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Ferrofluid lubrication of a parallel plate squeeze film bearing

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Abstract

We derived a Reynolds type equation for a ferrofluid lubrication in a squeeze film between two circular plates using Jenkins model and considering combined effects of rotation of the plates, anisotropic permeability in the porous matrix and slip velocity at the interface of porous matrix and film region. We used it to study the case of a parallel-plate squeeze film bearing. Expressions were obtained for dimensionless pressure, load capacity and response time. Computed values were displayed some in tabular form and some in graphical form. The load capacity decreased with increasing values of the radial permeability and attained a minimum when the plates rotated in the opposite directions with nearly the same speed. It increased with increasing values of the axial permeability or material constant of Jenkins model and attained a maximum when the value of the material constant was near unity. It increased or decreased for increasing values of the speed of rotation of the upper plate according as the value of the material constant is zero or not. The response time slowly decreased with increasing values of the radial permeability, speed of rotation of upper plate or the material constant. But, it increased with increasing values of the axial permeability and attained a maximum when the plates rotated in opposite directions with nearly the same speed. Anisotropic permeability affected the bearing characteristics considerably.

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Nomenclature

a	radius of each plate
F	defined in eq.(15)
F^*	defined in eq.(22)
G	defined in eq.(14)
G^*	defined in eq.(21)
h	general film thickness
$\bar{h}$	h/h_0
h_0	central film thickness
$\bar{h}_0$	h_0/h_2
h_2	initial film thickness
H_0	thickness of porous facing
$\bar{H}$	external magnetic field
H	magnitude of $\bar{H}$
k	defined in eq.(12)
p	film pressure
$\bar{p}$	defined in eq.(12)
P	porous pressure
r	radial coordinate
R	r/a
s	slip constant
$\bar{s}$	sh_0
s^*	sh_2
S	defined in eq.(12)
S_1	defined in eq.(20)
t	time
$\bar{t}$	defined in eq.(20)
u	radial component of film fluid velocity
$\bar{u}$	radial component of porous fluid velocity
w	axial component of film fluid velocity
$\bar{w}$	axial component of porous fluid velocity
W	load capacity
$\bar{W}$	defined in eq.(18)

z	axial coordinate
M	magnitude of $\bar{M}$
$\bar{M}$	magnetization vector
M_s	saturation magnetization
$\bar{M}^*$	corotational derivative of $\bar{M}$
$\bar{q}$	velocity vector of the film fluid
v	the tangential component of $\bar{q}$

Greek symbols

α^2	material constant of Jenkins model
β^2	defined in eq.(12)
$\gamma*$	defined in eq.(12)
γ_1^*	defined in eq.(20)
η	fluid viscosity
η_r	porosity of porous facing
μ_0	permeability of free space
$\bar{\mu}$	magnetic susceptibility
μ^*	defined in eq.(12)
μ_1^*	defined in eq.(20)
ρ	fluid density
ϕ	inclination to $\bar{H}$ to the radial direction
ϕ_r	permeability in the radial direction
ϕ_z	permeability in the axial direction
ψ_r	defined in eq.(12)
ψ_r^*	defined in eq.(20)
ψ_z	defined in eq.(12)
ψ_z^*	defined in eq.(20)
Ω_f	Ω_l/Ω_u
Ω_l	angular speed of lower plate
Ω_r	$\Omega_u - \Omega_l$
Ω_u	angular speed of the upper plate
γ	another material constant
χ_0	initial susceptibility of the fluid

1 Introduction

When studying porous bearing problems it was usual to assume no slip condition at the porous surface. Beavers et al [1] and Sparrow et al [2] not only showed that such an assumption need not hold at the nominal boundary of a naturally permeable material but provided the necessary boundary conditions also. Using these conditions Prakash and Vij [3] studied the squeeze film behaviour for porous discs considering slip velocity while Patel and Gupta [4] analysed a porous slider bearing considering slip velocity. The increase in the slip parameter decreased the load capacity of the bearings.

Another assumption usually made is that the permeability in the porous matrix is isotropic. Owing to manufacturing defects it may be different along the three axes. With this end in view Kulkarni and Vinay Kumar [5] obtained a new lubrication equation for porous slider bearings considering anisotropic permeability in the porous matrix as well as the slip velocity at the porous-film interface. Bhat [6] extended the equation [5] including the effect of a conducting lubricant under a transverse magnetic field. Puri and Patel [7] analysed anisotropic porous slider bearing considering slip velocity. Recently, Ram and Verma [8] studied the ferrofluid lubrication in a porous inclined slider bearing using Jenkins model for the ferrofluid flow while Shah and Bhat [9] analysed the effect of rotation on the curved squeeze film betweeen two circular plates using Neuringer-Rosensweig model.

In this paper, we use Jenkins model to derive a Reynolds type equation for ferrofluid lubrication of a squeeze film between two circular plates considering combined effects of rotation of the plates, anisotropic permeability in the porous matrix and slip velocity at the interface of porous matrix and film region. We use it to study the case of a parallel plate squeeze film bearing. This paper also includes the added advantage of inclusion of effect of material constant due to Jenkins model.

2 Analysis

The bearing consists of two circular plates, each of radius a. The upper plate has a porous facing of thickness H_0 which is backed by a solid wall. It moves normally towards an impermeable and flat plate with a uniform velocity

$$\dot{h}_0 = \frac{dh_0}{dt},$$

where h_0 is the central film thickness. The general film thickness is h. But the figure 1 is to discuss the case when $h = h_0$. The upper and lower plates rotate with angular velocities Ω_u and Ω_l respectively. We assume axisymmetric flow of the ferrofluid flowing as per Jenkins model. The external magnetic field $\bar{H}$ with magnitude H and inclination ϕ to the lower plate is taken,

$$\bar{H} = H(r)(\cos\phi(r,z), 0, \sin\phi(r,z))$$

i.e. $H(r)$ depends only on radial coordinate so it is axisymmetric.

The governing equation as derived in Appendix is

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta\left(1 - \frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\left[\frac{d}{dr}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right) - \rho r\left(\frac{z}{h}\Omega_r + \Omega_l\right)^2\right], \quad (1)$$

where p is the film pressure, u is the radial velocity of the fluid in the film, η is the fluid viscosity, ρ is the fluid density, α^2 is the material constant, $\bar{\mu}$ is the magnetic susceptibility, μ_0 is the permeability of the free space and $\Omega_r = \Omega_u - \Omega_l$,

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0, \quad (2)$$

w being the axial component of the film fluid velocity.

Solving eq.(1) under the slip boundary conditions [2]

$$u = 0 \quad \text{when} \quad z = 0, \quad u = \frac{1}{s}\frac{\partial u}{\partial z} \quad \text{when} \quad z = h, \quad \frac{1}{s} = \frac{\sqrt{\phi_r\eta_r}}{5}, \quad (3)$$

ϕ_r being the permeability of the fluid in the porous matrix in the radial direction and η_r being the porosity in the same direction, we obtain

$$u = \frac{z}{2\eta\,(sh-1)\left(1-\dfrac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\left[\left(hsz - sh^2 - z + 2h\right)\times\right.$$

$$\frac{d}{dr}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right) - \frac{\rho r}{6h^2}\left\{\left(hsz^3 - sh^4 - z^3 + 4h^3\right)\Omega_r^2 + \right.$$

$$4h\left(hsz^2 - sh^3 - z^2 + 3h^2\right)\Omega_r\Omega_l +$$

$$\left.\left.6h^2\left(hsz - sh^2 - z + 2h\right)\Omega_l^2\right\}\right] \,. \tag{4}$$

The radial and axial velocity components of the fluid in the porous matrix are given by a generalized Darcy's law considering contributions from the magnetic pressure and the material constant as in eq.(A.9) and rotation of the upper plate as [8]

$$\bar{u} = -\frac{\phi_r}{\eta}\left[\frac{\partial}{\partial r}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right) - \rho\,\Omega_u^2 r + \frac{\rho\alpha^2\bar{\mu}}{2}\frac{\partial}{\partial z}\left(H\frac{\partial u}{\partial z}\right)\right], \tag{5}$$

$$\bar{w} = -\frac{\phi_z}{\eta}\left[\frac{\partial}{\partial z}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right) - \frac{\rho\alpha^2\bar{\mu}}{2r}\frac{\partial}{\partial r}\left(rH\frac{\partial u}{\partial z}\right)\right], \tag{6}$$

ϕ_z is the permeability of the porous fluid in the axial direction and P is the fluid pressure there.

Substitution of eqs.(5) – (6) in the continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\bar{u}\right) + \frac{\partial\bar{w}}{\partial z} = 0\,, \tag{7}$$

and integrating it across the porous matrix, yields

$$\frac{\phi_r H_0}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left(p-\frac{1}{2}\mu_0\bar{\mu}H^2\right)\right\}-$$

$$2\rho\,\Omega_u^2\phi_r H_0-\phi_z\frac{\partial}{\partial z}\left(P-\frac{1}{2}\mu_0\bar{\mu}H^2\right)\Big|_{z=h}+ \tag{8}$$

$$\frac{\rho\alpha^2\bar{\mu}}{2}\left(\phi_r-\phi_z\right)\frac{1}{r}\frac{\partial}{\partial r}\left(rH\frac{\partial u}{\partial z}\right)\Big|_{z=h}^{h+H_0}\;=\;0\,,$$

using Morgan-Cameron approximation to get the first term and the fact that the surface $z=h+H_0$ is impermeable.

Using eq.(4), eq.(8) gives

$$\frac{\partial}{\partial z}\left(P-\frac{1}{2}\mu_0\bar{\mu}H^2\right)\Big|_{z=h}=\frac{\phi_r H_0}{\phi_z r}\frac{d}{dr}\left\{r\frac{d}{dr}\left(p-\frac{1}{2}\mu_0\bar{\mu}H^2\right)\right\}$$
$$-\frac{2\rho\Omega_u^2\phi_r H_0}{\phi_z}+\frac{\rho\alpha^2\bar{\mu}}{2\eta\phi_z}\left(\phi_r-\phi_z\right)\frac{H_0}{r} \tag{9}$$
$$\times\frac{d}{dr}\left(\frac{rH}{1-\dfrac{\rho\alpha^2\bar{\mu}H}{2\eta}}\left\{\frac{d}{dr}\left(p-\frac{1}{2}\mu_0\bar{\mu}H^2\right)-\rho r\Omega_u^2\right\}\right).$$

Owing to continuity of the fluid velocity components across the surfaces $z=h$, we have

$$w_{z=h}=\dot{h}_0+\bar{w}|_{z=h}=\dot{h}_0-\frac{\phi_z}{\eta}\left[\frac{\partial}{\partial z}\left(P-\frac{1}{2}\mu_0\bar{\mu}H^2\right)\Big|_{z=h}\right.$$
$$-\frac{s\rho\alpha^2\bar{\mu}}{2}\frac{1}{r}\frac{d}{dr}\left\{\frac{rHh^2}{2\eta\left(sh-1\right)\left(1-\dfrac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\right. \tag{10}$$
$$\left.\left.\times\left(\frac{d}{dr}\left(p-\frac{1}{2}\mu_0\bar{\mu}H^2\right)-\frac{\rho r}{6}\left(3\Omega_r^2+8\Omega_l\Omega_r+6\Omega_l^2\right)\right)\right\}\right],$$

using eqs.(4) and (6).

Integrating eq.(2) across the film thickness and using eqs.(9) and we obtain the Reynolds type equation in this case as

$$\frac{1}{r}\frac{d}{dr}\left[\left\{12\phi_r H_0+\frac{h^3(4-sh)+3s\rho\alpha^2\bar{\mu}\phi_z h^2 H/\eta}{(1-sh)\left(1-\frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\right.\right.$$

$$\left.\left.+\frac{6\rho\alpha^2\bar{\mu}(\phi_r-\phi_z)H_0 H}{\eta\left(1-\frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\right\} r\frac{d}{dr}\left(p-\frac{1}{2}\mu_0\bar{\mu}H^2\right)\right]$$

$$=12\eta\dot{h}_0+24\rho\,\Omega_u^2\phi_r H_0+6\rho\alpha^2\bar{\mu}(\phi_r-\phi_z)\frac{H_0}{\eta r}\frac{d}{dr}\left(\frac{\rho r^2\Omega_u^2 H}{1-\frac{\rho\alpha^2\bar{\mu}H}{2\eta}}\right) \quad (11)$$

$$-s\rho^2\alpha^2\bar{\mu}\frac{\phi_z}{2\eta r}\frac{d}{dr}\left\{\frac{r^2h^2H\left(3\Omega_r^2+8\Omega_l\Omega_r+6\Omega_l^2\right)}{(sh-1)\left(1-\frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\right\}$$

$$-\frac{\rho}{r}\frac{d}{dr}\left[\frac{r^2h^3}{(sh-1)\left(1-\frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\left\{\frac{3}{10}(6-sh)\Omega_r^2\right.\right.$$

$$\left.\left.+(5-sh)\Omega_r\Omega_l+(4-sh)\Omega_l^2\right\}\right],$$

Using the quantities

$$H^2 = \frac{kr^2\,(a-r)}{a}, \quad R = \frac{r}{a}, \quad \bar{h} = \frac{h}{h_0}, \;\; \psi_r = \frac{\phi_r H_0}{h_0^3}, \;\; \psi_z = \frac{\phi_z H_0}{h_0^3}$$

$$\bar{s} = sh_0\,, \qquad \beta^2 = \frac{\rho\alpha^2\bar{\mu}\sqrt{k}a}{2\eta}\,, \qquad \gamma^* = \frac{6\phi_z}{h_0^2}\,, \qquad \mu^* = -\frac{k\mu_0\bar{\mu}h_0^3}{\eta\dot{h}_0}$$

$$\bar{p} = -\frac{h_0^3 p}{\eta a^2\dot{h}_0}, \qquad S = -\frac{\rho\Omega_u^2 h_0^3}{\eta\dot{h}_0}\,, \qquad \Omega_f = \frac{\Omega_l}{\Omega_u}\;. \tag{12}$$

k being a quantity to suit the dimensions of both sides of the first eq. in (12), eq.(11) can be expressed as

$$\frac{1}{R}\frac{d}{dR}\left[GR\frac{d}{dR}\left\{\bar{p} - \frac{1}{2}\mu^* R^2\,(1-R)\right\}\right] = \frac{1}{R}\frac{d}{dR}\,(RF)\;, \tag{13}$$

where

$$G = \frac{\bar{h}^3\left(4-\bar{s}\bar{h}\right) + \beta^2\bar{s}\gamma^*\bar{h}^2 R\sqrt{1-R}}{\left(1-\bar{s}\bar{h}\right)\left(1-\beta^2 R\sqrt{1-R}\right)} + \frac{12\left(\psi_r - \beta^2\psi_z R\sqrt{1-R}\right)}{1-\beta^2 R\sqrt{1-R}} \tag{14}$$

and

$$\begin{aligned} F = &-6R + \frac{12SR\left(\psi_r - \beta^2\psi_z R\sqrt{1-R}\right)}{\left(1-\beta^2 R\sqrt{1-R}\right)} \\ &- \frac{S\beta^2\gamma^*\left(3+2\Omega_f+\Omega_f^2\right)\bar{s}\bar{h}^2 R^2\sqrt{1-R}}{6\left(\bar{s}\bar{h}-1\right)\left(1-\beta^2 R\sqrt{1-R}\right)} \\ &+ \frac{SR\bar{h}^3\left\{3\bar{s}\bar{h} - 18 + \left(4\bar{s}\bar{h}-14\right)\Omega_f + \left(3\bar{s}\bar{h}-8\right)\;\Omega_f^2\right\}}{10\left(\bar{s}\bar{h}-1\right)\left(1-\beta^2 R\sqrt{1-R}\right)}\,. \end{aligned} \tag{15}$$

3 Solutions

Since the atmospheric pressure is negligible compared to the film pressure, $\bar{p}(1) = 0$.

Solving eq.(13) under the boundary conditions

$$\bar{p}\,(1) = 0, \; \frac{d\bar{p}}{dR} = 0 \;\text{ when }\; R = 0 \tag{16}$$

yields

$$\bar{p} = \frac{1}{2}\mu^* \left(R^2 - R^3\right) + \int\limits_1^R \frac{F}{G} dR\,. \tag{17}$$

The load capacity W defined by the equation

$$w = 2\pi \int\limits_0^a prdr\,,$$

can be expressed in dimensionless form as

$$\bar{W} = -\frac{h_0^3 W}{2\pi\eta a^4 \dot{h}_0} = \frac{\mu^*}{40} - \frac{1}{2}\int\limits_0^1 \frac{R^2 F}{G} dR\,. \tag{18}$$

The response time t to reach a film thickness h_0 starting with an initial film thickness h_2 is given in dimensionless form by the equation

$$\frac{d\bar{t}}{d\bar{h}_0} = \frac{3\int\limits_0^1 \dfrac{R^3}{G^*} dR}{-\dfrac{1}{2\pi} + \dfrac{\mu_1^*}{40} - \dfrac{1}{2}\int\limits_0^1 \dfrac{R^2 F^*}{G^*} dR} \tag{19}$$

where

$$\left.\begin{array}{l} \bar{h}_0 = \dfrac{h_0}{h_2}, \quad \mu_1^* = \dfrac{k\mu_0\bar{\mu}a^4}{W}, \quad \psi_r^* = \dfrac{\phi_r H_0}{h_2^3}, \quad \psi_z^* = \dfrac{\phi_z H_0}{h_2^3}\,, \\[2ex] \gamma_1^* = \dfrac{6\phi_z}{h_2^2}, \quad \bar{t} = \dfrac{h_2^2 W t}{\eta a^4}, \quad s^* = sh_2\,, \quad S_1 = \dfrac{\rho a^4 \Omega_u^2}{W}\,, \end{array}\right\} \tag{20}$$

$$G^* = \frac{\bar{h}_0^3 \bar{h}^3 \left(4 - s^* \bar{h}_0 \bar{h}\right) + \beta^2 s^* \gamma_1^* \bar{h}_0^2 \bar{h}^2 R \sqrt{1-R}}{\left(1 - s^* \bar{h}_0 \bar{h}\right) \left(1 - \beta^2 R \sqrt{1-R}\right)} + \frac{12 \left(\psi_r^* - \beta^2 \psi_z^* R \sqrt{1-R}\right)}{1 - \beta^2 R \sqrt{1-R}}, \tag{21}$$

$$F^* = \frac{12 S_1 R \left(\psi_r^* - \beta^2 \psi_z^* R \sqrt{1-R}\right)}{\left(1 - \beta^2 R \sqrt{1-R}\right)} - \frac{S_1 \beta^2 \gamma_1^* \left(3 + 2\Omega_f + \Omega_f^2\right) s^* \bar{h}_0^2 \bar{h}^2 R^2 \sqrt{1-R}}{6 \left(s^* \bar{h}_0 \bar{h} - 1\right) \left(1 - \beta^2 R \sqrt{1-R}\right)} \tag{22}$$

$$+ \frac{S_1 R \bar{h}_0^3 \bar{h}^3 \left\{3 s^* \bar{h}_0 \bar{h} - 18 + \left(4 s^* \bar{h}_0 \bar{h} - 14\right) \Omega_f + \left(3 s^* \bar{h}_0 \bar{h} - 8\right) \Omega_f^2\right\}}{10 \left(s^* \bar{h}_0 \bar{h} - 1\right) \left(1 - \beta^2 R \sqrt{1-R}\right)}.$$

4 Results and discussion

The dimensionless pressure $\bar{p}$, load capacity $\bar{W}$, and response time $\bar{t}$ are given by equations (17)-(19). The main parameters appearing in them are the radial and axial permeability parameters ϕ_r/h_0^2 and ϕ_z/h_0^2, rotation parameters S and S_1, magnetization parameters μ^* and μ_1^* , material parameter β^2 of Jenkins model, rotational speeds ratio parameter Ω_f. By setting $\bar{h} = 1$ in eqs.(17)-(19) we obtain the results for a parallel plate squeeze film bearing.

We take the representative values $a = 0.05m$, $h_0 = 2.5 \times 10^{-5}m$, $\eta_r = 0.25$, $W = 50Kgms^{-2}$, $\eta = 2 \times 10^{-3}Kgm^{-1}s^{-1}$, $\bar{\mu} = 0.05$, $\mu_0 = 4\pi \times 10^{-7}Kgms^{-2}A^{-2}$, $\rho = 800Kgm^{-3}$, $maxH = 1.9 \times 10^5 Am^{-1}$, when $\Omega_u = 10rads^{-1}$, $S = 0.25$ and $S_1 = 0.01$ for computing $\bar{W}$ and $\bar{t}$. The computed values are displayed in Tables 1-2 and Figs. 2-7.

Table 1 shows that $\bar{W}$ decreases slowly with increasing values of ϕ_r/h_0^2 . However, $\bar{W}$ increases with increasing values of ϕ_z/h_0^2 and the increase in $\bar{W}$ is marked for smaller values of ϕ_z/h_0^2. The values of $\bar{W}$

on the downward diagonal corresponds to the isotropic case where it decreases slowly with increasing values of the permeability parameter. From fig.2, when S increases, $\bar{W}$ decreases slowly or increases markedly according as $\phi_z/h_0^2 < 10^{-4}$ or not.

In Fig.3 values of $\bar{W}$ for $\beta^2 = 0$ correspond to Neuringer-Rosensweig model [9] where $\bar{W}$ decreases slowly with increasing values of S. However, in Jenkins model $\bar{W}$ increases with increasing values of S and attains a maximum for a values of β^2 near 1. It can be seen from fig.4 that $\bar{W}$ attains a minimum for a value of Ω_f near -1, i.e. when the plates rotate with nearly the same speed in opposite directions.

Table 2 shows that $\bar{t}$ decreases slowly with increasing values of ϕ_r/h_0^2 and it increases slowly with increasing values of ϕ_z/h_0^2. Figures 5 – 6 show that $\bar{t}$ decreases slowly with increasing values of S_1 or β^2. From fig.7, $\bar{t}$ attains a maximum when the plates rotate in opposite directions with nearly the same speed. Since the slip parameter

$$\frac{1}{\bar{s}} = \frac{\sqrt{\eta_r \phi_r}}{5h_0},$$

increase in $1/\bar{s}$ causes decrease in both $\bar{W}$ and $\bar{t}$ as seen from Tables 1 and Fig.2 respectively. $\bar{W}$ and $\bar{t}$ depend on the strength k of the magnetic field via μ^* or μ_1^* and β^2.

Thus, anisotropic permeability of the porous facing can be used to increase the load capacity of a parallel plate squeeze film porous bearing.

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Table 1 Values of $\bar{W}$ for different values of ϕ_r/h_0^2 and ϕ_z/h_0^2 for $\beta^2 = 0.5$, μ^*=19.6, S=0.25, $\Omega_f = 1.0$, $H_0/h_0 = 10$.

ϕ_z/h_0^2 ϕ_r/h_0^2	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}
10^{-7}	2.953	1.287	1.120	1.098	1.038
10^{-6}	19.609	2.953	1.287	1.115	1.040
10^{-5}	186.201	19.613	2.952	1.280	1.055
10^{-4}	1855.58	186.557	19.636	2.933	1.205
10^{-3}	18902.691	1891.330	190.011	19.817	2.740

Table 2 Values of $\bar{t}$ for different values of ϕ_r/h_0^2 and ϕ_z/h_0^2 for β^2=0.5, μ_1^*=0.785, $S_1 = 0.01$, Ω_f= 1.0, $H_0/h_0 = 10$, $\bar{h}_0 = 0.8$.

ϕ_z/h_0^2 ϕ_r/h_0^2	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}
10^{-7}	1.2860	1.2861	1.2850	1.2680	1.1052
10^{-6}	1.2860	1.2861	1.2850	1.2680	1.1053
10^{-5}	1.2863	1.2864	1.2853	1.2683	1.1055
10^{-4}	1.2892	1.2893	1.2882	1.2711	1.1072
10^{-3}	1.3195	1.3196	1.3185	1.3005	1.1296

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Appendix

The equations of Jenkins model[8] are

$$\begin{aligned} \rho\big[\tfrac{\partial\bar{q}}{\partial t} + (\bar{q}\bullet\nabla)\bar{q}\big] &= -\nabla p + \eta\nabla^2\bar{q} + \\ +\mu_0(\bar{M}\bullet\nabla)\bar{H} &+ \rho\alpha^2\nabla\times\left(\tfrac{\bar{M}}{M}\times\bar{M}^*\right)\,, \end{aligned} \tag{A.1}$$

$$\nabla\bullet\bar{q} = 0\,, \tag{A.2}$$

$$\nabla\times\bar{H} = 0\,, \tag{A.3}$$

$$\nabla\bullet(\bar{H} + 4\pi\bar{M}) = 0\,, \tag{A.4}$$

$$\gamma\frac{D^2\bar{M}}{Dt^2} = -4\pi\rho\frac{M_s}{\chi_0}\frac{\bar{M}}{M_s - M} - \frac{2\alpha^2}{M}\bar{M}^* + \bar{H} \tag{A.5}$$

$$\text{with}\ \ \bar{M}^* = \frac{D\bar{M}}{Dt} + \frac{1}{2}(\nabla\times\bar{q})\times\bar{M}\ , \tag{A.6}$$

where $\bar{q}$, γ, $\bar{M}$, M, $M_{\rm s}$, χ_0 and $\bar{M}^*$ are the fluid velocity, another material constant, the magnetization vector, its magnitude, the saturation magnetization, initial susceptibility of the fluid and the corotational derivative of $\bar{M}$ respectively.

In the present analysis, we replace eq.(A.5) by

$$\bar{M} = \bar{\mu}\bar{H} \,. \tag{A.7}$$

Using eqs. (A.3) and (A.7) , eq. (A.1) yields

$$\begin{aligned} \rho\left[\frac{\partial\bar{q}}{\partial t} + (\bar{q}\bullet\nabla)\bar{q}\right] &= -\nabla p + \eta\nabla^2\bar{q} + \frac{1}{2}\mu_0\bar{\mu}\nabla H^2 + \\ &+ \frac{\rho\alpha^2\bar{\mu}}{2}\nabla\times\left(\frac{\bar{H}}{H}\times\left\{(\nabla\times\bar{q})\times\bar{H}\right\}\right) \,. \end{aligned} \tag{A.8}$$

Assume that the flow is quasi-steady, fully developed (i.e. sufficient time elapsed after the flow started so that there are no singularities), axially symmetric and incompressible. Then eq. (A.8) yields, with usual assumptions of lubrication,

$$-\rho\frac{v^2}{r} = -\frac{\partial p}{\partial r} + \eta\frac{\partial^2 u}{\partial z^2} + \frac{1}{2}\mu_0\bar{\mu}\frac{\partial H^2}{\partial r} - \frac{\rho\alpha^2\bar{\mu}H}{2}\frac{\partial^2 u}{\partial z^2} \,, \tag{A.9}$$

$$0 = \frac{\partial^2 v}{\partial z^2} \,, \tag{A.10}$$

$$0 = \frac{\partial p}{\partial z} \,. \tag{A.11}$$

Solving eq.(A.10) under the boundary conditions:

$$v = r\Omega_{\mathrm{u}} \quad \text{when} \quad z = h, \quad v = r\Omega_{\mathrm{l}} \quad \text{when} \quad z = 0$$

one obtains

$$v = r\left(\frac{z}{h}\Omega_r + \Omega_l\right) \,. \tag{A.12}$$

Eqs.(A.9)-(A.12) yield eq.(1) of the text.

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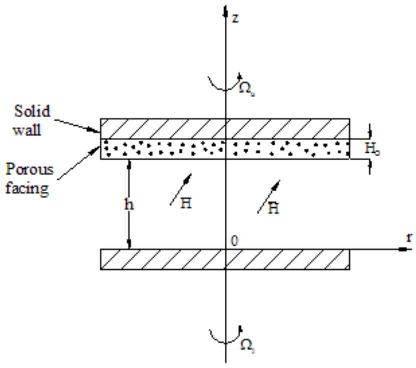

Figure 1: Configuration of the problem.

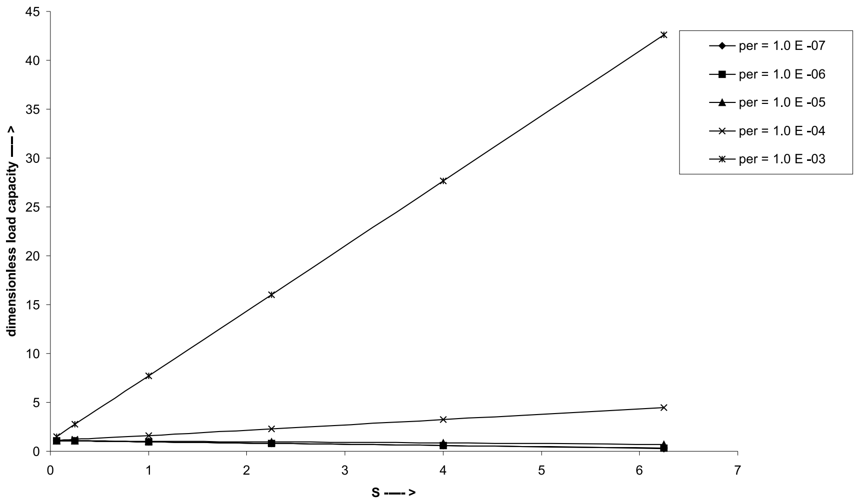

Figure 2: Dimensionless load capacity for different values of ϕ_z/h_0^2[per] and S for $\phi_r/h_0^2 = 10^{-3}$, $\beta^2 = 0.5$, $\mu^* = 19.6$, $\Omega_f = 1.0$, $H_0/h_0 = 10$.

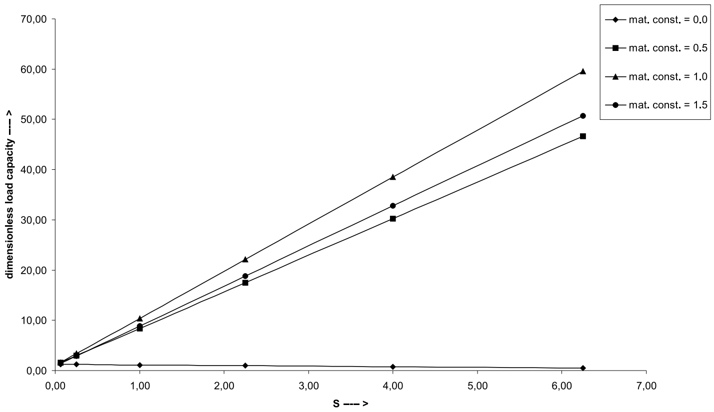

Figure 3: Dimensionless load capacity for different values of β^2 [mat.const.] and S for $\phi_r/h_0^2 = \phi_z/h_0^2 = 10^{-5}$, $\mu^* = 19.6$, $\Omega_f = 1.0$, $H_0/h_0 = 10$.

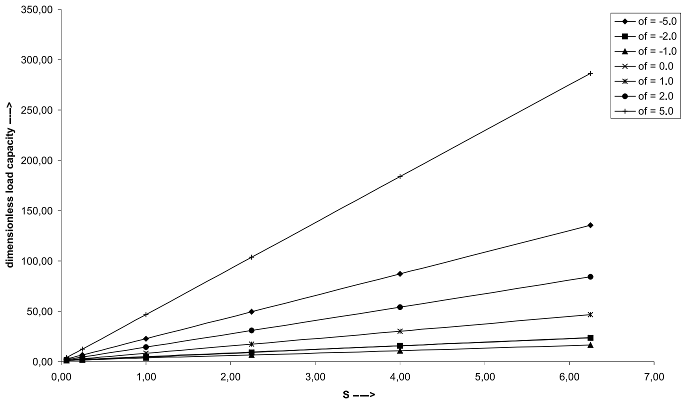

Figure 4: Dimensionless load capacity for different values of Ω_f [of] and S for $\phi_r/h_0^2 = \phi_z/h_0^2 = 10^{-5}$, $\mu^*= 19.6$, $\beta^2 = 0.5$, $H_0/h_0 = 10$.

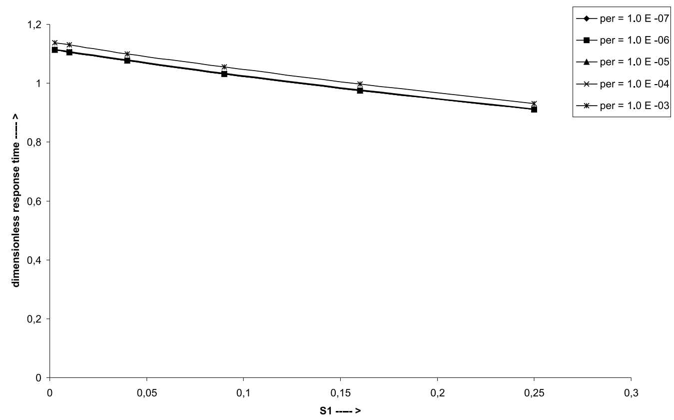

Figure 5: Dimensionless response time for different values of ϕ_z/h_0^2 [per] and S_1 for $\phi_r/h_0^2 = 10^{-3}$, $\beta^2 = 0.5$, $\mu_1^* = 0.785$, $\Omega_f = 1.0$, $H_0/h_0 = 10$, $\bar{h}_0 = 0.8$.

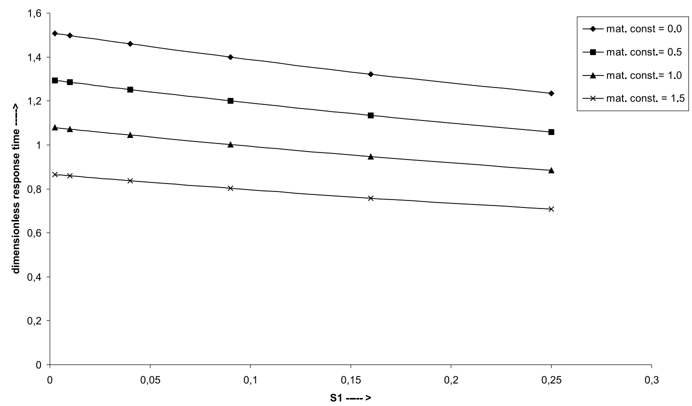

Figure 6: Dimensionless response time for different values of β^2 [mat.const.] and S_1 for $\phi_r/h_0^2 = \phi_z/h_0^2 = 10^{-5}$, $\mu_1^* = 0.785$, $\Omega_f = 1.0$, $H_0/h_0 = 10$, $\bar{h}_0 = 0.8$.

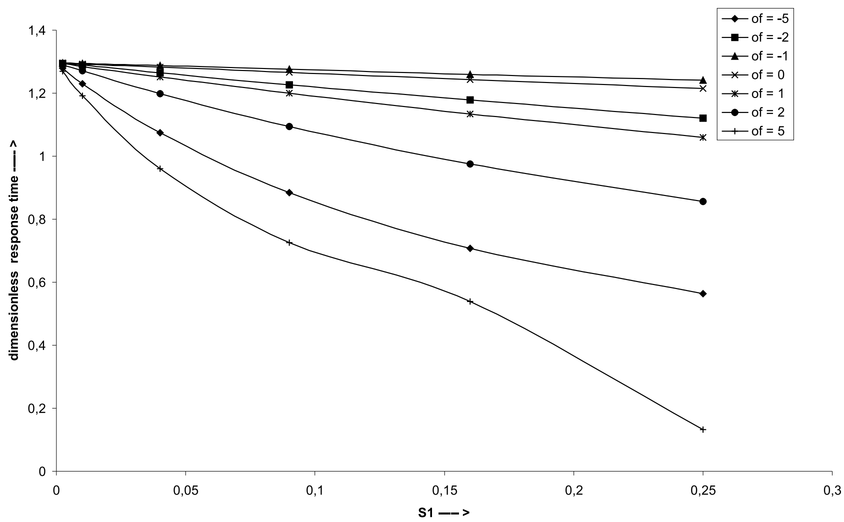

Figure 7: Dimensionless response time for different values of Ω_f [of] and S_1 for $\phi_r/h_0^2 = \phi_z/h_0^2 = 10^{-5}$, $\mu_1^* = 0.785$, $\beta^2 = 0.5$, $H_0/h_0 = 10$, $\bar{h}_0 = 0.8$.

Podmazivanje gvozdenim fluidom ležišta sa stisnutim filmom

UDK 531.783

Izvedena je jednačina Rejnoldsovog tipa za podmazivanje gvozdenim fluidom u stisnutom filmu izmedju dve kružne ploče. Pritom se koristi Dženkinsov model, a posmatraju se kombinovani efekti obrtanja ploča, anizotropne permeabilnosti u poroznoj matrici i brzini klizanja na medjupovrši oblasti porozne matrice i filma. Nju smo koristili za proučavanje clučaja nošenja stisnutog filma. Dobijeni su izrazi za bezdimenzioni pritisak, kapacitet nosivosti i vreme odgovora. Izračunate vrednosti su pokazane grafički i tabelarno. Kapacitet

nosivosti je opadao sa porastom vrednosti radijalne permeabilnosti i dostigao je minimum kada su se ploče obrtale u suprotnim smerovima približno istom brzinom. On je rastao sa porastom vrednosti aksijalne permeabilnosti ili materijalne konstante Dženkinsovog modela i dostigao je maksimum kada je vrednost te materijalne konstante bila približno jedanak jedinici. On je rastao ili opadao za rastuće vrednosti brzine obrtanja gornje ploče zavisno od toga da li je ova materijalna konstanta bila jednaka nuli ili ne. Vreme odgovora je sporo opadalo sa porastima radijalne permeabilnosti, brzine obrtanja gornje ploče ili pomenute materijalne konstante. Medjutim, ono je raslo sa porastom vrednosti aksijalne permeabilnosti i dostiglo je maksimum kada su se ploče obrtale u suprotnim smerovima približno istom brzinom. Anizotropna permeabilnost je uticala značajno na karakteristike nosivosti.