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Existence and globally exponential stability of equilibrium for fuzzy BAM neural networks with distributed delays and impulse

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Abstract

In this article, fuzzy bi-directional associative memory neural networks with distributed delays and impulses are considered. Some sufficient conditions for the existence and globally exponential stability of unique equilibrium point are established using fixed point theorem and differential inequality techniques. The results obtained are easily checked to guarantee the existence, uniqueness, and globally exponential stability of equilibrium point.
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Introduction

The bidirectional associative memory neural networks (BAM) models were first introduced by Kosko [1,2]. It is a special class of recurrent neural networks that can store bipolar vector pairs. The BAM neural network is composed of neurons arranged in two layers, the X-layer and Y-layer. The neurons in one layer are fully interconnected to the neurons in the other layer, while there are no interconnections among neurons in the same layer. Through iterations of forward and backward information flows between the two layers, it performs two-way associative search for stored bipolar vector pairs and generalize the single-layer autoassociative Hebbian correlation to two-layer pattern-matched heteroassociative circuits. Therefore, this class of networks possesses a good applications prospects in the areas of pattern recognition, signal and image process, automatic control. Recently, they have been the object of intensive analysis by numerous authors. In particular, many researchers have studied the dynamics of BAM neural networks with or without delays [1-23] including stability and periodic solutions. In Refs. [1-9], the authors discussed the problem of the stability of the BAM neural networks with or without delays, and obtained some sufficient conditions to ensure the stability of equilibrium point. Recently, some authors, see [10], [14,15] investigated another dynamical behaviors-periodic oscillatory, some sufficient conditions are obtained to ensure other solution converging the periodic solution. In this article, we would like to integrate fuzzy operations into BAM neural networks and maintain local connectedness among cells. Speaking of fuzzy operations, Yang et al.

[24-26] first combined those operations with cellular neural networks and investigated the stability of fuzzy cellular neural networks (FCNNs). Studies have shown that FCNNs has its potential in image processing and pattern recognition, and some results have been reported on stability and periodicity of FCNNs [24-30]. On the other hand, time delays inevitably occurs in electronic neural networks owing to the unavoidable finite switching speed of amplifiers. It is desirable to study the fuzzy BAM neural networks which has a potential significance in the design and applications of stable neural circuits for neural networks with delays.

Though the non-impulsive systems have been well studied in theory and in practice (e.g., see [1-3,5-30] and references cited therein), the theory of impulsive differential equations is now being recognized to be not only richer than the corresponding theory of differential equations without impulse, but also represents a more natural framework for mathematical modelling of many real-world phenomena, such as population dynamic and the neural networks. In recent years, the impulsive differential equations have been extensively studied (see the monographs and the works [4,31-35]). Up to now, to the best of our knowledge, dynamical behaviors of fuzzy BAM neural networks with delays and impulses are seldom considered. Motivated by the above discussion, in this article, we investigate the fuzzy BAM neural networks with distributed delays and impulses by the following system

$$\begin{cases} x'_i(t) = -a_i x_i(t) + \sum_{j=1}^{m} \int_0^{\tau} c_{ji}(s) f_j(y_j(t-s))\,\mathrm{d}s + \wedge_{j=1}^{m} \int_0^{\tau} \alpha_{ji}(s) f_j(y_j(t-s))\mathrm{d}s + A_i \\ \qquad + \vee_{j=1}^{m} \int_0^{\tau} \beta_{ji}(s) f_j(y_j(t-s))\mathrm{d}s + \wedge_{j=1}^{m} T_{ji} u_j + \vee_{j=1}^{m} H_{ji} u_j, \quad t > 0, t \neq t_k. \\ \Delta x_i(t_k) = I_k(x_i(t_k)), \quad i = 1, 2, \ldots, n.\ k = 1, 2, \ldots, \\ y'_j(t) = -b_j y_j(t) + \sum_{i=1}^{n} \int_0^{\sigma} d_{ij}(s) g_i(x_i(t-s))\mathrm{d}s + \wedge_{i=1}^{n} \int_0^{\sigma} p_{ij}(s) g_i(x_i(t-s))\,\mathrm{d}s + B_j \\ \qquad + \vee_{i=1}^{n} \int_0^{\sigma} q_{ij}(s) g_i(x_i(t-s)) ds + \wedge_{i=1}^{n} K_{ij} u_i + \vee_{i=1}^{n} L_{ij} u_i, \quad t > 0, t \neq t_k. \\ \Delta y_j(t_k) = J_k(y_j(t_k)), \quad j = 1, 2, \ldots, m.\ k = 1, 2, \ldots, \end{cases} \tag{1}$$

where n and m correspond to the number of neurons in X-layer and Y -layer, respectively. $x_i(t)$ and $y_j(t)$ are the activations of the ith neuron and the jth neurons, respectively, $a_i > 0$, $b_j > 0$ denote the rate with which the ith neuron and jth neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs; α_{ji}, β_{ji}, T_{ji}, and H_{ji} are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template, and fuzzy feed-forward MAX template in X-layer, respectively; p_{ij}, q_{ij}, K_{ij}, and L_{ij} are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template, and fuzzy feed-forward MAX template in Y -layer, respectively; $\wedge$ and $\vee$ denote the fuzzy AND and fuzzy OR operation, respectively; u_j and u_i denote external input of the ith neurons in X-layer and external input of the jth neurons in Y -layer, respectively; A_i and B_j represent bias of the ith neurons in X-layer and bias of the jth neurons in Y -layer,respectively; $c_{ji}(t)$ and $d_{ij}(t)$ are the delayed feedback. $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$, $\Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k^-)$ are the impulses at moments t_k and $t_1 < t_2 < \ldots$ is a strictly increasing sequences such that $\lim_{k \to \infty} t_k = +\infty$. $\tau > 0$ and $\sigma > 0$ are constants and correspond to the transmission delays, and $f_j(\cdot)$, $g_i(\cdot)$ are signal transmission functions.

The main purpose of this article is, employing fixed point theorem and differential inequality techniques, to give some sufficient conditions for the existence, uniqueness, and global exponential stability of equilibrium point of system (1). Our results extend and improve the corresponding works in the earlier publications.

The initial conditions associated with system (1) are of the form

$$x_i(s) = \phi_i(s),\ s \in (-\sigma, 0], \quad i = 1, 2, \ldots, n.$$
$$y_i(s) = \psi_j(s),\ s \in (-\tau, 0], \quad j = 1, 2, \ldots, m.$$

where $\varphi_i(\cdot)$ and $\psi_j(\cdot)$ are continuous bounded functions defined on [-σ, 0] and [-τ; 0], respectively.

Throughout this article, we always make the following assumptions.

(A1) The signal transmission functions $f_j(\cdot)$, $g_i(\cdot)(i = 1, 2, ..., n, j = 1, 2, ..., m)$ are Lipschitz continuous on R with Lipschitz constants μ_j and ν_i, namely, for $x, y \in R$

$$|f_j(x) - f_j(y)| \le \mu_j |x - y|, \quad |g_i(x) - g_i(y)| \le \nu_i |x - y| \tag{2}$$

(A2) For $i = 1,2, ..., n$, $j = 1,2, ..., m$, there exist nonnegative constants $c_{ji}^+, d_{ij}^+, \alpha_{ji}^+, \beta_{ji}^+, p_{ij}^+, q_{ij}^+$ such that

$$\int_0^\tau |c_{ji}(s)|\,\mathrm{d}s \le c_{ji}^+, \quad \int_0^\tau |\alpha_{ji}(s)|\mathrm{d}s \le \alpha_{ji}^+, \quad \int_0^\tau |\beta_{ji}(s)|\mathrm{d}s \le \beta_{ji}^+, \tag{3}$$

$$\int_0^\sigma |d_{ij}(s)|\,\mathrm{d}s \le d_{ij}^+, \quad \int_0^\sigma |p_{ij}(s)|\mathrm{d}s \le p_{ij}^+, \quad \int_0^\sigma |q_{ij}(s)|\mathrm{d}s \le q_{ij}^+, \tag{4}$$

As usual in the theory of impulsive differential equations, at the points of discontinuity t_k of the solution $t\ \alpha\ (x_1(\mathrm{t}), ..., x_n(t), y_1(t), ..., y_m(t))^{\mathrm{T}}$. We assume that $(x_1(t), ..., x_n(t), y_1(t), ..., y_m(t))^{\mathrm{T}} = (x_1(t - 0), ..., x_n(t - 0), y_1(t - 0), ..., y_m(t - 0))^{\mathrm{T}}$. It is clear that, in general, the derivatives $x_i'(t_k)$ and $y_j'(t_k)$ do not exist. On the other hand, according to system (1), there exist the limits $x_i'(t_k \mp 0)$ and $y_j'(t_k \mp 0)$. In view of the above convention, we assume that $x_i'(t_k) = x_i'(t_k - 0)$ and $y_j'(t_k) = y_j'(t_k - 0)$.

To be convenience, we introduce some notations. $x = (x_1, x_2, ..., x_l)^{\mathrm{T}} \in R^l$ denotes a column vector, in which the symbol ($^{\mathrm{T}}$) denotes the transpose of vector. For matrix $D = (d_{ij})_{l\times l}$, D^{T} denotes the transpose of D, and E_l denotes the identity matrix of size l. A matrix or vector $D \ge 0$ means that all entries of D are greater than or equal to zero. $D > 0$ can be defined similarly. For matrices or vectors D and E, $D \ge E$ (respectively $D > E$) means that $D - E \ge 0$ (respectively $D - E > 0$). Let us define that for any $\omega \in R^{n+m}$, $||\omega|| = \max_{1\le k\le n+m} |\omega_k|$.

Definition 1.1. Let $z^* = (x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_m^*)^{\mathrm{T}}$ be an equilibrium point of system (1) with $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^{\mathrm{T}}, y^* = (y_1^*, y_2^*, \ldots, y_m^*)^{\mathrm{T}}$. If there exist positive constants M, λ such that for any solution $z(t) = (x_1(t), x_2(t), ..., x_n(t), y_1(t), y_2(t), ..., y_m(t))^{\mathrm{T}}$ of system (1) with initial value (φ,ψ) and $\varphi = (\varphi_1(t), \varphi_2(t), ..., \varphi_n(t))^{\mathrm{T}} \in C([-\sigma, 0], R^n)$, $\psi = (\psi_1(t), \psi_2(t), ..., \psi_m(t))^{\mathrm{T}} \in C([-\tau, 0], R^m)$,

$$|x_i(t) - x_i^*| \le M||(\phi, \psi) - (x^*, y^*)||e^{-\lambda t},$$
$$|y_j(t) - y_j^*| \le M||(\phi, \psi) - (x^*, y^*)||e^{-\lambda t}$$

where i = 1, 2, ..., n, j = 1, 2, ..., m

$$||(\phi, \psi) - (x^*, y^*)|| \max\left\{\max_{1\le i\le n}\sup_{-\sigma\le t\le 0}|\phi_i(t) - x_i^*|, \max_{1\le j\le m}\sup_{-\tau\le t\le 0}|\psi_j(t) - y_j^*|\right\}$$

Then z^* is said to be globally exponentially stable.

Definition 1.2. If $f(t)$: $R \to R$ is a continuous function, then the upper left derivative of $f(t)$ is defined as

$$D^- f(t) = \lim_{h\to 0^-}\sup\frac{1}{h}(f(t+h) - f(t))$$

Definition 1.3 . A real matrix $A = (a_{ij})_{l\times l}$ is said to be an M-matrix if $a_{ij} \le 0$, i, j = 1,2, ..., l, $i \ne j$, and all successive principal minors of A are positive.

Lemma 1.1. *Let* $A = (a_{ij})$ *be an* $l \times l$ *matrix with non-positive off-diagonal elements. Then the*

following statements are equivalent:

(i) *A is an M-matrix;*

(ii) *the real parts of all eigenvalues of A are positive;*

(iii) *there exists a vector* $\eta > 0$ *such that* $A\eta > 0$*;*

(iv) *there exists a vector* $\xi > 0$ *such that* $\xi^{\mathrm{T}}A > 0$*;*

(v) *there exists a positive definite* $l \times l$ *diagonal matrix D such that* $AD + DA^{\mathrm{T}} > 0$.

Lemma 1.2 [24]. *Suppose × and y are two states of system (1), then we have*

$$\left|\bigwedge_{j=1}^{n}\alpha_{ij}g_j(x) - \bigwedge_{j=1}^{n}\alpha_{ij}g_j(y)\right| \le \sum_{j=1}^{n}|\alpha_{ij}||g_j(x) - g_j(y)|,$$

and

$$\left|\bigvee_{j=1}^{n}\beta_{ij}g_j(x) - \bigvee_{j=1}^{n}\beta_{ij}g_j(y)\right| \le \sum_{j=1}^{n}|\beta_{ij}||g_j(x) - g_j(y)|$$

Lemma 1.3 *Let* $A \ge 0$ *be an* $l \times l$ *matrix and* $\rho(A) < 1$, *then* $(E_l - A)^{-1} \ge 0$, *where* $\rho(A)$ *denotes the spectral radius of A.*

The remainder of this article is organized as follows. In next section, we shall give some sufficient conditions for checking the existence and uniqueness of equilibrium point, followed by some sufficient conditions for global exponential stability of the unique equilibrium point of (1). Then, an example will be given to illustrate effectiveness of our results obtained. Finally, general conclusion is drawn.

Existence and uniqueness of equilibrium point

In this section, we will derive some sufficient conditions for the existence and uniqueness of equilibrium point for fuzzy BAM neural networks model (1).

Theorem 2.1. *Suppose that (A1) and (A2) hold and* $\rho(D^{-1}EU) < 1$, *where* $D = diag(a_1, ..., a_n, b_1, ..., b_m)$, $U = diag(\mu_1, ..., \mu_n, v_1, ..., v_m)$

$$E = \begin{pmatrix} 0_{n\times n} & P^{\mathrm{T}} \\ Q^{\mathrm{T}} & 0_{m\times m}\end{pmatrix}, \quad P = (c_{ji}^+ + \alpha_{ji}^+ + \beta_{ji}^+)_{m\times n}, \quad Q = (d_{ij}^+ + p_{ij}^+ + q_{ij}^+)_{n\times m}$$

Then there exists a unique equilibrium point of system (1).

Proof. An equilibrium point $z^* = (x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_m^*)^{\mathrm{T}} \in R^{n+m}$is a constant vector satisfying system (1), i.e.,

$$\begin{cases} x_i^* = a_i^{-1}\left[\sum_{j=1}^m \int_0^\tau c_{ji}(s)f_j(y_j^*)\mathrm{d}s\right] + a_i^{-1}\left[\wedge_{j=1}^m \int_0^\tau \alpha_{ji}(s)f_j(y_j^*)\mathrm{d}s\right] \\ \qquad + a_i^{-1}\left[\vee_{j=1}^m \int_0^\tau \beta_{ji}(s)f_j(y_j^*)\mathrm{d}s\right] + a_i^{-1}\left[\wedge_{j=1}^m T_{ji}u_j + \vee_{j=1}^m H_{ji}u_j + A_i\right] \\ y_j^* = b_j^{-1}\left[\sum_{i=1}^n \int_0^\tau d_{ij}(s)g_i(x_i^*)\mathrm{d}s\right] + b_j^{-1}\left[\wedge_{i=1}^n \int_0^\sigma p_{ij}(s)g_i(x_i^*)\mathrm{d}s\right] \\ \qquad + b_j^{-1}\left[\vee_{i=1}^n \int_0^\sigma q_{ij}(s)g_i(x_i^*)\mathrm{d}s\right] + b_j^{-1}[\wedge_{i=1}^n K_{ij}u_i + \vee_{i=1}^n L_{ij}u_i + B_j] \end{cases} \tag{5}$$

To finish the proof, it suffices to prove that (5) has a unique solution. Consider a mapping $\Phi = (\Phi_i, \Psi_j)^{\mathrm{T}} : R^{n+m} \to R^{n+m}$ defined by

$$\begin{aligned} \Phi_i(h_i) &= a_i^{-1}\left[\sum_{j=1}^m \int_0^\tau c_{ji}(s)f_j(v_j)\mathrm{d}s\right] + a_i^{-1}\left[\bigwedge_{j=1}^m \int_0^\tau \alpha_{ji}(s)f_j(v_j)\mathrm{d}s\right] + a_i^{-1}\left[\bigvee_{j=1}^m \int_0^\tau \beta_{ji}(s)f_j(v_j)\mathrm{d}s\right] \\ &\quad + a_i^{-1}\left[\bigwedge_{j=1}^m T_{ji}u_j + \bigvee_{j=1}^m H_{ji}u_j + A_i\right], \quad i = 1, 2, \ldots, n. \end{aligned} \tag{6}$$

$$\begin{aligned} \Psi_j(v_j) &= b_j^{-1}\left[\sum_{i=1}^n \int_0^\sigma d_{ij}(s)g_i(h_i)\mathrm{d}s\right] + b_j^{-1}\left[\bigwedge_{i=1}^n \int_0^\sigma p_{ij}(s)g_i(h_i)\right] + b_j^{-1}\left[\bigvee_{i=1}^n \int_0^\sigma q_{ij}(s)g_i(h_i)\mathrm{d}s\right] \\ &\quad + b_j^{-1}\left[\bigwedge_{i=1}^n K_{ij}u_i + \bigvee_{i=1}^n L_{ij}u_i + B_j\right], \quad j = 1, 2, \ldots, m. \end{aligned} \tag{7}$$

We show that Φ: $R^{n+m} \to R^{n+m}$ is global contraction mapping on R^{n+m}. In fact, for $h = (h_1, h_2, ..., h_n, v_1, v_2, ..., v_m)^{\mathrm{T}}$, $\bar{h} = (\bar{h}_1, \bar{h}_2, \ldots, \bar{h}_n, \bar{v}_1, \bar{v}_1, \ldots, \bar{v}_m)^{\mathrm{T}} \in R^{n+m}$. Using (A1), (A2), and Lemma 1.2, we have

$$\begin{aligned} |\Phi_i(h_i) - \Phi_i(\bar{h}_i)| = &\left| a_i^{-1}\left[\sum_{j=1}^m \int_0^\tau c_{ji}(s)(f_j(v_j) - f_j(\bar{v}_j))\mathrm{d}s\right] + a_i^{-1}\left[\bigwedge_{j=1}^m \int_0^\tau \alpha_{ji}(s)f_j(v_j)\mathrm{d}s\right.\right. \\ &\left.\left. - \bigwedge_{j=1}^m \int_0^\tau \alpha_{ji}(s)f_j(\bar{v}_j)\mathrm{d}s\right] + a_i^{-1}\left[\bigvee_{j=1}^m \int_0^\tau \beta_{ji}(s)f_j(v_j)\mathrm{d}s - \bigvee_{j=1}^m \int_0^\tau \beta_{ji}(s)f_j(\bar{v}_j)\mathrm{d}s\right]\right| \\ \le\ & a_i^{-1}\sum_{j=1}^m (c_{ji}^+ + \alpha_{ji}^+ + \beta_{ji}^+)\mu_j|v_j - \bar{v}_j|, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{8}$$

$$\begin{aligned} |\Psi_j(v_j) - \Psi_j(\bar{v}_j)| = &\left| b_j^{-1}\left[\sum_{i=1}^n \int_0^\sigma d_{ij}(s)(g_i(h_i) - g_i(\bar{h}_i))\mathrm{d}s\right] + b_j^{-1}\left[\bigwedge_{i=1}^n \int_0^\sigma p_{ij}(s)g_i(h_i)\mathrm{d}s\right.\right. \\ &\left.\left. - \bigwedge_{i=1}^n \int_0^\sigma p_{ij}(s)g_i(\bar{h}_j)\mathrm{d}s\right] + b_j^{-1}\left[\bigvee_{i=1}^n \int_0^\sigma q_{ij}(s)g_i(h_i)\mathrm{d}s - \bigvee_{i=1}^n \int_0^\sigma q_{ij}(s)g_i(\bar{h}_i)\mathrm{d}s\right]\right| \\ \le\ & b_j^{-1}\sum_{i=1}^n (d_{ij}^+ + p_{ij}^+ + q_{ij}^+)\nu_i|h_i - \bar{h}_i|, \quad j = 1, 2, \ldots, m \end{aligned} \tag{9}$$

In view of (8)-(9), it follows that

$$|\Phi(h_1, h_2, \ldots, h_n, v_1, v_2, \ldots, v_m) - \Phi(\bar{h}_1, \bar{h}_2, \ldots, \bar{h}_n, \bar{v}_1, \bar{v}_2, \ldots, \bar{v}_m)| \le F \begin{pmatrix} |h_1 - \bar{h}_1| \\ \vdots \\ |h_n - \bar{h}_n| \\ |v_1 - \bar{v}_1| \\ \vdots \\ |v_m - \bar{v}_m| \end{pmatrix} \tag{10}$$

where $F = D^{-1}EU = (w_{ij})_{(n+m)\times(n+m)}$. Let ξ be a positive integer. Then from (10) it follows that

$$|\Phi^{\xi}(s) - \Phi^{\xi}(\bar{s})| \le F^{\xi}\begin{pmatrix} |h_1 - \bar{h}_1| \\ \vdots \\ |h_n - \bar{h}_n| \\ |v_1 - \bar{v}_1| \\ \vdots \\ |v_m - \bar{v}_m| \end{pmatrix} \tag{11}$$

Since $\rho(F) < 1$, we obtain $\lim_{\xi\to+\infty} F^{\xi} = 0$, which implies that there exist a positive integer N and a positive constant $r < 1$ such that

$$F^N = (D^{-1}EU)^N = (l_{ij})_{(n+m)\times(n+m)}, \quad \sum_{j=1}^{n+m} l_{ij} \le r, \quad i = 1, 2, \ldots, n+m. \tag{12}$$

Nothing that (11) and (12), it follows that

$$|\Phi^N(h) - \Phi^N(\bar{h})| \le F^N\begin{pmatrix} |h_1 - \bar{h}_1| \\ \vdots \\ |h_n - \bar{h}_n| \\ |v_1 - \bar{v}_1| \\ \vdots \\ |v_m - \bar{v}_m| \end{pmatrix} \le F^N\begin{pmatrix} ||h - \bar{h}|| \\ \vdots \\ ||h - \bar{h}|| \\ ||h - \bar{h}|| \\ \vdots \\ ||h - \bar{h}|| \end{pmatrix} = ||h - \bar{h}||\begin{pmatrix} \sum_{j=1}^{n+m} l_{1j} \\ \vdots \\ \sum_{j=1}^{n+m} l_{nj} \\ \sum_{j=1}^{n+m} l_{(n+1)j} \\ \vdots \\ \sum_{j=1}^{n+m} l_{(n+m)j} \end{pmatrix} \tag{13}$$

which implies that $||\Phi^N(h) - \Phi^N(\bar{h})|| \le r||h - \bar{h}||$. Since $r < 1$, it is obvious that the mapping $\Phi^N : R^{n+m} \to R^{n+m}$ is a contraction mapping. By the fixed point theorem of Banach space, Φ possesses a unique fixed point in R^{n+m} which is unique solution of the system (5), namely, there exist a unique equilibrium point of system (1). The proof of theorem 2.1 is completed.

Global exponential stability of equilibrium point

In this section, we shall give some sufficient conditions to guarantee global exponential stability of equilibrium point of system (1).

Theorem 3.1 *Suppose that (A1), (A2), and* $\rho(D^{-1}EU) < 1$. *Let* $z^* = (x_1^*, \ldots, x_n^*, y_1^*, \ldots, y_m^*)^{\mathrm{T}}$ *be a unique equilibrium point of system (1). Furthermore, assume that the impulsive operators* $I_k(\cdot)$ *and* $J_k(\cdot)$ *satisfy*

$$(A3)\quad \begin{cases} I_k(x_i(t_k)) = -\gamma_{ik}(x_i(t_k) - x_i^*),\ 0 \le \gamma_{ik} \le 2, & i = 1, 2, \ldots, n,\ k = 1, 2, \ldots \\ J_k(y_j(t_k)) = -\bar{\gamma}_{jk}(y_j(t_k) - y_j^*),\ 0 \le \bar{\gamma}_{ik} \le 2, & j = 1, 2, \ldots, m,\ k = 1, 2, \ldots \end{cases}$$

Then the unique equilibrium point z^* *of system (1) is globally exponentially stable.*

Proof. Let $z(t) = (x_1(t), x_2(t), \ldots, x_n(t), y_1(t), y_2(t), \ldots, y_m(t))^{\mathrm{T}}$ be an arbitrary solution of system (1) with initial value (φ,ψ) and $\varphi = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t))^{\mathrm{T}} \in C([-\sigma, 0]; R^n)$, $\psi = (\psi_1(t), \psi_2(t), \ldots, \psi_n(t))^{\mathrm{T}} \in C([-\tau, 0]; R^m)$. Set $\bar{x}_i(t) = x_i(t) - x_i^*, \bar{y}_j(t) = y_j(t) - y_j^*$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

From (1) and (5), for $t > 0$, $t \neq t_k$, $k = 1, 2, ...,$ we have

$$\begin{cases} \bar{x}_i'(t) = -a_i\bar{x}_i(t) + \sum_{j=1}^m \int_0^\tau c_{ji}(s)(f_j(y_j(t-s)) - f_j(y_j^*))\,ds + \left[\wedge_{j=1}^m \int_0^\tau \alpha_{ji}(s)f_j(y_j(t-s))\,ds \right. \\ \qquad \left. - \wedge_{j=1}^m \int_0^\tau \alpha_{ji}(s)f_j(y_j^*)\,ds\right] + \left[\vee_{j=1}^m \int_0^\tau \beta_{ji}(s)f_j(y_j(t-s))\,ds - \vee_{j=1}^m \int_0^\tau \beta_{ji}(s)f_j(y_j^*)\,ds\right] \\ \bar{y}_j'(t) = -b_j\bar{y}_j(t) + \sum_{i=1}^n \int_0^\sigma d_{ij}(s)(g_i(x_i(t-s)) - g_i(x_i^*))\,ds + \left[\wedge_{i=1}^n \int_0^\sigma p_{ij}(s)g_i(x_i(t-s))\,ds \right. \\ \qquad \left. - \wedge_{i=1}^n \int_0^\sigma p_{ij}(s)g_i(x_i^*)\,ds\right] + \left[\vee_{i=1}^n \int_0^\sigma q_{ij}(s)g_i(x_i(t-s))\,ds - \vee_{i=1}^n \int_0^\sigma q_{ij}(s)g_i(x_i^*)\,ds\right] \end{cases} \tag{14}$$

According to ($A3$), we get

$$\begin{aligned} x_i(t_k+0) - x_i^* &= x_i(t_k) + I_i(x_i(t_k)) - x_i^* \\ &= (1-\gamma_{ik})(x_i(t_k) - x_i^*), \quad i = 1, 2, \ldots n,\ k \in Z^+ \\ y_j(t_k+0) - y_j^* &= y_j(t_k) + J_j(y_j(t_k)) - y_j^* \\ &= (1-\bar{\gamma}_{jk})(y_j(t_k) - y_j^*), \quad j = 1, 2, \ldots, m,\ k \in Z^+. \end{aligned} \tag{15}$$

Using (A1), (A2), (A3), Definition 1.2, and Lemma 1.2, from (14) and (15), we have

$$\begin{cases} D^-|\bar{x}_i(t)| \le -a_i|\bar{x}_i(t)| + \sum_{j=1}^m \int_0^\tau |c_{ji}(s)|\mu_j|y_j(t-s) - y_j^*|ds \\ \qquad + \sum_{j=1}^m \int_0^\tau (|\alpha_{ji}(s)| + |\beta_{ji}(s)|)\mu_j|y_j(t-s) - y_j^*|ds \\ \qquad \le -a_i|\bar{x}_i(t)| + \sum_{j=1}^m (c_{ji}^+ + \alpha_{ji}^+ + \beta_{ji}^+)\mu_j\widetilde{y_j(t)} \\ D^-|\bar{y}_j(t)| \le -b_j|\bar{y}_j(t)| + \sum_{i=1}^n \int_0^\sigma |d_{ij}(s)|\nu_i|x_i(t-s) - x_i^*|ds \\ \qquad + \sum_{i=1}^n \int_0^\sigma (|p_{ij}(s)| + |q_{ij}(s)|)\nu_i|x_i(t-s) - x_i^*|ds \\ \qquad \le -b_j|\bar{y}_j(t)| + \sum_{i=1}^n (d_{ij}^+ + p_{ij}^+ + q_{ij}^+)\nu_i\widetilde{x_i(t)} \end{cases} \tag{16}$$

Where $\widetilde{x_i(t)} = \sup_{t-\sigma\le s\le t}|\bar{x}_i(s)|$, $\widetilde{y_j(t)} = \sup_{t-\tau\le s\le t}|\bar{y}_j(s)|$, $t > 0$, $t \neq t_k$, $k \in Z^+$, $i = 1,2, ..., n$; $j = 1,2, ..., m$. and

$$\begin{cases} |x_i(t_k+0) - x_i^*| = |1-\gamma_{ik}||x_i(t_k) - x_i^*| \le |x_i(t_k) - x_i^*|, \quad i = 1, 2, \ldots, n,\ k \in Z^+ \\ |y_j(t_k+0) - y_j^*| = |1-\bar{\gamma}_{jk}||y_j(t_k) - y_j^*| \le |y_j(t_k) - y_j^*|, \quad j = 1, 2, \ldots, m,\ k \in Z^+. \end{cases}$$

which implies that

$$\begin{cases} |\bar{x}_i(t_k+0)| = |x_i(t_k+0) - x_i^*| \le |x_i(t_k) - x_i^*| = |\bar{x}_i(t_k-0)|, \quad i = 1, 2, \ldots, n,\ k \in Z^+ \\ |\bar{y}_j(t_k+0)| = |y_j(t_k+0) - y_j^*| \le |y_j(t_k) - y_j^*| = |\bar{y}_j(t_k-0)|, \quad j = 1, 2, \ldots, m,\ k \in Z^+. \end{cases} \tag{17}$$

Since $\rho(D^{-1}EU) = \rho(F) < 1$, it follows from Lemmas 1.1 and 1.3 that $E_{n+m} - D^{-1}EU$ is an M- matrix, therefore there exists a vector $\eta = (\eta_1, \eta_2, ..., \eta_n, \zeta_1, \zeta_2, ..., \zeta_m)^T > (0, 0, ...,0, 0, 0, ..., 0)^T$ such that

$$(E_{n+m} - D^{-1}EU)\eta > (0, 0, \ldots, 0, 0, 0, \ldots, 0)^T.$$

Hence

$$\eta_i - \sum_{j=1}^m a_i^{-1}(c_{ji}^+ + \alpha_{ji}^+ + \beta_{ji}^+)\mu_j\varsigma_j > 0, \quad \varsigma_j - \sum_{i=1}^n b_j^{-1}(d_{ij}^+ + p_{ij}^+ + q_{ij}^+)\nu_i\eta_i > 0, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m,$$

which implies that

$$-a_i\eta_i + \sum_{j=1}^m (c_{ji}^+ + \alpha_{ji}^+ + \beta_{ji}^+)\mu_j\zeta_j < 0,\ -b_j\zeta_j + \sum_{i=1}^n (d_{ij}^+ + p_{ij}^+ + q_{ij}^+)\nu_i\eta_i < 0. \tag{18}$$

We can choose a positive constant $\lambda < 1$ such that, for $i = 1, 2, ...n$; $j = 1, 2, ..., m$

$$\lambda\eta_i+\left[-a_i\eta_i+\sum_{j=1}^{m}(c_{ji}^{+}+\alpha_{ji}^{+}+\beta_{ji}^{+})\mu_j\zeta_j e^{\lambda\tau}\right]<0, \lambda\zeta_j+\left[-b_j\zeta_j+\sum_{i=1}^{n}(d_{ij}^{+}+p_{ij}^{+}+q_{ij}^{+})\nu_i\eta_i e^{\lambda\sigma}\right]<0. \quad (19)$$

For all $t \in [-\sigma - \tau, 0]$, we can choose a constant $\gamma > 1$ such that

$$\gamma\eta_i e^{-\lambda t} > 1, \gamma\zeta_j e^{-\lambda t} > 1. \quad (20)$$

For $\forall\varepsilon > 0$, set

$$\Delta = \sum_{i=1}^{n}\widetilde{x_i(0)}+\sum_{j=1}^{m}\widetilde{y_j(0)}\ ;\ V_i(t)=\gamma\eta_i(\Delta+\varepsilon)\ e^{-\lambda t}, W_j(t)=\gamma\zeta_j(\Delta+\varepsilon)\ e^{-\lambda t} \quad (21)$$

Caculating the upper left derivative of $V_i\ (t)$ and $W_j\ (t)$, respecively, and noting that (19)

$$\begin{aligned} D^{-}V_i(t) &= -\lambda\gamma\eta_i(\Delta+\varepsilon)e^{-\lambda t} \\ &> \left[-a_i\eta_i+\sum_{j=1}^{m}(c_{ji}^{+}+\alpha_{ji}^{+}+\beta_{ji}^{+})\mu_j\zeta_j e^{\lambda\tau}\right]\gamma(\Delta+\varepsilon)e^{-\lambda t} \\ &= -a_i\gamma\eta_i(\Delta+\varepsilon)e^{-\lambda t}+\sum_{j=1}^{m}(c_{ji}^{+}+\alpha_{ji}^{+}+\beta_{ji}^{+})\mu_j\zeta_j\gamma(\Delta+\varepsilon)e^{-\lambda t}e^{\lambda\tau} \\ &= -a_iV_i(t)+\sum_{j=1}^{m}(c_{ji}^{+}+\alpha_{ji}^{+}+\beta_{ji}^{+})\mu_j\overline{W_j}(t) \end{aligned} \quad (22)$$

and

$$\begin{aligned} D^{-}W_j(t) &= -\lambda\gamma\zeta_j(\Delta+\varepsilon)e^{-\lambda t} \\ &> \left[-b_j\zeta_j+\sum_{i=1}^{n}(d_{ij}^{+}+p_{ij}^{+}+q_{ij}^{+})\nu_i\eta_i e^{\lambda\tau}\right]\gamma(\Delta+\varepsilon)e^{-\lambda t} \\ &= -b_j\gamma\zeta_j(\Delta+\varepsilon)e^{-\lambda t}+\sum_{i=1}^{n}(d_{ij}^{+}+p_{ij}^{+}+q_{ij}^{+})\nu_i\eta_i\gamma(\Delta+\varepsilon)e^{-\lambda t}e^{\lambda\tau} \\ &= -b_jW_j(t)+\sum_{i=1}^{n}(d_{ij}^{+}+p_{ij}^{+}+q_{ij}^{+})\nu_i\overline{V_i}(t) \end{aligned} \quad (23)$$

where $\overline{W_j}(t) = \sup_{t-\tau\le s\le t}W_j(s), \overline{V_i}(t) = \sup_{t-\tau\le s\le t}V_j(s)$. $i = 1, 2, ..., n$; $2, ..., m$. from (20) and (21), we have

$$V_i(t)=\gamma\eta_i(\Delta+\varepsilon)e^{-\lambda t}>|\bar{x}_i(t)|, t\in[-\sigma,0]\ ;\ W_j(t)=\gamma\zeta_j(\Delta+\varepsilon)e^{-\lambda t}>|\bar{y}_j(t)|, t\in[-\tau,0]. \quad (24)$$

On the other hand, we claim that for all $t > 0$, $t \neq t_k$, $k \lfloor Z^{+}$, $i = 1, 2,..., n$; $j = 1, 2,..., m$.

$$V_i(t)=\gamma\eta_i(\Delta+\varepsilon)e^{-\lambda t}>|\bar{x}_i(t)|\ ;\ \ W_j(t)=\gamma\zeta_j(\Delta+\varepsilon)e^{-\lambda t}>|\bar{y}_j(t)|, . \quad (25)$$

By contrary, from (17) one of the following two cases must occur

(i) there must exist $i \in \{1, 2,..., n\}$ and $t_i^{*} > 0(t_i^{*} \neq t_k, k \in Z^{+})$ such that for $l = 1, 2,..., n$, $k = 1, 2,..., m$.

$$|\bar{x}_i(t_i^{*})|=V_i(t_i^{*});\ \ |\bar{x}_l(t)|<V_l(t),\ \ \forall t\in[-\sigma,t_i^{*}];\ \ |\bar{y}_k(t)|<W_k(t),\ \ \forall t\in[-\tau,t_i^{*}]. \quad (26)$$

(ii) there must exist $j \llcorner$ {1, 2,..., m} and $t_j^* > 0(t_j^* \neq t_k, k \in Z^+)$ such that for l = 1, 2,..., n, k = 1, 2,..., m.

$$|\bar{y}_j(t_i^*)| = W_j(t_j^*); \quad |\bar{x}_l(t)| < V_l(t), \quad \forall t \in [-\sigma, t_j^*]; \quad |\bar{y}_k(t)| < W_k(t), \quad \forall t \in [-\tau, t_j^*]. \tag{27}$$

Suppose case (i) occurs, we obtain

$$\begin{aligned} 0 &\le D^-(|\bar{x}_i(t_i^*)| - V_i(t_i^*)) \\ &= \lim_{h\to 0^-} \sup \frac{[|\bar{x}_i(t_i^* + h)| - V_i(t_i^* + h)] - [|\bar{x}_i(t_i^*)| - V_i(t_i^*)]}{h} \\ &\le \lim_{h\to 0^-} \sup \frac{|\bar{x}_i(t_i^* + h)| - |\bar{x}_i(t_i^*)|}{h} - \lim_{h\to 0^-} \inf \frac{V_i(t_i^* + h) - V_i(t_i^*)}{h} \\ &= D^-|\bar{x}_i(t_i^*)| - D_-V_i(t_i^*). \end{aligned} \tag{28}$$

In view of (16), (22) and (25), we have

$$\begin{aligned} D^-|\bar{x}_i(t_i^*)| &\le -a_i|\bar{x}_i(t_i^*)| + \sum_{j=1}^{m} (c_{ji}^+ + \alpha_{ji}^+ + \beta_{ji}^+)\mu_j y_j(\widetilde{t_i^*}) \\ &= -a_i V_i(t_i^*) + \sum_{j=1}^{m} (c_{ji}^+ + \alpha_{ji}^+ + \beta_{ji}^+)\mu_j y_j(\widetilde{t_i^*}) \\ &\le -a_i V_i(t_i^*) + \sum_{j=1}^{m} (c_{ji}^+ + \alpha_{ji}^+ + \beta_{ji}^+)\mu_j \overline{W}_j(t_i^*) \\ &< D_-V_i(t_i^*) \end{aligned} \tag{29}$$

which contradicts (28).

Suppose case (ii) occurs, we obtain

$$\begin{aligned} 0 &\le D^-(|\bar{y}_j(t_j^*)| - W_j(t_j^*)) \\ &= \lim_{h\to 0^-} \sup \frac{[|\bar{y}_j(t_j^* + h)| - W_j(t_j^* + h)] - [|\bar{y}_j(t_j^*)| - W_j(t_j^*)]}{h} \\ &\le \lim_{h\to 0^-} \sup \frac{|\bar{y}_j(t_j^* + h)| - \bar{y}_j(t_j^*)|}{h} - \lim_{h\to 0^-} \inf \frac{W_j(t_j^* + h) - W_j(t_j^*)}{h} \\ &= D^-|\bar{y}_j(t_j^*)| - D_-W_j(t_j^*). \end{aligned} \tag{30}$$

In view of (16), (23) and (27), we have

$$\begin{aligned} D^-|\bar{y}_j(t_j^*)| &\le -b_j|\bar{y}_j(t_j^*)| \sum_{i=1}^{n} (d_{ji}^+ + p_{ij}^+ + q_{ij}^+)\nu_i x_i(\widetilde{t_j^*}) \\ &= -b_i\ W_j(t_j^*) + \sum_{i=1}^{n} (d_{ji}^+ + p_{ij}^+ + q_{ij}^+)\nu_i x_i(\widetilde{t_j^*}) \\ &\le -b_j\ W_j(t_j^*) + \sum_{i=1}^{m} (d_{ji}^+ + p_{ij}^+ + q_{ij}^+)\nu_i \overline{V}_i(t_j^*) \\ &< D_-W_j(t_j^*) \end{aligned} \tag{31}$$

which contradicts (30). Therefore (25) holds.

Furthermore, together with (17) and (25), we have

$$\begin{cases} |x_i(t_k + 0) - x_i^*| \ \le \ |x_i(t_k) - x_i^*| \ = \ |x_i(t_k - 0) - x_i^*| \ \le \ V_i(t_k) \\ |y_j(t_k + 0) - y_j^*| \ \le \ |y_j(t_k) - y_j^*| \ = \ |y_j(t_k - 0) - y_j^*| \ \le \ W_j(t_k) \end{cases} \tag{32}$$

where $i = 1, 2, ..., n$, $j = 1, 2,..., m$, $k \in Z^+$.

Let $\varepsilon \to 0^+$, $M = (n + m) \max \{\max_{1 \le i \le n} \{\gamma\eta_i\}, \max_{1 \le i \le m} \{\gamma\zeta_j\}\} + 1$, we have from (25) and (32) that

$$\begin{cases} |x_i(t) - x_i^*| \le \gamma\eta_i[\Delta + \varepsilon]\, e^{-\lambda t} \le M||(\phi, \psi) - (x^*, y^*)||e^{-\lambda t} \\ |y_j(t) - y_j^*| \le \gamma\zeta_j[\Delta + \varepsilon]\, e^{-\lambda t} \le M||(\phi, \psi) - (x^*, y^*)||e^{-\lambda t} \end{cases} \tag{33}$$

for all $t > 0$, $i = 1, 2, ..., n$; $j = 1, 2, ..., m$. The proof of theorem 3.1 is completed.

Corollary 3.1 *Suppose* ($A1$), ($A2$) *and* ($A3$) *hold, and if there exist some constants* $\eta_i > 0(i = 1, 2, ..., n)$; $\zeta_j > 0(j = 1, 2, ..., m)$, *such that*

$$-a_i\eta_i + \sum_{j=1}^{m} (c_{ji}^+ + \alpha_{ji}^+ + \beta_{ji}^+)\mu_j\zeta_j < 0, \quad -b_j\zeta_j + \sum_{i=1}^{n} (d_{ij}^+ + p_{ij}^+ + q_{ij}^+)\nu_i\eta_i < 0. \tag{34}$$

Then system (1) has a unique equilibrium point z which is globally exponentially stable.*

Corollary 3.2 *Let* ($A1$), ($A2$) *and* ($A3$) *hold, and suppose that* $E_{n+m} - D^{-1}EU$ *is an M-matrix. Then system (1) has a unique equilibrium point z* which is globally exponentially stable.*

An illustrative example

In this section, we give an example to illustrate the results obtained.

Example 4.1. Considering the following fuzzy BAM neural networks with constant delays.

$$\begin{cases} x_i'(t) = -a_i x_i(t) + \sum_{j=1}^{2} \int_0^1 c_{ji}(s) f_j(y_j(t-s)\mathrm{d}s + \wedge_{j=1}^{2} \int_0^1 \alpha_{ji}(s) f_j(y_j(t-s)\mathrm{d}s + A_i \\ \qquad + \vee_{j=1}^{2} \int_0^1 \beta_{ji}(s) f_j(y_j(t-s))\mathrm{d}s + \wedge_{j=1}^{2} T_{ji}u_j + \vee_{j=1}^{2} H_{ji}u_j, \ t > 0, t \ne t_k. \\ \Delta x_i(t_k) = -\gamma_{ik}(x_i(t_k) - 1, \ k = 1, 2, ... \\ y_i'(t) = -b_j y_j(t) + \sum_{j=1}^{2} \int_0^1 d_{ij}(s)\, g_i(x_i(t-s)\, \mathrm{d}s + \wedge_{j=1}^{2} \int_0^1 p_{ij}(s) g_i(x_i(t-s)\, \mathrm{d}s + B_j \\ \qquad + \vee_{i=1}^{2} \int_0^1 q_{ij}(s)\, g_i(x_i(t-s)\, \mathrm{d}s + \wedge_{i=1}^{2} K_{ij}u_i + \vee_{i=1}^{2} L_{ij}u_i, \ t > 0, t \ne t_k \\ \Delta y_i(t_k) = -\bar{\gamma}_{jk}(y_j(t_k) - 1), \ k = 1, 2, ... \end{cases} \tag{35}$$

where $i, j = 1, 2. f_i(x) = g_i(x) = \frac{1}{2}(|x + 1| - |x - 1|)$, and $t_1 < t_2 < ...$ is strictly increasing sequences such that $\lim_{k\to\infty} t_k = +\infty$, $\gamma_{ik} = 1 - \frac{1}{3}\sin(2 + k), \bar{\gamma}_{jk} = 1 + \frac{2}{3}\cos(3k). a_i = b_j = 1, c_{11}(s) = c_{12}(s) = \frac{1}{2}s, c_{21}(s) = c_{22}(s) = \frac{1}{3}s, d_{11}(s) = d_{12}(s) = \frac{1}{2}s, d_{21}(s) = d_{22}(s) = \frac{1}{3}s, \alpha_{11}(s) = \alpha_{12}(s) = \frac{1}{3}s, \alpha_{21}(s) = \alpha_{22}(s)\ \frac{1}{4}s, \beta_{11}(s) = \beta_{12}(s) = \frac{1}{5}s, \beta_{21}(s) = \beta_{22}(s) = \frac{1}{6}s, p_{11}(s) = p_{12}(s) = \frac{1}{4}s, p_{21}(s) = p_{22}(s) = \frac{1}{5}s, q_{11}(s) = q_{12}(s) = \frac{1}{6}s, q_{21}(s) = q_{22}(s) = \frac{1}{5}s, T_{ij} = H_{ji} = K_{ij} = L_{ij} = 1, \ u_i = u_j = 1, (i, j = 1, 2), \ A_i = B_j = 1, \ (i, j = 1, 2)$.

So, by easy computation, we can see that system (35) satisfy the conditions ($A1$), ($A2$), ($A3$), and $\rho(D^{-1}EU) = 0.8917 < 1$. Therefore, from Theorem 3.1, system (35) has an unique equilibrium point which is globally exponentially stable.

Conclusion

In this article, fuzzy BAM neural networks with distributed delays and impulse have been studied. Some sufficient conditions for the existence, uniqueness, and global exponential stability of equilibrium point have been obtained. The criteria of stability is simple and independent of time delay. It is only associated with the templates of system (1). Moreover, an example is given to illustrate the effectiveness of our results obtained.

Abbreviations
BAM: bi-directional associative memory; FCNNs: fuzzy cellular neural networks.

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Authors' contributions
The authors indicated in parentheses made substantial contributions to the following tasks of research: drafting the manuscript.(L.H.Y); participating in the design of the study (D.X.L); writing and revision of paper (Q.H.Z, L.H.Y).

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The authors declare that they have no competing interests.

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