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pre-g-bi-irresolute and pre-g-stable in ditopological texture spaces

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ABSTRACT: We introduce and study new notions of continuity, compactness and stability in ditopological texture spaces based on the notions of pre-g-open and pre-g-closed sets and some of their characterizations are obtained.

Key Words: Texture, difunction, pre-g-bi-irresolute, pre-g-stability.

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1. Introduction

Textures and ditopological texture spaces were first introduced by L. M. Brown as a point-based setting for the study fuzzy topology. The study of compactness and stability in ditopological texture spaces was started to begin in [6]. In this paper, we introduce and study the concepts of pre-g-bicontinuity, pre-g-bi-irresolute, pre-g-compactness and pre-g-stability in ditopological textures spaces.

2. Preliminaries

The following are some basic definitions of textures we will need later on.
Texture space: [6] Let S be a set. Then $\varphi \subseteq P(S)$ is called a texturing of S, and S is said to be textured by φ if

1. $(\varphi, \subseteq)$ is a complete lattice containing S and ϕ and for any index set I and $A_i \in \varphi$, $i \in I$, the meet $\bigwedge_{i\in I} A_i$ and the join $\bigvee_{i\in I} A_i$ in φ are related with the intersection and union in $P(S)$ by the equalities
   $\bigwedge_{i\in I} A_i = \bigcap_{i\in I} A_i$
   for all I, while
   $\bigvee_{i\in I} A_i = \bigcup_{i\in I} A_i$
   for all finite I.
2. φ is completely distributive.

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3. φ separates the points of S. That is, given $s_1 \neq s_2$ in S we have $L \in \varphi$ with $s_1 \in L$, $s_2 \notin L$, or $L \in \varphi$ with $s_2 \in L$, $s_1 \notin L$.

If S is textured by φ then (S, φ) is called a texture space, or simply a texture.
Complementation: [6] A mapping $\sigma : \varphi \to \varphi$ satisfying $\sigma(\sigma(A)) = A$, for all $A \in \varphi$ and $A \subseteq B$ implies that $\sigma(B) \subseteq \sigma(A)$, for all $A, B \in \varphi$ is called a complementation on (S, φ) and (S, φ, σ) is then said to be a complemented texture.

For a texture (S, φ), most properties are conveniently deńned in terms of the p-sets

$$P_s = \bigcap\{A \in \varphi : s \in A\}$$

and the q-sets,

$$Q_s = \bigvee\{A \in \varphi : s \notin A\}.$$

Ditopology: [6] A dichotomous topology on a texture (S, φ), or ditopology for short, is a pair (τ, k) of subsets of φ, where the set of open sets τ satisfies

1. $S, \phi \in \tau$,
2. $G_1, G_2 \in \tau$ implies that $G_1 \cap G_2 \in \tau$, and
3. $G_i \in \tau, i \in I$ implies that $\bigvee_i G_i \in \tau$,

and the set of closed sets k satisfies

1. $S, \phi \in k$,
2. $K_1, K_2 \in k$ implies that $K_1 \cup K_2 \in k$, and
3. $K_i \in k, i \in I$ implies that $\bigcap K_i \in k$.

Hence a ditopology is essentially a "topology" for which there is no a priori relation between the open and closed sets.
For $A \in \varphi$ we define the closure $[A]$ and the interior $]A[$ of A under (τ, k) by the equalities

$[A] = \bigcap\{K \in k : A \subseteq K\}$ and $]A[= \bigvee\{G \in \tau : G \subseteq A\}$

We refer to τ as the topology and k as the cotopology of (τ, k).

If (τ, k) is a ditopology on a complemented texture (S, φ, σ), then we say that (τ, k) is complemented if the equality $k = \sigma(\tau)$ is satisfied. In this study, a complemented ditopological texture space is denoted by $(S, \varphi, \tau, k, \sigma)$.
In this case we have $\sigma([A]) =]\sigma(A)[$ and $\sigma(]A[) = [\sigma(A)]$.

We denote by $O(S, \varphi, \tau, k)$, or when there can be no confusion by $O(S)$, the set of open sets in φ. Likewise, $C(S, \varphi, \tau, k)$, $C(S)$ will denote the set of closed sets.

Let (S_1, φ_1) and (S_2, φ_2) be textures. In the following definition we consider the product texture [3] $P(S_1) \otimes \varphi_2$, and denote by $\overline{P}_{(s,t)}$, $\overline{Q}_{(s,t)}$, respectively the p-sets and q-sets for the product texture $(S_1 \times S_2, P(S_1) \otimes \varphi_2)$.
Direlation: [5] Let (S_1, φ_1) and (S_2, φ_2) be textures. Then

1. $r \in P(S_1) \otimes \varphi_2$ is called a relation from (S_1, φ_1) to (S_2, φ_2) if it satisfies
   R1 $r \not\subseteq \overline{Q}_{(s,t)}$, $P_{s'} \not\subseteq Q_s$ implies that $r \not\subseteq \overline{Q}_{(s',t)}$.
   R2 $r \not\subseteq \overline{Q}_{(s,t)}$ implies that there exists $s^{'} \in S_1$ such that $P_s \not\subseteq Q_{s'}$ and $r \not\subseteq \overline{Q}_{(s',t)}$.
2. $R \in P(S_1) \otimes \varphi_2$ is called a corelation from (S_1, φ_1) to (S_2, φ_2) if it satisfies
   CR1 $\overline{P}_{(s,t)} \not\subseteq R$, $P_s \not\subseteq Q_{s'}$ implies that $\overline{P}_{(s',t)} \not\subseteq R$.
   CR2 $\overline{P}_{(s,t)} \not\subseteq R$ implies that there exists $s^{'} \in S_1$ such that $P_{s'} \not\subseteq Q_s$ and $\overline{P}_{(s',t)} \not\subseteq R$.
3. A pair (r, R), where r is a relation and R a corelation from (S_1, φ_1) to (S_2, φ_2) is called a direlation from (S_1, φ_1) to (S_2, φ_2).

One of the most useful notions of (ditopological) texture spaces is that of difunction. A difunction is a special type of direlation.
Difunctions: [5] Let (f, F) be a direlation from (S_1, φ_1) to (S_2, φ_2). Then (f, F) is called a difunction from (S_1, φ_1) to (S_2, φ_2) if it satisfies the following two conditions.
DF1 For $s, s^{'} \in S_1$, $P_s \not\subseteq Q_{s'}$ implies that there exists $t \in S_2$ such that $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s',t)} \not\subseteq F$.
DF2 For $t, t^{'} \in S_2$ and $s \in S_1$, $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s,t')} \not\subseteq F$ implies that $P_{t'} \not\subseteq Q_t$.
Image and Inverse Image: [5] Let $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be a difunction.

1. For $A \in \varphi_1$, the image $f^{\to}A$ and the co-image $F^{\to}A$ are defined by
$$f^{\to}A = \bigcap\{Q_t : \text{for all } s, f \not\subseteq \overline{Q}_{(s,t)} \text{ implies that } A \subseteq Q_s\},$$
$$F^{\to}A = \bigvee\{P_t : \text{for all } s, \overline{P}_{(s,t)} \not\subseteq F \text{ implies that } P_s \subseteq A\}.$$
2. For $B \in \varphi_2$, the inverse image $f^{\leftarrow}B$ and the inverse co-image $F^{\leftarrow}B$ are defined by
$$f^{\leftarrow}B = \bigvee\{P_s : \text{for all } t, f \not\subseteq \overline{Q}_{(s,t)} \text{ implies that } P_t \subseteq B\},$$
$$F^{\leftarrow}B = \bigcap\{Q_s : \text{for all } t, \overline{P}_{(s,t)} \not\subseteq F \text{ implies that } B \subseteq Q_t\}.$$

For a difunction, the inverse image and the inverse co-image are equal, but the image and co-image are usually not.
Bicontinuity: [4] The difunction $(f, F) : (S_1, \varphi_1, \tau_1, k_1) \to (S_2, \varphi_2, \tau_2, k_2)$ is called continuous if $B \in \tau_2$ implies that $F^{\leftarrow}B \in \tau_1$, cocontinuous if $B \in k_2$ implies that $f^{\leftarrow}B \in k_1$, and bicontinuous if it is both continuous and cocontinuous.
Surjective difunction: [5] Let $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be a difunction. Then (f, F) is called surjective if it satisfies the condition
SUR. For $t, t^{'} \in S_2$, $P_t \not\subseteq Q_{t'}$ implies that there exists $s \in S_1$ with $f \not\subseteq \overline{Q}_{(s,t')}$ and $\overline{P}_{(s,t)} \not\subseteq F$.

If (f, F) is surjective then $F^{\to}(f^{\leftarrow}B) = B = f^{\to}(F^{\leftarrow}B)$ for all $B \in \varphi_2$ [[5], Corollary 2.33]

Definition 2.1. *[5] Let (f, F) be a difunction between the complemented textures $(S_1, \varphi_1, \sigma_1)$ and $(S_2, \varphi_2, \sigma_2)$. The complement $(f, F)^{'} = (F^{'}, f^{'})$ of the difunction (f, F) is a difunction, where $f^{'} = \bigcap\{\overline{Q}_{(s,t)} | \exists u, v$ with $f \not\subseteq \overline{Q}_{u,v}, \sigma_1(Q_s) \not\subseteq Q_u$ and $P_v \not\subseteq \sigma_2(P_t)\}$ and $F^{'} = \bigvee\{\overline{P}_{(s,t)} | \exists u, v$ with $\overline{P}_{u,v} \not\subseteq F, P_u \not\subseteq \sigma_1(P_s)$ and $\sigma_2(Q_t) \not\subseteq Q_v\}$.*

If $(f, F) = (f, F)^{'}$ then the difunction (f, F) is called complemented.

Definition 2.2. *[7] Let (S, φ, τ, k) be a ditopological texture space. A set $A \in \varphi$ is called pre-open (pre-closed) if $A \subseteq][A][$ ($[]A[] \subseteq A$).*

We denote by $PO(S, \varphi, \tau, k)$, or when there can be no confusion by $PO(S)$, the set of pre-open sets in φ. Likewise, $PC(S, \varphi, \tau, k)$, or $PC(S)$ will denote the set of pre-closed sets.

Definition 2.3. *[2] Let (S, φ, τ, k) be a ditopological texture space. A subset A of a texture φ is said to be generalized closed (g-closed for short) if $A \subseteq G \in \tau$ then $[A] \subseteq G$.*

Definition 2.4. *[2] Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. A subset A of a texture φ is said to be generalized open (g-open for short) if $\sigma(A)$ is g-closed.*

We denote by $gc(S, \varphi, \tau, k)$, or when there can be no confusion by $gc(S)$, the set of g-closed sets in φ. Likewise, $go(S, \varphi, \tau, k, \sigma)$, or $go(S)$ will denote the set of g-open sets.

Definition 2.5. *[1] Let (S, φ, τ, k) be a ditopological texture space. A subset A of a texture φ is said to be pre-g-closed if $A \subseteq G \in PO(S)$ then $[A] \subseteq G$.*

We denote by $pregc(S, \varphi, \tau, k)$, or when there can be no confusion by $pregc(S)$, the set of pre-g-closed sets in φ.

Definition 2.6. *[1] Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. A subset A of a texture φ is called pre-g-open if $\sigma(A)$ is pre-g-closed.*

We denote by $prego(S, \varphi, \tau, k, \sigma)$, or when there can be no confusion by $prego(S)$, the set of pre-g-open sets in φ.

Definition 2.7. *[1] Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. For $A \in \varphi$, we define the pre-g-closure $[A]_{pre-g}$ and the pre-g-interior $]A[_{pre-g}$ of A under (τ, k) by the equalities*

$$[A]_{pre-g} = \bigcap\{K \in pregc(S) : A \subseteq K\} \text{ and }]A[_{pre-g} = \bigcup\{G \in prego(S) : G \subseteq A\}.$$

3. pre-g-bicontinuous, pre-g-bi-irresolute, pre-g-compact and pre-g-stable

Definition 3.1. *The difunction $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ is called:*

1. *pre-g-continuous (pre-g-irresolute), if* $F^{\leftarrow}(G) \in prego(S_1)$, *for every* $G \in O(S_2)$ $(G \in prego(S_2))$.
2. *pre-g-cocontinuous (pre-g-co-irresolute), if* $f^{\leftarrow}(G) \in pregc(S_1)$, *for every* $G \in k_2$ $(G \in pregc(S_2))$.
3. *pre-g-bicontinuous, if it is pre-g-continuous and pre-g-cocontinuous.*
4. *pre-g-bi-irresolute, if it is pre-g-irresolute and pre-g-co-irresolute.*

Corollary 3.2. *Let* $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ *be a difunction. Then:*

1. *Every continuous is pre-g-continuous.*
2. *Every cocontinuous is pre-g-cocontinuous.*
3. *Every pre-g-irresolute is pre-g-continuous.*
4. *Every pre-g-co-irresolute is pre-g-cocontinuous.*

Proof. Clear.

Theorem 3.3. *Let* $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ *be a difunction. Then:*

1. *The following are equivalent:*
   (a) (f, F) *is pre-g-continuous.*
   (b) $]F^{\to}A[^{S_2} \subseteq F^{\to}]A[^{S_1}_{pre-g}$, *for all* $A \in \varphi_1$.
   (c) $f^{\leftarrow}]B[^{S_2} \subseteq]f^{\leftarrow}B[^{S_1}_{pre-g}$, *for all* $B \in \varphi_2$.
2. *The following are equivalent:*
   (a) (f, F) *is pre-g-cocontinuous.*
   (b) $f^{\to}[A]^{S_1}_{pre-g} \subseteq [f^{\to}A]^{S_2}$, *for all* $A \in \varphi_1$.
   (c) $[F^{\leftarrow}B]^{S_1}_{pre-g} \subseteq F^{\leftarrow}[B]^{S_2}$, *for all* $B \in \varphi_2$.

Proof. We prove (1), leaving the dual proof of (2) to the interested reader.
$(a) \Rightarrow (b)$. Let $A \in \varphi_1$. From [[5], Theorem 2.24 (2a)] and the definition of interior,

$$f^{\leftarrow}]F^{\to}(A)[^{S_2} \subseteq f^{\leftarrow}(F^{\to}(A)) \subseteq A.$$

Since inverse image and co-image under a difunction is equal, $f^{\leftarrow}]F^{\to}(A)[^{S_2} = F^{\leftarrow}]F^{\to}(A)[^{S_2}$. Thus, $f^{\leftarrow}]F^{\to}(A)[^{S_2} \in prego(S_1)$, by pre-g-continuity. Hence

$$f^{\leftarrow}]F^{\to}(A)[^{S_2} \subseteq]A[^{S_1}_{pre-g}$$

and applying [[5], Theorem 2.24 (2b)] gives

$$]F^{\rightarrow}(A)[^{S_2} \subseteq F^{\rightarrow}(f^{\leftarrow}(]F^{\rightarrow}(A)[^{S_2}) \subseteq F^{\rightarrow}]A[^{S_1}_{pre-g},$$

which is the required inclusion.
$(b) \Rightarrow (c)$. Take $B \in \varphi_2$. Applying inclusion (b) to $A = f^{\leftarrow}(B)$ and using [[5], Theorem 2.24 (2b)] gives

$$]B[^{S_2} \subseteq]F^{\rightarrow} f^{\leftarrow}(B)[^{S_2} \subseteq F^{\rightarrow}]f^{\leftarrow}(B)[^{S_1}_{pre-g}.$$

Hence, we have $f^{\leftarrow}]B[^{S_2} \subseteq f^{\leftarrow}F^{\rightarrow}]f^{\leftarrow}(B)[^{S_1}_{pre-g} \subseteq]f^{\leftarrow}(B)[^{S_1}_{pre-g}$ by [[5], Theorem 2.24 (2a)].
$(c) \Rightarrow (a)$. Applying (c) for $B \in O(S_2)$ gives

$$f^{\leftarrow}(B) = f^{\leftarrow}]B[^{S_2} \subseteq]f^{\leftarrow}(B)[^{S_1}_{pre-g},$$

so $F^{\leftarrow}(B) = f^{\leftarrow}(B) =]f^{\leftarrow}(B)[^{S_1}_{pre-g} \in prego(S_1)$. Hence, (f, F) is pre-g-continuous.

Theorem 3.4. *Let* $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ *be a difunction. Then:*

1. *The following are equivalent:*
   (a) (f, F) *is pre-g-irresolute.*
   (b) $]F^{\rightarrow}A[^{S_2}_{pre-g} \subseteq F^{\rightarrow}]A[^{S_1}_{pre-g}$, *for all* $A \in \varphi_1$.
   (c) $f^{\leftarrow}]B[^{S_2}_{pre-g} \subseteq]f^{\leftarrow}B[^{S_1}_{pre-g}$, *for all* $B \in \varphi_2$.
2. *The following are equivalent:*
   (a) (f, F) *is pre-g-co-irresolute.*
   (b) $f^{\rightarrow}[A]^{S_1}_{pre-g} \subseteq [f^{\rightarrow}A]^{S_2}_{pre-g}$, *for all* $A \in \varphi_1$.
   (c) $[F^{\leftarrow}B]^{S_1}_{pre-g} \subseteq F^{\leftarrow}[B]^{S_2}_{pre-g}$, *for all* $B \in \varphi_2$.

Proof. We prove (1), leaving the dual proof of (2) to the interested reader.
$(a) \Rightarrow (b)$. Take $A \in \varphi_1$. Then

$$f^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{pre-g} \subseteq f^{\leftarrow}(F^{\rightarrow}A) \subseteq A$$

by [[5], Theorem 2.24 (2a)]. Now $f^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{pre-g} = F^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{pre-g} \in prego(S_1)$ by pre-g-irresolute, so $f^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{pre-g} \subseteq]A[^{S_1}_{pre-g}$ and applying [[5], Theorem 2.24 (2b)] gives

$$]F^{\rightarrow}A[^{S_2}_{pre-g} \subseteq F^{\rightarrow}(f^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{pre-g} \subseteq F^{\rightarrow}]A[^{S_1}_{pre-g},$$

which is the required inclusion.
$(b) \Rightarrow (c)$. Take $B \in \varphi_2$. Applying inclusion (b) to $A = f^{\leftarrow}B$ and using [[5], Theorem 2.24 (2b)] gives

$$]B[^{S_2}_{pre-g} \subseteq]F^{\rightarrow}(f^{\leftarrow}B)[^{S_2}_{pre-g} \subseteq F^{\rightarrow}]f^{\leftarrow}B[^{S_1}_{pre-g}.$$

Hence, $f^{\leftarrow}]B[^{S_2}_{pre-g} \subseteq f^{\leftarrow}F^{\rightarrow}]f^{\leftarrow}B[^{S_1}_{pre-g} \subseteq]f^{\leftarrow}B[^{S_2}_{pre-g}$ by [[5], Theorem 2.24 (2a)].
$(c) \Rightarrow (a)$. Applying (c) for $B \in prego(S_2)$ gives

$$f^{\leftarrow}B = f^{\leftarrow}]B[^{S_2}_{pre-g} \subseteq]f^{\leftarrow}B[^{S_1}_{pre-g},$$

so $F^{\leftarrow}B = f^{\leftarrow}B =]f^{\leftarrow}B[^{S_1}_{pre-g} \in prego(S_1)$. Hence, (f, F) is pre-g-irresolute.

Theorem 3.5. *Let $(S_j, \varphi_j, \tau_j, k_j, \sigma_j)$, $j = 1, 2$, complemented ditopology and $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be complemented difunction. If (f, F) is pre-g-continuous then (f, F) is pre-g-cocontinuous.*

Proof. Since (f, F) is complemented, $(F^{'}, f^{'}) = (f, F)$. From [[5], Lemma 2.20], $\sigma_1((f^{'})^{\leftarrow}(B)) = f^{\leftarrow}(\sigma_2(B))$ and $\sigma_1((F^{'})^{\leftarrow}(B)) = F^{\leftarrow}(\sigma_2(B))$ for all $B \in \varphi_2$. The proof is clear from these equalities.

Corollary 3.6. *Let $(S_j, \varphi_j, \tau_j, k_j, \sigma_j)$, $j = 1, 2$, complemented ditopology and $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be complemented difunction. If (f, F) is pre-g-irresolute then (f, F) is pre-g-co-irresolute.*

Proof. Clear.

Definition 3.7. *A complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$ is called pre-g-compact if every cover of S by pre-g-open sets has a finite subcover. Here we recall that $C = \{A_j : j \in J\}$, $A_j \in \varphi$ is a cover of S if $\bigvee C = S$.*

Corollary 3.8. *Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. Then:*

1. *Every pre-g-compact is compact.*
2. *Every g-compact is pre-g-compact.*

Proof. Clear.

Theorem 3.9. *If $(S, \varphi, \tau, k, \sigma)$ is pre-g-compact and $L = \{F_j : j \in J\}$ is a family of pre-g-closed sets with $\cap L = \phi$, then $\cap\{F_j : j \in J^{'}\} = \phi$ for $J^{'} \subseteq J$ finite.*

Proof. Suppose that $(S, \varphi, \tau, k, \sigma)$ is pre-g-compact and let $L = \{F_j : j \in J\}$ be a family of pre-g-closed sets with $\cap L = \phi$. Clearly $C = \{\sigma(F_j) : j \in J\}$ is a family of pre-g-open sets. Moreover,

$$\bigvee C = \bigvee\{\sigma(F_j) : j \in J\} = \sigma(\cap\{F_j : j \in J\}) = \sigma(\phi) = S,$$

and so we have $J^{'} \subseteq J$ finite with $\bigvee\{\sigma(F_j) : j \in J^{'}\} = S$. Hence $\cap\{F_j : j \in J^{'}\} = \phi$.

Theorem 3.10. *Let $(f,F):(S_1,\varphi_1,\tau_1,k_1,\sigma_1)\to(S_2,\varphi_2,\tau_2,k_2,\sigma_2)$ be an pre-g-irresolute difunction. If $A\in\varphi_1$ is pre-g-compact then $f^{\to}A\in\varphi_2$ is pre-g-compact.*

Proof. Take $f^{\to}A\subseteq\bigvee_{j\in J}G_j$, where $G_j\in prego(S_2)$, $j\in J$. Now by [[5], Theorem 2.24 (2a) and Corollary 2.12 (2)] we have

$$A\subseteq F^{\leftarrow}(f^{\to}A)\subseteq F^{\leftarrow}(\bigvee_{j\in J}G_j)=\bigvee_{j\in J}F^{\leftarrow}G_j.$$

Also, $F^{\leftarrow}G_j\in prego(S_1)$ because (f,F) is pre-g-irresolute. So by the pre-g-compactness of A there exists $J^{'}\subseteq J$ finite such that $A\subseteq\cup_{j\in J'}F^{\leftarrow}G_j$. Hence

$$f^{\to}A\subseteq f^{\to}(\cup_{j\in J'}F^{\leftarrow}G_j)=\cup_{j\in J'}f^{\to}(F^{\leftarrow}G_j)\subseteq\cup_{j\in J'}G_j$$

by [[5], Corollary 2.12 (2) and Theorem 2.24 (2b)]. This establishes that $f^{\to}A$ is pre-g-compact.

Corollary 3.11. *Let $(f,F):(S_1,\varphi_1,\tau_1,k_1,\sigma_1)\to(S_2,\varphi_2,\tau_2,k_2,\sigma_2)$ be a surjective pre-g-irresolute difunction. Then, if $(S_1,\varphi_1,\tau_1,k_1,\sigma_1)$ is pre-g-compact so is $(S_2,\varphi_2,\tau_2,k_2,\sigma_2)$.*

Proof. This follows by taking $A=S_1$ in Theorem 3.10 and noting that $f^{\to}S_1=f^{\to}(F^{\leftarrow}S_2)=S_2$ by [[5], Proposition 2.28 (1c) and Corollary 2.33 (1)].

Definition 3.12. *A complemented ditopological texture space $(S,\varphi,\tau,k,\sigma)$ is called pre-g-stable if every pre-g-closed set $F\in\varphi\setminus\{S\}$ is pre-g-compact in S.*

Corollary 3.13. *Let $(S,\varphi,\tau,k,\sigma)$ be a complemented ditopological texture space. Then:*

1. *Every pre-g-stable is stable.*
2. *Every g-stable is pre-g-stable.*

Proof. Clear.

Theorem 3.14. *Let $(S,\varphi,\tau,k,\sigma)$ be pre-g-stable. If G is an pre-g-open set with $G\neq\phi$ and $D=\{F_j:j\in J\}$ is a family of pre-g-closed sets with $\cap_{j\in J}F_j\subseteq G$ then $\cap_{j\in J'}F_j\subseteq G$ for a finite subsets $J^{'}$ of J.*

Proof. Let $(S,\varphi,\tau,k,\sigma)$ be pre-g-stable, let G be an pre-g-open set with $G\neq\phi$ and $D=\{F_j:j\in J\}$ be a family of pre-g-closed sets with $\cap_{j\in J}F_j\subseteq G$. Set $K=\sigma(G)$. Then K is pre-g-closed and satisfies $K\neq S$. Hence K is pre-g-compact. Let $C=\{\sigma(F)|F\in D\}$. Since $\cap D\subseteq G$ we have $K\subseteq\bigvee C$, that is C is an pre-g-open cover of K. Hence there exists $F_1,F_2,...,F_n\in D$ so that

$$K\subseteq\sigma(F_1)\cup\sigma(F_2)\cup...\cup\sigma(F_n)=\sigma(F_1\cap F_2\cap...\cap F_n).$$

This gives $F_1\cap F_2\cap...\cap F_n\subseteq\sigma(K)=G$, so $\cap_{j\in J'}F_j\subseteq G$ for a finite subsets $J^{'}=\{1,2,...,n\}$ of J.

Theorem 3.15. *Let* $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$, $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ *be two complemented ditopological texture spaces with* $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$ *is pre-g-stable, and* $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ *be an pre-g-bi-irresolute surjective difunction. Then* $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ *is pre-g-stable.*

Proof. Take $K \in pregc(S_2)$ with $K \neq S_2$. Since (f, F) is pre-g-co-irresolute, so $f^{\leftarrow} K \in pregc(S_1)$. Let us prove that $f^{\leftarrow} K \neq S_1$. Assume the contrary. Since $f^{\leftarrow} S_2 = S_1$, by [[5], Lemma 2.28 (1c)] we have $f^{\leftarrow} S_2 \subseteq f^{\leftarrow} K$, whence $S_2 \subseteq K$ by [[5], Corollary 2.33 (1 ii)] as (f, F) is surjective. This is a contradiction, so $f^{\leftarrow} K \neq S_1$. Hence $f^{\leftarrow}(K)$ is pre-g-compact in $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$ by pre-g-stability. As (f, F) is pre-g-irresolute, $f^{\rightarrow}(f^{\leftarrow} K)$ is pre-g-compact for the ditopology (τ_2, k_2) by Theorem 3.10, and by [[5], Corollary 2.33 (1)] this set is equal to K. This establishes that $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ is pre-g-stable.

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