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A COMPARISON OF THE EFFICIENCY

OF TECHNIQUES FOR DIGITAL ENCODING

OF CONTOUR MAPS

by

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ABSTRACT

A COMPARISON OF THE EFFICIENCY OF TECHNIQUES FOR DIGITAL ENCODING OF
CONTOUR MAPS

There are two digitizing techniques which are operable without human
interpretation of contour-mapped data and which thus may be performed
semi-automatically; line following and raster scanning. After a
comparison of these techniques the author concentrates on raster
scanning, as this technique facilitates creation of matrix data, the data
structure most widely used in spatial analysis performed by digital
computers.

The author proposes the quantization method of computing matrix data
and shows that quantized data may be used not only for overall computations
but also to determine geometrical properties of surfaces - giving the
possibility of error estimation-shown by the contour map. In particular
such an approach enables pre-determination of a scanning parameter - track
spacing - before digitizing to satisfy the required accuracy of analysis.

The different aspects of utilising data obtained by raster scanning
are discussed; storage, display in contour map form - the algorithm to
produce the type of map where the contour lines pass exactly through the
sample points taken during scanning is included.

TO MY WIFE AND PARENTS

ACKNOWLEDGEMENTS

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GLOSSARY OF TERMS

flying spot scanner
- a system that encodes a picture by scanning in strips across the image and recording the brightness at each incremental point along the scan line.

gantry
- structure (a part of a digitizer) supporting a travelling arm or bridge to which a recording device is attached.

line following
- a technique of digitizing of lines by measurement of absolute or incremental rectangular co-ordinates of points along the lines, each measurement being taken at pre-determined intervals of time or distance.

raster scanning
- a technique of recording or generating an image with an intensity controlled, line-by-line sweep across the entire display surface.

sample point
- point on the map which is registered (i.e. its co-ordinates and the value at this point) during digitizing; when raster scanning is utilised the sample points are located at places where the tracks cross the contour lines.

track
- (also scan line or raster line) a line (usually generated by hardware) along which the sample points are located during raster scanning.

'One cannot create something from nothing (although the reverse is true); therefore, it is extremely important that the <u>initial</u> external data structure for digitizing data should contain all of the information necessary to satisfy anticipated internal and external data structure requirements.'

A.H.Schmidt (1976)

INTRODUCTION

In the earth sciences one of the most frequently used media for the display and storage of spatial data is the contour map. Such a map consists of a set of lines, each of them joining the points on the plane where the function describing the physical phenomenon has a constant value.

The development of different methods and techniques of computerised spatial data analysis created the need to have input data in a form readable by digital computers, namely a stream of alpha-numeric characters. Thus, if one wishes to use data mapped by contours for computer analysis the first operation one has to perform is to convert the information contained in the contour maps into a sequence of numbers. In this work some aspects of the digitization of contour maps (i.e. the process of converting them into digital form) are discussed.

To begin with the following question should be asked: is it necessary to use existing maps to produce a digital surface model?

For topographic surfaces, digital data can be obtained directly from air-photographs by use of a stereoplotter (Evans, 1972). At the moment, however, such facilities are not very commonly available and production of data is rather expensive. So, even if Boyle's statement 'The future of automated cartography is dominated by the cost and time of digitization of underline existing maps' (Boyle, 1976) may seem to be too categorical, the fact is that, even in fields in which there is an alternative way to obtain digital data, the need for a method that enables the creation of a digital surface model from existing maps without special expense and using available hardware is very great.

On the other hand, in sciences like geology or meteorology, one cannot use the orthophotomap method to produce digital data but the question mentioned at the beginning is still relevant. In geology, for example,

the existing maps were produced from field measurements - by definition

digital. Is it possible to use these data in further computations and

analysis instead of data from contour maps? If the contour map was

produced only by means of mathematical operations the answer is, Yes,

it is. Such operations can be repeated very easily (if it is necessary)

by means of the computer. But more often the contour map is not only

the result of an interpolation method but also reflects the experience

of the (often very many) people who drew it. In this instance it would

be a waste of potential information if we started from the measurement

points again.

Without forgetting the alternative ways of obtaining digital data,

this work will concentrate on looking at methods of digitizing contour

maps which may be performed very quickly using devices no more complicated

than the commonly available table-digitizer.

CHAPTER 1

DIGITIZATION OF CONTOUR MAPS

1.1 A contour map as a source of information

A contour map is a graphical image of a function of two variables

describing certain physical phenomena. Since a contour map portrays

three-dimensional phenomena on a two-dimensional paper sheet it is

impossible to describe the function in a full manner and some information

is lost. In fact, a contour map is not a representation of any particular

surface (function), it is a representation of a class of surfaces (functions).

The common property of the functions belonging to this class is that at

points on particular contour lines, all functions have the same value.

In the intercontour regions the value of the functions may be different.

If such functions are denoted by $f(x,y)$ then the contour lines making

up the map satisfy the equation

$$f(xy) = Z_k$$

where Z_k (k= 1, 2,...,K) is a given set of values.

Although the only points on the map at which the function has an

explicit value are those situated on the contour lines, the values of the

function at any point on the map sheet may be estimated. Provided that

the function in question $f(x,y)$ is continuous for any arbitrary point

(x,y) situated between two contour lines (let us say Z_k and Z_{k+1}) the

value of the function at this point must satisfy the following condition:

$$(f(x,y) - Z_k)(f(x,y) - Z_{k+1}) < 0$$

To prove this it is sufficient to notice that this inequality is true when,

$$f(x,y) - Z_k > 0 \text{ and } f(x,y) - Z_{k+1} < 0 \tag{1}$$

or

$$f(x,y) - Z_k < 0 \text{ and } f(x,y) - Z_{k+1} > 0 \tag{2}$$

From (1) we obtain

$$Z_k < f(x,y) < Z_{k+1}$$

and from (2)

$$Z_k > f(x,y) > Z_{k+1}$$

which are the only possible estimations of a value of the function at

points situated between the contour lines Z_k and Z_{k+1}.

Up to now it has been implicit in the discussion that determining

the value of the function at points not situated on contour lines

involves some method of approximation.

As mentioned earlier in this thesis it has been assumed that the

function $f(x,y)$ is single-valued and continuous, i.e. the surface is

without vertical cliffs or faults and it is determined in the whole area

given by the map sheet.   Morse (1968) considers a more general mathematical

model of contour-mapped data.

1.2 Digitizing

A contour map can be considered as a graphical image of a function

of two variables.   Before such a function can be processed by a digital

computer the contour map needs to be digitized or in other words converted

from its analogue structure into a discrete form as a finite string of

numbers.   This may be done by several methods and may be performed

manually or semi-automatically.   The choice of the digitizing method should

be determined not only by the available hardware but also by the subsequent

use of the data.

The general purpose of diɡitizing is to create a bank of spatial data

in machine readable form which may later be searched, displayed (perhaps

in its initial form, i.e. as contour maps plotted by computer) or analysed

mathematically.

The digitization method must guarantee that, in spite of the loss of

information that usually accompanies analogue-digital conversion, our aims

will be achieved.   In addition it should allow the possibility of error

estimation.

A general classification of digitizing methods is shown in Figure 1.
The diagram is based on a classification introduced by Wesley (1974).
The problems connected with classification of di̇tizing methods are
discussed also by Yoeli (1975) and Türke (1978).

DIGITIZING

DIRECT

CARTOGRAPHIC

Stereoplotter
with a numerical
output device

survey
instruments
(Zeiss Req Elta,
Aqa 700)

manual

semi-automatic

line
following

raster
scanning
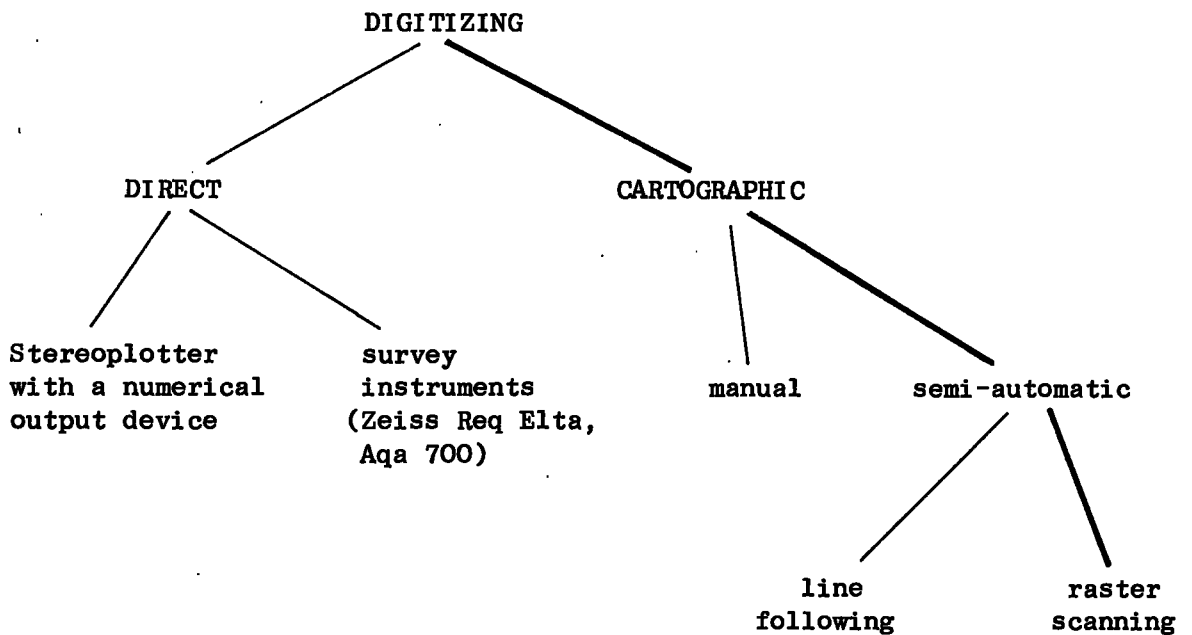
FIGURE 1.   Classification of digitizing methods

The thick lines on the diagram show the aspects which are emphasised
in this thesis.

1.3   Manual methods of digitizing

The division of cartographic digitizing into manual and semi-automatic
methods needs some explanation.   Manual methods are those involving human
interpretation of contour map data and visual interpolation during digitizing.
Clearly such a process cannot be performed by machines.   On the other hand,
although both of the semi-automatic techniques of digitizing can be
performed manually without any special equipment, by using hardware like

the table digitizer the speed and accuracy of digitizing can be increased enormously.

There are several methods of manual digitizing. They can be classified into two groups defined by the positioning of the sample points. The first group consists of methods in which the location of the sample points is not related to the positioning of the contour lines; in other words, the co-ordinates of the sample points are generated and the same sample pattern may be applied to different contour maps.

The most frequently used methods of generating sample points are:

a) regular grids, usually square grids; triangular grids and rectangular grids may be also used,

b) random points - in the domain given by the map area a set of randomly distributed points is generated (Mark, 1975; MacDougall, 1976),

c) mixed patterns like the stratified systematic unaligned pattern (MacDougall,1976).

The second approach is different, the set of sample points being dependent on the contour map to be digitized. It is called a surface specific point pattern (Mark, 1975). By interpretation of surface behaviour places with special significance are selected, for example ridges, valleys, peaks and pits and so on. The sample points are located in these places.

In all manual methods, however, the result depends ultimately on the operator, for he has to estimate the value of the function at a point by inspection and visual interpolation from surrounding contour lines.

1.4 <u>Semi-automatic techniques of digitizing</u>

There are two semi-automatic techniques of digitizing; line following and raster scanning, and it is often emphasised that for contour digitizing
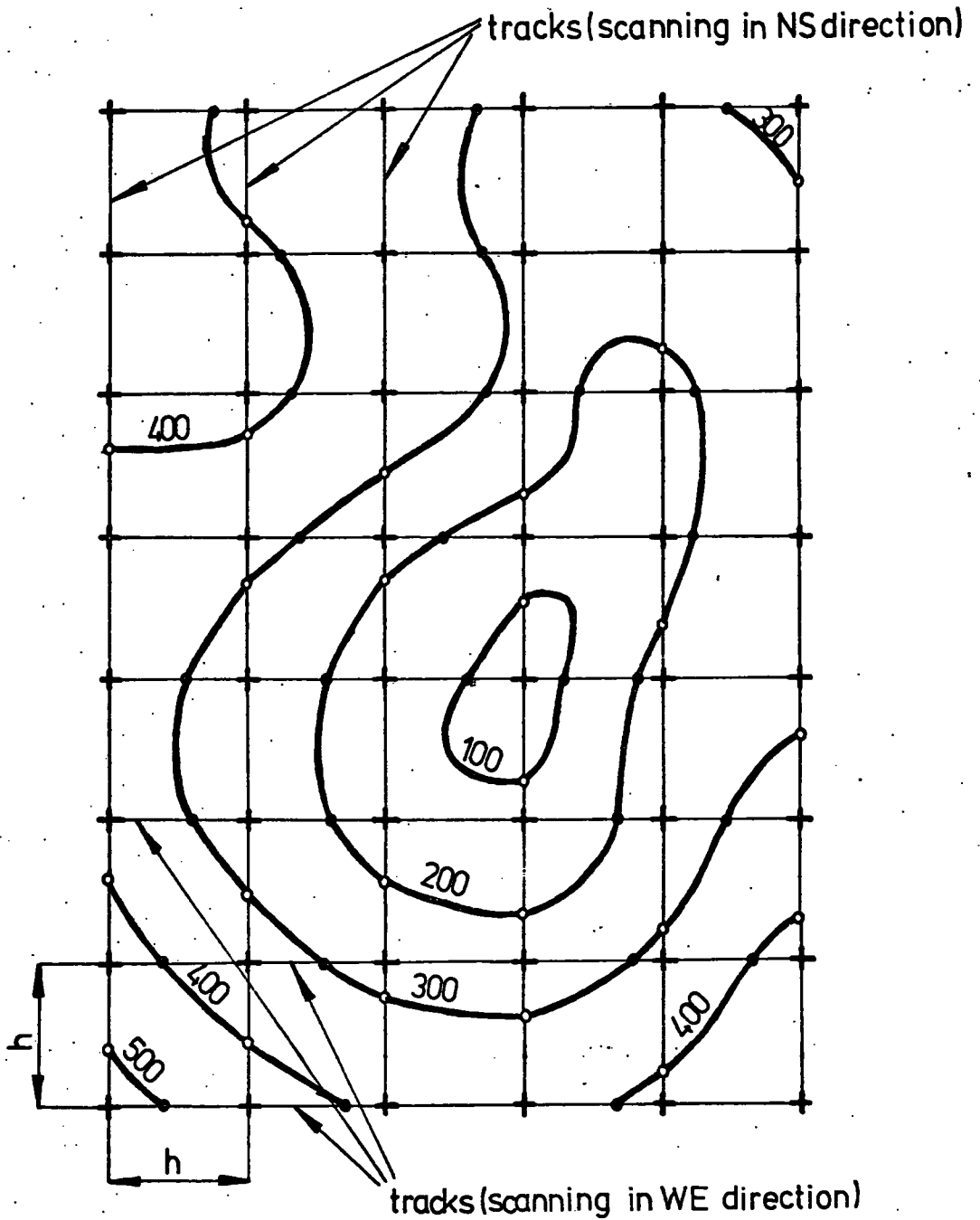
Figure 2. Raster Scanning

closed circles ● indicate points to be registered during raster
digitizing in W-E direction.

open circles o indicate points to be registered during raster
digitizing in N-S direction.

only the line following and raster scanning are practical e.g. Yoeli (1975).

Line following:

When the contour lines on the map to be digitized are followed one by one and points along them are registered, then the map is being digitized by the line following technique. Usually the sample points on the contour lines are taken in predetermined intervals of time or distance and co-ordinates at the points are registered either absolutely or incrementally. To represent the contours efficiently, the sample points are spaced along the contour in inverse proportion to its curvature e.g.Boehm (1967), Rhind (1974). Different algorithms have been developed to generate such a sequence of points; usually it is done using so-called sagittal tolerance, e.g. Hunting Surveys Ltd. (1977), and also MacDougall (1976) - algorithm for digitizing the boundaries of regions.

Raster scanning:

Raster scanning is the process of digitizing along straight lines superimposed on the contour map. The set of lines is called a raster and the lines themselves are called tracks, raster lines or scan lines. The points where the tracks cross contour lines are registered during digitizing (Figure 2). The principle of analysing the picture in raster scanning is similar to the generation of the image on a television screen.

Two parameters have to be determined before digitization by the raster scanning technique; namely the direction and the spacing of the tracks. When using table digitizers the tracks are usually located parallel to one of the co-ordinate axes because of the hardware implementation. When electronic devices are used (Flying Spot Scanners) the direction of the tracks may be different (Figure 3). Although the direction of the tracks is important - to minimise the errors the tracks should be perpendicular to the contour lines - it is practically impossible to determine for a relatively complicated contour map the optimal direction
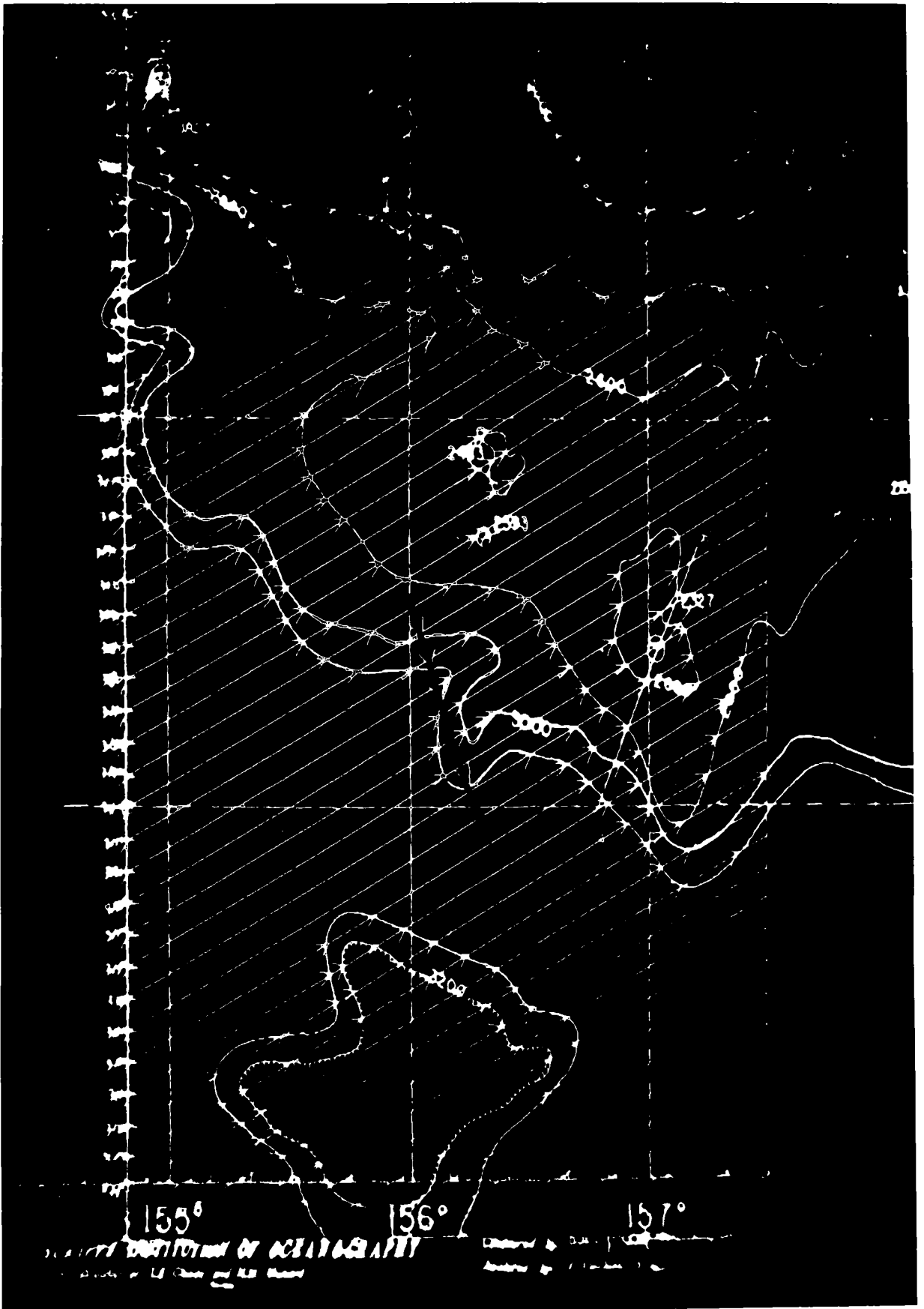
Figure 3. Location of tracks in the SEASCAPE system (from Ryan, 1976).

of scanning.   For choropleth map digitization when the data are to be

stored in chain   mode the right choice of scanning direction may reduce

the required memory space.   Amidon and Akin (1971) discuss relationships

between the direction of scanning and required memory space.

The proper choice of track density is of greater importance.   It

should be determined by the purpose and scale of further analysis.

1.5 A Comparison between the line following technique and the raster

scanning technique

Although line following and raster scanning are both based on the

same principle of digitization of contour maps - they both give us a

sequence of points situated on contour lines - there is a difference in

the amount of information they convey.

The line following technique seems to be a natural method of digitizing

line-drawn pictures such as contour maps.   It has a long history and

its various aspects are discussed in several papers.   Freeman (1974)

gives a review of applications using data derived by this technique,

Evans (1972) describes the creation of inter-contour slope lines, Boyle

(1976) gives some information about the mass digitization of contour

maps using this technique automatically and Rhind (1974) discusses the

choice of location of sample points along a contour line.

The popularity of the line following technique is due to two factors;

it is logical in its construction from the formal point of view, i.e. if

we were not limited by technical restrictions (memory space, hardware

accuracy) it would allow the possibility of a complete reconstruction of

the initial contour lines or, in other words, by reducing the distance

between sample points we can always produce an adequate approximation to

the contour lines.   Secondly, the line following technique seems to

convey all the information from the contour map.   It seems logical that,

if the contour map consists of contour lines, then by digitizing them one

digitizes the map. However, the line following technique does not

convey the spatial information necessary for much analysis of spatial

problems. Consider an arbitrary point on the map sheet not situated

on any contour line. The value of the function at this point can be

estimated by inspecting the surrounding contour lines: this information

is implicit in the contour map. However, a first problem is that the

value at that point cannot be estimated from the sequence of points

obtained by the line following technique. Increasing the number of

sample points along the contour lines does not help. Secondly, nothing

can be said regarding the nature of a function along a straight line

joining subsequent sample points, even though that information is

contained in the contour map.

Before the nature of the spatial information conveyed by the raster

scanning technique can be discussed it has to be proved that this method

is correct from the formal point of view and allows complete restoration

of the initial data. Clearly, if we were not limited by hardware, we

could space the tracks at an arbitrarily small distance. The next step

is constructing the grid and computing the values at the nodes of a grid

square by the quantization method. It may be assumed that the grid size

is small enough to satisfy the postulate that no contour line crosses the

side of the grid square more than once - otherwise the track spacing could

be reduced further.

This situation is shown in Figure 4.

Let R be the distance between the real contour line and the line

obtained by linear approximation from quantized matrix data (this line

on Figure 4 is shown as a dashed line). The maximum value of R is then

$\frac{3\sqrt{2}}{4}h$. As h tends to zero so R tends to zero, or in other words, the lines

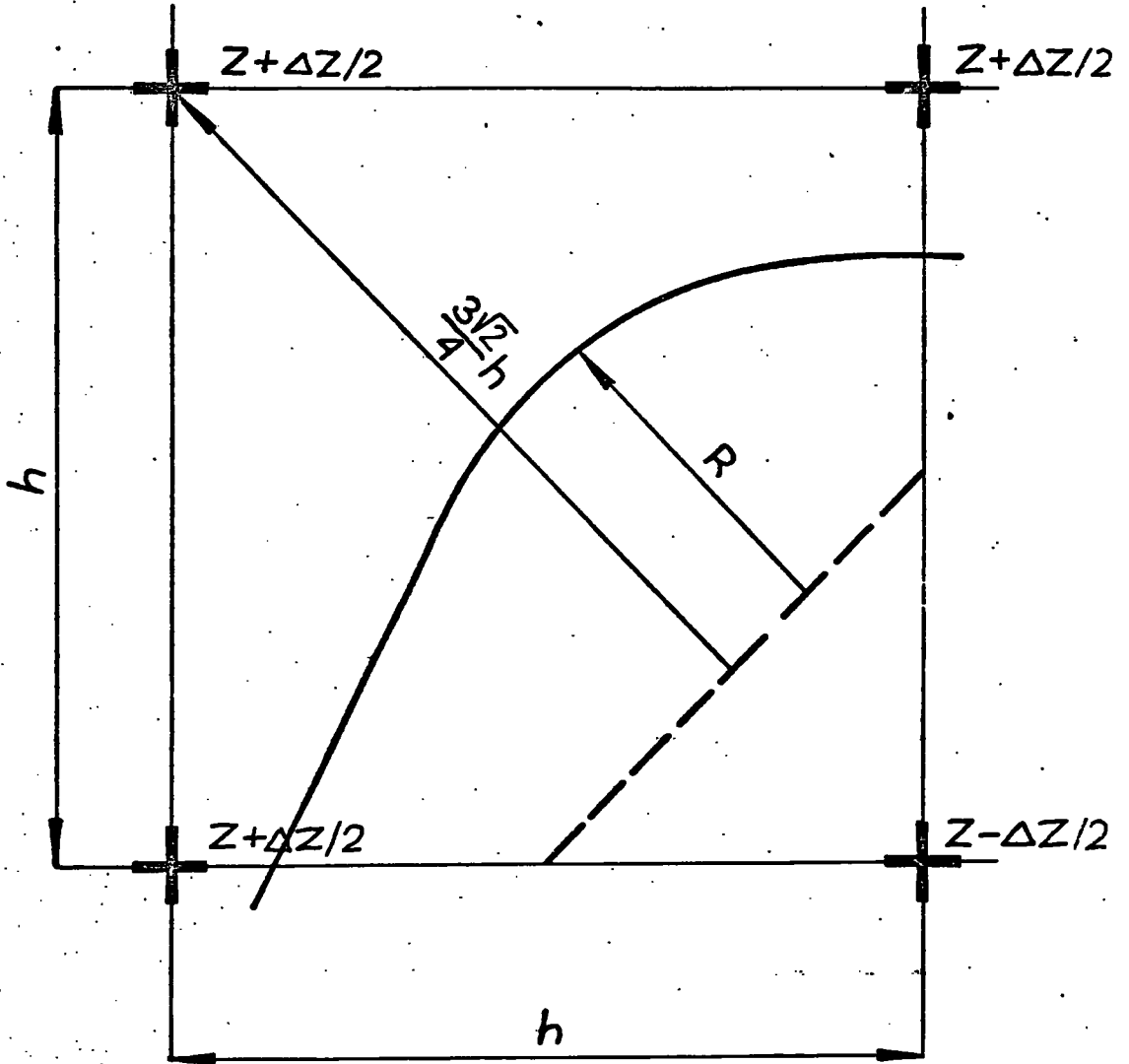derived from quantized matrix data tend to the real contour lines.

Figure 4. Estimation of maximum distance between the contour line on the map (thick line) and the contour line drawn from quantized data using linear interpolation (dashed line).

Having shown that the raster scanning technique is formally correct, the solution to the first problem posed earlier is that, for any arbitrary point, the track spacing can be chosen so that this point will be situated on the track so the value at this point can be estimated by inspecting surrounding sample points on the track. The solution to the second problem is obvious; the values the function takes between two successive points along the track must lie between the values of the function at those two points.

The additional advantage of using the raster scanning technique is that the process of digitizing may be performed in more than one step. If, for example, it turns out that for certain applications the track spacing h is too wide one can digitize again with new tracks also spaced h apart. By shifting their location by h/2 in relation to the previous ones, we obtain a situation equivalent to a digitization with track spacing h/2 and we can continue this process if desired. No work or information from the previous steps is wasted.

CHAPTER II

PROCESSING OF DATA OBTAINED BY RASTER SCANNING

2.1 Creation of the data matrix

The most convenient structure for analysis of a function of two

variables by means of digital computers is a data matrix, i.e. the set

of values of the function given at the nodes of a square grid. The two

main reasons for that are: firstly, the numerical analysis of spatial

data is usually based on this particular structure; and secondly,

programming of spatial problems is made easier because all the commonly

used computer languages allow handling of two-dimensional arrays.

To obtain matrix data when raster scanning has been used for

digitizing, the contour map values are approximated at points equally

spaced along the scanning tracks. If the distance between the nodes

of approximation is the same as the track spacing, we obtain matrix data

equal-spaced in x and y.

Three different approaches were considered and the efficiency of

their interpolation methods was compared.

Linear interpolation: for each successive pair of sample points

(where tracks cross contours) the equation of a straight line is found

and from this equation the values at nodes situated between those points

are calculated. Using linear interpolation one cannot determine values

at the nodes situated between the edge of a map and the first sample

point, nor at those situated between the last sample point and the edge.

Linear interpolation fails when two successive sample points have the

same value; then a constant equal to the value at the sample points is

obtained. With this method the value at any node cannot be greater (less)

than the greatest (smallest) value of contour lines. In other words,

if matrix data obtained by linear interpolation are displayed on a

contour map the reconstituted map will lose the extreme-valued peaks and pits present on the original document. When the values of the function at the nodes are determined by linear interpolation the maximum error is not greater than a contour interval.

Spline interpolation: for each four successive sample points the equation of a spline function (Appendix A) can be found. It is valid for the segment between two middle points. At the nodes situated there, the values are calculated from the equation. For this method the edge effect is even larger; at each end of a track value at nodes situated between the edge and the second sample point cannot be determined. In fact, the more sophisticated the interpolation method, the bigger is the edge effect.

It is difficult to give a quantitative estimate of maximum error for this method, but it may exceed Δz(contour interval). The greatest errors will occur mainly at plateau tops. In fact, if the change in slope is sharp enough, the reconstituted map may well have contour lines which were not on the original map.

Quantization method: the values the function takes between two successive sample points on the track P_k and P_{k+1} with contour values Z_k and Z_{k+1} ($Z_k \neq Z_{k+1}$) are limited by values Z_k, Z_{k+1}. At all nodes situated between P_k and P_{k+1} we estimate values at the function as $(Z_k + Z_{k+1})/2$. If the contour values are the same, i.e. $Z_k = Z_{k+1}$ then we attach to all nodes the value $Z_k + \Delta Z/2$ if the function in the segment (P_{k-2}, P_k) is increasing in value, or $Z_k - \Delta Z/2$ if the function is decreasing in value in this segment. In the same way we can attach values to nodes situated between the edges of a map and the first sample point (they will be $Z_1 + \Delta Z/2$ or $Z_1 - \Delta Z/2$ depending on the behaviour of the function in the segment (P_1, P_2)). A formal description of the algorithm to compute the quantized data is included in Appendix B.

In every case the maximum error generated by this method is not

greater than $\Delta Z/2$. In other words the set of contour lines on the reconstituted map and the initial document will be the same. To aid visualization of the effects of these methods a track with approximated values plotted is shown on Figure 5 by way of example.

To examine the influence of the direction of scanning the test map (Map 1) was scanned twice (from North to South and from West to East). During digitizing only 100 m contours were taken into account. The co-ordinates of sample points were read with an accuracy of $^{+}_{-}$ 0.25 mm.

As a basis for comparison a set of matrix data digitized by hand on a 200 x 200 m grid were used and the values of the function were estimated to thenearest 5 m from 20 m contours. These values were compared with values computed by the three methods mentioned above. Results of the error analysis are presented in Table 1.

As can be seen from the table, only the quantization method permits values to be estimated at all nodes, and it minimizes the maximum error. Although the mean absolute error is smallest for spline interpolation, only about 62% of the nodes have determined values. The results obtained by the quantization method do not depend on the direction of scanning.

The quantization approach, i.e. dividing of the range of values into classes and attaching all members of a class with the same value unique to that class, derives from digital picture processing where the picture must be quantized (Rosenfeld and Kak, 1976). For a contour map the number of quantizing levels needed is normally determined by the contour spacing and cannot be arbitrarily set by the operator unless he can accept much lower resolution than is present in the initial map.

2.2 Spatial analysis based on the quantized matrix data

Although quantization of matrix data seems to yield a direct representation of the contour map and is characterised by the minimization of maximum error, it cannot always be directly used in its original form
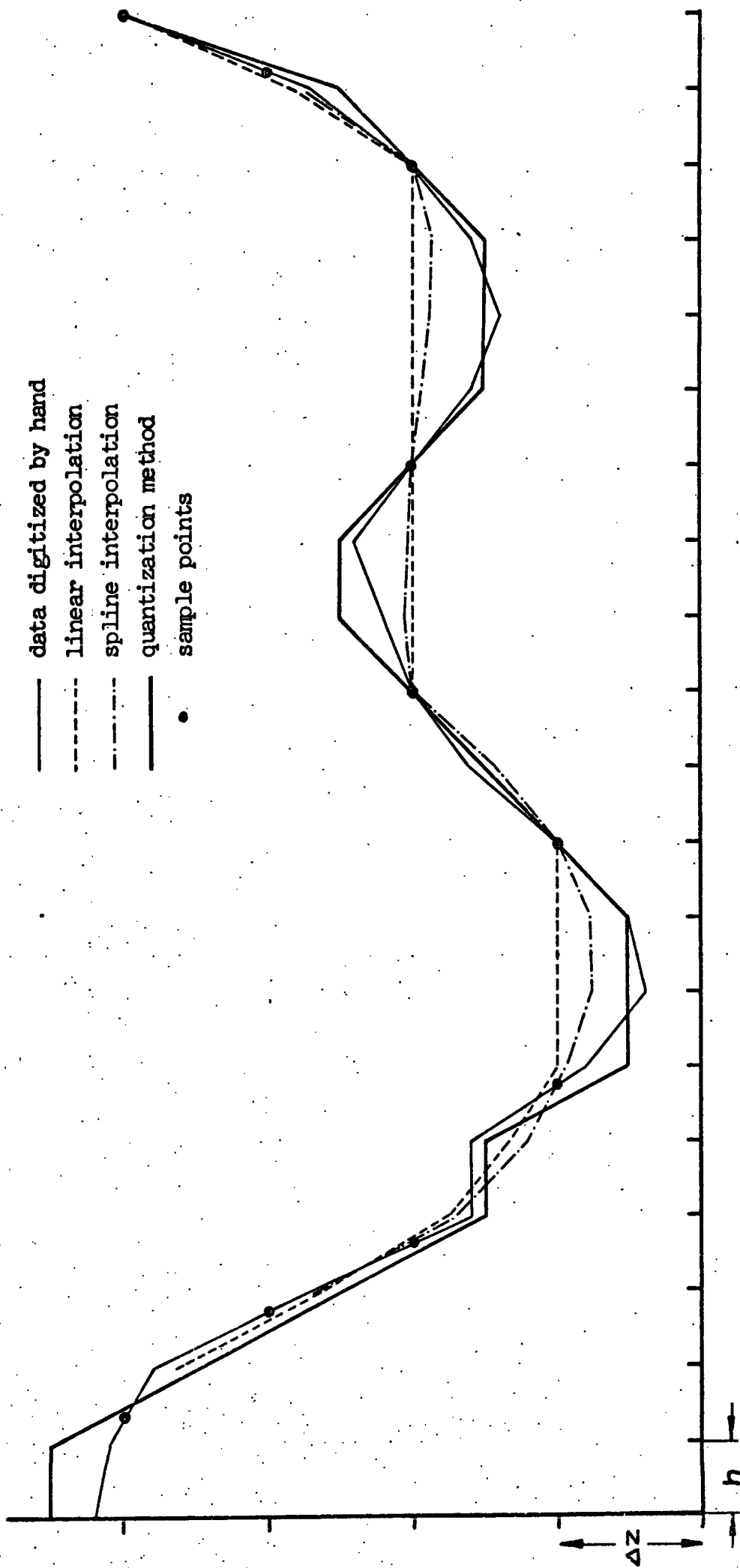
Figure 5. Methods of approximations of values at grid points along track

data digitized by hand
linear interpolation
spline interpolation
quantization method
sample points

Table 1   -19-

	Linear interpolation		Spline interpolation		Quantization method	
Direction of scanning	W – E	N – S	W – E	N – S	W – E	N – S
Total number of points registered during scanning	256	234	256	234	256	234
Total number of mesh points N	441	441	441	441	441	441
Total number of mesh points with undeterminable value N_W	94	76	164	156	0	0
Ratio $\frac{N - N_W}{N}$	0.787	0.828	0.628	0.646	1.0	1.0
Mean error	0.089	0.478	-0.882	1.378	0.760	0.760
Standard deviation of mean error	29.574	29.285	24.556	22.877	25.689	25.689
Mean absolute error	20.417	20.348	16.824	15.859	21.168	21.168
Standard deviation of mean absolute error	21.367	21.039	17.880	16.520	14.540	14.540
Maximum absolute error	100.0	90.0	91.021	79.410	50.0	50.0

to solve spatial problems.   As can be seen from Table 1, the mean error
obtained during quantization tends to zero.   It may be also assumed that
in the domain under question (i.e. given by the map sheet) the errors are
randomly distributed.   When values of the errors are small relative to
values of the function to be processed, the quantized matrix data can be
used directly for global computations.

For example, the results of trend surface analysis should not be
affected by quantization errors.   Maps 4 and 5 both show fourth order
polynomials found by the least squares method.   The former (Map 4) uses
matrix data digitized by hand from 20 m contours, the latter (Map 5) uses
quantized matrix data from digitization by raster scanning of 100 m contours.
For these, as for lower order polynomials, differences in coefficient value
are negligible; moreover, the maps appear visually to be highly similar.

Problems such as computation of the total volume between surfaces
given by their contour maps may be solved easily by using quantized
matrix data.   This sort of computation occurs in the estimation of natural
resources and changes over time.

Computation of local geometrical properties of the surface (such as
gradient, aspect, convexity) directly from a contour map is based on
measurements of distances between contour lines.   The process of
quantization changes these distances, therefore the values of parameters
derived from the quantized data will incorporate some errors.   It is
obviously important, if one wishes to broaden the scope of the quantization
method, to investigate the possibility of estimating these errors knowing
only the variability of the contour spacing and the track spacing;
determining the relationship between the errors and the track spacing
enables choice of an appropriate track spacing before digitizing to
ensure that the parameters are estimated to the required accuracy.

To this end, the distances between the 100 m contour lines were measured

on the tracks (i.e. the distances between sample points) and the results are given in Table 2 (for W-E direction of the tracks) and Table 3 (for N-S direction of the tracks). For each track the number of contours, minimum, maximum and mean distance between contours and the standard deviation of mean distance are given. All distances are given as a multiple of the track spacing h i.e. for example 1.5 denotes that distance is 1.5 h.

In determining the contour spacing, allowance must be made for the edge effect, i.e. it is not known where the next contour line would be if the track were to be extended (off the map sheet). The smallest possible maximum error is obtained by assuming that the first contour off the map is situated at a point whose distance from the edge of the map is equal to the distance between the last contour on the map and the edge. If the maximum or minimum distances have been estimated by this method they are indicated by '*' on Tables 2 and 3. Figure 6 shows the histograms of mean distances between contours on one track.

The variance of an arbitrary function g(x,y) may be estimated by variances of the variables in the following way (see, e.g. Davies, 1961)

$$\text{Var (g)} = \left(\frac{\partial g}{\partial x}\right)^2 \text{Var(x)} + \left(\frac{\partial g}{\partial y}\right)^2 \text{Var(y)} \tag{1}$$

where the values of the derivatives are calculated using mean values of x and y. As an example this formula will be applied to estimate the error obtained during computation of gradient S from the quantized data.

$$S(u,v) = \sqrt{\left(\frac{\Delta z}{u}\right)^2 + \left(\frac{\Delta z}{v}\right)^2} \tag{2}$$

where Δz is a contour interval and u,v are the distances between two successive contours in the x and y directions respectively (Figure 6).

Therefore the variance of gradient Var(S) is given by

$$\text{Var(S)} = \frac{(\Delta z)^2}{(\bar{u} + \bar{v})} \left(\frac{\bar{v}^2 \text{Var(u)}}{\bar{u}^4} + \frac{\bar{u}^2 \text{Var(v)}}{\bar{v}^4} \right) \tag{3}$$

Table 2

Track number	Number of contours	Minimum distance between contours	Maximum distance between contours	Mean distance between contours	Standard deviation of mean distance	$\overline{U - Uq}$	$Var(U - Uq)$
1	13	0.25*	9.37	1.48	2.33	0.0	0.13
2	14	0.25*	5.87	1.35	1.59	0.02	0.14
3	13	0.5	5.37	1.47	1.45	-0.01	0.08
4	5	1.25	7.50	4.08	2.34	0.0	0.03
5	5	0.62	14.25*	4.87	4.78	0.18	0.15
6	7	0.75	13.25*	3.60	4.08	0.02	0.21
7	7	0.62	12.50*	3.81	4.36	0.08	0.12
8	8	0.62	6.00*	2.76	1.74	-0.05	0.02
9	10	0.50	7.50*	2.20	2.03	-0.02	0.05
10	13	0.12	2.50	1.49	0.80	-0.03	0.16
11	12	0.37	4.25	1.66	1.26	-0.01	0.10
12	14	0.125	5.25	1.40	1.36	-0.01	0.10
13	13	0.25	2.87	1.50	0.94	-0.02	0.07
14	15	0.50	4.37	1.35	1.09	0.01	0.03
15	17	0.37	4.50	1.13	0.96	0.0	0.06
16	15	0.5	3.75	1.33	0.94	0.0	0.08
17	19	0.12	3.62	1.05	0.78	0.04	0.18
18	17	0.37	4.50	1.14	0.97	0.0	0.11
19	16	0.37	6.00*	1.35	1.39	0.0	0.03
20	14	0.5	15.25*	1.87	3.72	0.0	0.04
21	9	0.37	19.25*	3.21	5.80	-0.01	0.04

TABLE 3

Track number	Number of contours	Minimum distance between contours	Maximum distance between contours	Mean distance between contours	Standard deviation of mean distance	$\overline{\upsilon - \upsilon q}$	Var $(\upsilon - \upsilon q)$
1	8	0.62	6.12	2.56	2.03	-0.01	0.11
2	7	0.37	7.25*	3.03	2.11	-0.06	0.06
3	9	0.87	5.00*	2.45	1.42	0.0	0.12
4	13	0.37	5.25*	1.75	1.41	-0.01	0.08
5	11	0.62	4.25	1.93	1.22	-0.02	0.08
6	13	0.50	3.75*	1.65	0.98	-0.01	0.20
7	9	0.37	5.50	2.33	1.78	0.01	0.10
8	9	0.50	5.50	2.31	1.89	0.02	0.11
9	9	1.00	4.75	2.16	1.33	0.04	0.15
10	11	0.50	6.00*	2.00	1.73	0.0	0.07
11	14	0.25	4.62	1.34	1.28	-0.03	0.09
12	10	0.75	6.50	1.97	1.92	-0.02	0.03
13	10	0.25	7.37	2.07	2.60	-0.01	0.14
14	13	0.25	5.50*	1.62	1.54	-0.02	0.09
15	10	0.25*	8.50*	2.21	2.50	0.04	0.05
16	10	0.50	5.00*	2.13	1.47	0.0	0.08
17	11	0.37	6.00*	1.95	1.72	0.0	0.06
18	13	0.37	3.75	1.56	0.90	-0.03	0.12
19	13	0.37	4.50	1.50	1.16	0.0	0.03
20	16	0.25	5.12	1.30	1.25	0.01	0.06
21	15	0.25	3.75	1.38	1.11	-0.02	0.03

WE direction

$\bar{X} = 2.10h$

$S_x = 1.13h$

NS direction

$\bar{X} = 1.96h$

$S_x = 0.44h$
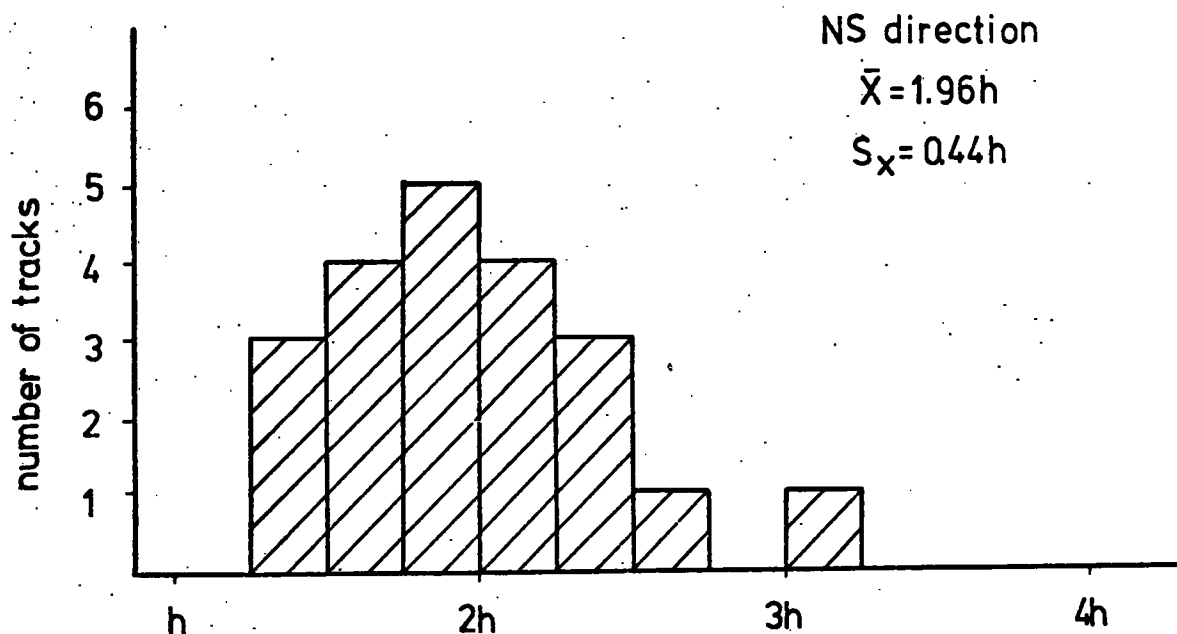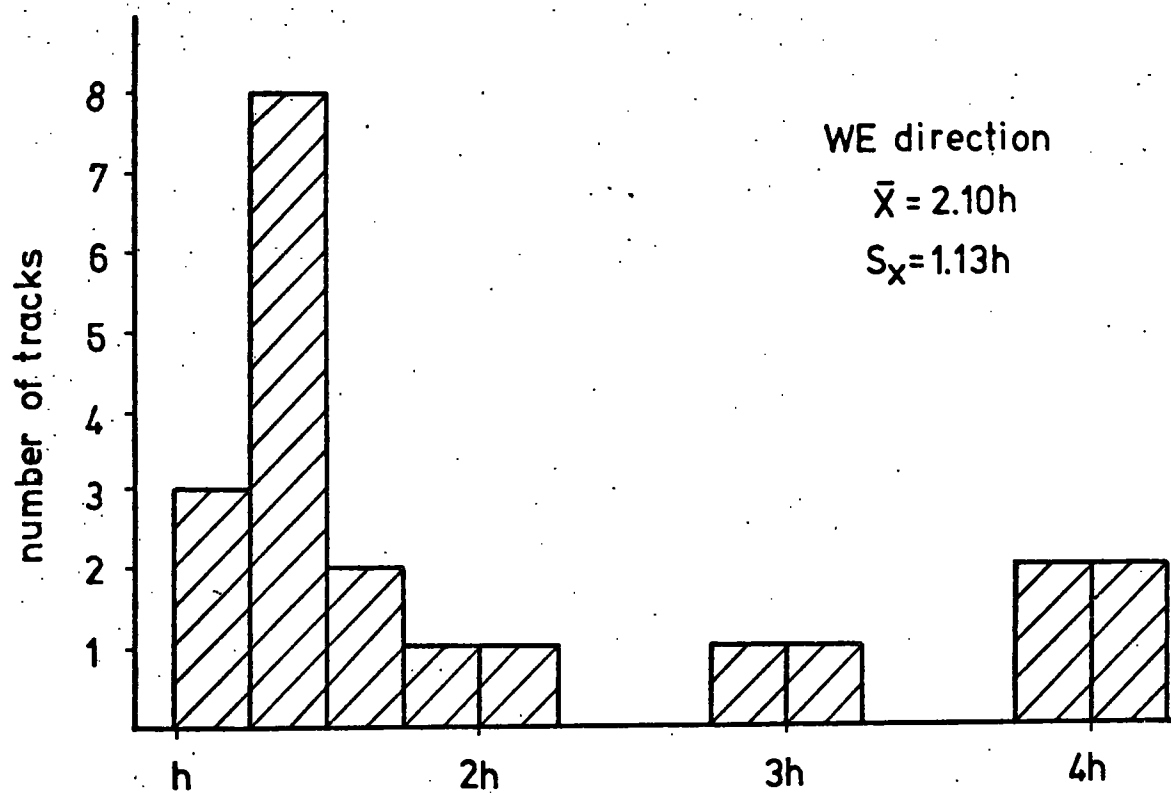
average distance between contours on one track

Figure 6. Histograms of average distance between contours on one track

To estimate the variances $Var(u)$ and $Var(v)$, the distances between

contours obtained by linear interpolation from quantized data (on Figure

7, u_q, v_q) were obtained and subtracted from the distances between the

contours (on Figure 7 u,v).   In the two last columns of Tables 2 and

3 the mean difference and the variance of mean difference for each track

are given for W-E scanning in Table 2 and for N-S scanning in Table 3.

The histograms of the variance of mean difference are given on Figure 8.

For the test data $\bar{u} = 2.10h$, $\bar{v} = 1.96h$, $\Delta z = 100$ m, $Var(u) \approx Var(v) =$

$0.09h^2$, h = 200 m, thus

$$Var(S) = 0.0013$$

A ratio of standard deviation $Var(S)$ to a mean gradient $S(\bar{u},\bar{v})$

$$\frac{\sqrt{Var(S)}}{S(\bar{u},\bar{v})} = 0.105 \tag{4}$$

gives an estimate of average error during computation of gradient from

quantized data for the test area.   For a track spacing of 200 m the error

is about 10%.   For a given pair (u,v) where u,v = 1h, 2h,...,6h the error

may be estimated from Table 4.   For any pair the value of standard

deviation (squared value from (3)), the value of gradient computed from

(2) and the ratio (4) are given.

The error obtained in computing any parameter from quantized data

may be estimated using a similar procedure.

These considerations have a wider aspect, since the variability of

spacing of contours may be estimated from the map before digitizing (by

measurement of distances between contours in randomly chosen places),

by use of Table 4 it is possible to deduce the track spacing necessary

to satisfy the required accuracy of computation of gradient.   For example,

if the error is to be about 5% then the average distance between contours

should be about 4h in both directions or, in other words, in the case of

TABLE 4

V

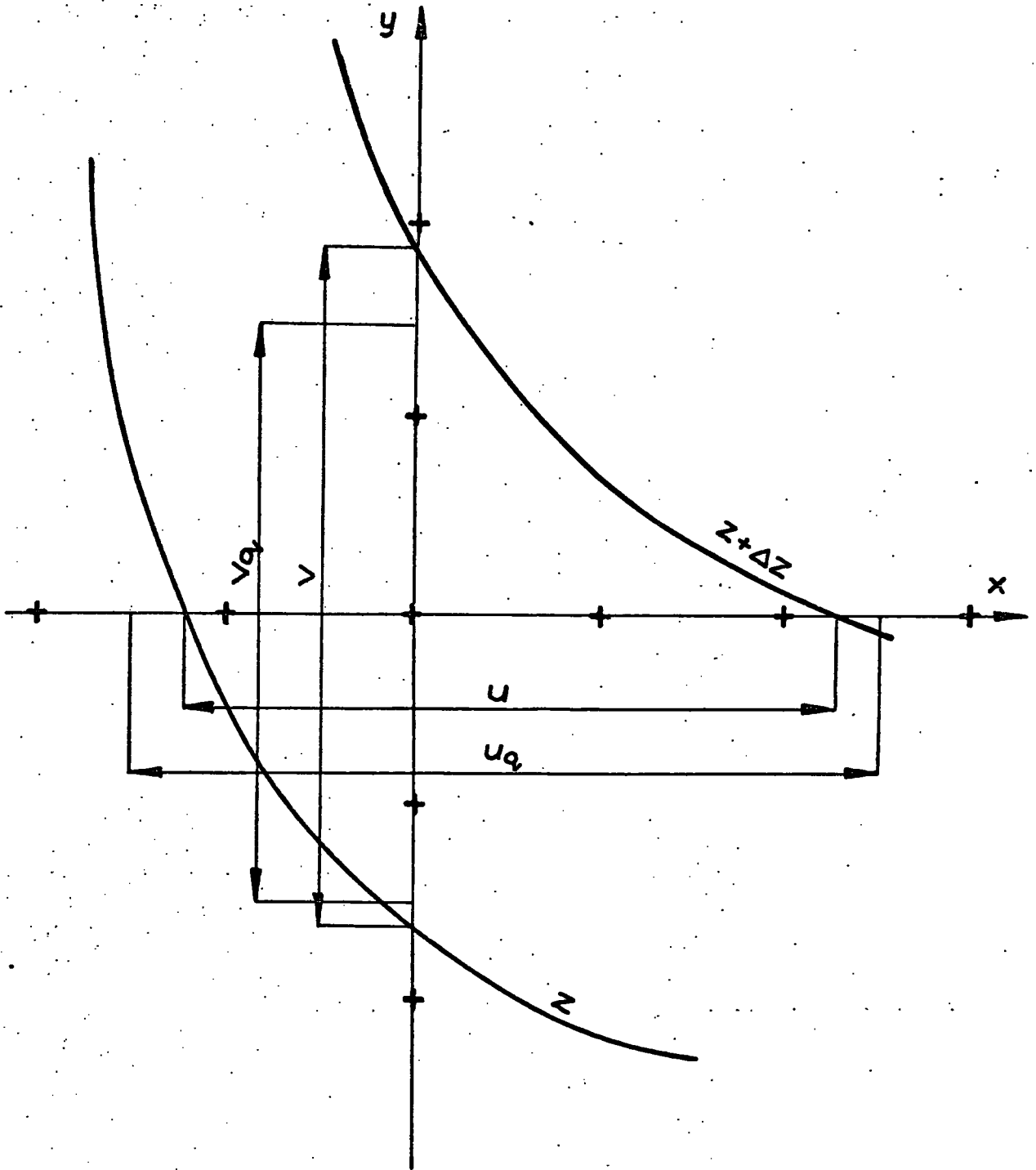|	1h	2h	3h	4h	5h	6h
6h	0.1480 0.5069 0.2919	0.0356 0.2635 0.1351	0.0150 0.1863 0.0806	0.0081 0.1502 0.0542	0.0053 0.1302 0.0409	0.0042 0.1179 0.0354
5h	0.1471 0.5099 0.2885	0.0349 0.2693 0.1296	0.0146 0.1944 0.0752	0.0082 0.1601 0.0514	0.0060 0.1414 0.0424	0.0053 0.1302 0.0409
4h	0.1455 0.5154 0.2824	0.0338 0.2795 0.1209	0.0145 0.2083 0.0695	0.0o94 0.1768 0.0530	0.0082 0.1601 0.0514	0.0081 0.1502 0.0542
3h	0.1424 0.5270 0.2702	0.0325 0.3005 0.1083	0.0167 0.2357 0.0707	0.0145 0.2083 0.0695	0.0146 0.1944 0.0752	0.0150 0.1863 0.0806
2h	0.1352 0.5590 0.2419	0.0375 0.3536 0.1061	0.0325 0.3005 0.1083	0.0338 0.2795 0.1209	0.0349 0.2693 0.1296	0.0356 0.2635 0.1351
1h	0.1500 0.7071 0.2121	0.1352 0.5590 0.2419	0.1424 0.5270 0.2702	0.1455 0.5154 0.2824	0.1471 0.5099 0.2885	0.1480 0.5069 0.2919

U

Figure 7. Figure to show how inter-contour distances are measured using quantized data (u_q, v_q) compared with real distances (u, v)
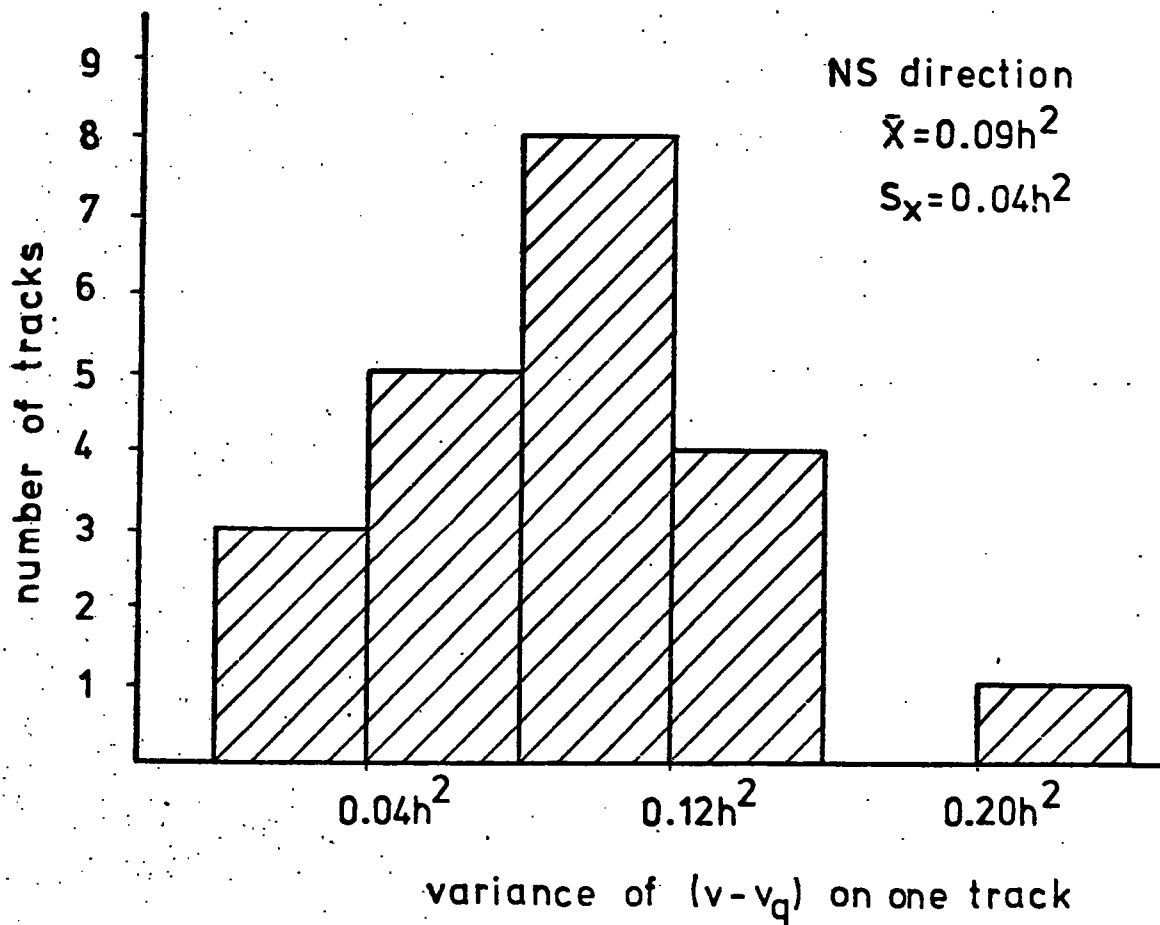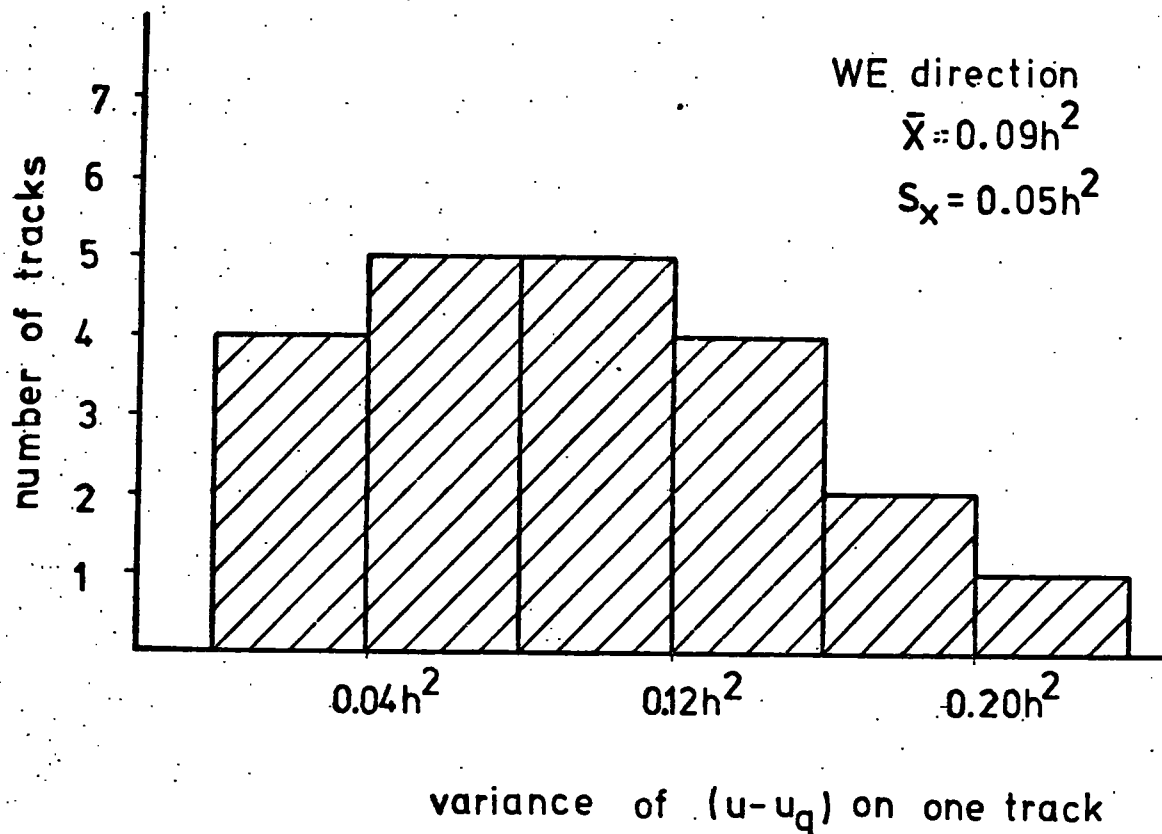
Figure 8. Histograms of the variance of $(u-u_q)$ and $(v-v_q)$

the test data to obtain this accuracy the track spacing should be halved; then the average distance between contours would be 4.20h in W-E direction and 3.92h in N-S direction.

Park and others (1971) give a simplified formula to compute a gradient to avoid taking square roots

$$S(u,v) = \sqrt{u^2 + v^2} \approx \max (u,v) + 0.5 \min (u,v)$$

If this simplification had been used in the test case it would have introduced an additional 6% error.

The geometrical properties of the surface may be calculated in a different way, namely, before computations some generalisation of surface (e.g. smoothing of quantized data to reduce the influence of quantization noise) is done. There are two main strategies available:

(a)   the least squares method is used to determine a polynomial over a certain area. Usually it is a quadratic applied to each 3 x 3 matrix position square. The values of the required parameters may be derived directly from the equation of the polynomial. This method is used by Evans and Young (1977).

(b)   Alternatively, to generalise the surface the local weighted averaging of matrix elements, usually called filtering may be used. A 3 x 3 filter is denoted in the following form:

$$\frac{1}{\sum\limits_{i=n-1}^{n+1} \sum\limits_{j=m-1}^{m+1} W_{i,j}} Z_{nm} \left\{ \begin{array}{ccc} W_{n-1,m+1} & W_{n,m+1} & W_{n+1,m+1} \\ W_{n-1,m} & W_{n,m} & W_{n+1,m} \\ W_{n-1,m-1} & W_{n,m-1} & W_{n+1,m-1} \end{array} \right\}$$

where w_{ij} is a weight of value Z_{ij}. In an abbreviated form only a matrix of weights is given.

Values of the derivatives at the point (n,m) in the formulae for required parameters are usually estimated by a finite difference method

as follows:

$$\frac{\partial f}{\partial x} \approx \frac{G_{n+1,m} - G_{n-1,m}}{2h}$$

$$\frac{\partial f}{\partial y} \approx \frac{G_{n,m+1} - G_{n,m-1}}{2h}$$

$$\frac{\partial^2 f}{\partial x^2} \approx \frac{G_{n+1,m} - 2G_{n,m} + G_{n-1,m}}{h^2}$$

$$\frac{\partial^2 f}{\partial y^2} \approx \frac{G_{n,m+1} - 2G_{n,m} + G_{n,m-1}}{h^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} \approx \frac{G_{n+1,m+1} + G_{n-1,m-1} - G_{n+1,m-1} - G_{n-1,m-1}}{4h^2}$$

where h denotes a grid size and $G_{n,m}$ are the smoothed values of the grid points.

For any given number of grid values taken into account during computations the quadratic is more efficient - that is the generalisation is smaller - than filtering because this sort of filtering is in fact the (weighted) least square approximation by the O-th order polynomial.

Map 6 shows the result of generalisation of matrix data digitized by hand by a quadratic computed for a 3x3 neighbourhood. Map 7 shows the results of the same operation when quantized matrix data were used. As can be seen, some of the quantization noise is still present. The reason for these seems to be the size of the neighbourhood. In fact, for this data 5% of the 3x3 squares do not contain any contour lines, 32% contain only one, 37% two and 26% of squares contain more than two contour lines. In order to calculate parameters dependent upon spatial relationships for each square, at least two contour lines should cross the x-axis and two contour lines should cross the y-axis (the origin of the local co-ordinate system here is in the middle of squares). The correct size of the neighbourhood when data are obtained by the raster scanning technique may be estimated from the average spacing of the contour lines in the directions

of both axes. For the test data, the average spacing of the contour
lines in x-direction (W-E) is 2.10h and 1.96 h in y-direction (N-S).
It can be expected that the segment with length 4h will be crossed
twice by the contour lines. But to obtain the same level of smoothing
with an enlarged neighbourhood, the order of the polynomial should be
increased.

Map 8 shows the result of generalisation of matrix data digitized
by hand by a cubic polynomial computed for the neighbourhood pattern below:

```
          x
        x x x
    x  x  x  x  x
        x x x
          x
```

Map 9 shows the results of the same operation for quantized matrix data.
A comparison with the original contour map indicates that now the
generalisation of the contour lines in both cases is equally good.

Maps 10, 11, 12 show the generalisation of quantized matrix data
by three different local filters. For the test data the filter $\begin{pmatrix} 010 \\ 141 \\ 010 \end{pmatrix}$
(Map 11) gives the most satisfactory results, in the two remaining cases
the smoothing causes a loss of the extreme-valued contour lines.

2.3 Display of data when digitizing by raster scanning

One of the functions of a spatial data base is the provision of data
to create a display in contour map form. If the data in this bank are
derived from the digitization of contour maps, it is important to know
the difference between the initial contour map and the map constructed
by the computer from stored values. Very often the statement that both
maps are similar is not enough and it has to be determined at which places
(or points) both maps are identical. For example, if the contour map
constructed by computer is to be used in the location of a new borehole
a set of control points, i.e. the points at which the contour lines on

both maps are the same has to be known.   This set of control points

consists of the sample points taken during digitization.

The display of data in contour map form when they were obtained by

the line following technique and have been stored in ordered form is a

relatively easy task.   When the raster scanning technique is applied

to digitize the contour map the matter is more complicated.   It has already

been stated that if quantized matrix data are used to produce a contour

map the quantization causes deformation of the contour lines (Map 3) and

it can only be said that the contour lines on the reconstructed map are

situated not more than $\frac{3\sqrt{2}}{4}$ h (where h denotes scan-line spacing) from the

original contour lines.   To visualise the quantization noise more

effectively, a test picture (Map 13) was scanned (with the same track

spacing as was used for scanning Map 1) and quantized, then reconstructed

by the Calcomp GPCP (General Purpose Contouring Program) considering the

data as a regular grid.   The result is shown on Map 14.   The effect of

quantization noise depends on the track spacing (the grid size), the angle

between the data lines and the direction of scanning.

Although the quantization noise can be reduced by generalisation of

the surface, at the same time the estimation of the maximum error, i.e.

the maximum distance between the contour lines on original and reconstructed

map$\left(\frac{3\sqrt{2}}{4}h \right)$ is lost.

The next question which should be answered is that, given a set of

points obtained by digitization, and given contouring programs that can

operate on irregularly spaced data, what sort of contour map will these

programs produce?   Map 15 was produced by using GPCP on data obtained by

raster scanning and treated as a set of irregularly spaced points.

Comparison with the original map (Map 1) shows that the differences (in

location, shape and number of the contour lines) are large.

To overcome the problem of how to display data in contour map form when the initial document was digitized by raster scanning a special algorithm was written. A formal description of this algorithm is included in Appendix C. The algorithm permits production of a contour map directly from the sequences of sample points along the tracks and guarantees that contour lines on the reconstructed map pass through the sample points. Map 16 shows the result of the program for the test data.

2.4 Storage of data when digitizing by raster scanning

Although matrix data (i.e. data taken over a square grid) are the most convenient form of representing a surface in terms both of computation - this is because numerical analysis of two-variable functions is based on that structure of data - and of programming - that is because commonly used computer languages allow the handling of two-subscripted arrays - the problem of how to store the spatial data is still open. If, for example, the cost of storage of a large matrix is higher than the cost of generating it, plus the cost of the storage of the original data, it would be unwise to store data in the matrix form.

In the case of spatial data obtained by raster scanning the best method of storage seems to be the raw data, i.e. the sequences of sample points (where the tracks cross the contour lines). First of all the data carry very accurate information about the original document whereas the matrix data are 'second hand' and it is impossible to extract from them information about exact positions of the contour lines. When it is desired to increase the accuracy of the data base by further scanning, creating a new denser grid, the initial data are necessary.

The matrix form of storage is convenient for displaying spatial data in contour map form because that structure of data is used by most of the contouring programs. Regarding the display of spatial data when the contour

map has been digitized by raster scanning, storage in matrix form has no advantage since it is possible to display the data in contour map form directly from the raw sample points (see Appendix C).

The memory space required for storing the data from raster scanning may be reduced in the following ways::

(a)   it is not necessary to store the co-ordinates of the tracks; knowing the co-ordinate of the first track and the track spacing the co-ordinates of all the tracks can be determined,

(b)   the set of contour line values can be transformed into a set of integer numbers. Let us say that Z_o is the lowest contour line on the map sheet to be digitized and ΔZ is the contour interval then $Z_1 = Z_0 + \Delta Z$, $Z_2 = Z_0 + 2\Delta Z, \ldots)$ $Z_k = Z_o + k \Delta Z, \ldots$ are the values of contours on the map.

The transformation

$$T(Z_k) = \frac{Z_k - Z_o}{\Delta Z}$$

gives the set of the integer numbers $o, 1 \ldots, K$ where Z_k is a maximum value of the contour lines on the map sheet. Now, instead of using the whole machine word to store the real value of the contour line, one can use a part of it to store an integer value. Let us say that the total number of the different values of the contour lines on the map sheet is not greater than 256 which is a reasonable assumption for a great many maps. Any integer in the range 0,255 can be represented with only 8 bits. Most computer systems permit operations on parts of the machine word (so-called bytes) consisting of 6 or 8 bits.

A similar principle of storage reduction is used by Pfaltz (1975) in bit plane representation of surfaces.

CHAPTER III

TECHNICAL IMPLEMENTATION OF DIGITIZING METHODS

3.1 Digitizing errors

There are two main problems causing errors during digitizing:

(a)   the thickness of the contour lines as a source of errors

Up to now it has been assumed that the contour lines are

dimensionless and hence that the raster scanning technique provides exactly

one point from each crossing of a contour line by a track. Usually it can

be assumed that the thickness of the tracks is negligible but very often this

assumption cannot be made about the thickness of the contour lines; in this

case a segment rather than a point is being sampled. The problem then

arises as to which point from that segment should be chosen as a sample

point. Although the middle one tends to be chosen it seems unlikely that

the 'real' contour lines go exactly through the middle of the contour lines

on the map. If it is assumed that the contour lines on the map cover the

'real' contours, taking the middle point of the segment will cause the

smallest error. The amount of error caused by the thickness of the

contour lines depends on local gradients and the thickness of the contour

line itself. If, for example, the thickness of the 100 m contour line

corresponds to a height difference of 1 m then the error is insignificant.

On the other hand, it may be relatively big. On the test map (Map 1)

the thickness of the 100 m contours corresponds in certain parts of the

map to a 20 m height difference. Assuming that the 'real' contour lines

are underneath the contours on the map and that one samples in the middle

of the contour line, is still possible to have errors of plus or minus half

the height difference of the area covered by the contour lines.

The thickness of contour lines may be a source of error not only

when the raster scanning technique is applied, but also when the line

following technique is used. The following problems, however, only arise

during raster scanning.

(b)   the_tangency_problem

So far it has been assumed that the scanning tracks cross the contour

lines.   However, sometimes situations arise when the track and the contour

line are collinear for a certain distance (as was mentioned previously, the

thickness of the track is relatively small compared with the thickness of

the contour line): for example, the situations shown on Figure 9b or when

the track touches a contour line but does not cross it (Figure 9c). In

fact, both cases may occur simultaneously as on Figure 9a.

The case when the track follows the contour line is referred to by

Mor and Lamdon (1972) as the tangency problem. In their paper, they describe a

two-level test to detect whether the track passes from one intercontour region

to another (crossing type) or remains within the same intercontour region

(tangent type). These tests may be used when the raster scanning technique

is computer controlled. When a digitizing table is used the type can be

deduced directly from the map. A general rule is that, with the crossing

type an odd number of sample points has to be registered along the track,

whereas with the tangent type there are an even number of sample points.

(c)   mismatching_the_tracks

Before digitizing by the raster scanning technique, the spacing of the

tracks must be decided.   Usually equal spacing of the tracks is assumed.

However, the physical location of the tracks may be different from that

planned and thus sample points may be incorrectly located (Figure 10a).

The amount of deviation depends firstly on the error in location of the

track, and secondly, on the angle between the contour line and the track

(Figure 10b).   Usually errors caused by mismatching the tracks are not large

hardware accuracy enables fitting of the tracks to an accuracy of a
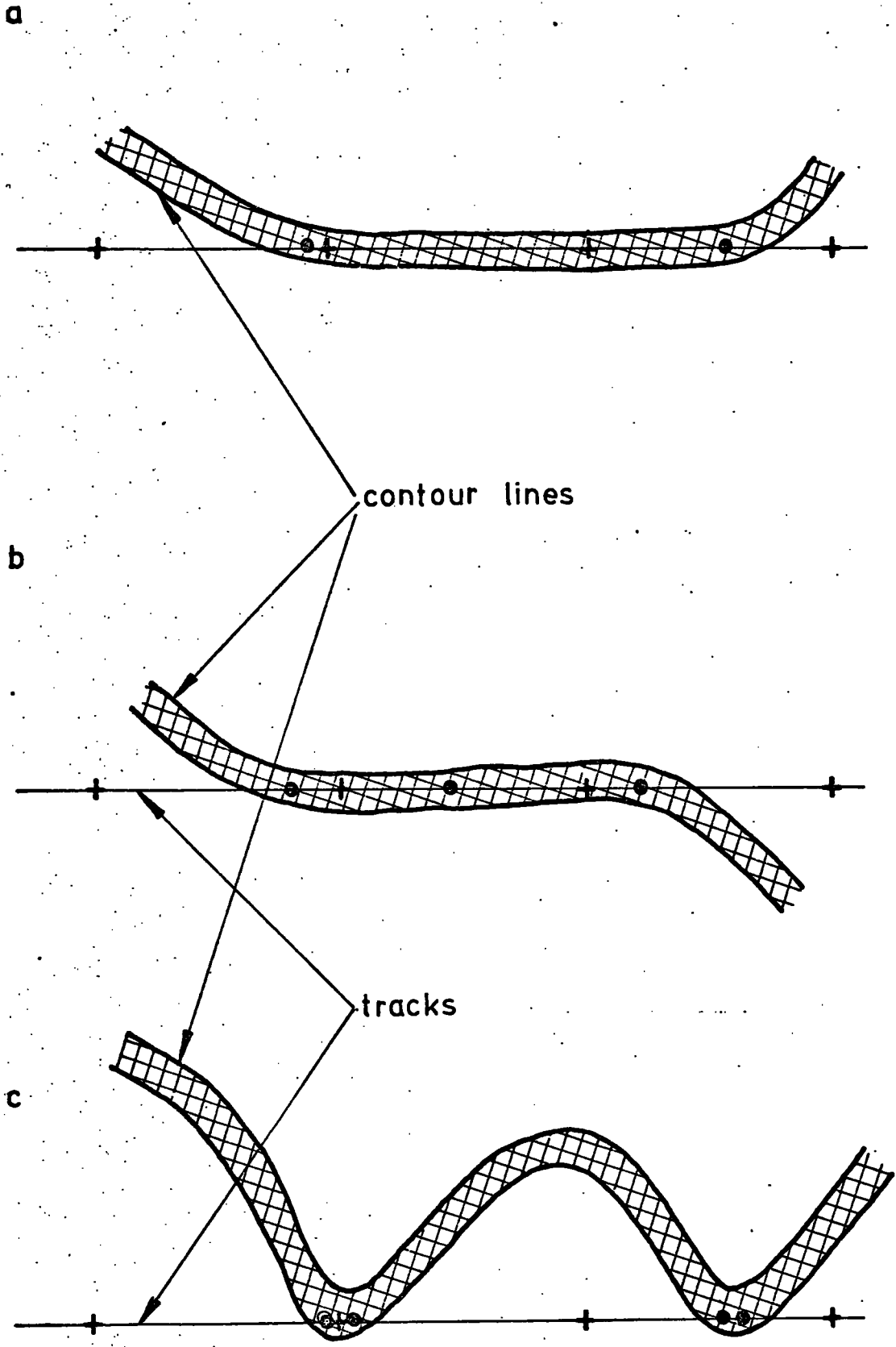
few hundredths of a millimetre.

a

b

contour lines

tracks

c

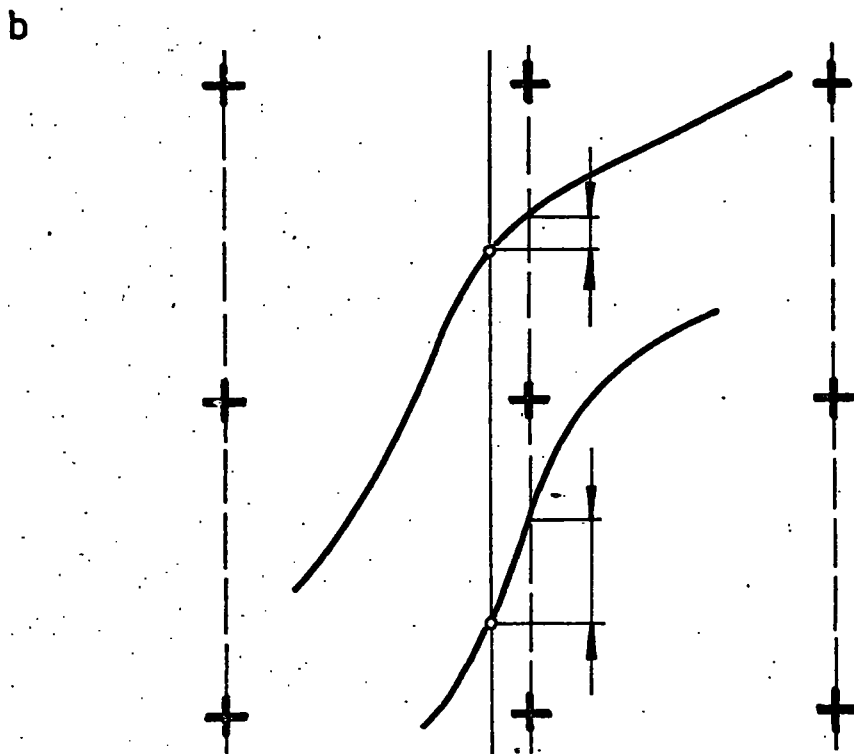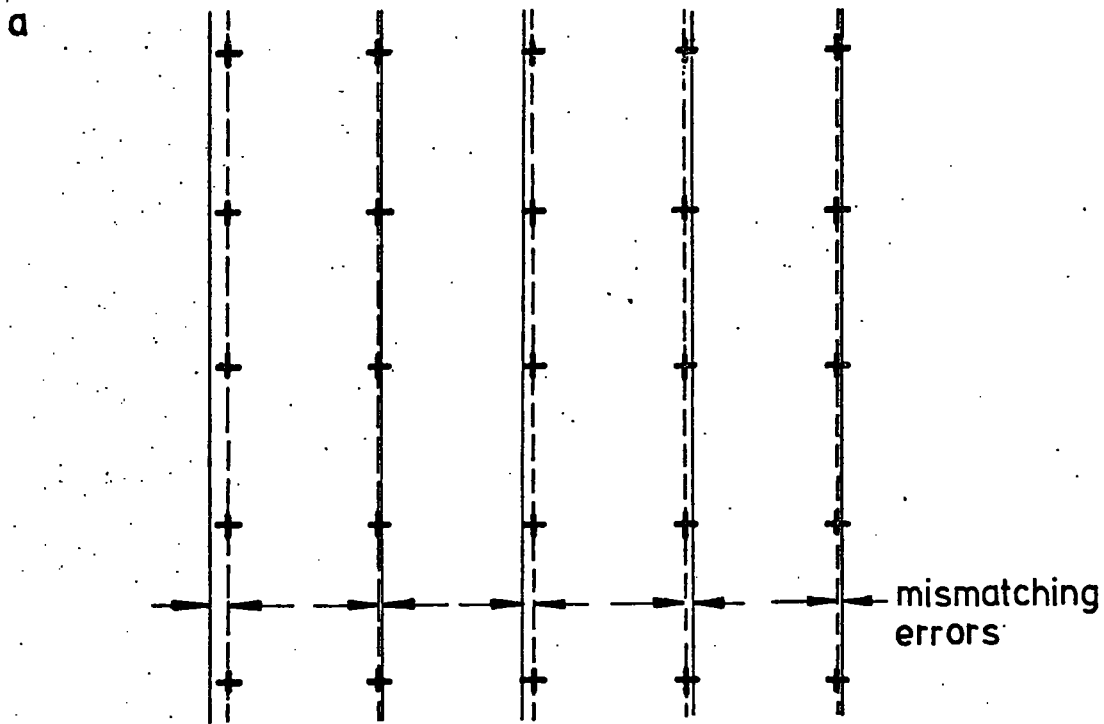Figure 9. The tangency problem

a

mismatching
errors

b

Figure 10. Mismatching the tracks

This kind of error may also arise when the contour map is scanned again, i.e. when new tracks are located to extend the digitization and the initial alignment of the scanner differs from that in the second scanning.

3.2 Hardware for digitizing

Although the term 'automatic digitizing system' is commonly used to describe the various systems used for digitizing of contour maps, in fact there is no system which could be used entirely automatically. The problem is that, during digitizing, it is necessary not only to locate certain lines, i.e. determine their co-ordinates, but also to determine the values attached to these lines. This last step cannot be performed without a human operator. First, not all contour lines on the map have an explicit value, as usually only selected contour lines (every fourth, fifth or tenth) are labelled. Secondly, even if the line has a printed value, the instrumentation to read it is very complex and so far has not been entirely successful in cartographic implementation. Lastly, even the location of lines sometimes requires an operator decision since there may be dashed lines on the map sheet.

The hardware for digitizing can be broken down into two main groups:

(a)   table digitizers which enable automatic read out of the

co-ordinates of points on the map to be digitized. Usually

the sequence of points is registered on magnetic or paper

tape. They may be classified as follows:

1. pencil follower digitizers

2. solid state tables

3. gantry tables

(b)   automatic systems which enable automatic location of lines

on the document to be digitized. Very often the document

Figure 11. Calspan's SEASCAPE digitizing facility (from Ryan, 1976)

has been photocopied onto transparent film so that lines may
be detected by variations in the transmitted light.   Depend-
ing on the technique involved, automatic systems may be
classified thus:

1. scanners - flying spot scanners (Figure 11)

(Ryan, 1976; Davey et al, 1974)

drum scanners (McDonough, 1976)

2. automatic line followers (Laserscan, 1977)

Kelk (1974) gave a description of different types of table digitizers.
An excellent review of all automatic systems available in Europe and North
America is given by Davey et al (1977).

The problem of attaching the values to the lines might be overcome
by using coloured lines.   Mor and Lamdon (1972) described the logic of such
a method based on two-coloured lines, where one colour is used for even-
valued contour lines and the other for odd-valued ones.   Knowing the value
of the first line on the track and the initial trend, the values of all the
lines on the track can be determined by noting their colours.   The
disadvantage of this method is that the map must be preprocessed by hand
before digitizing.   However, according to Davey et al (1977) there is no
system for cartographic digitizing making use of coloured lines.

Among table digitizers, the most convenient type for raster scanning
seems to be the gantry table, i.e. with the gantry above the table top and
the cursor attached to it, especially when the position of the gantry (i.e.
location of the tracks) may be automatically changed.

CONCLUSIONS

The aim of this thesis was to determine and compare the efficiency of techniques for the digital encoding of contour maps. Various aspects were considered: problems in the creation of matrix data, e.g. the most convenient structure for computations, the facility and accuracy of reconstructing digitized data in contour map form, storage requirements and error estimations. In particular, the raster scanning technique was stressed: this technique can be a good alternative to manual methods which are very time consuming and always subjective. Raster scanning can also be utilised without any special hardware.

The quantization method seems to be very useful for producing matrix data. Compared with linear and spline interpolation it minimizes the maximum error and permits the estimation of values at all nodes.

The specially written algorithm enables the reconstruction of digitized data obtained by raster scanning into contour map form, giving a set of control points at which the contour lines on the reconstructed map and the initial document are the same.

Although in the creation of quantized matrix data it is sufficient to scan a contour map once it seems worthwhile to stress that two-fold scanning in orthogonal directions creates new possibilities:

1. having the set of sample points on the tracks from two perpendicular directions one can estimate all parameters at the nodes with the same accuracy as is obtained when estimating directly from a map;

2. because the values encoded by quantization methods do not depend on the direction of scanning, the two data matrices obtained by orthogonal scanning may be compared with one another for a check on possible mistakes, be they on the map or due to the operator;

3. a comparison of the distributions of density and spacing of contours
    crossing the tracks in both directions may improve understanding of
    surface properties.

Although this thesis concentrates on digitizing of contour maps, there
is an alternative way to obtain a digital terrain model, namely the
orthophotomap method.   The state of the art in orthophotmap techniques is
presented by Petrie (1977).   However, because many map features, such as
public land surveys and political boundaries, do not appear in stereophotographs,
in addition to that technique, thousands of existing maps must be digitized
either manually or semi-automatically (Southard, 1978).   For example, in
Sweden the high density terrain elevation data base is created from three
different sources (Ottoson, 1978):   photogrammetric measurements of aerial
photos; digitizing of existing contour lines; and digitizing of existing
profile plates.   Also in West Germany data for cartographic data bases are
partly captured by digitizing of contour maps (Uhrig, 1978).

The process of digitizing of contour maps cannot be performed entirely
automatically.   Although operator influence during semi-automatic digitizing
is significantly less than in manual digitizing, one should remember that
a contour map is a product of human subjectivity and fallibility.

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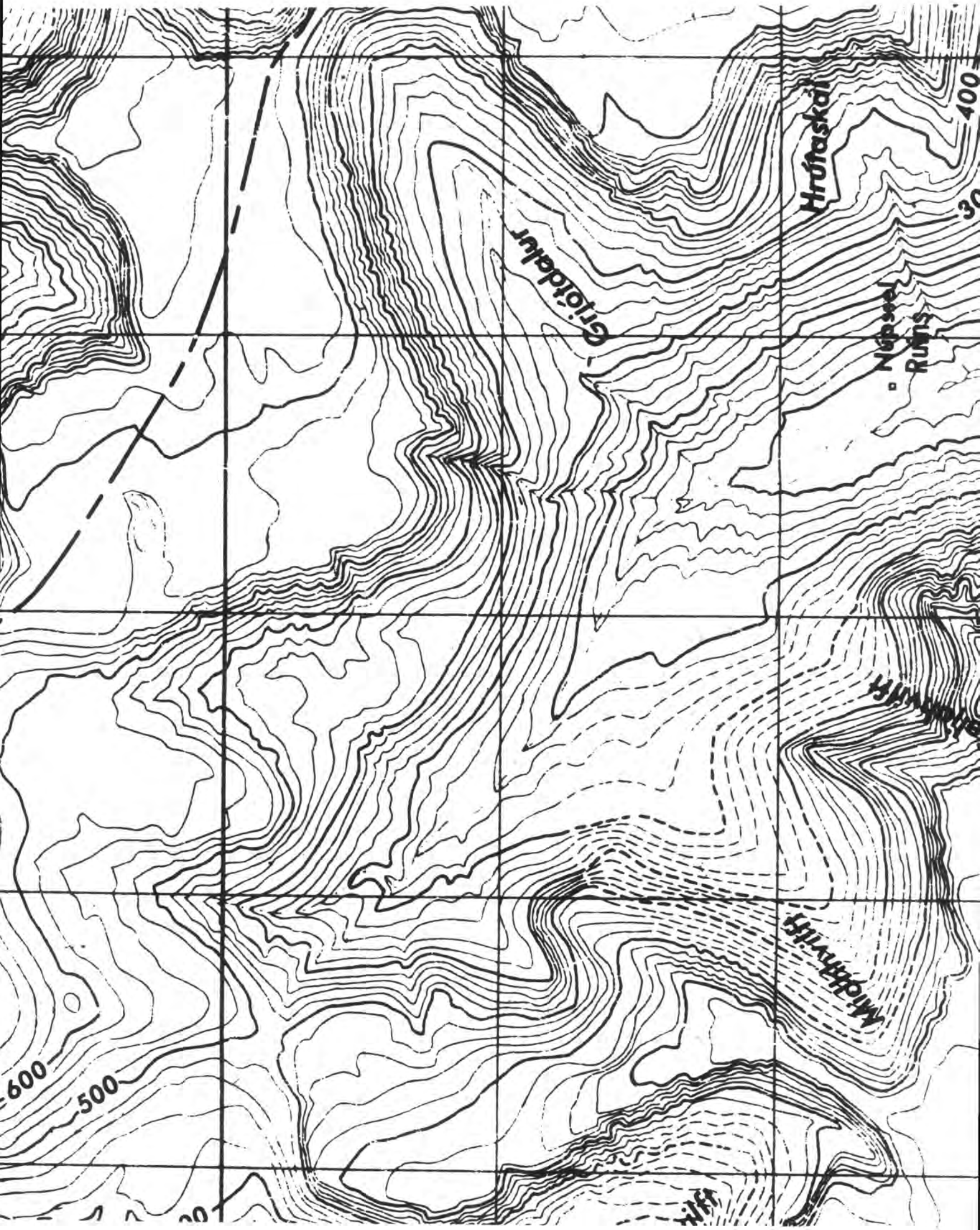MacDougall, E.B. (1976) Computer programming for spatial problems, Arnold, London, 106-115.

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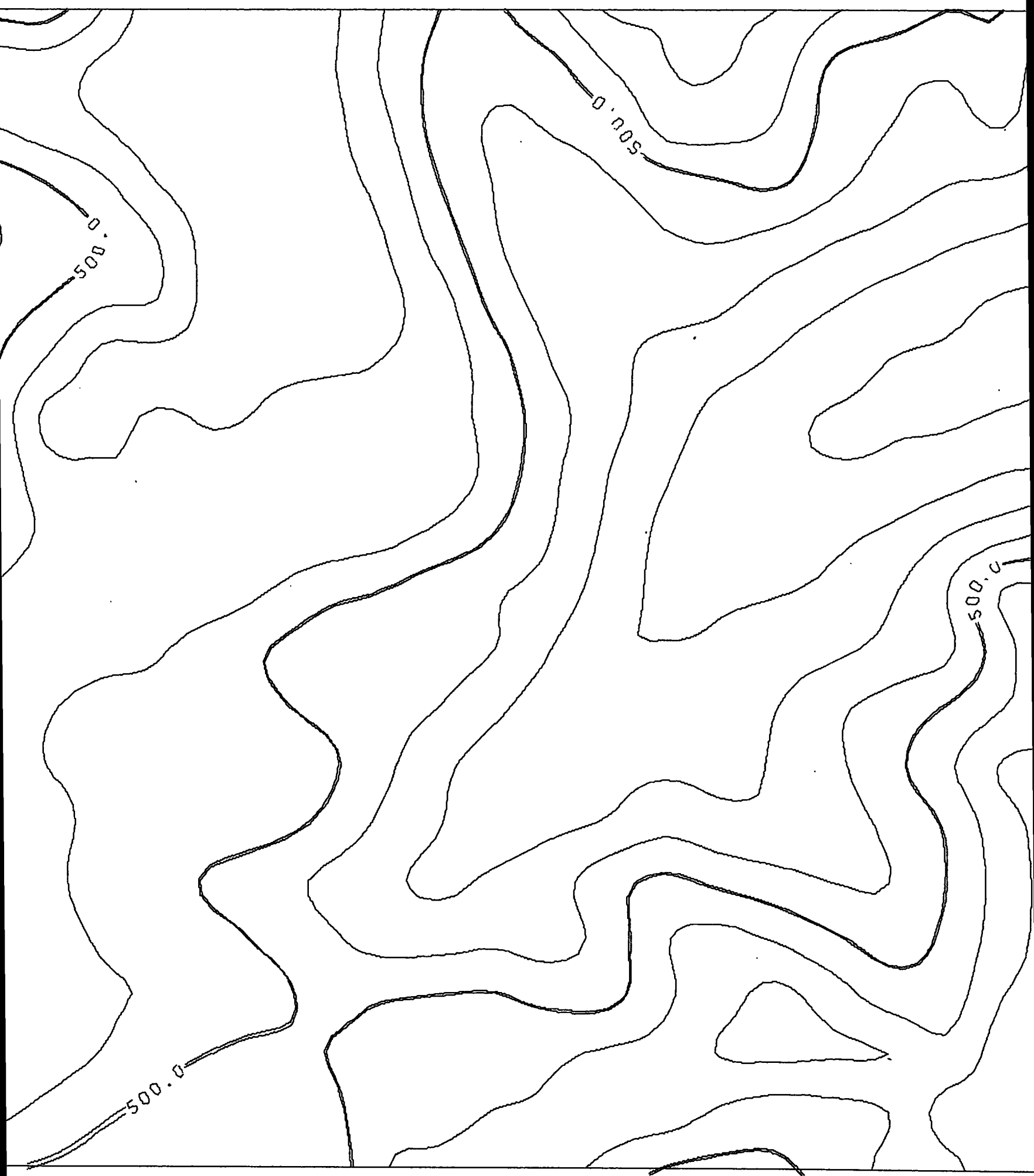
Description of maps

Contour lines on the maps 4 -12, 14 were drawn using Calcomp's GPCP (General Purpose Contouring Program) from matrix data and copied on the test map (Map 1).

Map 1.   Test map – Nupur area (Iceland) enlarged from 1:50 000 scale, contour interval 20 m.

Map 2.   A map obtained by using GPCP from data digitized by hand at 200 x 200 m. grid.

Map 3.   A map obtained by using GPCP from quantized matrix data.

Map 4.   A polynomial of 4th order computed on data digitized by hand in 200 x 200 m grid.

Map 5.   A polynomial of 4th order computed on quantized matrix data.

Map 6.   A map of the surface smoothed by a quadratic (input data were the same as for Map 2.)

Map 7.   A map of the surface smoothed by a quadratic (input data were the same as for Map 3).

Map 8.   A map of the surface smoothed by a cubic (input data were the same as for Map 2).

Map 9.   A map of the surface smoothed by a cubic (input data were the same as for Map 3).

Map 10. A map of the surface smoothed by the filter

$$\begin{array}{ccc} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{array} \quad \text{(data as in Map 3)}$$

Map 11. A map of the surface smoothed by the filter

$$\begin{array}{ccc} 0 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 0 \end{array} \quad \text{(data as in Map 3)}$$

Map 12. A map of the surface smoothed by the filter

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 1 & 2 & 8 & 2 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array} \quad \text{(data as in Map 3)}$$

Map 13. Test data to visualize quantization noise.

Griðdalur

Hrútaská

Nupsel
Rusir

Miðhvilft

Miðhvilft

400

30

600
500

29

MAP 2

500.0

500.0

500.0

500.0

500.0

500.0

MAP 3

MAP 4

MAP 5

Gilddelur

Hrútaskál

Nupsel
Ruins

400

500

600
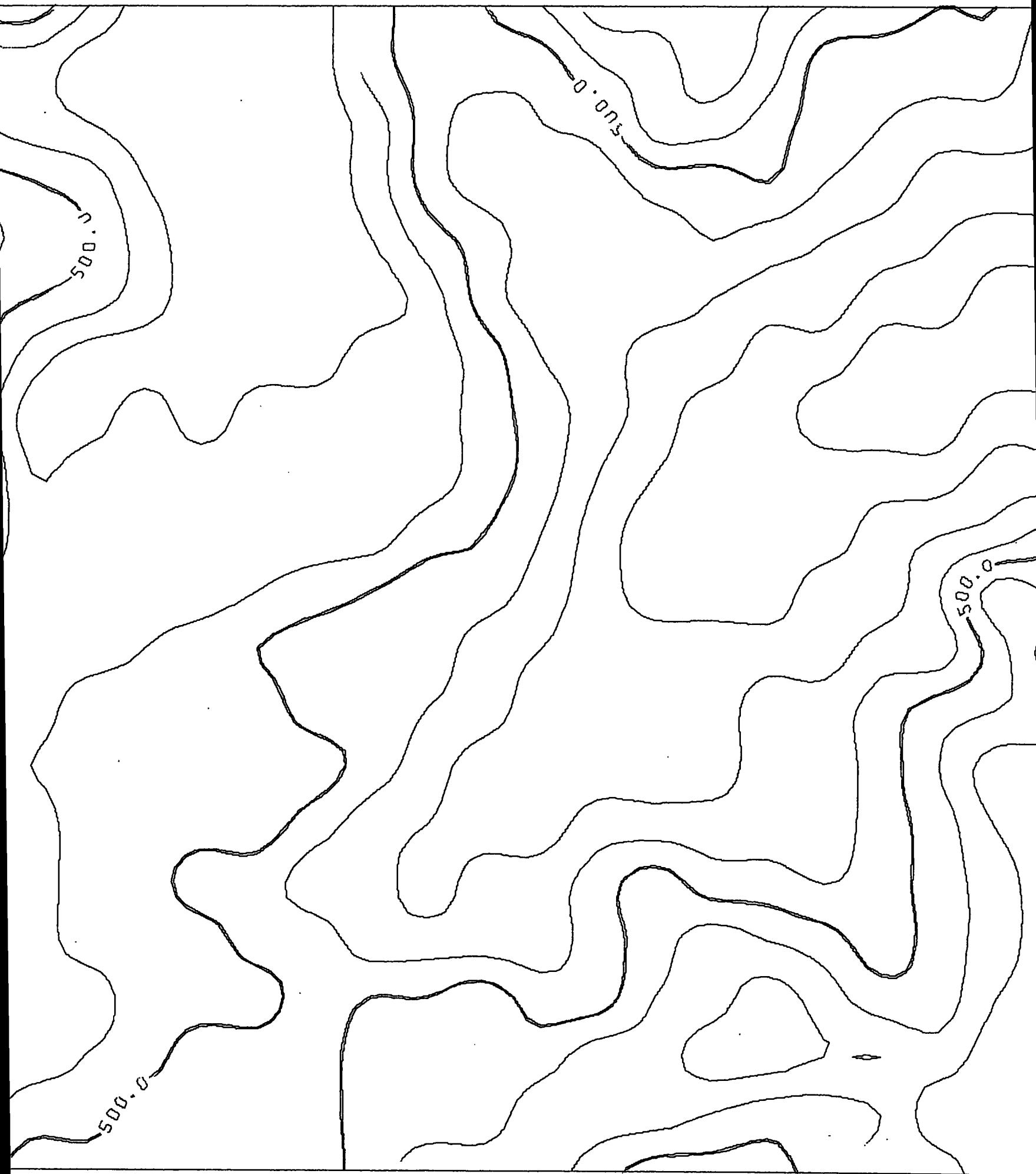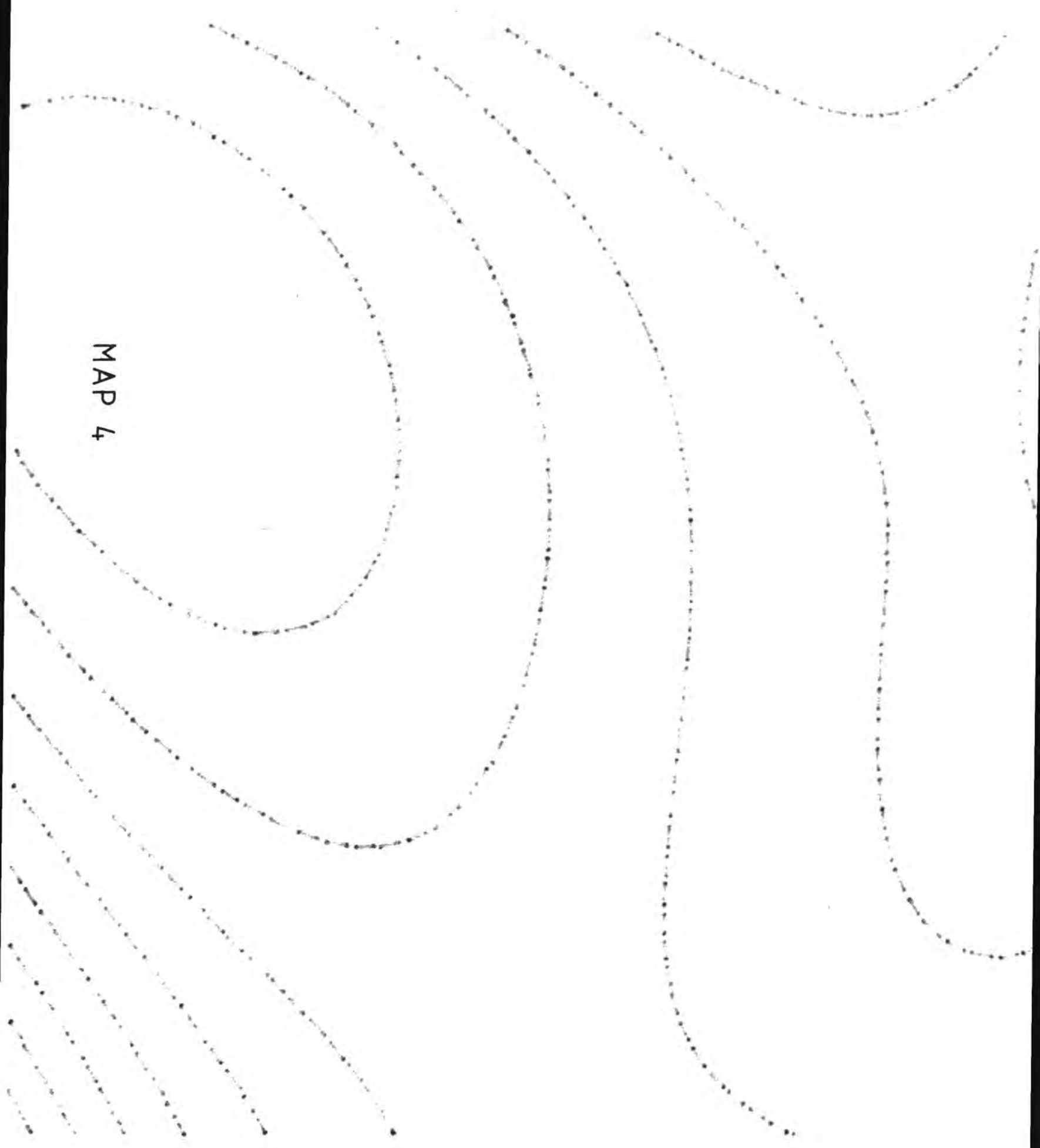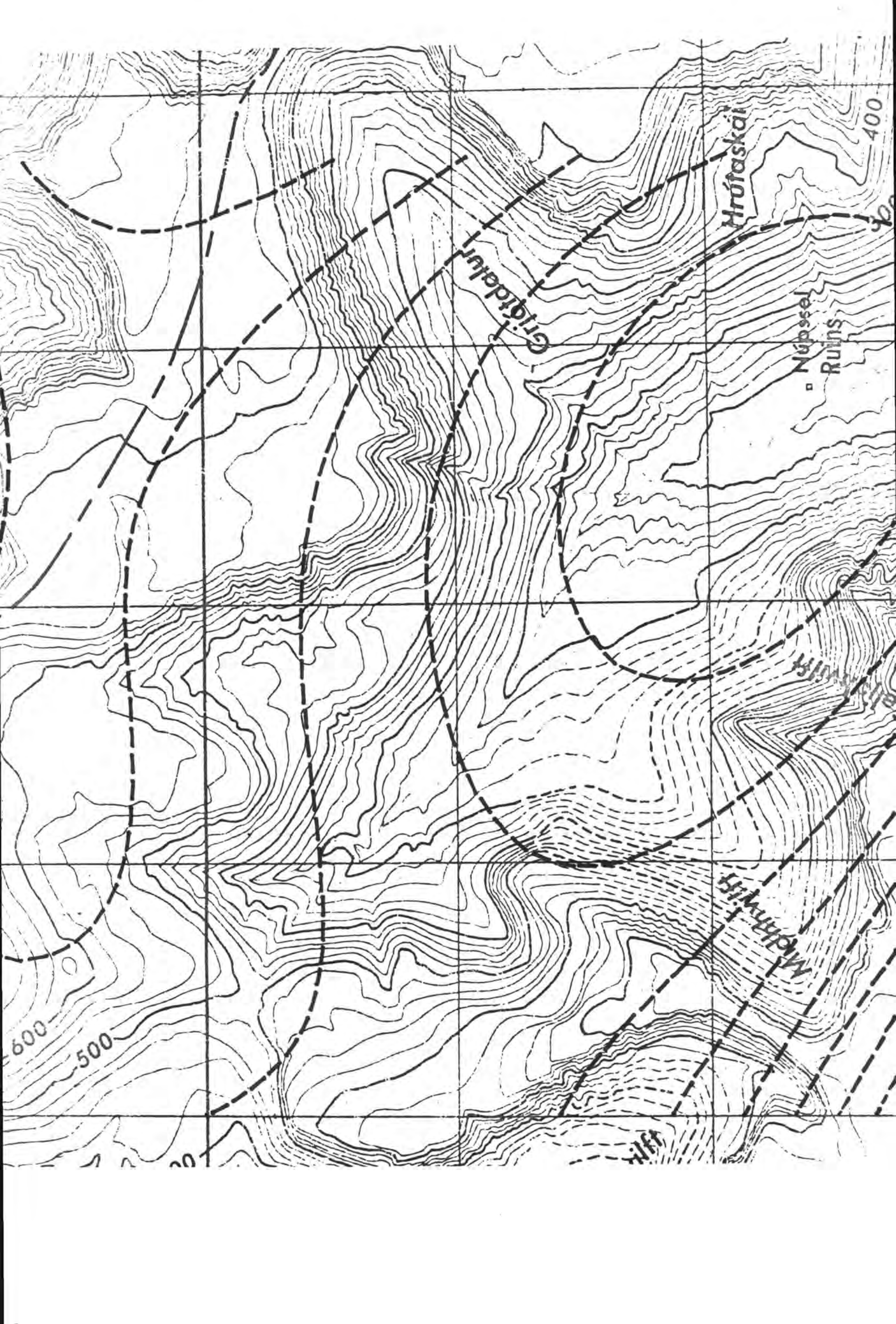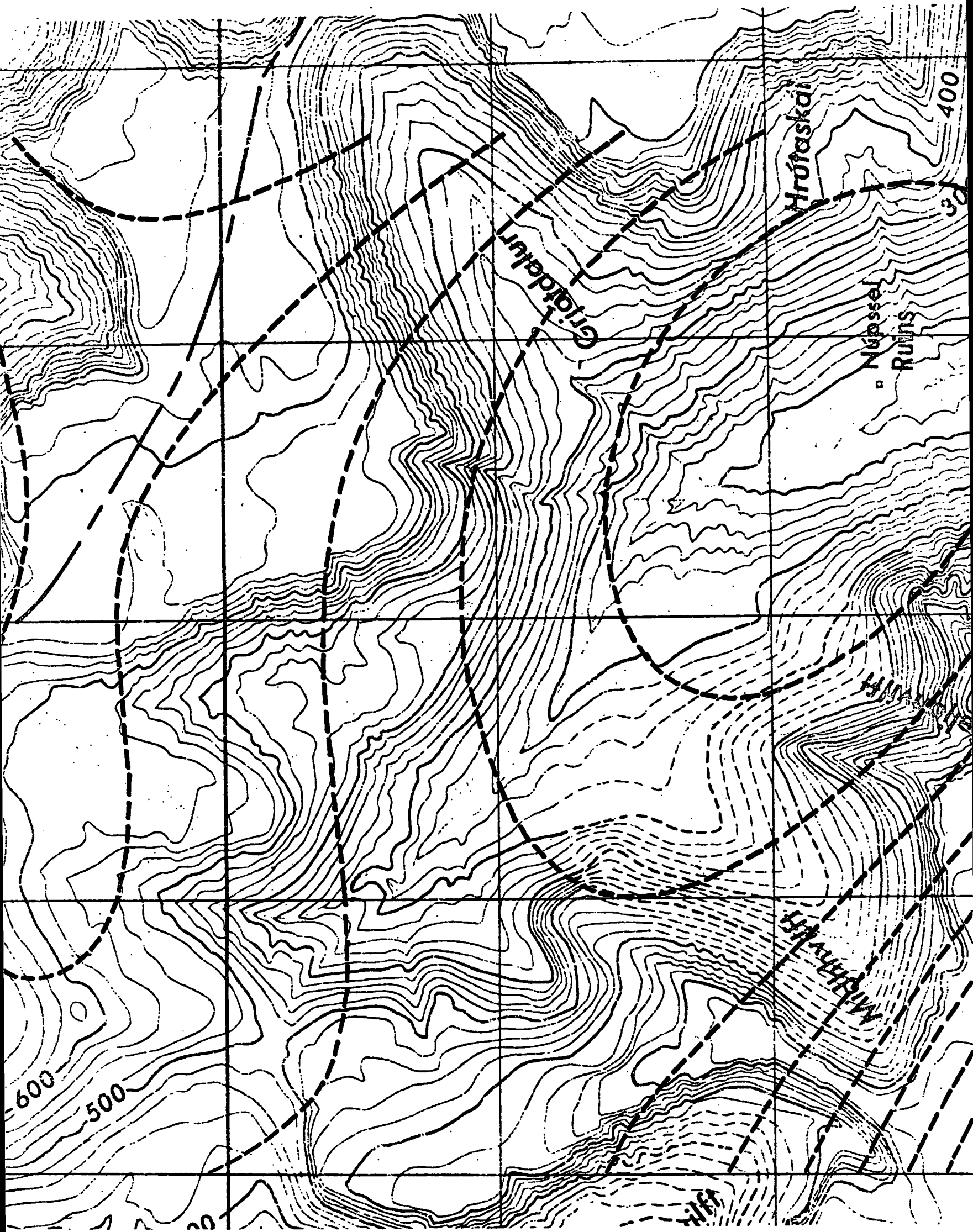
MAP 6
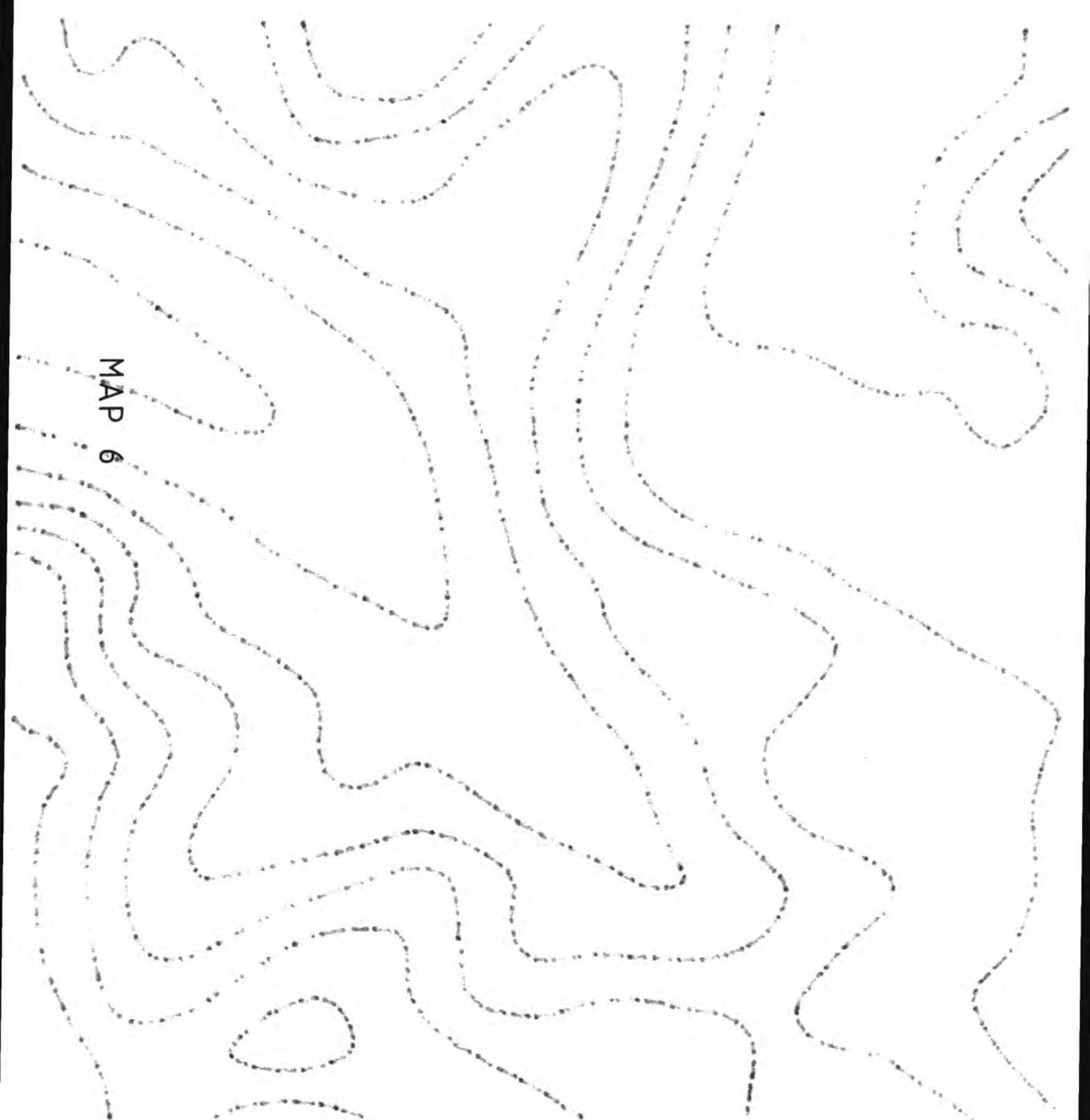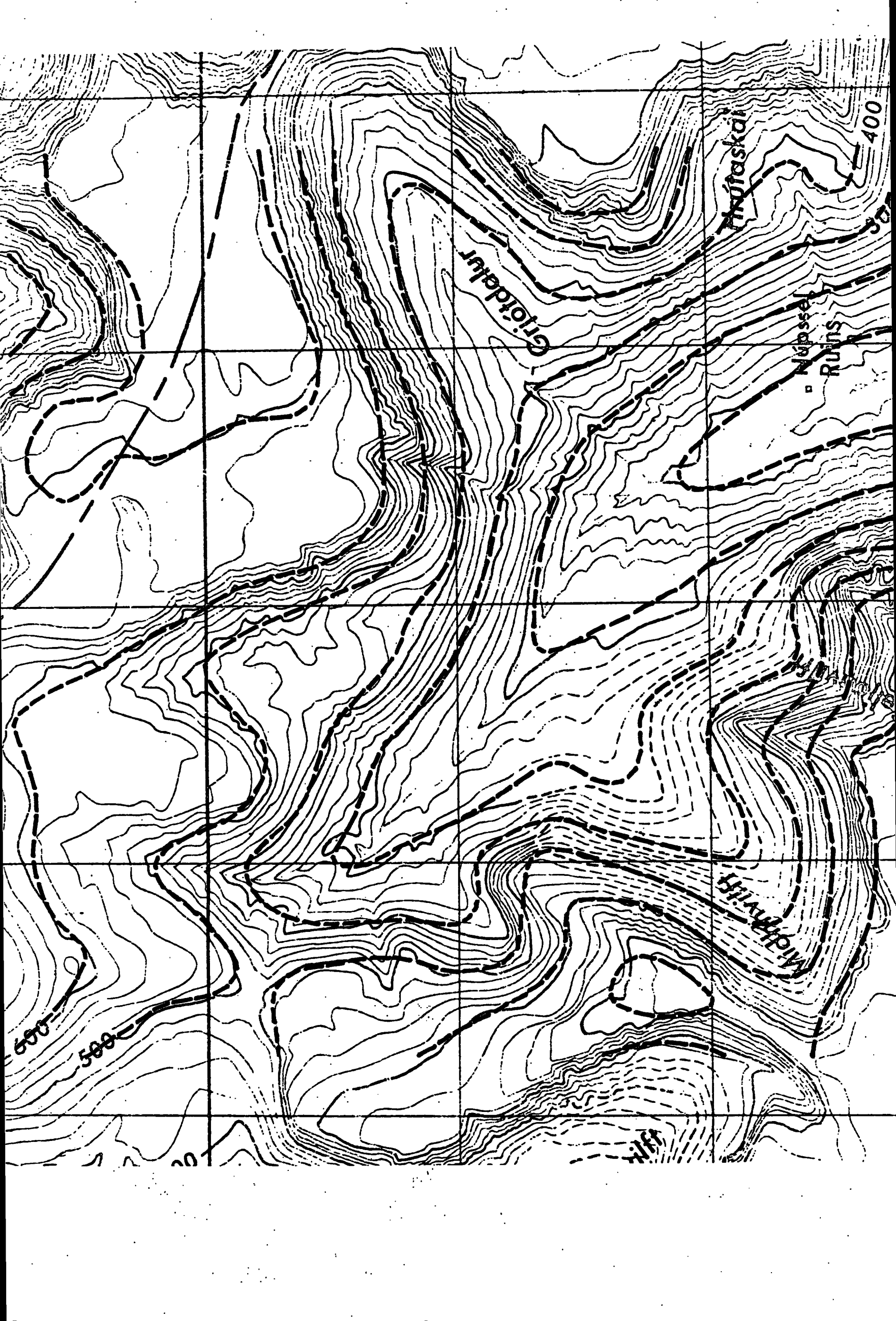
MAP 7

MAP 8

MAP 9

MAP 10

MAP 11

Gró()dalur

Nykraskál

Nupsseð
Ruins

Norðvik

400

600   500
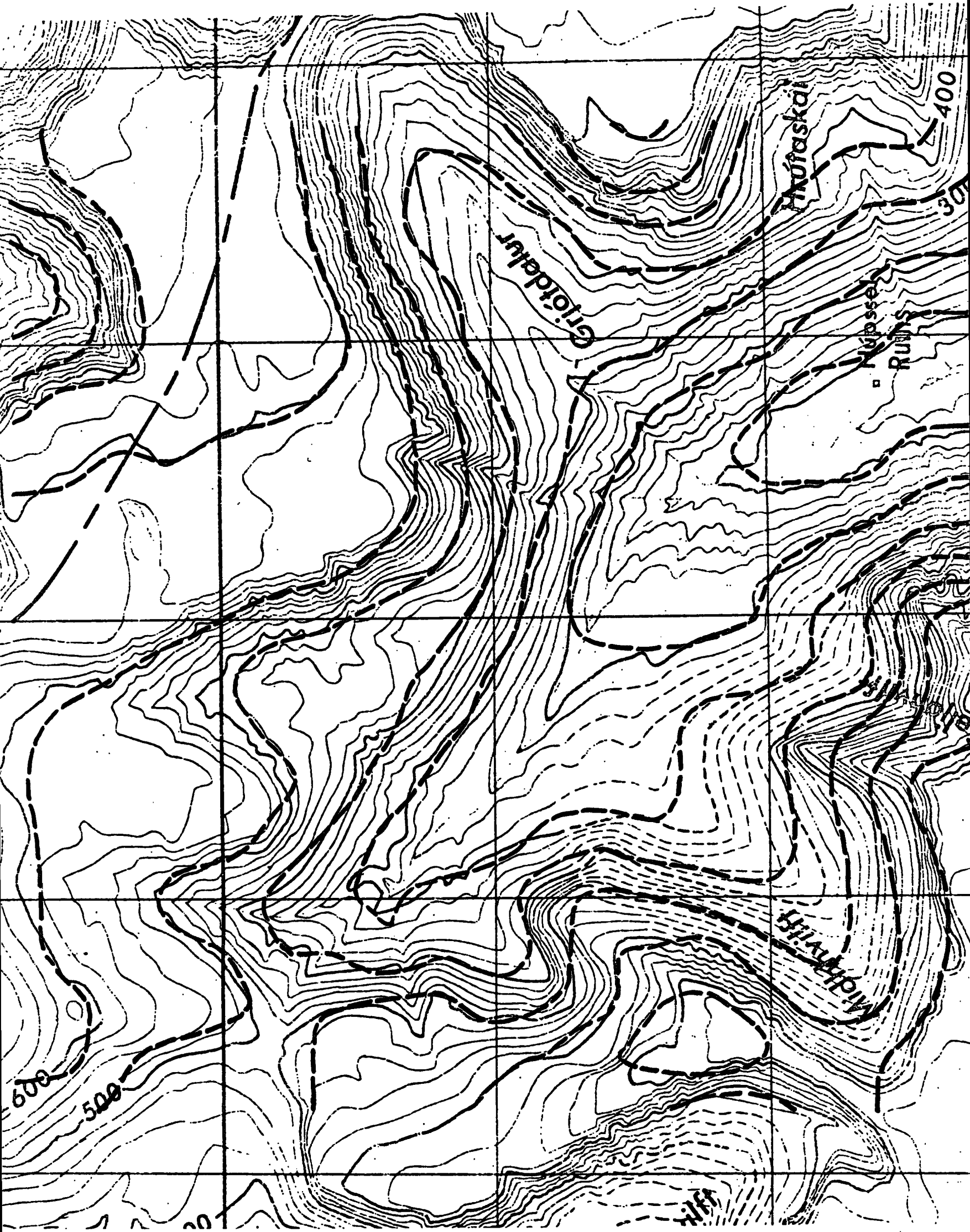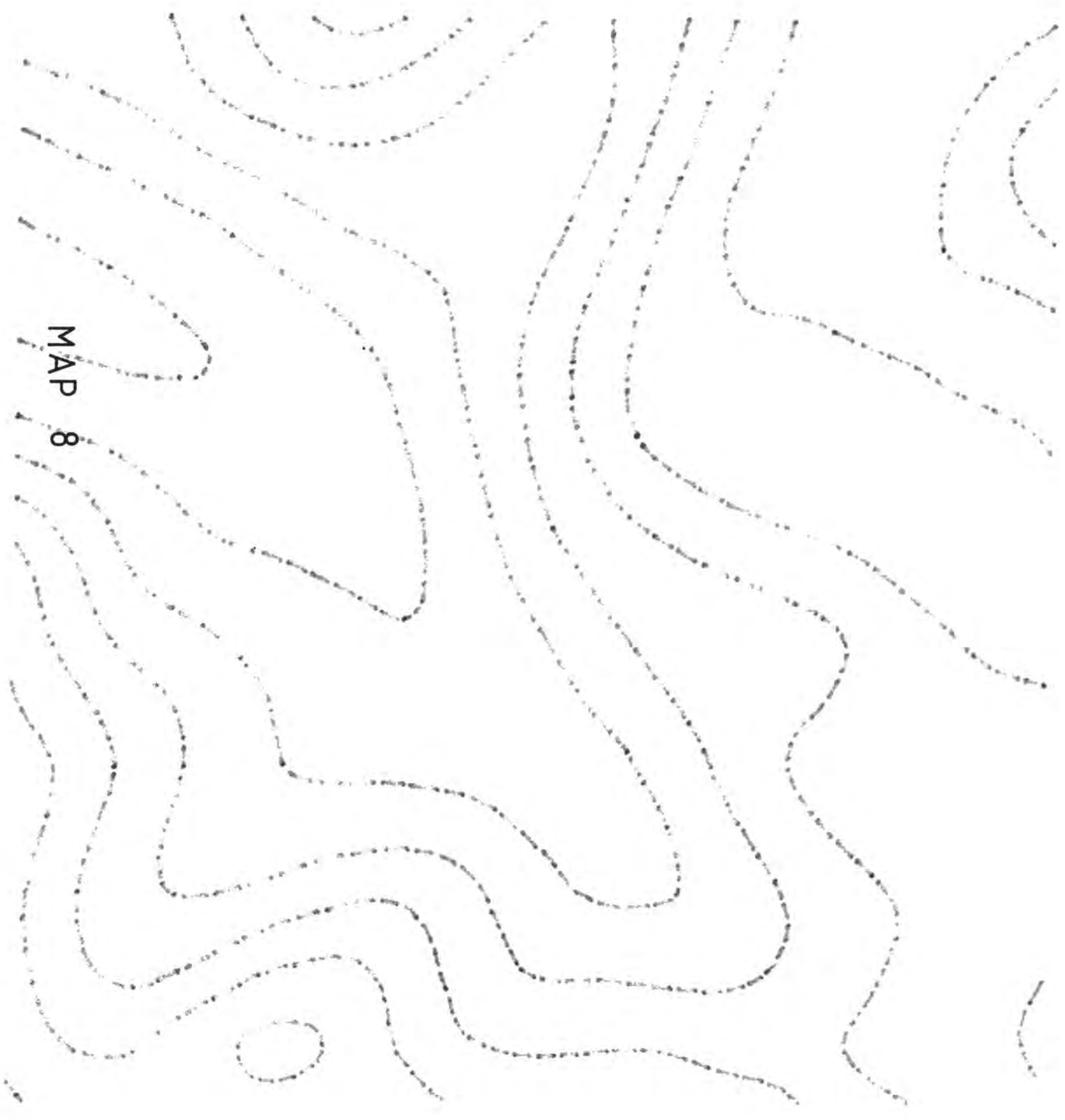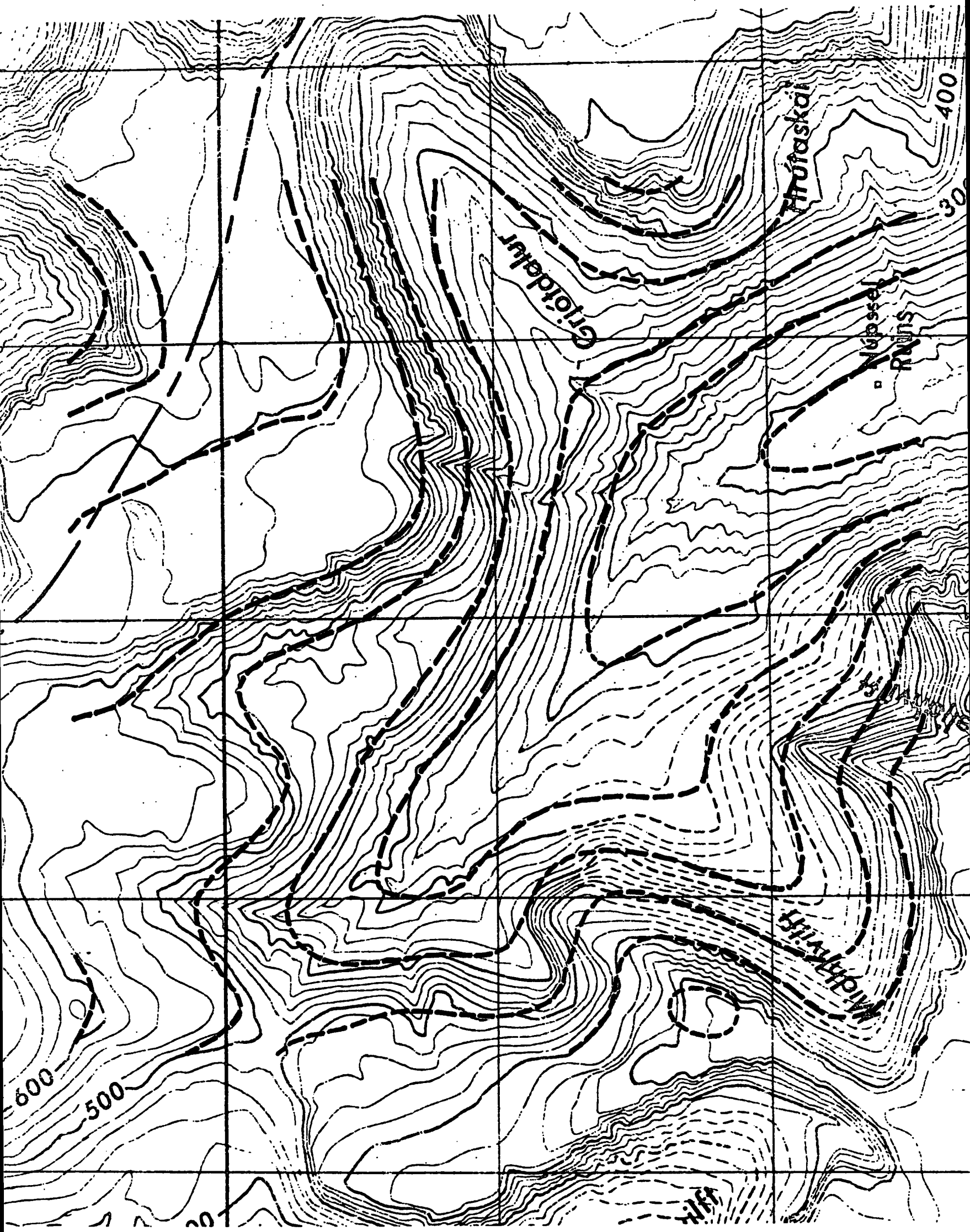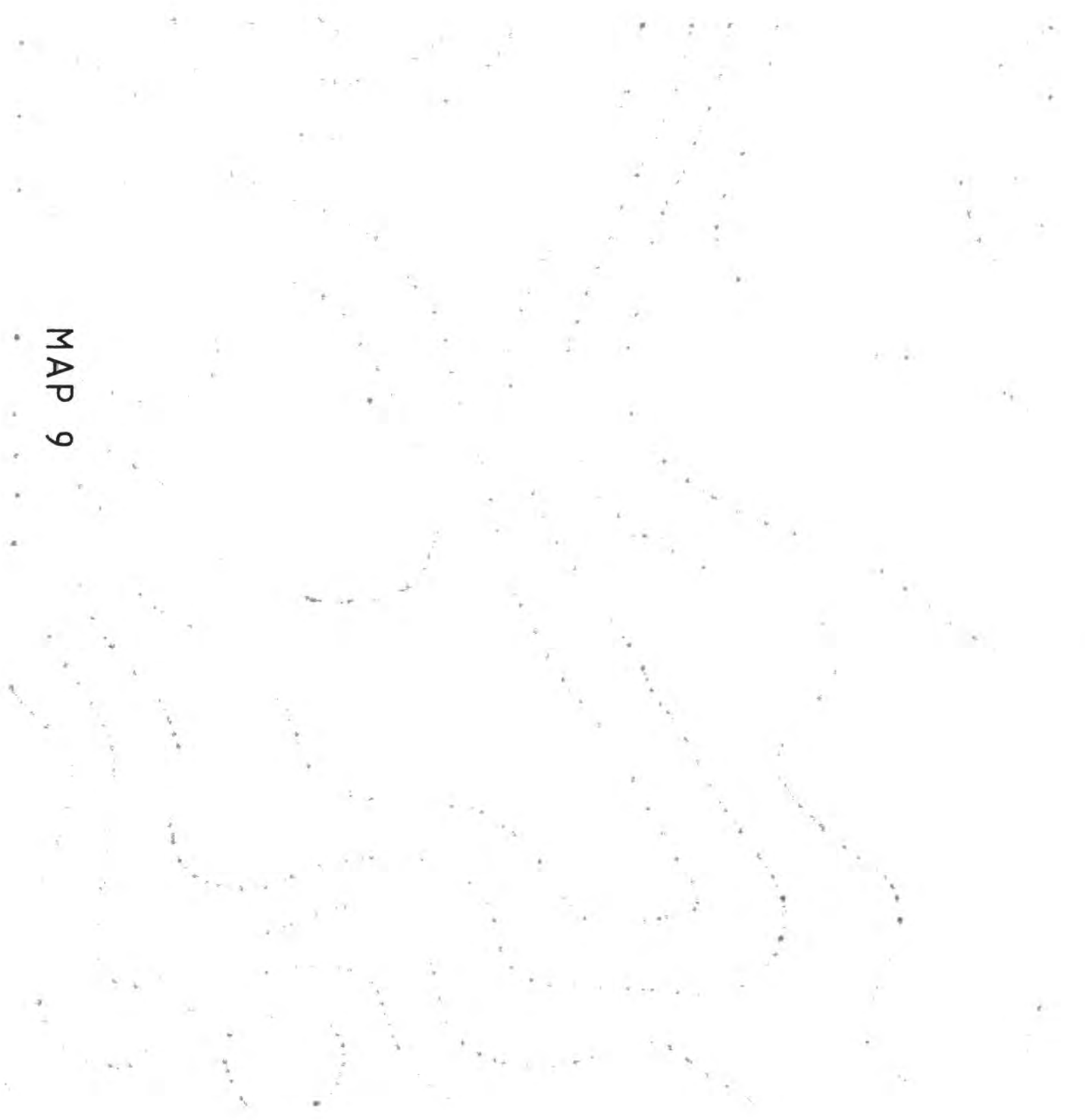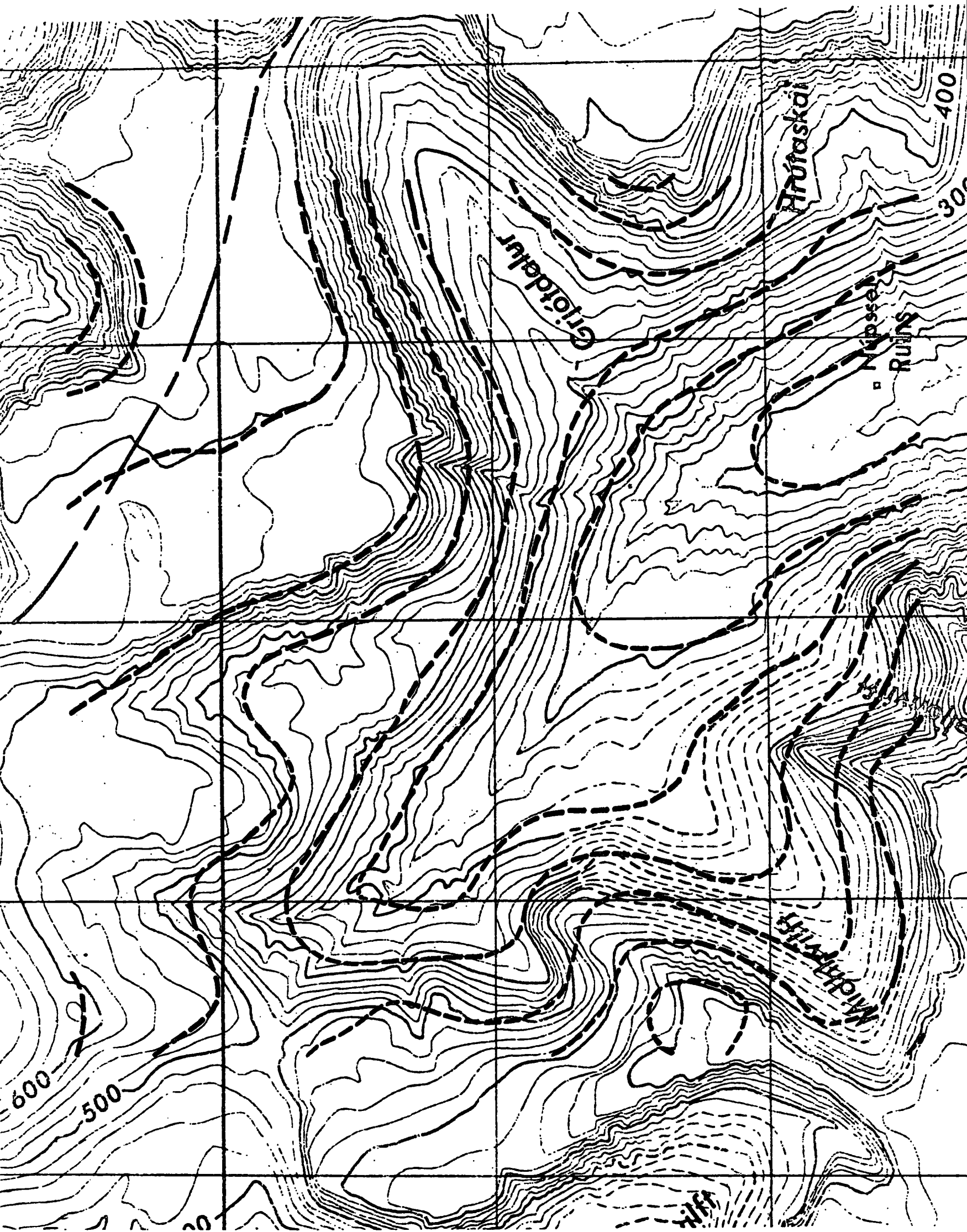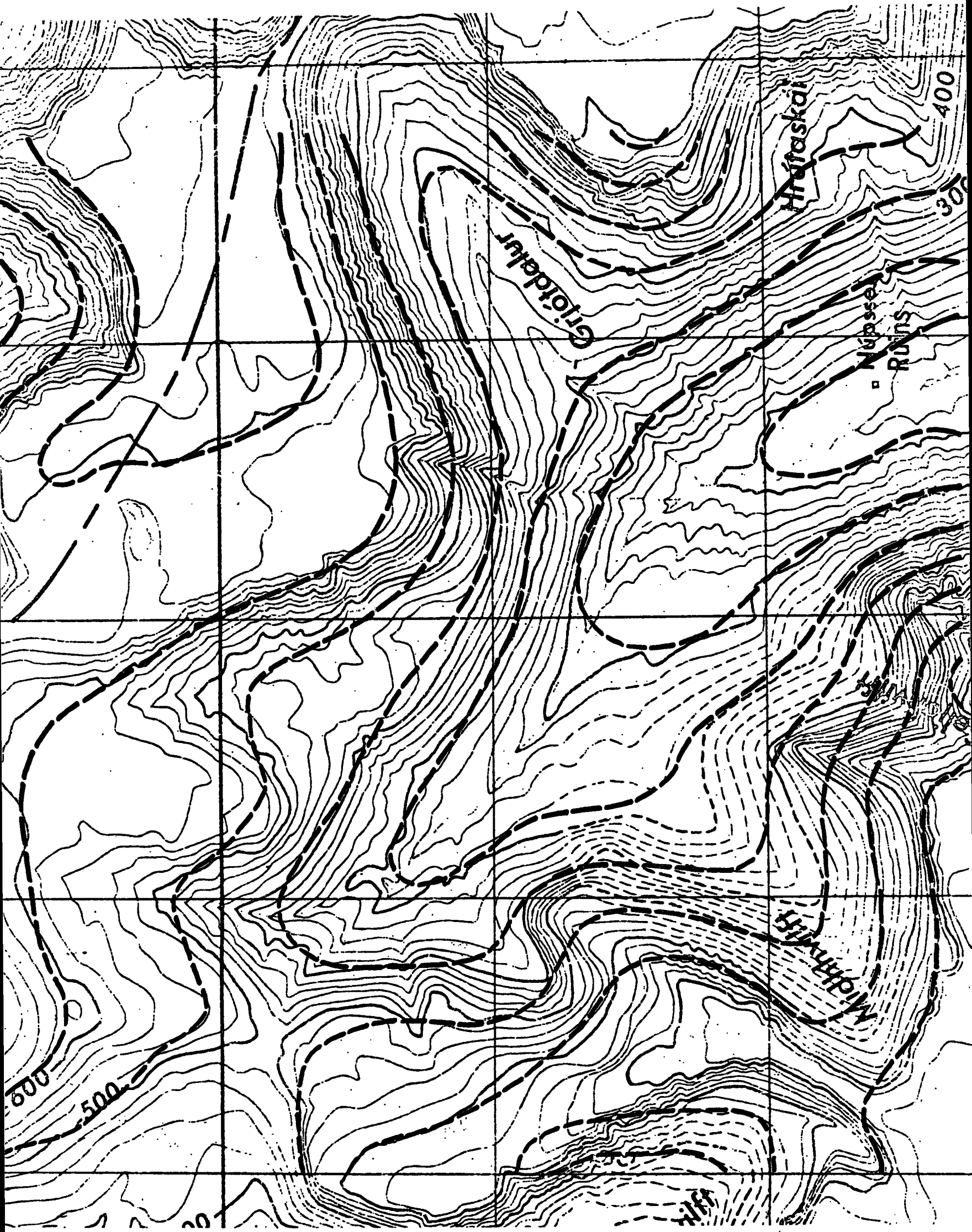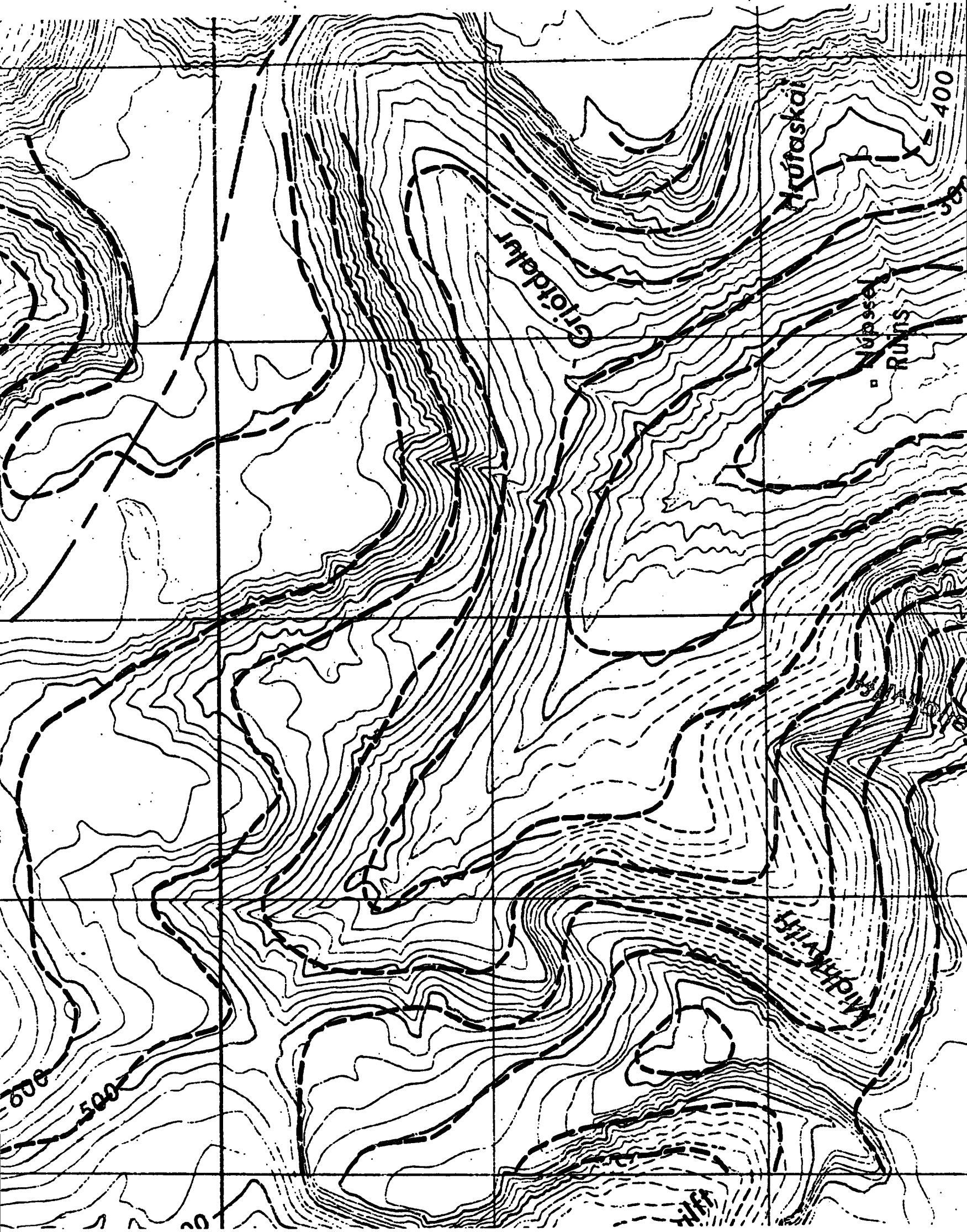
MAP 12

MAP 13

MAP 14

MAP 15

MAP 16

Appendix A. SPLINE INTERPOLATION

Let $f(x)$ be a function of one variable with a given set of values at points $\{x_i\}$ $(i = 1,2,\ldots,N)$ (it may be, for example, a set of sample points with the values of the contour lines, obtained from digitizing by raster scanning). The function $f(x)$ is approximated by a sequence of auxiliary functions $\{g_i(x)\}$ where the function $g_i(x)$ is valid over an interval $[x_i, x_{i+1}]$ for $i = 2,3,\ldots,N-2$. The functions $g_i(x)$ are to be defined in such a manner that over two adjacent closed intervals $[x_i, x_{i+1}]$ and $[x_{i+1}, x_{i+2}]$ both the functions $g_i(x)$ and $g_{i+1}(x)$ and their first derivatives $\dfrac{dg_i}{dx}$ and $\dfrac{dg_{i+1}}{dx}$ are continuous at the point x_{i+1}. Taking as a class of auxiliary functions the third degree polynomial i.e.

$$g_i(x) = \sum_{k=0}^{3} a_{ki} x^k \quad \text{for} \quad x_i \ll x \ll x_{i+1}$$

the coefficients a_{ki} are computed such that

$$g_i(x_i) = f(x_i) = y_i$$

$$g_i(x_{i+1}) = f(x_{i+1}) = y_{i+1}$$

$$\left.\frac{dg_i}{dx}\right|_{x=x_{i+1}} = \left.\frac{df}{dx}\right|_{x=x_i} \qquad (1)$$
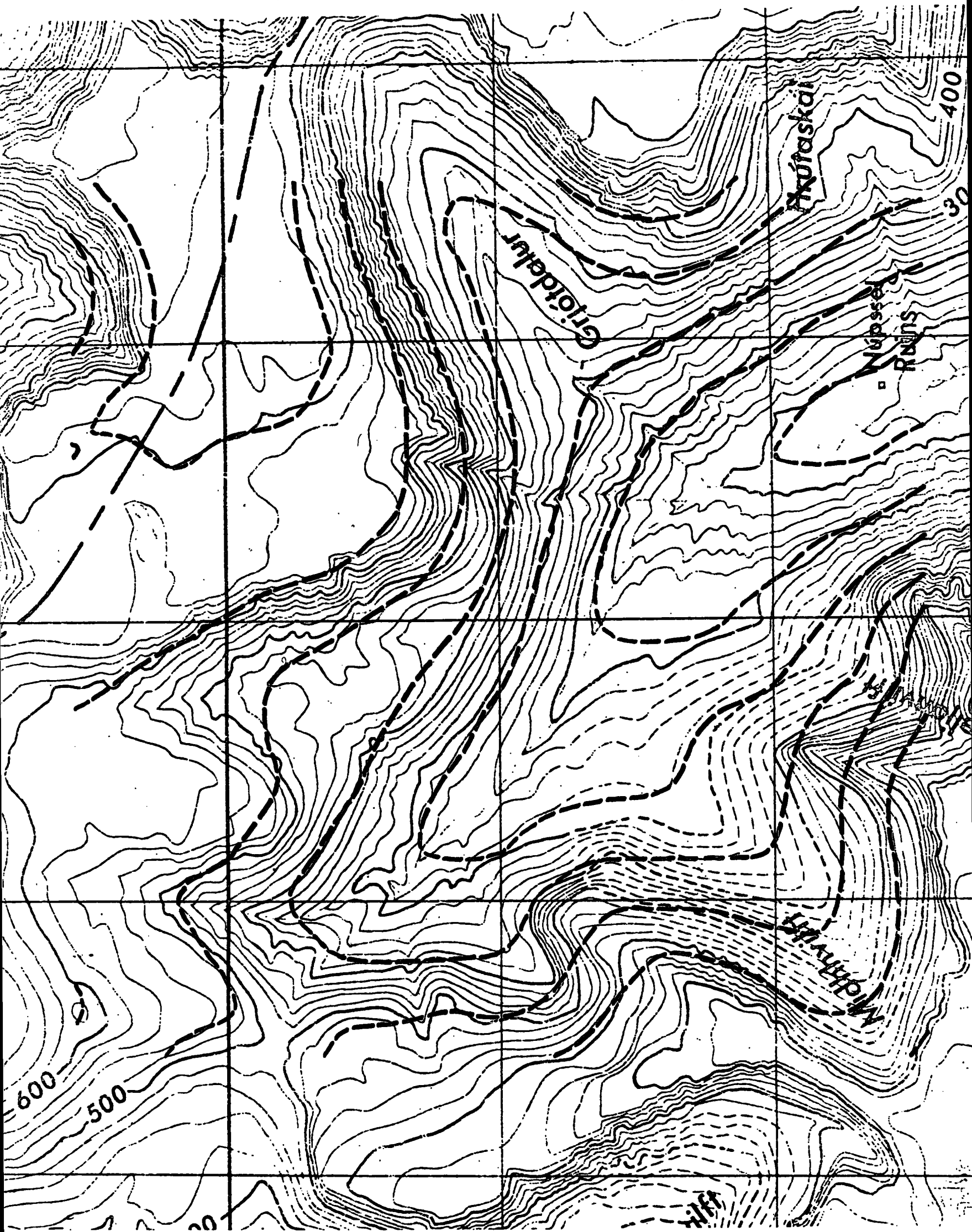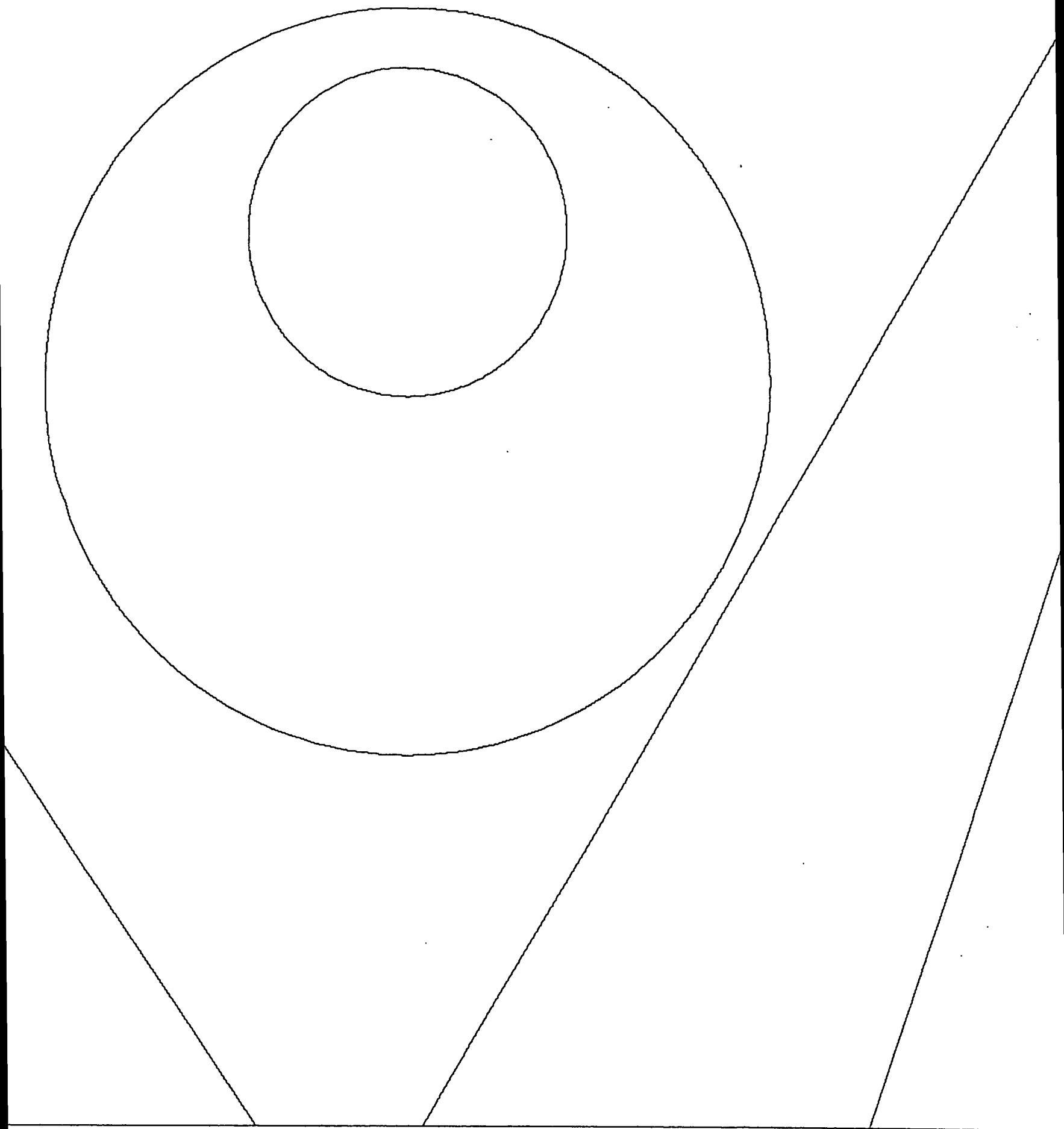
$$\left.\frac{dg_i}{dx}\right|_{x=x_{i+1}} = \left.\frac{df}{dx}\right|_{x=x_{i+1}}$$

Because the values of derivative $\dfrac{df}{dx}$ are usually unknown at the points x_i, x_{i+1} they may be estimated as follows

$$\left.\frac{df}{dx}\right|_{x=x_i} \approx \frac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}} = y_i'$$

$$\left.\frac{df}{dx}\right|_{x=x_{i+1}} \approx \frac{f(x_{i+2}) - f(x_i)}{x_{i+2} - x_i} = y_{i+1}' \qquad (2)$$

Substituting equations (2) into equations (1) we compute the
values of coefficients a_{ki}. To simplify the computations the interval
$[x_i, x_{i+1}]$ can be transformed into the interval $[0, \overset{*}{x_i}]$ where
$\overset{*}{x_i} = x_{i+1} - x_i$. Because it is a shifting it does not affect the
values of derivatives. Thus we obtain the following set of equations

$$a_{oi} = y_i$$
$$a_{oi} + a_{1i}\overset{*}{x_i} + a_{2i}\overset{*}{x_i}{}^2 + a_{3i}\overset{*}{x_i}{}^3 = y_{i+1}$$

$$a_{1i} = y_i'$$
$$a_{1i} + 2a_{2i}\overset{*}{x_i} + 3a_{3i}\overset{*}{x_i}{}^2 = y_{i+1}'$$

Thus

$$a_{oi} = y_i$$

$$a_{1i} = y_i'$$

$$a_{2i} = \frac{3}{\overset{*}{x_i}{}^2}(y_i' - y_i) - \frac{1}{\overset{*}{x_i}}(2y_{i+1} + y_{i+1}')$$

$$a_{3i} = \frac{2}{\overset{*}{x_i}{}^3}(y_i - y_i') + \frac{1}{\overset{*}{x_i}{}^2}(y_{i+1} + y_{i+1}')$$

It is easy to show that the functions $g_i(x)$ and $g_{i+1}(x)$ and
their first derivatives are continuous at the point x_{i+1}.

Appendix B. A MACHINE ALGORITHM TO CREATE A QUANTIZED DATA MATRIX FOR DATA OBTAINED BY RASTER SCANNING (with FORTRAN subroutine)

Definition of symbols

P_k^m — the k-th point situated along the m-th track, k=1,2,....,K_m

where K_m is total number of points located on the m-th track.

$Z(P_k^m)$ — the value of the contour line at the point P_k^m

M — total number of tracks

G_{nm} — the n-th grid point situated on the m-th track

N — total number of grid points situated along one track

ΔZ — contour spacing

This algorithm is for the determination of quantized data for m-th track where m= 1,2,....,M

Step 1. Find two successive points P_k^m and P_{k+1}^m such that $Z(P_k^m) \neq Z(P_{k+1}^m)$

Step 2. Determine the trend T, i.e. if $Z(P_{k+1}^m) \neq Z(P_k^m)$ then T= -1, otherwise T=1, store value of T.

Step 3. At points G_{nm} situated between P_k^m and P_{k+1}^m assign value $(Z(P_k^m) + Z(P_{k+1}^m))/2$.

Step 4. Move to the segment (P_{k+1}^m, P_{k+2}^m). When $Z(P_{k+1}^m) \neq Z(P_{k+2}^m)$ then assign the value $(Z(P_{k+1}^m) + Z(P_{k+2}^m))/2$ to the points G_{nm} situated in this segment; when $Z(P_{k+1}^m) = Z(P_{k+2}^m)$ assign to the points G_{nm} the value $Z(P_{k+1}^m) + T \cdot \Delta Z/2$, and multiply T by -1 to obtain a new trend.

Step 5. Repeat step 4 up to $P_{K_m}^m$.

Step 6. At the points G_{nm} situated in $(P_{K_m}, G_{N_m}]$ assign value $Z(P_{K_m}^m) - \Delta Z/2$ if $Z(P_{K_m-1}^m) > Z(P_{K_m}^m)$ or $Z(P_{K_m}^m) + \Delta Z/2$ if $Z(P_{K_m-1}^m) < Z(P_{K_m}^m)$. If $Z(P_{K_m-1}^m) = Z(P_{K_m}^m)$ then assign value $Z(P_{K_m}^m) + T\Delta Z/2$.

Step 7. Take the initial value of trend T (step 2) and multiply by -1.
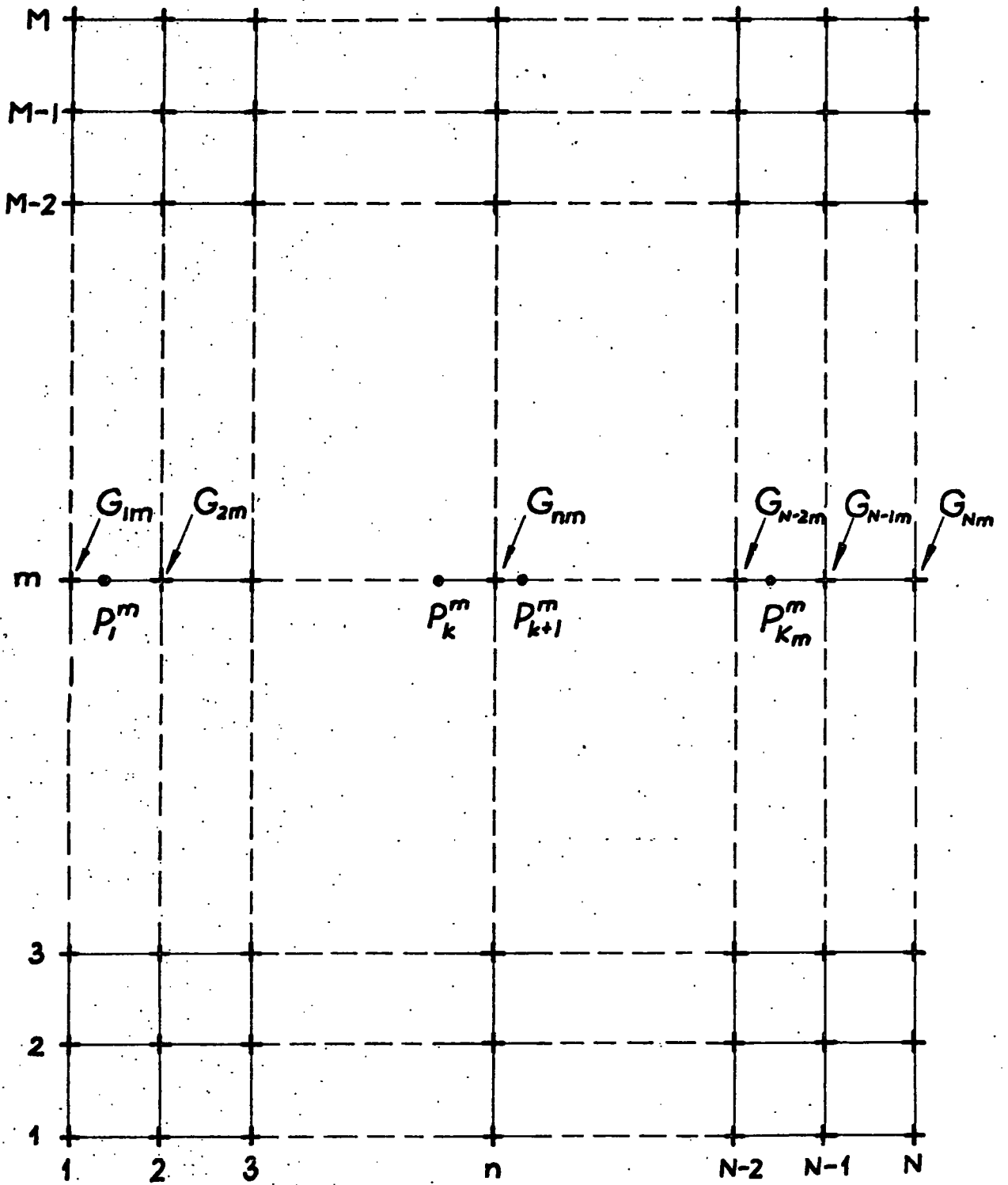
Figure B

Step 8. Move to the segment (P_{k-1}^m, P_k^m). When $Z(P_{k-1}^m) \neq Z(P_k^m)$ then assign the value $(Z(P_{k-1}^m) + Z(P_k^m))/2$ to the points G_{nm} situated in this segment; when $Z(P_{k-1}^m) = Z(P_k^m)$ assign to the points G_{nm} the value $Z(P_{k-1}^m) + T\Delta Z/2$, and multiply T by -1 to obtain a new trend.

Step 9. Repeat step 8 up to P_1^m.

Step 10. At the points G_{nm} situated in $[G_{1m}, P_1^m)$ assign value $Z(P_1^m) - \Delta Z/2$ if $Z(P_2^m) > Z(P_1^m)$ or $Z(P_1^m) + \Delta Z/2$ if $Z(P_2^m) < Z(P_1^m)$. If $Z(P_1^m) = Z(P_2^m)$ then at the points G_{nm} assign value $Z(P_1^m) + T. \Delta Z/2$.

Step 11. At the points G_{nm} if there is such a k that $G_{nm} = P_k^m$ then assign value $Z(P_k^m)$.

```
1.000              SUBROUTINE QUANT(XMIN,GRID,G,N,P,Z,K,DELZ)
2.000              DIMENSION G(N),P(K),Z(K)
3.000     C
4.000     C   SUBROUTINE COMPUTES QUANTIZED DATA /FOR CNE TRACK/ WHICH ARE
5.000     C   STORED IN ARRAY G /N - NUMBER OF GRID POINTS CN ONE TRACK/
6.000     C   P(I) - CO-ORDINATE OF THE I-TH SAMPLE POINT,
7.000     C   Z(I) - VALUE OF CONTOUR LINE AT P(I);   /I=1,2,...,K/
8.000     C   K - NUMBER CF SAMPLE POINTS,
9.000     C   XMIN - CO-ORDINATE OF THE FIRST GRID POINT,
10.000    C   GRID - GRID SIZE /TRACK SPACING/,
11.000    C   DELZ - CONTOUR INTERVAL.
12.000    C
13.000             DO 11 I=1,N
14.000             XB=XMIN+FLOAT(I-1)*GRID
15.000             DO 11 J=1,K
16.000             IF(XB.EQ.P(J)) G(I)=Z(J)
17.000     11 CONTINUE
18.000             KK=K-1
19.000             DO 1 J=1,KK
20.000             IF(Z(J).NE.Z(J+1)) GO TO 2
21.000     1 CONTINUE
22.000             J=KK
23.000     2 JP=J
24.000             TT=1
25.000             IF(Z(J+1).LT.Z(J)) TT=-1
26.000             T=TT
27.000             DO 3 J=JP,KK
28.000             IF(Z(J).NE.Z(J+1)) GO TO 4
29.000             ZP=Z(J)+T*DELZ
30.000             T=T*(-1)
31.000             GO TO 6
32.000     4 ZP=(Z(J)+Z(J+1))/2.0
33.000     6 CONTINUE
34.000             DO 7 I=1,N
35.000             XB=XMIN+FLOAT(I-1)*GRID
36.000             IF(XB.LE.P(J).OR.XE.GE.P(J+1)) GO TO 7
37.000             G(I)=ZP
38.000     7 CONTINUE
39.000     3 CONTINUE
40.000             IF(Z(K).EQ.Z(K-1)) GO TO 8
```

-69-

```
41.000          IF(Z(K).GT.Z(K-1)) ZP=Z(K)+DELZ
42.000          IF(Z(K).LT.Z(K-1)) ZP=Z(K)-DELZ
43.000          GO TO 9
44.000        8 ZP=Z(K)+T*DELZ
45.000        9 CONTINUE
46.000          DO 10 I=1,N
47.000          XB=XMIN+FLOAT(I-1)*GRID
48.000          IF(XB.LE.P(K)) GO TO 10
49.000          G(I)=ZP
50.000       10 CONTINUE
51.000          IF(JP.EQ.1) GO TO 20
52.000          KK=JP-1
53.000          T=TT*(-1)
54.000          DO 20 JJ=1,KK
55.000          J=JP-JJ
56.000          IF(Z(J).NE.Z(J+1)) GO TO 14
57.000          ZP=Z(J)+T*DELZ
58.000          T=T*(-1)
59.000          GO TO 16
60.000       14 ZP=(Z(J)+Z(J+1))/2.0
61.000       16 CONTINUE
62.000          DO 20 I=1,N
63.000          XB=XMIN+FLOAT(I-1)*GRID
64.000          IF(XB.LE.P(J).OR.XB.GE.P(J+1)) GO TO 20
65.000          G(I)=ZP
66.000       20 CONTINUE
67.000          IF(Z(1).EQ.Z(2)) GO TO 18
68.000          IF(Z(1).LT.Z(2)) ZP=Z(1)-DELZ
69.000          IF(Z(1).GT.Z(2)) ZP=Z(1)+DELZ
70.000          GO TO 19
71.000       18 ZP=Z(1)+T*DELZ
72.000       19 CONTINUE
73.000          DO 21 I=1,N
74.000          XB=XMIN+FLOAT(I-1)*GRID
75.000          IF(XB.GE.P(1)) GO TO 21
76.000          G(I)=ZP
77.000       21 CONTINUE
78.000          RETURN
79.000          END
```

Appendix C.   A MACHINE ALGORITHM TO DISPLAY DATA OBTAINED BY RASTER
SCANNING IN CONTOUR MAP FORM (with FORTRAN subroutine).

The map is generated in raster mode and only two tracks are
used at a time. All variables referring to the lower track (Figure C a)
bearing the subscript 1 for example $G_1(1), \ldots, G_1(M)$ denote the quantized
values at the grid points on the lower track. $P_1(k)$ denotes co-ordinate
of the k-th sample point on the lower track and $Z_1(k)$ is the value of
the contour line at this point. All variables connected with the top
track have the subscript 2 for example $P_2(k)$, $Z_2(k)$, $G_2(m)$.

1. From the sequence of points on the lower track take a point which
has not been already used - if all have been used go to 10. Let us
denote this point by $P_1(k)$; then the value of contour line at this
point is $Z_1(k)$. Find the co-ordinates of grid points such that $P_1(k)$
is situated between them (Figure C b). Denote them by X_m and X_{m+1}.

2. For all vertical sides of grid squares determine whether or not they
are crossed by the contour line with value $Z_1(k)$. Store the results
of the test in matrix V as follows:

if $(Z_1(k) - G_1(m))(Z_1(k) - G_2(m)) < 0$ i.e. contour line crosses
this side then $V(m) = 0$

3.If $P_1(k)$ is not the first point in the segment (X_m, X_{m+1}) or $V(m) = 0$
go to 5.

If $V(1) = V(2) = \ldots = V(m) = 1$ then draw the line joining the point
$P_1(k)$ with the point on the left side of the first square (co-ordinate
of this point determined using linear interpolation), indicate that
the point $P_1(k)$ has been used and go to 1, otherwise go to 4.

1 $P_1(k)$

$G_2(1)$ $G_2(2)$ $G_2(M-1)$ $G_2(M)$

$G_1(1)$ $G_1(2)$ $G_1(M-1)$ $G_1(M)$

$G_2(m)$ $G_2(m+1)$

$G_1(m)$ $Z_1(k)$ $G_1(m+1)$

X_m $P_1(k)$ X_{m+1}
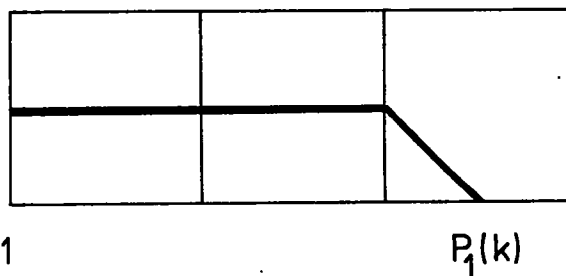
Figure C

4. From a set of points on the top track choose a point so that:

        a. the value of the contour line is the same as at $P_1(k)$ and

        b. it is situated in (X_1, X_m) and

        c. it is closest to $P_1(k)$

then join the point $P_1(k)$ with this point, indicate that both points

have been used and go to 1.

$P_1(k)$

5. If among the points on the top track there is such a point that it

is situated in (X_m, X_{m+1}) and the value of the contour line is the

same as in $P_1(k)$ then join this point with $P_1(k)$, indicate that both

points have been used and go to 1; otherwise go to 6.

$P_1(k)$

6. If $V(m+1) = 0$ go to 9 otherwise if $V(m+2) = \ldots = V(M) = 1$ then join

the point $P_1(k)$ with the point on the right side of the last square

(co-ordinate of this point determined using linear interpolation),

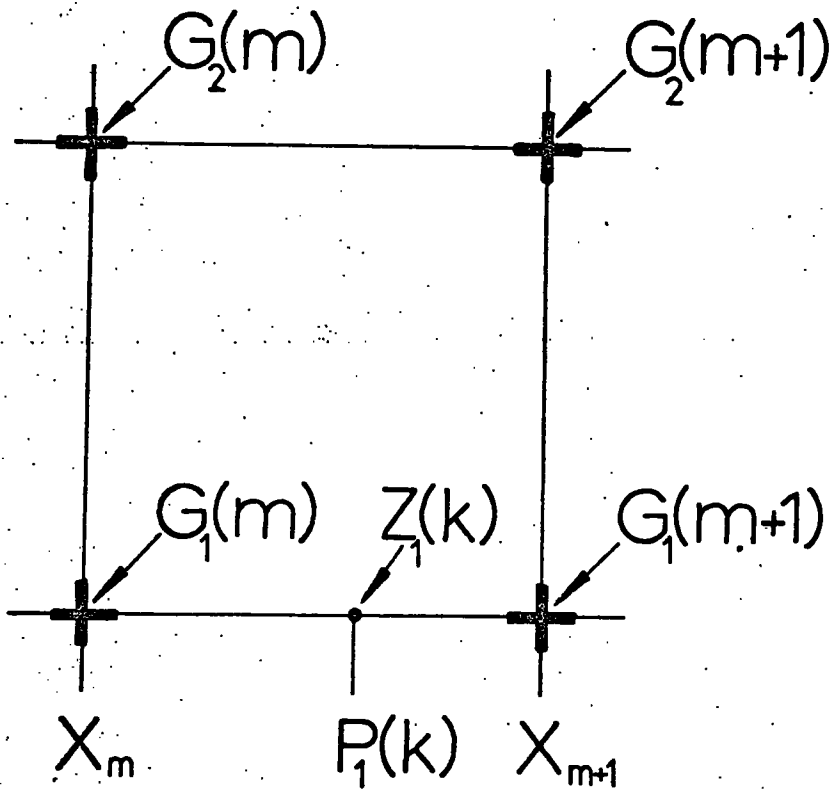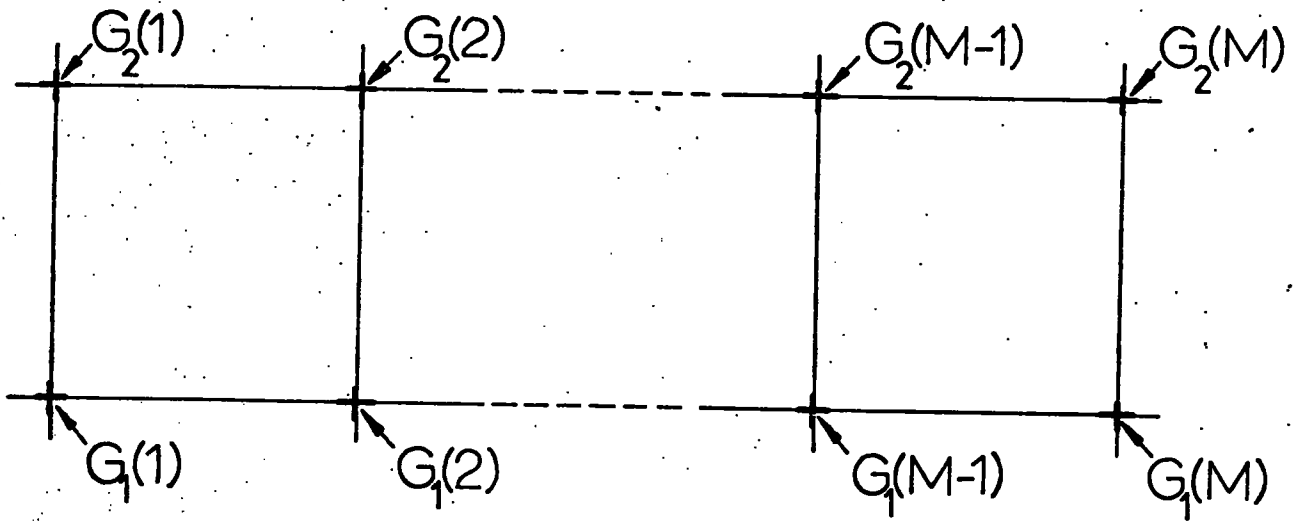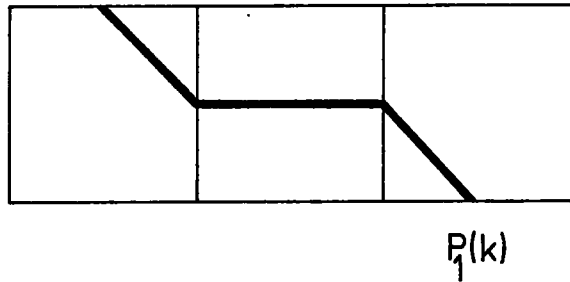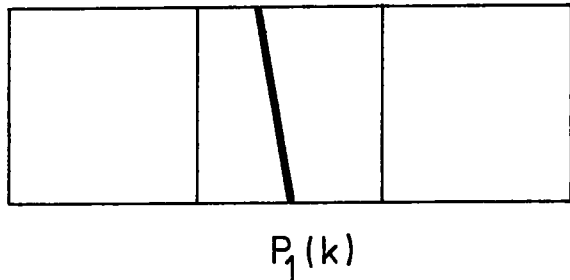indicate that the point $P_1(k)$ has been used and go to 1; otherwise

go to 7.

$P_1(k)$ M
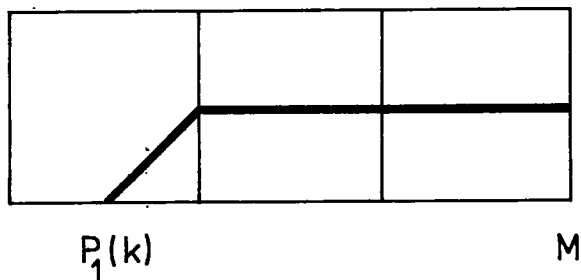
7. If V(1) = 0 for 1 > m+1 then find if among the points on the top track there is such a point that:

    a. it is situated in (X_{1-1}, X_1) and

    b. the value at the contour line at this point is the same as at $P_1(k)$

if so then join both points, indicate that they have been used and go to 1; otherwise go to 8.
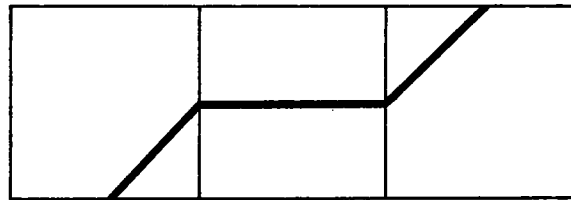
$P_1(k)$

8. Find among the points on the lower track a point such that:

    a. it is situated in (X_{1-1}, X_1) and

    b. the value of the contour line is the same as at $P_1(k)$

then join both points, indicate that they have been used and go to 1.

$P_1(k)$

9. Among the points on the lower track find such a point that:

    a. it is situated in (X_m, X_{m+1}) and

    b. the value of the contour line is the same as at $P_1(k)$

then join both points, indicate that they have been used and go to 1.
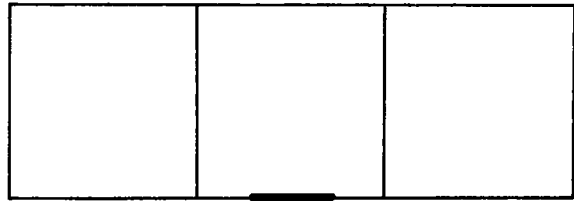
$P_1(k)$

10. From the points situated on the top track take the first point that has not been already used – if all have been used then STOP – determine the segment in which it is situated. Let us denote this segment by (X_m, X_{m+1}) and the point by $P_2(k)$, determine which vertical sides of grid squares are crossed with the contour line $Z_2(k)$, store the results of the test in matrix V (cf. 2) and go to 11.

11. If $V(m) = 0$ go to 12; otherwise join the point $P_2(k)$ with the point on the left side of the first square, indicate that the point $P_2(k)$ has been used and go to 10.
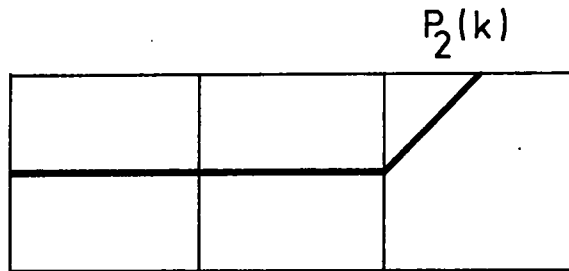
$P_2(k)$

1

12. If $V(m+1) = 0$ go to 14 otherwise if $V(m+2) = V(m+3) = \ldots = V(M) = 1$ then join the point $P_2(k)$ with the point on the right side of the last square (co-ordinate of this point determined using linear interpolation) indicate that $P_2(k)$ has been used and go to 10.

$P_2(k)$

M
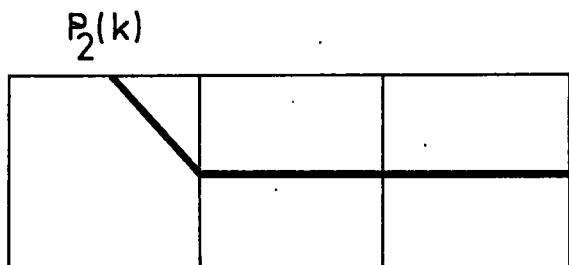
13. If $V(1) = 0$ for $1 > m+1$ then find among the point on the top track a point such that:

      a. it is situated in (X_{1-1}, X_1) and

      b. the value of the contour line at this point is the same as at $P_2(k)$

thenjoin them, indicate that they have been used and go to 10.

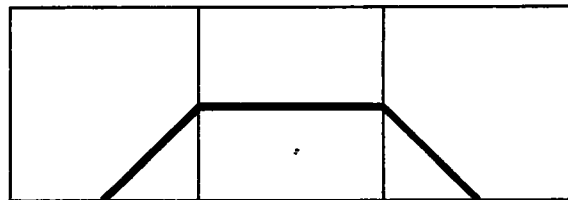$$P_2(k)$$
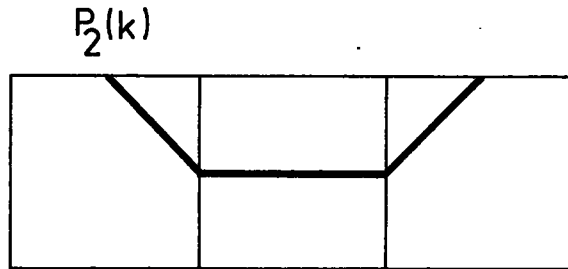
14. Among the points on the top track find a point such that:

      a. it is situated in (X_m, X_{m+1}) and

      b. the value of the contour line at this point is the same as at $P_2(k)$

thenjoin them, indicate that they have been used and go to 10.

$$P_2(k)$$
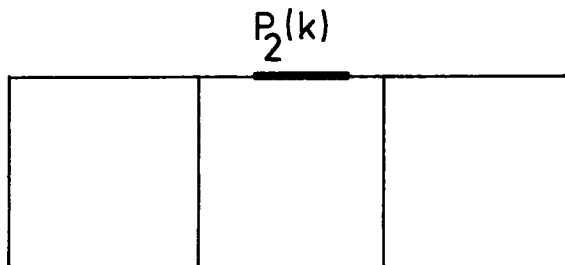
```
LINE NUMBER                                    TEXT                                    PAGE    1

1.000         SUBROUTINE DISPSC(XMIN,XMAX,GRID,YMIN,YMAX,DELZ)
2.000         DIMENSION P1(48),Z1(48),IP1(48),P2(48),Z2(48),IP2(48)
3.000         DIMENSION G1(21),G2(21),IV(21),IT(21)
4.000   C
5.000   C     SUBROUTINE DISPLAYS SCAN-MODE DATA IN CONTOUR MAP FORM
6.000   C     XMIN - CO-ORDINATE OF THE FIRST GRID POINT ON THE TRACKS,
7.000   C     XMAX - CO-ORDINATE OF THE LAST GRID POINT ON THE TRACKS,
8.000   C     GRID - GRID SIZE /TRACK SPACING/,
9.000   C     YMIN - CO-ORDINATE OF THE LOWEST TRACK,
10.000  C     YMAX - CO-ORDINATE OF THE HIGHEST TRACK,
11.000  C     DELZ - CONTOUR INTERVAL,
12.000  C     INPUT DATA ARE READ FROM UNIT 3 AND THEY SHOULD BE
13.000  C     WRITTEN IN THE FOLLOWING FORM - PAIRS OF NUMBERS:
14.000  C     CO-ORDINATE OF A SAMPLE POINT /F4.1/ AND VALUE OF
15.000  C     CONTOUR LINE AT THIS POINT /F4.0/, EIGHT PAIRS PER ROW
16.000  C     VALUE 99.9 AS A CO-ORDINATE OF A SAMPLE POINT INDICATES
17.000  C     THAT THE PREVIOUS SAMPLE POINT IS THE LAST ONE ON THE
18.000  C     TRACK.
19.000  C
20.000        CALL PAPER(1)
21.000        CALL PSPACE(0.0,1.0,0.0,1.0)
22.000        CALL MAP(XMIN,XMAX,YMIN,YMAX)
23.000        CALL BORDER
24.000        N=(XMAX-XMIN)/GRID+1.
25.000        MM=(YMAX-YMIN)/GRID
26.000        DO 73 K=1,6
27.000        KL=(K-1)*8+1
28.000        KR=KL+7
29.000        READ(3,74)(P2(KK),Z2(KK),KK=KL,KR)
30.000  74    FORMAT(8(F4.1,F4.0))
31.000        DO 75 KK=KL,KR
32.000        IF(P2(KK).EQ.99.9) GO TO 78
33.000  75    CONTINUE
34.000  73    CONTINUE
35.000        KK=49
36.000  78    K2=KK-1
37.000        CALL QUANT(XMIN,GRID,G2,N,P2,Z2,K2,DELZ)
38.000        Y=YMAX
39.000        DO 111 K=1,K2
40.000        CALL POSITN(P2(K),Y)
```

```
41.000        111 CALL CIRCLE(0.5)
42.000            DO 105 J=1,MM
43.000            YG=Y
44.000            Y=Y-GRID
45.000            CALL POSITN(0.,Y)
46.000            DO 63 K=1,6
47.000            KL=(K-1)*6+1
48.000            KR=KL+7
49.000            READ(3,74)(P1(KK),Z1(KK),KK=KL,KR)
50.000            DO 65 KK=KL,KR
51.000            IF(P1(KK).EQ.99.9) GO TO 68
52.000        65  CONTINUE
53.000        63  CONTINUE
54.000            KK=49
55.000        68  K1=KK-1
56.000            CALL QUANT(XMIN,GRID,G1,N,P1,Z1,K1,DELZ)
57.000            DO 50 K=1,K1
58.000        50  IP1(K)=0
59.000            DO 51 K=1,K2
60.000        51  IP2(K)=0
61.000            DO 52 K=1,N
62.000        52  IT(K)=0
63.000            DO 110 K=1,K1
64.000            CALL POSITN(P1(K),Y)
65.000        110 CALL CIRCLE(0.5)
66.000            CALL POSITN(0.,Y)
67.000            DO 100 K=1,K1
68.000            IF(IP1(K).EQ.1) GO TO 100
69.000            CALL POSITN(P1(K),Y)
70.000            LL=(P1(K)-XMIN)/GRID+1
71.000            LR=LL
72.000            IF(P1(K).GT.FLOAT(LL-1)*GRID+XMIN) LR=LR+1
73.000            DO 1 L=1,N
74.000            IV(L)=0
75.000            IF((Z1(K)-G1(L))*(Z1(K)-G2(L)).LT.0.) IV(L)=1
76.000        1   CONTINUE
77.000            XL=XMIN+FLOAT(LL-1)*GRID
78.000            XR=XL+GRID
79.000            IF(LR.GT.LL) GO TO 5
80.000            LL=LL-1
```

```
 81.000              LR=LR+1
 82.000              XL=XL-GRID
 83.000            5 CONTINUE
 84.000              IF(LL.LT.1) GC TO 7
 85.000              ITEST=0
 86.000              IF(K.EQ.1) GO TC 9
 87.000              KM=K-1
 88.000              DO 9 KB=1,KM
 89.000              IF(Z1(KB).NE.Z1(K)) GO TO 9
 90.000              IF(P1(KB).LT.XL.OR.P1(KB).GT.XR) GC TC 9
 91.000              ITEST=1
 92.000              KS=KB
 93.000            9 CONTINUE
 94.000              IF(IV(LL).EQ.1.ANC.ITEST.EQ.1) GO TO 7
 95.000              IF(IV(LL).EQ.C) GO TO 7
 96.000              ITEST=0
 97.000              DO 2 KK=1,K2
 98.000              IF(IP2(KK).EQ.1) GC TC 2
 99.000              IF(Z2(KK).NE.Z1(K)) GC TC 2
100.000              IF(P2(KK).GT.XL) GO TO 3
101.000              ITEST=1
102.000              KS=KK
103.000            2 CONTINUE
104.000            3 CONTINUE
105.000              IF(ITEST.EQ.1) GO TO 20
106.000              GO TO 11
107.000            7 CONTINUE
108.000              ITEST=0
109.000              DYST1=2.*(XMAX-XMIN)
110.000              DO 12 KK=1,K2
111.000              IF(IP2(KK).EQ.1) GC TC 12
112.000              IF(Z2(KK).NE.Z1(K)) GC TC 12
113.000              IF(P2(KK).LT.XL.OR.P2(KK).GT.XR) GO TO 12
114.000              ITEST=1
115.000              DYST=SQRT((P1(K)-P2(KK))**2+GRID*GRID)
116.000              IF(DYST.GE.DYST1) GC TC 12
117.000              DYST1=DYST
118.000              KS=KK
119.000           12 CCNTINUE
120.000              IF(ITEST.EQ.C) GO TO 30
```

```
121.000   13  IP1(K)=1
122.000       CALL JOIN(P2(KS),YG)
123.000       CALL POSITN(P2(KS),YG)
124.000       IP2(KS)=1
125.000       CALL POSITN(0.,Y)
126.000       GO TO 100
127.000   11  IF(IV(1).EQ.0) GO TO 45
128.000       YB=((Z1(K)-G1(1))/(G2(1)-G1(1)))*GRIC
129.000       YB=Y+YB
130.000       CALL JOIN(0.,YB)
131.000       IP1(K)=1
132.000       CALL POSITN(0.,Y)
133.000       GO TO 100
134.000   20  CALL JOIN(P2(KS),YG)
135.000       CALL POSITN(P2(KS),YG)
136.000       IP1(K)=1
137.000       IP2(KS)=1
138.000       CALL POSITN(0.,Y)
139.000       GO TO 100
140.000   30  IF(IV(LR).EQ.1) GO TO 37
141.000       ITEST=0
142.000       KM=K+1
143.000       DO 38 KB=KM,K1
144.000       IF(Z1(KB).NE.Z1(K)) GO TO 38
145.000       IF(P1(KB).LT.XL.OR.P1(KB).GT.XR) GO TO 38
146.000       CALL POSITN(P1(K),Y)
147.000       CALL JOIN(P1(KB),Y)
148.000       CALL POSITN(P1(KB),Y)
149.000       IP1(KB)=1
150.000       IP1(K)=1
151.000       GO TO 45
152.000   38  CONTINUE
153.000   37  XL=XR
154.000       XR=XR+GRIC
155.000       KM=K+1
156.000       ITEST=0
157.000       DO 31 KB=KM,K1
158.000       IF(Z1(KB).NE.Z1(K)) GO TO 31
159.000       IF(P1(KB).LT.XL.OR.P1(KB).GT.XR) GO TO 31
160.000       ITEST=1
```

```
161.000              KS=KB
162.000              GC TO 33
163.000         31   CONTINUE
164.000              IF(ITEST.EQ.0) GO TO 34
165.000         33   IF(IV(LR).NE.1) GO TO 45
166.000              YB=((ZI(K)-G1(LR))/(G2(LR)-G1(LR)))*GRIC
167.000              YB=Y+YB
168.000              CALL JOIN(XL,YB)
169.000              CALL JOIN(P1(KS),Y)
170.000              CALL POSITN(P1(KS),Y)
171.000              IP1(KS)=1
172.000              CALL POSITN(0.,Y)
173.000              GO TO 100
174.000         34   DO 35 KK=1,K2
175.000              IF(Z2(KK).NE.ZI(K)) GO TO 35
176.000              IF(P2(KK).LT.XL.OR.P2(KK).GT.XR) GO TO 35
177.000              ITEST=1
178.000              KS=KK
179.000              GO TO 36
180.000         35   CONTINUE
181.000              IF(IV(LR).NE.1) GO TO 45
182.000              YB=((ZI(K)-G1(LR))/(G2(LR)-G1(LR)))*GRIC
183.000              YB=Y+YB
184.000              CALL JOIN(XL,YB)
185.000              LL=LR
186.000              LR=LR+1
187.000              IF(LR.GE.22) GC TO 45
188.000              GO TO 37
189.000         36   CALL JCIN(P2(KS),YG)
190.000              CALL POSITN(P2(KS),YG)
191.000              IP2(KS)=1
192.000              CALL POSITN(0.,Y)
193.000              GO TO 100
194.000         45   CALL PCSITN(0.,Y)
195.000         100  CONTINUE
196.000              DO 200 KK=1,K2
197.000              IF(IP2(KK).EQ.1) GC TC 200
198.000              LL=(P2(KK)-XMIN)/GRID+1
199.000              LR=LL
200.000              XL=XMIN+(LL-1)*GRIC
```

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```
201.000          XR=XL+GRIC
202.000          IF(P2(KK).GT.FLOAT(LL-1)*GRID) LR=LR+1
2C3.000          IF(LR.GT.LL) GO TO 250
2C4.000          LL=LL-1
2C5.000          LR=LR+1
206.000          XL=XL-GRIC
2C7.000    250   CONTINUE
2C8.000          DO 201 L=1,N
2C9.000          IV(L)=0
210.000          IF((Z2(KK)-G1(L))*(Z2(KK)-G2(L)).LT.0) IV(L)=1
211.000    201   CONTINUE
212.000          ITEST=0
213.000          IF(KK.EQ.1) GO TO 202
214.000          KM=KK-1
215.000          DO 202 KB=1,KM
216.000          IF(KB.EQ.KK.OR.IP2(KB).EQ.1) GO TO 202
217.000          IF(P2(KB).LT.XL.OR.P2(KB).GT.XR) GO TO 202
218.000          IF(Z2(KK).NE.Z2(KB)) GO TC 2C2
219.000          ITEST=1
220.000          KS=KB
221.000          GO TO 205
222.000    202   CONTINUE
223.000          IF(IV(LL).EQ.1) GO TC 210
224.000    206   CONTINUE
225.000          IF(IV(LR).EQ.1) GO TO 220
226.000          GO TO 200
227.000    210   IF(IV(1).EQ.0) GO TO 245
228.000          YB=((Z2(KK)-G1(1))/(G2(1)-G1(1)))*GRID
229.000          CALL POSITN(P2(KK),YG)
230.000          YB=Y+YB
231.000          CALL JOIN(0.,YB)
232.000          CALL POSITN(O.,Y)
233.000          GO TO 200
234.000    205   CALL POSITN(P2(KS),YG)
235.000          CALL JCIN(P2(KK),YG)
236.000          CALL PCSITN(P2(KK),YG)
237.000          IF(Z2(KK+1).NE.Z2(KK).OR.IP2(KK+1).EQ.1) GC TO 209
238.000          CALL JGIN(P2(KK+1),YG)
239.000          IP2(KK+1)=1
240.000          KK=KK+1
```

```
241.000         LL=(P2(KK)-XMIN)/GRID+1
242.000         LR=LL+1
243.000   205   CONTINUE
244.000         CALL POSITN(0.,Y)
245.000         IP2(KK)=1
246.000         IP2(KS)=1
247.000         GO TO 206
248.000   220   IF(P2(KK).GE.XMAX) GO TO 245
249.000         CALL POSITN(P2(KK),YG)
250.000         YB=((Z2(KK)-G1(LR))/(G2(LR)-G1(LR)))*GRID+Y
251.000         CALL JOIN(XR,YB)
252.000         IF(LR.EQ.21.OR.KK.EQ.K2) GO TO 245
253.000         ITEST=0
254.000         KM=KK+1
255.000         DO 221 KB=KM,K2
256.000         IF(IP2(KB).EQ.1) GO TO 221
257.000         IF(Z2(KK).NE.Z2(KB)) GO TO 221
258.000         ITEST=1
259.000         KS=KB
260.000         GO TO 222
261.000   221   CONTINUE
262.000         IF(IV(21).EQ.0) GO TO 245
263.000         YB=((Z2(KK)-G1(21))/(G2(21)-G1(21)))*GRID+Y
264.000         CALL JOIN (XMAX,YB)
265.000         CALL POSITN(0.,Y)
266.000         GO TO 200
267.000   222   LL=(P2(KB)-XMIN)/GRID+1
268.000         XL=XMIN+FLOAT(LL-1)*GRID
269.000         IF(P2(KB).EQ.XL)  LL=LL-1
270.000         IF(P2(KB).EQ.XL) XL=XL-GRIC
271.000         IF(IV(LL).NE.1) GO TO 245
272.000         YB=((Z2(KK)-G1(LL))/(G2(LL)-G1(LL)))*GRIC+Y
273.000         CALL JOIN(XL,YB)
274.000   223   CALL JCIN(P2(KB),YC)
275.000         CALL PCSITN(P2(KB),YG)
276.000         IP2(KB)=1
277.000         CALL PCSITN(0.,Y)
278.000         GO TO 200
279.000   245   CALL POSITN(0.,Y)
280.000   200   CONTINUE
```

```
281.000          DO 101 K=1,K1
282.000          P2(K)=P1(K)
283.000          Z2(K)=Z1(K)
284.000      101 CONTINUE
285.000          K2=K1
286.000          DO 102 K=1,N
287.000      102 G2(K)=G1(K)
288.000      105 CONTINUE
289.000      500 CALL GRATSI(GRID,GRID)
290.000          CALL GREND
291.000          RETURN
292.000          END
```

The subroutine DISPSC is written in ANSI FORTRAN IV on IBM computers (360/67 and 370/168) working on NUMAC (Northumbrian Universities Multiple Access Computer) system. It uses subroutine QUANT (Appendix B) and graphic routines from *GHOST package.