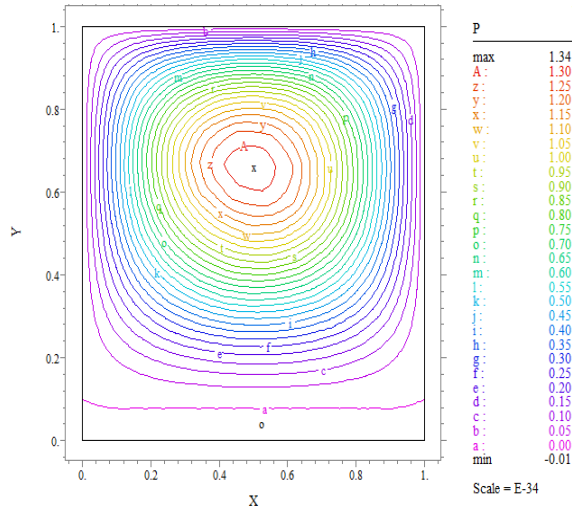


Fig.9 Pressure distribution in the oven after 4hrs 30mins

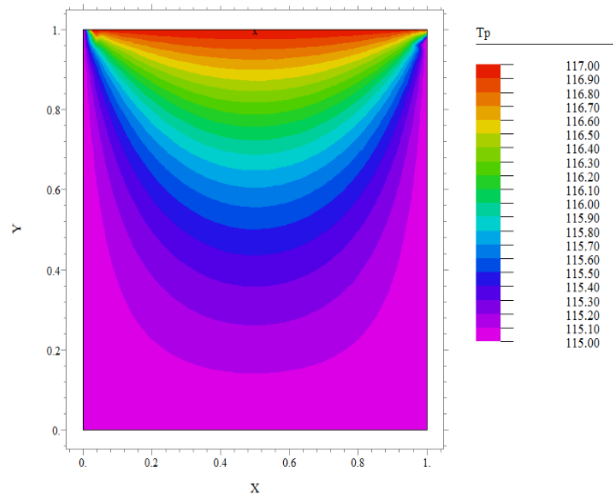


Fig.10 Temperature distribution in the oven after 6hrs

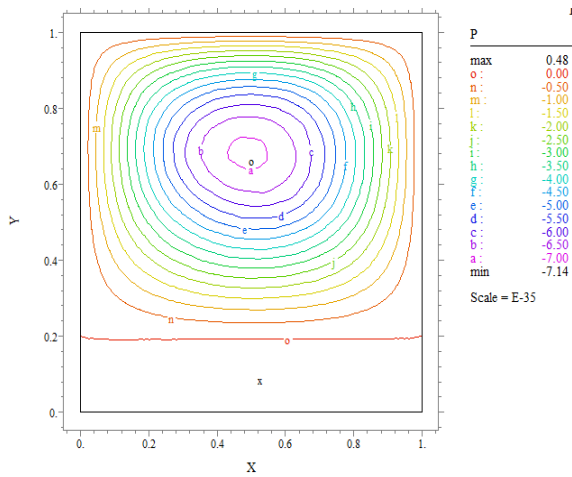


Fig.11 Pressure distribution in the oven after 6hrs

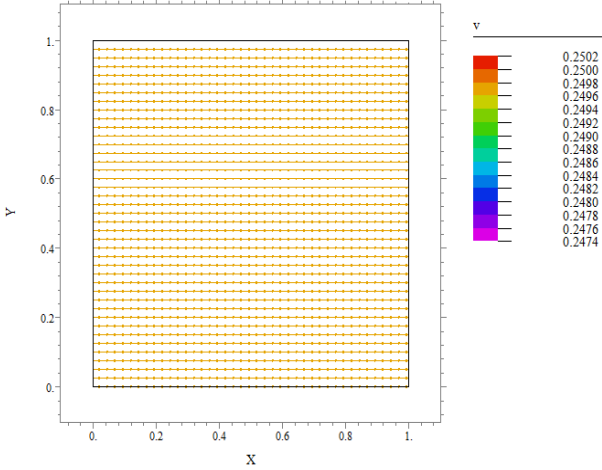


Fig.12 Velocity distribution in the oven after 6hrs

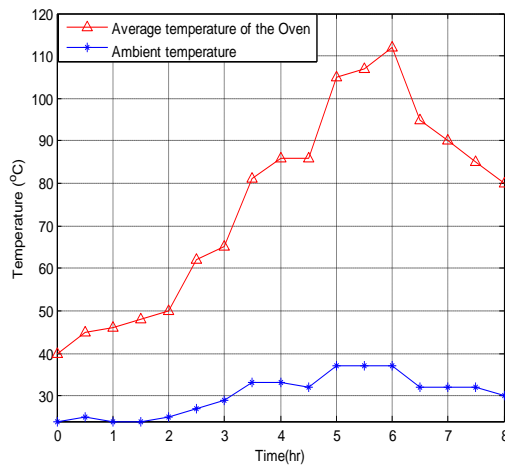


Fig.13 Experimental results of the solar oven

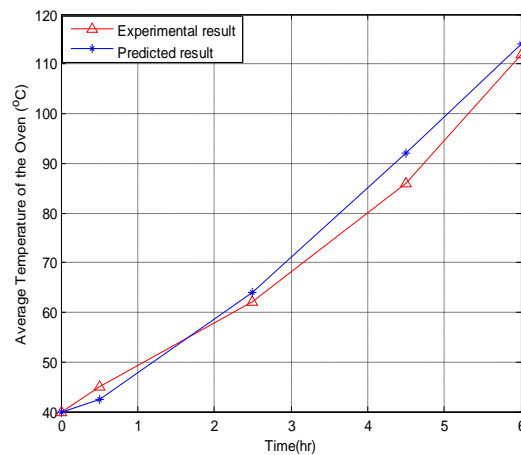


Fig.14 Comparison of Experimental results with model results



Fig. 3-12 show the temperature pressure, and velocity distributions as predicted from the numerical analysis of the solar oven. The figures display the variation and the distributions in colours and contours. The simulated results shown above are the results of what happened in the Solar oven chamber during a period of 6hours of maximum insolation.

Fig.13 shows the experimental results of the average temperature of the solar oven and the ambient. The test was set out around 8am of 8th October, 2005. From the results, it was discovered that the time taken for the oven to attain a maximum temperature of 112°C was 6hours without the average insolation on no load. Fig. 14 shows the comparison of the predicted results with the experimental results. As it could be seen from the figure, a good agreement was achieved between the predicted results of the numerical analysis and the experimental results.

5. CONCLUSION

In this work, mathematical models were developed and solved by using Characteristics-Based Split (CBS) Finite Element Method to numerically investigate the temperature, velocity and pressure distributions in solar oven. The numerical results were compared with the Experimental results and a good agreement were found between the two results. The work will greatly assist in developing

and optimising the design solar oven especially for the developing countries.

Nomenclature

- Gr Grashof number
- Pr Prandtl number
- Re Reynold Number
- Pe Peclet number
- t time, s
- T temperature, °C
- T_w Wall Temperature, °C
- T_∞ Atmospheric temperature, °C
- U dimensionless velocity along x-axis
- u_∞ Free stream velocity, m/s
- V dimensionless velocity along x-axis
- x length of the drying chamber, m
- X dimensionless length of the drying chamber
- y breadth of the drying chamber, m
- Y dimensionless breadth of the drying chamber

Symbol

- α thermal diffusivity, m²/s
- θ dimensionless temperature
- τ dimensionless time
- ρ density, kg/m³
- μ Coefficient of dynamic viscosity, kg/ms
- β Volumetric expansion coefficient

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