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An application of measures of noncompactness in the investigation of boundedness of solutions of second-order neutral difference equations

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Abstract

The purpose of this paper is to investigate a nonlinear second-order neutral difference equation of the form

$$\Delta(r_n \Delta(x_n + p_n x_{n-k})) + a_n f(x_n) = 0,$$

where $x : \mathbb{N}_0 \rightarrow \mathbb{R}$, $a : \mathbb{N}_0 \rightarrow \mathbb{R}$, $p, r : \mathbb{N}_0 \rightarrow \mathbb{R} \setminus \{0\}$, $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function, and k is a given positive integer. Sufficient conditions for the existence of a bounded solution of this equation are obtained. Also, stability and asymptotic stability of this equation are studied. Additionally, the Emden-Fowler difference equation is considered as a special case of the above equation. The obtained results are illustrated by examples.

MSC: 39A10; 39A22; 39A30

Keywords: difference equation; measures of noncompactness; Darbo's fixed point theorem; boundedness; stability; Emden-Fowler equation

1 Introduction

In presented paper we study a nonlinear second-order difference equation of the form

$$\Delta(r_n \Delta(x_n + p_n x_{n-k})) + a_n f(x_n) = 0, \quad (1)$$

where $x : \mathbb{N}_0 \rightarrow \mathbb{R}$, $a : \mathbb{N}_0 \rightarrow \mathbb{R}$, $p, r : \mathbb{N}_0 \rightarrow \mathbb{R} \setminus \{0\}$, and $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function. Here $\mathbb{N}_0 := \{0, 1, 2, \dots\}$, $\mathbb{N}_k := \{k, k+1, k+2, \dots\}$, where k is a given positive integer and \mathbb{R} is a set of all real numbers. By a solution of equation (1), we mean a sequence $x : \mathbb{N}_0 \rightarrow \mathbb{R}$ which satisfies (1) for every $n \in \mathbb{N}_0$.

Putting $f(x) = x^\gamma$, where $\gamma < 1$ is a quotient of two odd integers, $r_n \equiv 1$ and $p_n \equiv p \in (0, \infty)$, $p \neq 1$ in equation (1), we get an Emden-Fowler difference equation of the form

$$\Delta^2(x_n + p x_{n-k}) + a_n x_n^\gamma = 0. \quad (2)$$

In the last years many authors have been interested in studying the asymptotic behavior of solutions of difference equations, in particular, second-order difference equations (see,

for example, papers of Medina and Pinto [1], Migda [2], Migda and Migda [3], Migda *et al.* [4], Musielak and Popena [5], Popena and Werbowski [6], Schmeidel [7], Schmeidel and Zbąszyniak [8] and Thandapani *et al.* [9]).

Neutral difference equations were studied in many other papers by Grace and Lalli [10] and [11], Lalli and Zhang [12], Migda and Migda [13], Luo and Bainov [14], and Luo and Yu [15].

Some relevant results related to this topic can be found in papers by Baštinec *et al.* [16], Baštinec *et al.* [17], Berezansky *et al.* [18], Diblík and Hlavičková [19], and Diblík *et al.* [20].

For the reader's convenience, we note that the background for difference equations theory can be found, *e.g.*, in the well-known monograph by Agarwal [21] as well as in those by Elaydi [22], Kocić and Ladas [23], or Kelley and Peterson [24].

The theory of measures of noncompactness can be found in the book of Akhmerov *et al.* [25] and in the book of Banaś and Goebel [26]. In our paper, we used axiomatically defined measures of noncompactness as presented in paper [27] by Banaś and Rzepka.

2 Measures of noncompactness and Darbo's fixed point theorem

Let $(E, \|\cdot\|)$ be an infinite-dimensional Banach space. If X is a subset of E , then \bar{X} , $\text{Conv } X$ denote the closure and the convex closure of X , respectively. Moreover, we denote by \mathcal{M}_E the family of all nonempty and bounded subsets of E and by \mathcal{N}_E the subfamily consisting of all relatively compact sets.

Definition 1 A mapping $\mu : \mathcal{M}_E \rightarrow [0, \infty)$ is called a measure of noncompactness in E if it satisfies the following conditions:

- 1° $\ker \mu = \{X \in \mathcal{M}_E : \mu(X) = 0\} \neq \emptyset$ and $\ker \mu \subset \mathcal{N}_E$,
- 2° $X \subset Y \Rightarrow \mu(X) \leq \mu(Y)$,
- 3° $\mu(\bar{X}) = \mu(X) = \mu(\text{Conv } X)$,
- 4° $\mu(\alpha X + (1 - \alpha)Y) \leq \alpha \mu(X) + (1 - \alpha)\mu(Y)$ for $0 \leq \alpha \leq 1$,
- 5° if $X_n \in \mathcal{M}_E$, $X_{n+1} \subset X_n$, $X_n = \bar{X}_n$ for $n = 1, 2, 3, \dots$ and $\lim_{n \rightarrow \infty} \mu(X_n) = 0$, then $\bigcap_{n=1}^{\infty} X_n \neq \emptyset$.

The following Darbo's fixed point theorem given in [27] is used in the proof of the main result.

Theorem 1 Let M be a nonempty, bounded, convex, and closed subset of the space E , and let $T : M \rightarrow M$ be a continuous operator such that $\mu(T(X)) \leq k\mu(X)$ for all nonempty subset X of M , where $k \in [0, 1)$ is a constant. Then T has a fixed point in the subset M .

We consider the Banach space l^∞ of all real bounded sequences $x : \mathbb{N}_0 \rightarrow \mathbb{R}$ equipped with the standard supremum norm, *i.e.*,

$$\|x\| = \sup_{n \in \mathbb{N}_0} |x_n| \quad \text{for } x \in l^\infty.$$

Let X be a nonempty, bounded subset of l^∞ , $X_n = \{x_n : x \in X\}$ (it means X_n is a set of n th terms of any sequence belonging to X), and let

$$\text{diam } X_n = \sup\{|x_n - y_n| : x, y \in X\}.$$

We use the following measure of noncompactness in the space l^∞ (see [26]):

$$\mu(X) = \limsup_{n \rightarrow \infty} \text{diam } X_n.$$

3 Main result

In this section, sufficient conditions for the existence of a bounded solution of equation (1) are derived. Further, stable solutions of (1) are studied. We start with the following theorem.

Theorem 2 *Let*

$$f : \mathbb{R} \rightarrow \mathbb{R} \text{ be a continuous function,} \tag{3}$$

and let there exist constants L and M such that for all $x \in \mathbb{R}$,

$$|f(x)| \leq M|x| + L, \tag{4}$$

the sequence $p : \mathbb{N}_0 \rightarrow \mathbb{R} \setminus \{0\}$ satisfies the following condition:

$$-1 < \liminf_{n \rightarrow \infty} p_n \leq \limsup_{n \rightarrow \infty} p_n < 1, \tag{5}$$

sequences $a : \mathbb{N}_0 \rightarrow \mathbb{R}$, $r : \mathbb{N}_0 \rightarrow \mathbb{R} \setminus \{0\}$ are such that

$$\sum_{n=0}^{\infty} \left| \frac{1}{r_n} \right| \sum_{i=n}^{\infty} |a_i| < \infty. \tag{6}$$

Then there exists a bounded solution $x : \mathbb{N}_0 \rightarrow \mathbb{R}$ of equation (1).

Proof Condition (5) implies that there exist $n_1 \in \mathbb{N}_0$ and a constant $P \in [0, 1)$ such that

$$|p_n| \leq P < 1 \quad \text{for } n \geq n_1. \tag{7}$$

The remainder of a series is the difference between the n th partial sum and the sum of a series. Let us denote by α_n the remainder of series $\sum_{n=0}^{\infty} \left| \frac{1}{r_n} \right| \sum_{i=n}^{\infty} |a_i|$ so that

$$\alpha_n = \sum_{j=n}^{\infty} \left| \frac{1}{r_j} \right| \sum_{i=j}^{\infty} |a_i|. \tag{8}$$

From (6), the remainder α_n tends to zero. Therefore, we can denote

$$\lim_{n \rightarrow \infty} \alpha_n = 0. \tag{9}$$

Let us denote that C is a given positive constant. Condition (6) implies that there exists a positive integer n_2 such that

$$\alpha_n \leq C \frac{1 - P}{2(CM + L)} \tag{10}$$

for $n \geq n_2$.

We define a set B as follows:

$$B := \{(x_n)_{n=0}^\infty : |x_n| \leq C \text{ for } n \in \mathbb{N}_{n_3}\}, \tag{11}$$

where $\mathbb{N}_{n_3} := \{n_3, n_3 + 1, n_3 + 2, \dots\}$ and $n_3 = \max\{n_1, n_2\}$.

It is not difficult to prove that B is a nonempty, bounded, convex, and closed subset l^∞ .

Let us define a mapping $T : B \rightarrow l^\infty$ as follows:

$$(Tx)_n = -p_n x_{n-k} - \sum_{j=n}^\infty \frac{1}{r_j} \sum_{i=j}^\infty a_i f(x_i) \tag{12}$$

for any $n \in \mathbb{N}_{n_3}$.

We will prove that the mapping T has a fixed point in B .

Firstly, we show that $T(B) \subset B$. Indeed, if $x \in B$, then by (12), (7), (11), and (10), we have

$$\begin{aligned} |(Tx)_n| &\leq |p_n| |x_{n-k}| + \sum_{j=n}^\infty \left| \frac{1}{r_j} \right| \sum_{i=j}^\infty |a_i| |f(x_i)| \\ &\leq PC + \sum_{j=n}^\infty \left| \frac{1}{r_j} \right| \sum_{i=j}^\infty |a_i| (M|x_i| + L) \\ &\leq CP + (MC + L) \sum_{j=n}^\infty \left| \frac{1}{r_j} \right| \sum_{i=j}^\infty |a_i| \\ &\leq CP + (CM + L)\alpha_n = C \frac{P+1}{2} \leq C \quad \text{for } n \in \mathbb{N}_{n_3}. \end{aligned}$$

Next, we prove that T is continuous. Let $x^{(p)}$ be a sequence in B such that $\|x^{(p)} - x\| \rightarrow 0$ as $p \rightarrow \infty$. Because of (3), we have $\|f(x^{(p)}) - f(x)\| \rightarrow 0$. Since B is closed, $x \in B$. Now, utilizing (12), we get

$$|(Tx^{(p)})_n - (Tx)_n| \leq |p_n| |x_{n-k}^{(p)} - x_{n-k}| + \sum_{j=n}^\infty \left| \frac{1}{r_j} \right| \sum_{i=j}^\infty |a_i| |f(x_i^{(p)}) - f(x_i)|.$$

Hence, by (7) and (8),

$$|(Tx^{(p)})_n - (Tx)_n| \leq P|x_{n-k}^{(p)} - x_{n-k}| + \alpha_n \sup_{i \geq n} |f(x_i^{(p)}) - f(x_i)|, \quad n \in \mathbb{N}_{n_3}.$$

Therefore, by (10),

$$\|Tx^{(p)} - Tx\| \leq P\|x^{(p)} - x\| + C \frac{1-P}{2(CM+L)} \|f(x_i^{(p)}) - f(x_i)\| \rightarrow 0$$

and

$$\lim_{p \rightarrow \infty} \|Tx^{(p)} - Tx\| = 0.$$

This means that T is continuous.

Now, we need to compare a measure of noncompactness of any subset X of B and $T(X)$. Let us take a nonempty set $X \subset B$. For any sequences $x, y \in X$, we get

$$|(Tx)_n - (Ty)_n| \leq P|x_n - y_n| + CM\alpha_n, \quad n \in \mathbb{N}_{n_3}.$$

Hence, we obtain

$$\text{diam}(T(X))_n \leq k \text{diam} X_n + CM\alpha_n.$$

This yields

$$\limsup_{n \rightarrow \infty} \text{diam}(T(X))_n \leq k \limsup_{n \rightarrow \infty} \text{diam} X_n.$$

From the above, for any $X \subset B$, we have $\mu(T(X)) \leq k\mu(X)$, where $k = \frac{P+1}{2} \in [0, 1)$.

By virtue of Theorem 1, we conclude that T has a fixed point in the set B . It means that there exists $x \in B$ such that $x_n = (Tx)_n$. Thus

$$x_n = -p_n x_{n-k} + \sum_{j=n}^{\infty} \frac{1}{r_j} \sum_{i=j}^{\infty} a_i f(x_i), \quad n \in \mathbb{N}_{n_3} \tag{13}$$

for any $n \in \mathbb{N}_{n_3}$. To show that there exists a connection between the fixed point $x \in B$ and the existence of a solution of equation (1), we use the operator Δ for both sides of the following equation:

$$x_n + p_n x_{n-k} = \sum_{j=n}^{\infty} \frac{1}{r_j} \sum_{i=j}^{\infty} a_i f(x_i),$$

which is obtained from (13). We find that

$$\Delta(x_n + p_n x_{n-k}) = -\frac{1}{r_n} \sum_{i=n}^{\infty} a_i f(x_i), \quad n \in \mathbb{N}_{n_3}.$$

Using again the operator Δ for both sides of the above equation, we get equation (1) for $n \in \mathbb{N}_{n_3}$. The sequence x , which is a fixed point of the mapping T , is a bounded sequence which fulfills equation (1) for large n . If $n_3 \leq k$, the proof is ended. If $n_3 > k$, then we find previous $n_3 - k + 1$ terms of the sequence x by the formula

$$x_{n-k+l} = \frac{1}{p_{n+l}} \left(-x_{n+l} + \sum_{j=n+l}^{\infty} \frac{1}{r_j} \sum_{i=j}^{\infty} a_i f(x_i) \right), \quad \text{where } l \in \{0, 1, 2, \dots, k-1\},$$

the results of which follow directly from (1). It means that equation (1) has at least one bounded solution $x : \mathbb{N}_0 \rightarrow \mathbb{R}$.

This completes the proof. □

Example 1 Let us consider the equation

$$\Delta \left((-1)^n \Delta \left(x_n + \left(\frac{1}{2} + \frac{1}{2^n} \right) x_{n-2} \right) \right) + \frac{3(-1)^{n+1}}{2^{n+2}} (x_n)^{\frac{1}{3}} = 0.$$

All the assumptions of Theorem 2 are fulfilled. Then there exists a bounded solution x of the above equation. So, the sequence $x_n = (-1)^n$ is such a solution.

Remark 1 Assume that

$$p_n \equiv p \in (0, 1) \tag{14}$$

and

$$\sum_{n=0}^{\infty} \sum_{i=n}^{\infty} |a_i| < \infty \tag{15}$$

in an Emden-Fowler difference equation of the form (2). Then there exists a bounded solution of equation (2).

Proof Here all the assumptions of Theorem 2 are satisfied, e.g., the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by formula $f(x) = x^\gamma$ is a continuous function, and $|f(x)| = |x^\gamma| \leq \gamma|x| + 1 - \gamma$. So, taking $M = \gamma$ and $L = 1 - \gamma$, we obtain condition (4). The thesis follows directly from Theorem 2. \square

Finally, sufficient conditions for the existence of an asymptotically stable solution of equation (1) will be presented. We recall the following definition which can be found in [27].

Definition 2 Let x be a real function defined, bounded, and continuous on $[0, \infty)$. The function x is an asymptotically stable solution of the equation

$$x = Fx. \tag{16}$$

It means that for any $\varepsilon > 0$, there exists $T > 0$ such that for every $t \geq T$ and for every other solution y of equation (16), the following inequality holds:

$$|x(t) - y(t)| \leq \varepsilon.$$

Theorem 3 Assume that there exists a positive constant D such that

$$|f(x) - f(y)| \leq D|x - y| \tag{17}$$

for any $x, y \in \mathbb{R}$, and conditions (3)-(6) hold. Then equation (1) has at least one asymptotically stable solution $x : \mathbb{N}_0 \rightarrow \mathbb{R}$.

Proof From Theorem 2, equation (1) has at least one bounded solution $x : \mathbb{N}_0 \rightarrow \mathbb{R}$ which can be rewritten in the form

$$x_n = (Tx)_n, \tag{18}$$

where a mapping T is defined by (12).

Because of Definition 2, the sequence x is an asymptotically stable solution of the equation $x_n = (Tx)_n$, which means that for any $\varepsilon > 0$, there exists $n_4 \in \mathbb{N}_0$ such that for every $n \geq n_4$ and for every other solution y of equation (1), the following inequality holds:

$$|x_n - y_n| \leq \varepsilon. \tag{19}$$

From (12), by (7), we have

$$|(Tx)_n - (Ty)_n| \leq P|x_{n-k} - y_{n-k}| + \sum_{j=n}^{\infty} \left| \frac{1}{r_j} \right| \sum_{i=j}^{\infty} |a_i| |f(x_i) - f(y_i)|$$

for $n \geq n_3$. The above and (17) yield

$$|(Tx)_n - (Ty)_n| \leq P|x_{n-k} - y_{n-k}| + D \sum_{j=n}^{\infty} \left| \frac{1}{r_j} \right| \sum_{i=j}^{\infty} |a_i| |x_i - y_i|$$

for $n \geq n_5 = \max\{n_3, n_4\}$. Hence, by (8) and (19), we obtain

$$|(Tx)_n - (Ty)_n| \leq P|x_{n-k} - y_{n-k}| + D \sup_{i \geq n} |x_i - y_i| \alpha_n$$

for $n \geq n_5$. Thus, linking the above inequality and (18), we have

$$|x_n - y_n| \leq P|x_{n-k} - y_{n-k}| + D \sup_{i \geq n} |x_i - y_i| \alpha_n. \tag{20}$$

Let us denote

$$\limsup_{n \rightarrow \infty} |x_n - y_n| = l.$$

Because of

$$\limsup_{n \rightarrow \infty} |x_n - y_n| = \limsup_{n \rightarrow \infty} |x_{n-k} - y_{n-k}|,$$

and (20), we get

$$l \left(1 - P - D \lim_{n \rightarrow \infty} \alpha_n \right) \leq 0.$$

From the above and (9), we obtain

$$l(1 - P) \leq 0 \quad \text{for enough large } n.$$

Suppose to the contrary that $l > 0$. Thus, we obtain a contradiction with the fact that $0 < P < 1$. Therefore we get $\limsup_{n \rightarrow \infty} |x_n - y_n| = 0$. This completes the proof. \square

Remark 2 Under conditions (3)-(6) and (17), any bounded solution of equation (1) is asymptotically stable.

Proof If boundedness of a solution of equation (1) is assumed, then by virtue of the same arguments as in Theorem 3, the thesis of the above remark is obtained. \square

Example 2 Let us consider equation (1) with $f(x) = x$, $a_n = \Delta^2 p_n$ and $\sum_{n=0}^{\infty} \sum_{i=n}^{\infty} |a_i| < \infty$. Such an equation has infinitely many solutions of the form $x_n \equiv c$, where c is a real constant. All the assumptions of Theorem 3 are fulfilled, then each of such solutions is asymptotically stable.

Theorem 4 Assume that $L = 0$ in (4). Under conditions (3)-(6) and (17), if there exists a zero solution of equation (1), then it is asymptotically stable.

Proof If $L = 0$, then condition (4) takes the form $|f(x)| \leq M|x|$. This implies that $f(0) = 0$. Hence, the sequence $x \equiv 0$ is a bounded solution of equation (1). By Remark 2, the zero solution is asymptotically stable. \square

Competing interests

The author declares that they have no competing interests.

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