OPTIMAL CONTROL and FORECASTING of COMPLEX DYNAMICAL SYSTEMS

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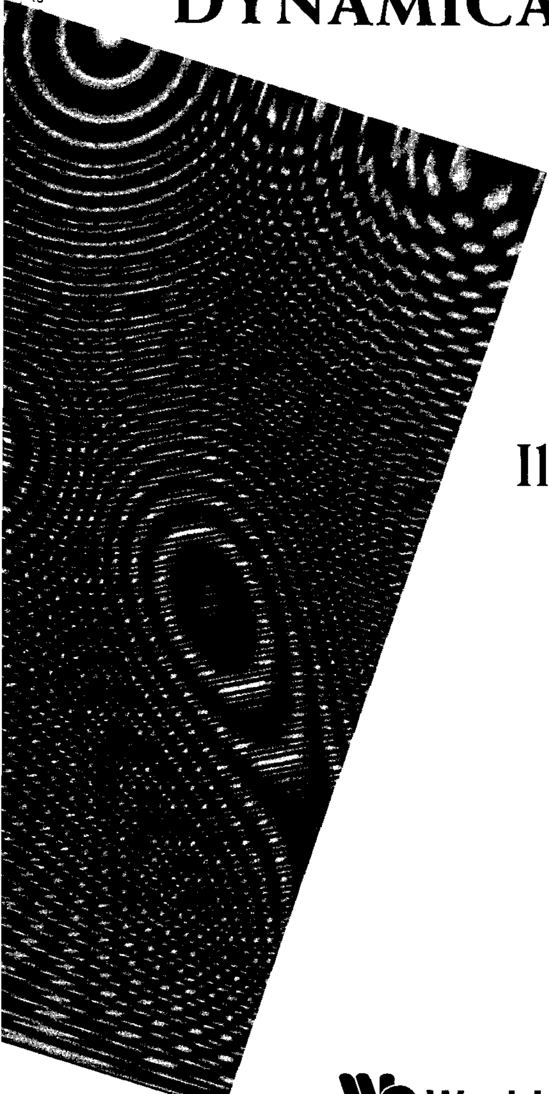

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To my beautiful wife Elena

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Preface

Chance, however, is the governess of life.
Palladas, 5th Century A.D.
Anthologia Palatina 10.65

This book has appeared by choice but also by some chance. I gave a talk in summer 2003 at Max Plank Institute of Complex Systems in Dresden, Germany, where I was kindly invited by Prof. Dr. Jan-Michael Rost. It happened that among the people who has visited my talk, was a representative of the World Scientific Inc. One month later I received an invitation to write this book.

The purpose of this text is to summarize and share with a reader the author's experience in the field of complex systems. The title of this book were constructed to maximally cover different topics of the author's recent research, as fully as it is possible to do in few words. The main target of this book is to show the variety of the problems in modern physics which be formulated in terms of optimization and optimal control. This idea is not new. From the 18th century it is known, that almost any physical (or mechanical) problem one can formulate as an extremum problem. Such approach is called Lagrangian formalism, after the great French mathematician Lagrange.

The text is written in such a way, that all the chapters are logically coupled to each other, so the reader should be ready to be referred to different parts of the book.

In this book the author tried to adopt a naive division of the complexity hierarchy. The simplest case is control and forecast of the systems, which one can describe with the help of linear differential equations, and control fields enter in these equations as additive inhomogeneous terms. The situation becomes more complicated, when control appears as not additive, but multiplicative. It leads to a nonlinear problem for the search of control fields. A typical example-control of a quantum system, where a control field enters into the Schrödinger equation as a product with the system's wave-function. The next level of complexity appears when the nonlinearity of controlled system is taken into account. Such problems are still tractable. As an example one can consider control of Bose-Einstein condensate (BEC), which dynamics is described by the Gross-Pitaevsky equation. Note, it is assumed, that we still *know* the explicit form of the mathematical equations governing the system's dynamics. However, the dynamics of the controlled system could be very complicated (chaotic). Additional complexity can be achieved, if dynamics of the system becomes not deterministic, with addition of some stochastic component. And the most difficult situation happens when we need to control a black box system (like biological, financial or social systems), for which *ab initio* evolution equations are unknown.

Chapter 1 provides an introduction to the long mathematical history of the calculus of variations, starting from the Ferma's variational principle, the famous Bernoulli *brachistochrone* problem and the beginning of the calculus of variations. Despite of the limited applicability of analytical methods, the calculus of variations remains a vital instrument to solve various variational problems. The author could go deeper into the ancient times and start his story with the princess Dido's problem, but he has feeling that the brachistochrone problem belongs to the *scientific* history, and Dido's problem is just a beautiful ancient legend based on Virgil's *Aeneid*, without clear proof of the Dido's priority.

In chapter 2 we discuss different aspects of numerical optimization, including effectiveness of the optimization algorithms and multiobjective optimization. We make a brief review of some popular numerical methods which could be useful for solution of various problems in optimization, control and forecasting. We give a broader review of the so-called "Quantum Genetic Algorithm", which operates with smooth differentiable functions with a limited absolute value of their gradient. As an example, we demonstrate its ability to solve few-body quantum statistical problems in 1D and 2D as a problem of minimization of the ground state energy, or maximization of the partition function. Different scenarios of the formation and melting of a "Wigner molecule" in a quantum dot.

Chapter 3 outlines some elements of the chaos theory and a deep connection between nonlinearity and complexity in different systems. In this chapter we give a generalization of the Lorenz system using fractional derivatives. We show, how the "effective dimension" of the system controls its dynamical behavior, including a transition from chaos to a regular motion.

In chapter 4 we discuss a problem of optimal control in application to nanoscale quantum systems. We introduced a novel approach, which permits us to obtain new analytical solutions of different optimal control problems. We also solve a problem for optimal control of the induced photo-current between two quantum dots using genetic algorithm. We analyze how decoherence processes, which result in non-unitary evolution of a quantum system, change the optimal control fields. This question is very significant for future design of nanoscale devices, since decoherence, in general, significantly limits optimal control.

In chapter 5 we continue to consider control of quantum systems with particular application to quantum computing. We have shown, that an optimal design of artificial quantum bits can decrease by an order of magnitude the number of errors due to quantum decoherence processes, and leads to a faster performance of basic quantum logical operations.

In chapter 6 we briefly discuss different aspects of forecasting and its connection with optimization and chaos theory, which we discuss in the previous chapters.

I would like to conclude this introduction with acknowledge of my teachers and colleagues, who helped and governed my research last years. The most of the results which were presented or mentioned in this book were done in a close collaboration with these nice people. I would like to thank my scientific supervisors and colleagues: Prof. Dr. B. G. Matisov and Dr. I. E. Mazets, Prof. Dr. K. H. Bennemann, Prof. Dr. M. E. Garcia, Prof. Dr. D. V. Khveshchenko, Prof. Dr. S. Haas and Prof. Dr. A. F. J. Levi.

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