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Improved positioning algorithm based on linear constraints on scatterers

Fei Zhou*, Xin-yue Fan and Ting-Ying Chen

Abstract

Non-line-of-sight (NLOS) error is a bottleneck problem influencing positioning accuracy. However, a large number of scatterers distribute randomly in the surrounding of the mobile station (MS) in the dense multipath environment, such as urban. In most cases, there is no obstacle between scatterer and MS. So, the geographic information of scatterers around MS can be used to restraint NLOS errors and improve the positioning accuracy. If scatterer can be regarded as the virtual base station (BS), the precondition of the positioning algorithm is easier to satisfy than the traditional positioning algorithm, such as circle positioning algorithm. The algorithm proposed in this article selects suitable scatterers with linear constraints by analyzing the Doppler frequency shift of received signals which reflected by scatterers. Thereby, the selected scatterers and only two real BSs form a complete positioning system. In addition, because MS is motionless in most scenarios, BS must be moving to acquire the Doppler frequency shift. The algorithm proposed in this article is adjusted for the scenarios. And the scatterers with linear constraint can also be utilize fully. Simulation results show the algorithm proposed in this article in two different scenarios, not only simplifies the traditional algorithm, but also achieves the higher positioning accuracy.

Keywords: Linear constraint, Non-line-of-sight, Doppler frequency shift, Virtual base station

1. Introduction

Non-line-of-sight (NLOS) error badly influences the accuracy of wireless positioning technology in the dense multipath environment, such as urban. In recent years, some algorithms using the information of scatterer have been developed to restraint NLOS errors. Their advantages of performance have attracted the attention of many researchers. These algorithms are mainly divided into two categories. The first category of the positioning algorithm utilizes different scatterer models to obtain some statistic information, e.g., probability density function of time-of-arrival (TOA) or angle-of-arrival (AOA). These statistic information are used to compute parameters which related to positioning. Scatterer models commonly assume that scatterers distribute in the surrounding of mobile station (MS) or base station (BS) [1,2], giving an Elliptical scattering model which suited for the Microcellular communications system. The model assumed that scatterers uniformly distributed with the ellipse which the focus of ellipse are BS and MS [3] gives a disk model which suited for the macrocell communications system. In the model, scatterers are assumed to distribute with the disk around MS, and BS is out of the disk [4,5], giving a type of geographical scattering model with non-uniform distribution of scatterers, for example, Gaussian distribution scattering model, Conical scattering model, Eccentro scattering model, and so on. According to these models, the statistic function, such as AOA, power azimuth spectrum, TOA, and power delay spectrum, can be acquired. Finally, some positioning algorithms based on scatterer model can apply TOA, AOA, or other parameters to compute the position of target [6-8].

The second category of positioning algorithm is independent of scatterer model, and utilizes the position of scatterer to compute the position of MS. The key of these algorithms is that NLOS error can be transformed into fixed factors. And positioning accuracy mainly depends on the accuracy of measurement parameters. Thereby, it is possible to achieve high positioning accuracy of scatterers and MS [9,10], giving some scatterer positioning algorithms based on nonlinear least square theory. These algorithms utilize synthetically spatial–time-

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frequency information of received signal to compute the position of scatterer and MS. Because the received multipath signals reflected by scatterers include lots of information about scatterers, the position of scatterer can be computed in theory [11], giving a network grid search algorithm based on single reflection geometry model, which utilizes the information of multiple BS to restraint NLOS error. In [12-14], several possible results of positioning and partial information of scatterer can be acquired from the priori information of BSs, and the selection of the final correct result can be fulfilled by scatterer information. If BS can measure the AOA and the rate of AOA variation of the measurement signal from scatterer, the position of scatterer can be computed. So, the constraint relation among BS, scatterer, and MS can be designed and can also be used to restraint positioning error. In most cases, the scatterer is assumed to be static. If the position of scatterer is known, scatterer can be regarded as virtual BS. Thereby, a circle fitting algorithm can be proposed based on virtual BS. In fact, the relation among scatterers, MS and BS can be obtained by analyzing AOA and angle-of-departure of signal. Thereby, system of linear equations can be designed from TOA. Positioning algorithms based on the linear equations have much advantage. For example, scatters and MS can be positioned simultaneously. And positioning accuracy only depends on measurement parameter. But the constraint condition of the category algorithm is hard to satisfy. In addition, if we can successfully identify NLOS propagation or LOS propagation from related parameters, positioning accuracy would be improved very much. If the number of LOS is enough, positioning algorithms is independent of NLOS error [15,16]. However, in a real environment, the number of LOS is too small or zero. So, the category of algorithms is hard to apply in real application. The algorithm proposed in this article is based on the second category of positioning algorithm. Referring to the previous study, multiple scatterers and MS in the same line are assumed to have the same absolute Doppler frequency shifts. Many methods that utilize linear constraint between scatterers and MS are introduced in [10]. Three cases are commonly considered in positioning algorithm: (i) one scatterer and MS are in the same line; (ii) two scatterers and MS are in the same line; (iii) three or more scatterers and MS are in the same line. In the first case, if only one scatterer is in the same line with MS, it is impossible to acquire enough scatterer by multipath signal pairing. So, the positioning algorithm proposed in this article is not effective. The third case is studied by so many researchers. However, the second case is more general than the third case. Thereby, the second case is more valuable in implementing the proposed positioning algorithm. Due to high performance of line constraint positioning and inappeasable precondition, this article proposed an improved positioning algorithm using linear constraint on two BSs [17-20].

The remainder of this article is organized as follows. Section 2 presents and analyzes traditional positioning algorithm, which uses linear constraint on scatterers. The key points multipath pairing and TOA reconstruction are analyzed. Section 3 introduces an improved positioning algorithm based on linear constraint on only two scatterers. Section 4 gives a special and valuable case for the moving BS situation. Then the proposed positioning algorithm in Section 3 is adjusted to suit for the new case. Section 5 provides the simulation and results. Finally, conclusion and the further work are given.

2. Traditional positioning algorithm using linear constraints on scatterers

Positioning algorithms using linear constraints on scatterers are mainly based on the single reflect model. When scatterer and MS are in the same line, their Doppler frequency shifts on multipath signals are the same. Thus, these multipath signals can be extracted by filtering all received signals. Assuming that the coordinate of MS is (x_0, y_0), and the polar coordinates of the scatterer is (ρ_k, θ_k), $k = 1, 2, \ldots, N$, then the following linear Equations (1) and (2) are derived when the scatter and the MS are in the same line [21,22]:

$$y_0 = ax_0 + b \tag{1}$$

$$\mathbf{s} = a\mathbf{c} + b\mathbf{e} \tag{2}$$

where a and b are real numbers; $\mathbf{s}$, $\mathbf{c}$, and $\mathbf{e}$ are $N \times 1$ vectors, $s_k = \rho_k sin(\theta_k)$, $c_k = \rho_k cos(\theta_k)$, $e_k = 1$, $k = 1, 2, \ldots, N$. By using the following equation, we can derive TOA of l_k of the multipath signal reflected by kth scatter:

$$l_k = \rho_k + \sqrt{\left(\rho_k cos(\theta_k) - x_0\right)^2 + \left(\rho_k sin(\theta_k) - y_0\right)^2} \tag{3}$$

Equation (4) is then derived from Equation (3)

$$x_0^2 + y_0^2 = l_k^2 - 2\rho_k[l_k - x_0\cos(\theta_k) - y_0\sin(\theta_k)] \tag{4}$$

By combining Equations (1), (2), and (4), we can obtain

$$aP_k + 2bl_k - gc_k - hs_k = q_k \tag{5}$$

where

$$c_k = cos(\theta_k), s_k = cos(\theta_k), P_k = l_k^2 cos(\theta_k), q_k = l_k^2 sin(\theta_k), g = 2bx_0 + a\left(x_0^2 + y_0^2\right), h = 2by_0 - \left(x_0^2 + y_0^2\right);$$

Thus, from Equation (5)

$$ah + g = 2bx_0 + 2aby_0 \tag{6}$$

By substituting the TOA of l_k and the AOA of θ_k, which is measured by BS, into Equation (5), we can obtain Equation (7)

$$AX = B \tag{7}$$

where

$$X = [a, b, g, h]^T, B = [q_1, q_2, \ldots, q_k]^T,$$
$$A = \begin{bmatrix} P_1 & l_1 & -c_1 & -s_1 \\ P_2 & l_2 & -c_2 & -s_2 \\ \vdots & \vdots & \vdots & \vdots \\ P_N & l_N & -c_N & -s_N \end{bmatrix}$$

We can solve Equation (7) by using the least square (LS) algorithm.

$$X = \left(A^T A\right)^{-1} A^T B \tag{8}$$

By substituting a,b, g, and h into Equations (1) and (6), we can obtain the position (x_0, y_0) of MS.

$$\begin{cases} x_0 = \dfrac{ah + g - 2ab^2}{2b(1 + a^2)} \\ y_0 = \dfrac{a(ah + g - 2ab^2)}{2b(1 + a^2)} + b \end{cases} \tag{9}$$

3. Improved positioning algorithm based on linear constraints on scatterers in two BSs

Figure 1 shows a brief frame of the improved algorithm, which comprises three steps.

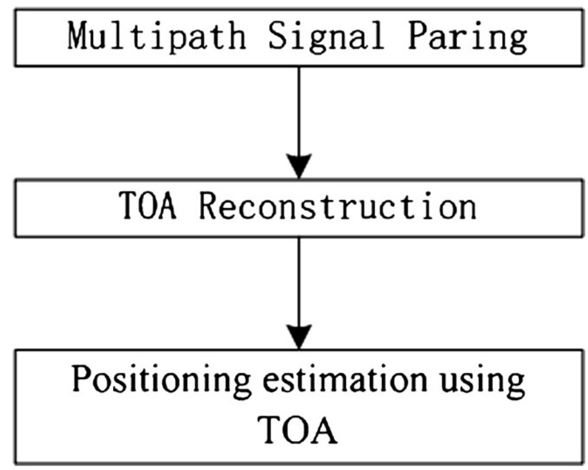

Figure 1 Positioning algorithm frame.

Step 1: *Multipath signal pairing*: The BS receives the multipath signal and measures the Doppler frequency shift of the signal. The signals that reflected from scatterers to BS are then paired. Two scatterers that lie on the same line with MS are selected.

Step 2: *TOA reconstruction*: After multipath signal pairing and scatterer selection, the chosen scatterers can be positioned based on the AOA measured by the two BSs. The distances between MS and the two scatterers can be computed. Finally, by applying the linear constraints of two scatterers, we can implement TOA reconstruction.

Step 3: *Positioning Estimation Using TOA*: Two scatterers are regarded as virtual BS. At the same time, the positions of two real BSs are known, four TOAs are obtained. Some existing algorithms can be used to compute for the position of the MS.

3.1. Multipath signal pairing

If two scatterers and one MS are in the same line, then the multipath components, the Doppler frequency shifts, reflected from two scatterers and received by a certain BS are all equivalent. Thus, the key to multipath signal pairing is distinguishing multipath signals with the same Doppler frequency shifts received from the same BS. As shown in Figure 2, the position parameter ($x_{si}y_{si}$) of the ith scatterer is acquired by using Equation (10)

$$\begin{aligned} x_{Si} &= \frac{x_2 - x_1 tan\beta_{2i} tan\beta_{1i} + (y_1 - y_2) tan\beta_{2i}}{1 - tan\beta_{1i}} \\ y_{Si} &= (x_{Si} - x_1) tan\beta_{1i} + y_1 \end{aligned} \tag{10}$$

where (x_1, y_1) and (x_2, y_2) are the position parameters of two BSs, and β_{ji} is the direction of arrival of the ith scatterer measured by the jth BS. So, β_{1i} and β_{2i} can be obtained by Equation (11)

$$\begin{aligned} tan\beta_{1i} &= \frac{x_{Si} - x_1}{y_{Si} - y_1} \\ tan\beta_{2i} &= \frac{x_{Si} - x_2}{y_{Si} - y_2} \end{aligned} \tag{11}$$

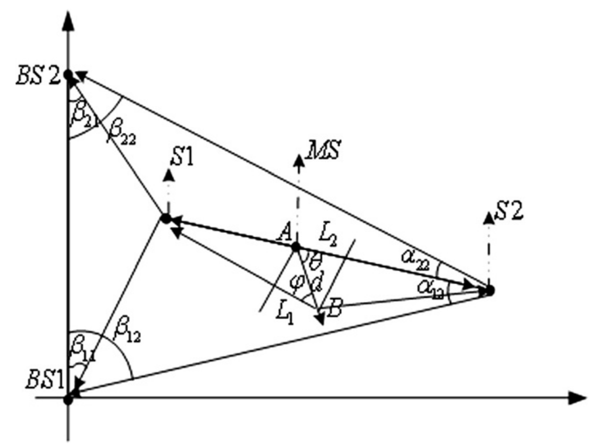

Figure 2 Layout of mobile terminal.

The linear distance between the ith BS and the jth BS is derived by Equation (12).

$$L_{ji} = \sqrt{\left(x_{Si} - x_j\right)^2 + \left(y_{Si} - y_j\right)^2} \tag{12}$$

If the measurement error is zero, then Equation (13) is logical.

$$L_i = l_{1i} - L_{1i} = l_{2i} - L_{2i} \tag{13}$$

where l_{ji} is TOA between jth scatter and ith BS. Equation (14) is the measurement error.

$$EN = \left[(l_{2i} - L_{2i}) - (l_{1i} - L_{1i})\right]^2 \tag{14}$$

Two datasets are paired when EN is close to minimum. Thus, four datasets from two BSs (l_{1i}, β_{1i}) *and* $(l_{2i}, \beta_{2i}), (l_{1i}, \beta_{1i})$ *and* $(l_{2j}, \beta_{2j}), (l_{1j}, \beta_{1j})$ *and* (l_{2i}, β_{2i}), *and* (l_{1j}, β_{1j}) *and* (l_{2j}, β_{2j}), are grouped into four pairs. The ENs of the datasets can be computed using Equation (14). The dataset with the smallest EN must be reflected by the same scatterer and must be measured by two BSs separately; that is, the set must have correct pairing. So, the other sets of data measured by the same two BSs are also correct pairing.

3.2. TOA reconstruction

If the multipath signals of the different BSs are successfully paired, then the positioning parameter of scatterers S_i and S_j, (x_{si}, y_{si}) and (x_{sj}, y_{sj}) can be computed by using Equation (10). The distance between the two scatterers can be derived by using Equation (15).

$$L_S = \sqrt{\left(x_{Si} - x_{Sj}\right)^2 + \left(y_{Si} - y_{Sj}\right)^2} \tag{15}$$

Thus, the line relation among S_i, S_j, and MS can be determined by L_i, L_j, and L_s.

If $L_j > L_s$ and $L_j > bL_i$, then S_i is located between S_j and MS.

If $L_i > L_s$ and $L_i > L_j$, then S_j is located between S_i and MS.

If $L_s > L_i$ and $L_s > L_j$, then MS is located between S_j and S_i.

In case (1), S_i, S_j, and BS1 form a triangle. Thus, Equation (16) can be derived.

$$cos\alpha_{1j} = \frac{L_S^2 + L_{1j}^2 - L_{1i}^2}{2L_S L_{1j}} \tag{16}$$

In another triangle formed by S_j, MS, and BS1, the line distance L_{line1} between MS and BS1 can be obtained by using Equation (17).

$$L_{line1} = \sqrt{r_j^2 + L_{1j}^2 - 2r_j L_{1j} cos\alpha_{1j}} \tag{17}$$

Similarly, the distance L_{line2} between MS and BS2 can be derived by Equation (18).

$$L_{line2} = \sqrt{r_j^2 + L_{2j}^2 - \frac{\left(L_S^2 + L_{2j}^2 - L_{2i}^2\right)r_j}{L_S}} \tag{18}$$

In case (2), on the basis of the triangle relations among S_i, S_j, and BSi ($i = 1,2$), we can calculate for L_{line1} and L_{line2} by using Equation (19). Thus, TOA reconstruction can be implemented.

$$\begin{aligned} L_{line1} &= \sqrt{r_i^2 + L_{1i}^2 - \frac{\left(L_S^2 + L_{1i}^2 - L_{1j}^2\right)r_i}{L_S}} \\ L_{line2} &= \sqrt{r_i^2 + L_{2i}^2 - \frac{\left(L_S^2 + L_{2i}^2 - L_{2j}^2\right)r_i}{L_S}} \end{aligned} \tag{19}$$

Similarly, the resolution of cases (1) and (2) is still in effect in case (3).

3.3. Positioning parameter of mobile computation

Figure 2 shows that S_i and S_j can be considered as virtual BS, and r_i and r_j are the TOAs in the line-of-sight (LOS) path of an MS to two scatterers S_i, S_j. The TOA in the LOS of BS1 and BS2 can be reconstructed. Thus, four TOAs from different BSs to MSs can be utilized to compute the positioning parameter of MS. The LS algorithm is the preferred algorithm.

$$(x_j - x_1)x + (y_j - y_1)y = \frac{1}{2}\left[x_j^2 + y_j^2 - \left(x_1^2 + y_1^2\right) + r_1^2 - r_j^2\right] \tag{20}$$

where $r_j^2 = (x_j - x)^2 + (y_j - y)^2$. Equation (20) can be derived and represented by the following vector matrix form, Equation (21)

$$AX = B \tag{21}$$

where

$$X = \begin{bmatrix} x \\ y \end{bmatrix} \tag{22}$$

$$A = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ x_{Si} - x_2 & y_{Si} - y_2 \\ x_{Sj} - x_{Si} & y_{Sj} - y_{Si} \end{bmatrix} \tag{23}$$

$$B = \begin{bmatrix} \frac{1}{2}\left[x_2^2 + y_2^2 - (x_1^2 + y_1^2) + L_{line1}^2 - L_{line2}^2\right] \\ \frac{1}{2}\left[x_{Si}^2 + y_{Si}^2 - (x_2^2 + y_2^2) + L_{line2}^2 - r_i^2\right] \\ \frac{1}{2}\left[x_{Sj}^2 + y_{Sj}^2 - (x_{Si}^2 + y_{Si}^2) + r_i^2 - r_j^2\right] \end{bmatrix} \tag{24}$$

The position of MS can also be computed by using the LS algorithm, as shown in Equation (8).

4. Improved algorithm on scatter linear restriction based on moving BS

The above algorithm is inefficient when the MS is static, because the Doppler frequency shift cannot be obtained. However, the Doppler frequency shift can be obtained if the BS is mobile. Thereby, a novel positioning algorithm based on mobile BS is proposed by utilizing two multipath signals. The following are the basic steps of the proposed algorithm:

(1) The BS receives a multipath signal and measures the Doppler frequency shift of that signal. Signals reflected from scatterers to BS are then paired. Among the pairings, two scatterers that lie in the same line with the MS are chosen.
(2) After the paired multipath signal that is reflected by two scatterers is chosen, the positions of the paired signal can be computed by utilizing the circle fitting algorithm [7,8] on the basis of the trajectory information of a single BS, TOA, and AOA.
(3) The TOA is reconstructed in each measurement point of any mobile trajectory of the BS.
(4) The measurement points of two scatterers and BS in the mobile trajectory are regarded as virtual BSs, and the reconstructed TOA is the TOA of LOS.

Thus, the position of the MS can be computed by employing the LS algorithm. The improved algorithm has two key points: multipath signals pairing and TOA/AOA reconstruction.

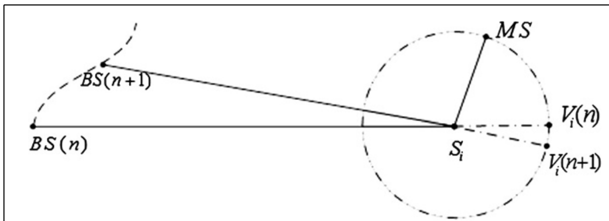

Figure 3 Circle motion trajectory of a pseudo-target.

4.1. Scatterer position estimation by circle fitting algorithm

4.1.1 Assumption

During the entire motion estimation, the positions of the two scatterers are regarded as scatter points and are assumed stationary.

Considering that the measured distance is greater than the perturbation motion of the scatterer, this perturbation is regarded as the measurement error of TOA. Therefore, the above assumption is reasonable, and scatterers have transient stationary characteristics. Assuming that N times measurement is executed in the motion trajectory of BS, as shown in Figure 3, the pseudo-target coordinate of the nth point $V(n) = (x_i(n), y_i(n))$ can be computed by using Equation (25).

$$\begin{cases} x_i(n) = x_n + l_i(n)\cos(\alpha_i(n)) \\ y_i(n) = y_n + l_i(n)\sin(\alpha_i(n)) \end{cases} \tag{25}$$

Here, (x_n, y_n) is the coordinate of the nth measurement point, $l_i(n)$ is the TOA of the measurement point, and $\alpha_i(n)$ is the AOA of the measurement point.

The MS shown in Figure 3 is a real mobile terminal. The MS is stationary during the entire process; thus, regardless of the motion trajectory of the BS, the distance r_i between the pseudo-target $V_i(n)$ to the measurement point BS(n) and the ith scatterer is the same. The nth pseudo-target to the measurement point BS(n) and MS are in the same circle, and the center of the circle is the

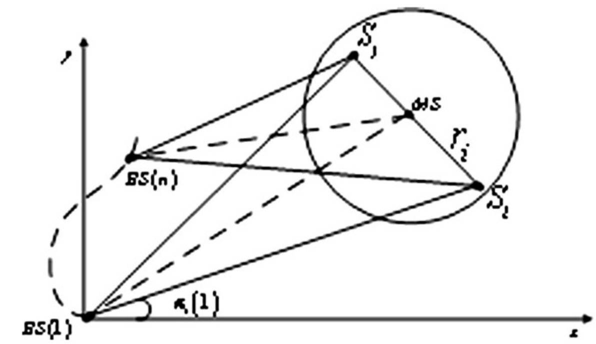

Figure 4 Single reflect model based on mobile BS.

Table 1 Simulation parameters

Coordinate of MS	**(800,800)**
Velocity along the *x*-axis of MS	10 m/s
Velocity along the *y*-axis of MS	10 m/s
Simulation number	2000
Multipath number	2

ith scatterer. The coordinate and radius of the circle are $S_i = (x_{si}, y_{si})$ and r_i, respectively.

$$(x_i(n) - x_{si})^2 + (y_i(n) - y_{si})^2 = r_i^2 \tag{26}$$

Equation (27) can be derived by using the LS algorithm:

$$A_i X_i = B_i \tag{27}$$

where

$$A_i = \begin{bmatrix} x_i(1) & y_i(1) & 1 \\ x_i(2) & y_i(2) & 1 \\ \vdots & \vdots & \vdots \\ x_i(N) & y_i(N) & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} x_i^2(1) + y_i^2(1) \\ x_i^2(2) + y_i^2(2) \\ \vdots \\ x_i^2(N) + y_i^2(N) \end{bmatrix}, X_i = \begin{bmatrix} -2x_{si} \\ -2y_{si} \\ x_{si}^2 + y_{si}^2 - r_i^2 \end{bmatrix}$$

When the number of measurement points is not less than three, the solution of the LS algorithm is as follows:

$$X_i = \left(A_i^T A_i\right)^{-1} A_i^T B_i \tag{28}$$

Thus,

$$\begin{cases} x_{si} = -X_i(1)/2 \\ y_{si} = -X_i(2)/2 \\ r_i = \sqrt{X_i^2(1) + X_i^2(2) - 4X_i(3)}/2 \end{cases} \tag{29}$$

4.2. TOA reconstruction of measurement point

The two scatter positioning coordinates (x_{si}, y_{si}) and (x_{sj}, y_{sj}) as well as the distances r_i and r_j between two scatterers and MS can be calculated by using Equation (29). The coordinate of the N measurement points can be computed by utilizing the initial coordinate and mobile trajectory during BS motion. The distance L_s between two scatterers can be derived by the following equation:

$$L_S = \sqrt{\left(x_{Si} - x_{Sj}\right)^2 + \left(y_{Si} - y_{Sj}\right)^2} \tag{30}$$

If $L_i > L_s$ and $L_i > L_j$, then the LOS distance of the nth measurement point L_n^{los} can be obtained from Equation (31).

$$L_n^{Los} = \left(\sqrt{r_i^2 + (l_i(n) - r_i)^2 - \left(\frac{\left(L_S^2 + (l_i(n) - r_i)^2 - (l_j(n) - r_j)^2\right) r_i}{L_S}\right)}\right)$$

$$ah + g = 2bx_0 + 2aby_0 \tag{31}$$

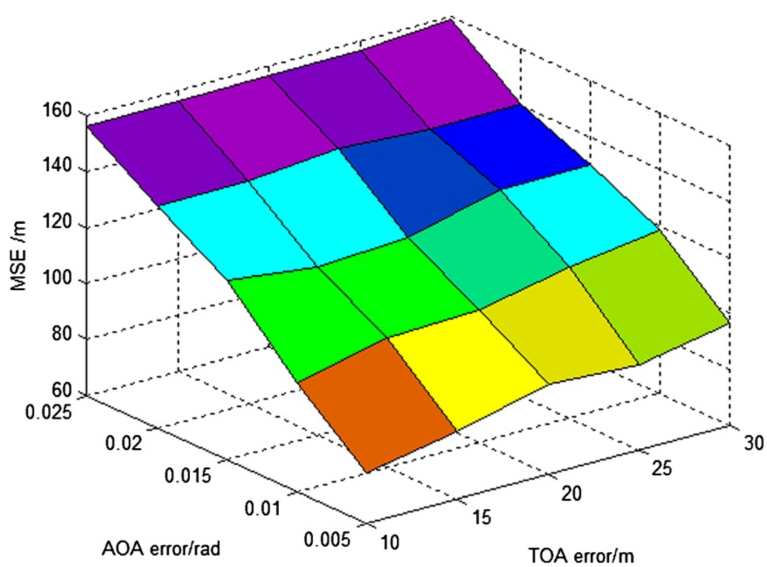

Figure 5 MSE of the different AOA and TOA measurement errors.

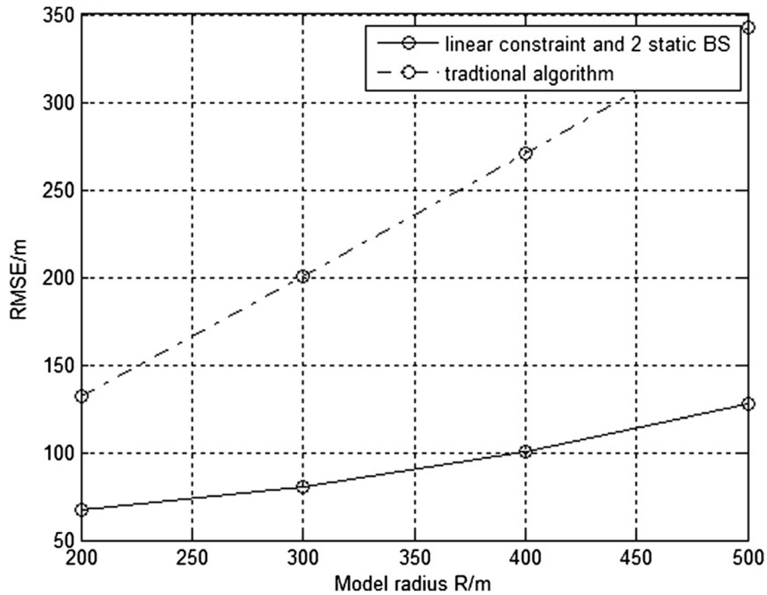

Figure 6 RMSE of the model under different radii.

If $L_j > L_s$ and $L_j > L_i$, then the LOS distance of the nth measurement point L_n^{LOS} can be obtained from Equation (32).

$$L_n^{Los} = \sqrt{r_j^2 + \left(l_j(n) - r_j\right)^2 - \left(\frac{\left(L_S^2 - (l_i(n) - r_i)^2 + \left(l_j(n) - r_j\right)^2\right) r_j}{L_S}\right)} \tag{32}$$

If $L_s > L_i$ and $L_s > L_j$, then L_n^{LOS} can be obtained by Equations (31) or (32).

4.3. Computation of MS position

In Figure 4, S_i and S_j are viewed as virtual BSs, and r_i and r_j are the LOS TOAs between two virtual BSs and MSs. Assuming that the measurement is executed N times during BS motion; thus, $N + 2$ measurement equations can be designed to compute for the position of MS by utilizing the LS algorithm and by combining the reconstructed TOA.

$$X = \left(A^T A\right)^{-1} A^T B \tag{33}$$

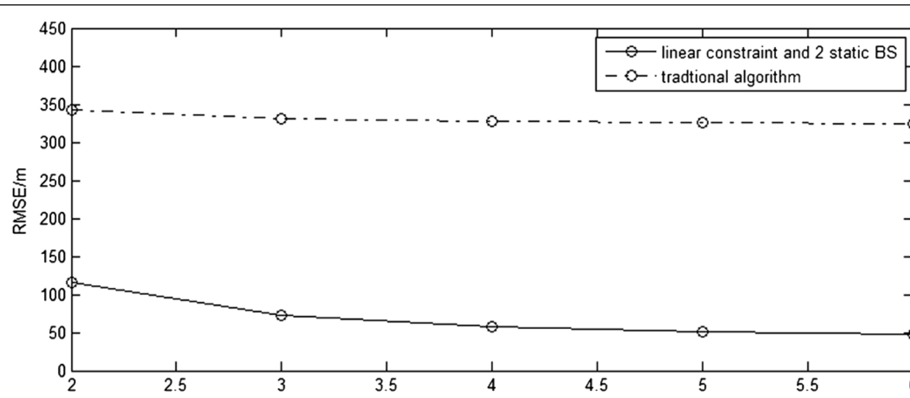

Figure 7 RMSE comparison of different numbers of multipath.

Table 2 Simulation parameters

Coordinate of MS	(800,800)
Velocity along the *x*-axis of BS	0 m/s
Velocity along the *y*-axis of BS	10 m/s
Simulation number	2000
Multipath number	2

where

$$X = \begin{bmatrix} x \\ y \end{bmatrix},$$

$$A = \begin{bmatrix} x_{Sj} - x_{Si} & y_{Sj} - y_{Si} \\ x_{Si} - x_1 & y_{Si} - y_1 \\ x_1 - x_2 & y_1 - y_2 \\ \vdots & \vdots \\ x_{N-1} - x_N & y_{N-1} - y_N \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{1}{2}\left[x_{Sj}^2 + y_{Sj}^2 - \left(x_{Si}^2 + y_{Si}^2\right) + r_i^2 - r_j^2\right] \\ \frac{1}{2}\left[x_{Si}^2 + y_{Si}^2 - \left(x_1^2 + y_1^2\right) + \left(L_1^{Los}\right)^2 - r_i^2\right] \\ \frac{1}{2}\left[x_1^2 + y_1^2 - \left(x_2^2 + y_2^2\right) + \left(L_2^{Los}\right)^2 - \left(L_1^{Los}\right)^2\right] \\ \vdots \\ \frac{1}{2}\left[x_{N-1}^2 + y_{N-1}^2 - \left(x_N^2 + y_N^2\right) + \left(L_N^{Los}\right)^2 - \left(L_{N-1}^{Los}\right)^2\right] \end{bmatrix}$$

5. Simulation and analysis

The improved algorithm in Section 3 is simulated and researched, and a macrocell district is assumed. BS1 is located in the original point of the Cartesian coordinate system, whereas BS2 is located in (0, 2000)m. BS is static and MS is moving. The remaining simulation parameters are shown in Table 1.

Figure 5 shows the mean-squared error (MSE) of the different TOA and AOA measurement errors in the disk of scatterers (DOS) model with a radius of 500 m. The AOA measurement error has a greater influence on positioning accuracy than that of the TOA measurement error. Thus, the effect of the TOA measurement error is evident when the AOA measurement error is minimal. However, the positioning accuracy is mainly influenced by the AOA measurement error when the AOA measurement error is large.

Figure 6 shows the comparison of the root-mean-square error (RMSE) of the model with different radii. The standard TOA and AOA measurement errors are 1.5 m and 0.005 rad, respectively. This finding concludes that the performance of the improved algorithm is better than that of the traditional algorithm. The performance of the traditional algorithm degrades with increasing radius. Consequently, the improved positioning algorithm, which is based on linear constraints on scatterers in two BSs, performs better than the traditional algorithm. The positioning error of the proposed algorithm is also stable.

Figure 7 compares the positioning error of simulation parameters in the DOS model for different numbers of multipaths. Figure 7 concludes that the RMSE of positioning decreases with increasing number of multipath.

Simultaneously, we analyze the improved algorithm in Section 5, in which the BS is mobile and the MS is stationary. The simulation parameters are shown in Table 2. The distance of the two measurement points is 50 m.

Figure 8 shows the RMSE of the positioning result under different TOA and AOA measurement errors.

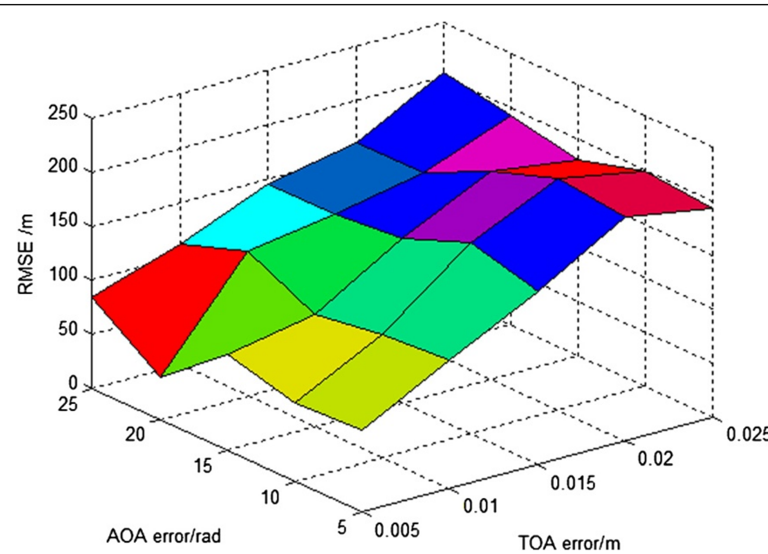

Figure 8 RMSE of different AOA and TOA measurement errors.

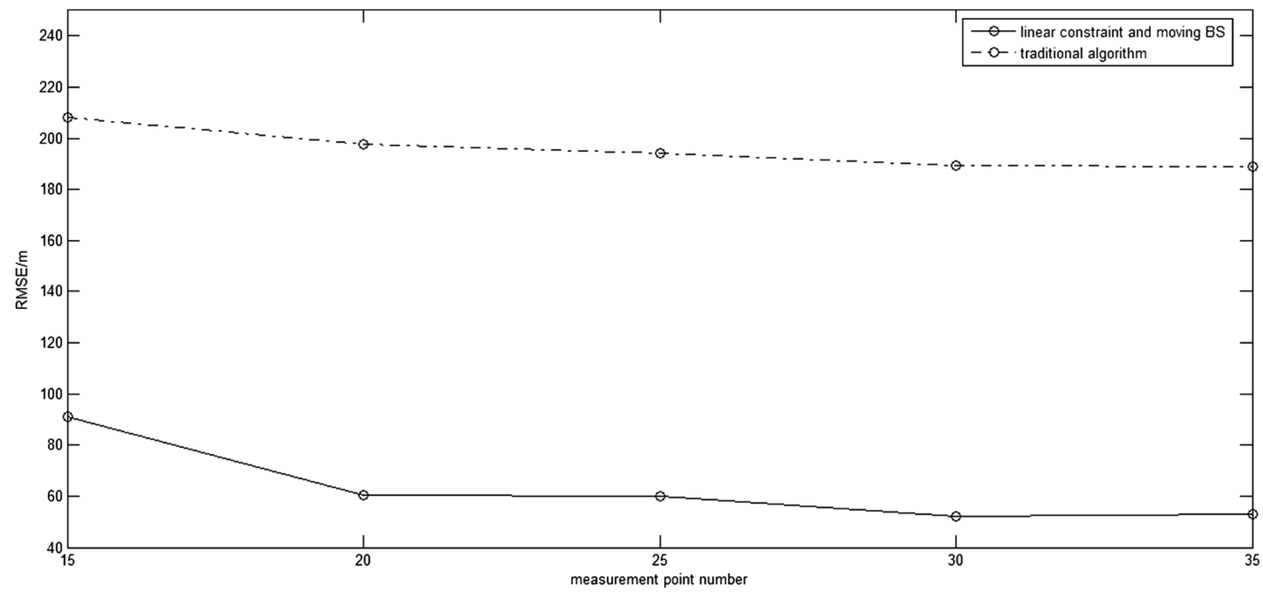

Figure 9 RMSE of different measurement point numbers.

Figure 8 denotes that the positioning accuracy is influenced mainly by the AOA measurement error, which agrees with the result of Figure 5.

Figure 9 denotes that the improved algorithm for scatter linear restriction, which is based on mobile BS, performs better than the traditional algorithm. The performance of the above algorithms improves with increased measurement points because more geographic information is provided with more measurement points.

If the TOA measurement standard error is 1.5 m, then the AOA measurement standard error is 0.005 rad. Table 3 shows the RMSE comparison of the traditional algorithm and the two improved algorithms proposed in this article.

This article proposes an improved traditional positioning algorithm by using scatter constraint information. The improved algorithm suits more common cases and performs better than the traditional algorithm. Further research is necessary to improve this study.

Table 3 RMSE comparison of different algorithms (m)

Multipath signal number	Traditional algorithm	Improved algorithm based on static BS	Improved algorithm based on moving BS
2	Invalid	116.15	77.75
3	Invalid	71.70	46.14
4	259.80	56.90	43.20
5	116.62	51.60	40.23
6	86.87	48.10	38.31

6. Conclusion

This article mainly analyzes the positioning algorithm based on linear constraint on scatterers and gives the drawback that the precondition of traditional algorithm is hard to be satisfied. So, an improved algorithm is proposed to reduce the impact of the drawback. This article first analyzes the key technology of the algorithm, such as multipath signal pairing and TOA reconstruction. Then in the algorithm proposed this article, the number of scatterer for positioning is reduced to two. The improvement can enhance the application range of algorithm, and would not deteriorate the positioning performance. Second, when MS is stationary, the proposed algorithm is adjusted to suit a special case which BS is moving. Both improved algorithms have the same advantage and the positioning performance. These algorithms mainly focus on the design of positioning algorithm, and the restriction of NLOS error. There are many other factors which influenced positioning accuracy, such as signal form and signal band. We will continue the research in the further work.

Competing interests
The authors declare that they have no competing interests.

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