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Steady flow of an incompressible perfectly conducting fluid past a thin airfoil

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Abstract

We consider the linearized Euler and Maxwell equations and Ohm's law. We calculate the fundamental matrix and give integral representations for the velocity, magnetic induction and pressure. We use the boundary (slip) condition to obtain an integral equation for the jump of the pressure. We give some graphic representations of the velocity and magnetic induction for the case of the flat plate.

MSC: Primary 76X05; secondary 45E05; 35E05

Keywords: linearized system; fundamental matrix; integral representation

Introduction

In papers dedicated to the motion of a wing in an electro-conductive fluid, the lift, drag and moment coefficients were calculated. Recent technological advances claim also for the study of the velocity and electromagnetic fields. We mention two examples: the plasma actuators for aircraft flow control (see [1]) and concealing aircrafts from radar using interaction between ionized gas and electromagnetic radiation. In the present paper, we study the steady two-dimensional flow of an ideal perfectly conducting incompressible fluid around a thin insulating airfoil. We consider the linearized partial differential equations of magnetohydrodynamics (consisting of Euler's and Maxwell's equations and Ohm's law) and calculate the corresponding fundamental matrix. In order to obtain the integral representations for the velocity, the magnetic induction and the pressure fields (which represent the original result of this work), we perform the convolution of the components of the fundamental matrix with the simple layer distributions determined by the jump of the functions we are looking for. We notice that every integral representation has an elliptic as well as a hyperbolic part, this last one being determined by the presence of simple waves bounded by straight characteristics (weak shocks). From the integral representation of the velocity and the boundary conditions (linearized slipping condition and the continuity of the magnetic induction), we rediscover the singular integral equation for the jump of the pressure across the airfoil. We consider the particular case of the flat plate for which the solution of the integral equation is known. Then we perform some numerical integrations to calculate the velocity and the magnetic induction at the points of a two-dimensional grid in order to provide graphic representations for the velocity and magnetic induction fields.

Fundamental matrix of the linearized equations for the two-dimensional incompressible flow of perfectly conducting fluids

Let **v**, **b** and p designate the nondimensional perturbations of the velocity, magnetic induction and pressure, respectively, determined by the presence of a thin insulating airfoil whose equation is

$$y = h_{\pm}(x), \quad x \in [0,1], \qquad \left|h_{\pm}(x)\right| \ll 1, \qquad \left|h'_{\pm}(x)\right| \ll 1. \tag{1}$$

At infinity, we assume that the unperturbed motion is uniform and parallel to the Ox-axis and that there exists a homogeneous magnetic field whose nondimensional expression is

$$\mathbf{B}_0 = (\alpha_x, \alpha_y), \qquad \alpha_x = \cos(\alpha), \qquad \alpha_y = \sin(\alpha), \quad \alpha \in (-\pi/2, \pi/2). \tag{2}$$

As it is shown in [2], Section 5.2, $\mathbf{v} = (v_x, v_y)$, $\mathbf{b} = (b_x, b_y)$ and p satisfy the following system of linear partial differential equations obtained by means of the small perturbations technique:

$$\begin{aligned}
&\frac{\partial v_x}{\partial x} + \frac{\partial p}{\partial x} + \frac{\alpha_y}{A}\left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right) = 0,\\
&\frac{\partial v_y}{\partial x} + \frac{\partial p}{\partial y} - \frac{\alpha_x}{A}\left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right) = 0,\\
&\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0,\\
&\frac{\partial b_x}{\partial x} + \frac{\partial b_y}{\partial y} = 0,\\
&b_y + \alpha_y v_x - \alpha_x v_y = 0
\end{aligned} \tag{3}$$

with $A = \frac{1}{\sqrt{Rh}}$ being Alfvén's number (Rh is the magnetic pressure number). We introduce like in [3] the fundamental matrix of linear system (3)

$$\begin{pmatrix}
v_x^{(1)} & v_y^{(1)} & b_x^{(1)} & b_y^{(1)} & p^{(1)}\\
v_x^{(2)} & v_y^{(2)} & b_x^{(2)} & b_y^{(2)} & p^{(2)}\\
v_x^{(3)} & v_y^{(3)} & b_x^{(3)} & b_y^{(3)} & p^{(3)}\\
v_x^{(4)} & v_y^{(4)} & b_x^{(4)} & b_y^{(4)} & p^{(4)}
\end{pmatrix}, \tag{4}$$

whose components are the fundamental solutions of the systems

$$\begin{aligned}
&\frac{\partial v_x^{(j)}}{\partial x} + \frac{\partial p^{(j)}}{\partial x} + \frac{\alpha_y}{A}\left(\frac{\partial b_y^{(j)}}{\partial x} - \frac{\partial b_x^{(j)}}{\partial y}\right) = \delta_{j1}\delta(x,y),\\
&\frac{\partial v_y^{(j)}}{\partial x} + \frac{\partial p^{(j)}}{\partial y} - \frac{\alpha_x}{A}\left(\frac{\partial b_y^{(j)}}{\partial x} - \frac{\partial b_x^{(j)}}{\partial y}\right) = \delta_{j2}\delta(x,y),\\
&\frac{\partial v_x^{(j)}}{\partial x} + \frac{\partial v_y^{(j)}}{\partial y} = \delta_{j3}\delta(x,y),
\end{aligned} \tag{5}$$

$$\frac{\partial b_x^{(j)}}{\partial x} + \frac{\partial b_y^{(j)}}{\partial y} = \delta_{j4}\delta(x,y),$$
$$b_y^{(j)} + \alpha_y v_x^{(j)} - \alpha_x v_y^{(j)} = 0,$$

where $\delta(x,y)$ is Dirac's distribution and $\delta_{ji} = \begin{cases} 1, & j = i, \\ 0, & j \neq i. \end{cases}$

Some components of the fundamental matrix

We shall use the Fourier transform

$$F[f](\xi_1,\xi_2) = \iint_{\mathbf{R}^2} \exp\bigl[i(x\xi_1 + y\xi_2)\bigr] f(x,y)\,dx\,dy. \tag{6}$$

Taking into account that

$$F\left[\frac{\partial f}{\partial x}\right](\xi_1,\xi_2) = -i\xi_1 F[f](\xi_1,\xi_2), \qquad F\left[\frac{\partial f}{\partial y}\right](\xi_1,\xi_2) = -i\xi_2 F[f](\xi_1,\xi_2), \tag{7}$$
$$F[\delta] = 1, \tag{8}$$

and using the notation $F[f] = \tilde{f}$, we obtain from (5)

$$\begin{aligned}
&-i\xi_1\tilde{v}_x^{(j)} - i\xi_1\tilde{p}^{(j)} + \frac{i\alpha_y\xi_2}{A^2}\tilde{b}_x^{(j)} - \frac{i\alpha_y\xi_1}{A^2}\tilde{b}_y^{(j)} = \delta_{j1},\\
&-i\xi_1\tilde{v}_y^{(j)} - i\xi_2\tilde{p}^{(j)} - \frac{i\alpha_x\xi_2}{A^2}\tilde{b}_x^{(j)} + \frac{i\alpha_x\xi_1}{A^2}\tilde{b}_y^{(j)} = \delta_{j2},\\
&-i\xi_1\tilde{v}_x^{(j)} - i\xi_2\tilde{v}_y^{(j)} = \delta_{j3},\\
&-i\xi_1\tilde{b}_x^{(j)} - i\xi_2\tilde{b}_y^{(j)} = \delta_{j4},\\
&\tilde{b}_y^{(j)} + \alpha_y\tilde{v}_x^{(j)} - \alpha_x\tilde{v}_y^{(j)} = 0.
\end{aligned} \tag{9}$$

Solving system (9), we get for $j = 2$

$$\begin{aligned}
\tilde{v}_x^{(2)} &= \frac{i\xi_1^2\xi_2}{(\xi_1^2+\xi_2^2)((\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2)},\\
\tilde{v}_y^{(2)} &= \frac{-i\xi_1^3}{(\xi_1^2+\xi_2^2)((\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2)},\\
\tilde{b}_x^{(2)} &= \frac{i\alpha_x\xi_1^2\xi_2 + i\alpha_y\xi_1\xi_2^2}{(\xi_1^2+\xi_2^2)((\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2)},\\
\tilde{p}^{(2)} &= -i\frac{\xi_1^2\xi_2}{(\xi_1^2+\xi_2^2)((\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2)} + i\frac{\alpha_y}{A^2}\frac{\xi_1\alpha_x+\xi_2\alpha_y}{(\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2}.
\end{aligned} \tag{10}$$

For $(l,k) \in \{(3,0);(2,1);(1,2)\}$, we have

$$\frac{\xi_1^l\xi_2^k}{(\xi_1^2+\xi_2^2)((\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2)} = \frac{a_{lk}\xi_1 + b_{lk}\xi_2}{\xi_1^2+\xi_2^2} + \frac{c_{lk}\xi_1 + d_{lk}\xi_2}{(\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2} \tag{11}$$

with

$$\begin{aligned} a_{30} &= -\frac{A^2(A^2+1-2\alpha_x^2)}{1+2A^2(1-2\alpha_x^2)+A^4}, \qquad b_{30} = -\frac{2A^2\alpha_x\alpha_y}{1+2A^2(1-2\alpha_x^2)+A^4}, \\ c_{30} &= \frac{\alpha_y^2(A^2+1+2\alpha_x^2)}{1+2A^2(1-2\alpha_x^2)+A^4}, \qquad d_{30} = \frac{2\alpha_x\alpha_y^3}{1+2A^2(1-2\alpha_x^2)+A^4}, \end{aligned} \tag{12}$$

$$\begin{aligned} a_{21} &= \frac{2A^2\alpha_x\alpha_y}{1+2A^2(1-2\alpha_x^2)+A^4}, \qquad b_{21} = -\frac{A^2(A^2+1-2\alpha_x^2)}{1+2A^2(1-2\alpha_x^2)+A^4}, \\ c_{21} &= \frac{2\alpha_x\alpha_y(A^2-\alpha_x^2)}{1+2A^2(1-2\alpha_x^2)+A^4}, \qquad d_{21} = \frac{\alpha_y^2(A^2+1-2\alpha_x^2)}{1+2A^2(1-2\alpha_x^2)+A^4}, \end{aligned} \tag{13}$$

$$\begin{aligned} a_{12} &= \frac{A^2(A^2+1-2\alpha_x^2)}{1+2A^2(1-2\alpha_x^2)+A^4}, \qquad b_{12} = \frac{2A^2\alpha_x\alpha_y}{1+2A^2(1-2\alpha_x^2)+A^4}, \\ c_{12} &= \frac{(A^2-\alpha_x^2)(A^2+1-2\alpha_x^2)}{1+2A^2(1-2\alpha_x^2)+A^4}, \qquad d_{12} = -\frac{2\alpha_x\alpha_y^3}{1+2A^2(1-2\alpha_x^2)+A^4}. \end{aligned} \tag{14}$$

Hence

$$\begin{aligned} \tilde{v}_x^{(2)} &= \frac{ia_{21}\xi_1 + ib_{21}\xi_2}{\xi_1^2+\xi_2^2} + \frac{ic_{21}\xi_1 + id_{21}\xi_2}{(\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2}, \\ \tilde{v}_y^{(2)} &= \frac{-ia_{30}\xi_1 - ib_{30}\xi_2}{\xi_1^2+\xi_2^2} + \frac{-ic_{30}\xi_1 - id_{30}\xi_2}{(\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2}, \\ \tilde{b}_x^{(2)} &= \alpha_x\tilde{v}_x^{(2)} + \alpha_y\tilde{v}_y^{(2)} + \frac{i\alpha_y\xi_1}{(\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2}, \\ \tilde{p}^{(2)} &= -\tilde{v}_x^{(2)} + \frac{i\alpha_y}{A^2}\frac{\xi_1\alpha_x+\xi_2\alpha_y}{(\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2}. \end{aligned}$$

Using the inverse Fourier transforms ([4], Appendix 1)

$$\begin{aligned} F^{-1}\left[\frac{\xi_1}{\xi_1^2+\xi_2^2}\right] &= \frac{-i}{2\pi}\frac{\partial}{\partial x}\ln\sqrt{x^2+y^2}, \\ F^{-1}\left[\frac{\xi_2}{\xi_1^2+\xi_2^2}\right] &= \frac{-i}{2\pi}\frac{\partial}{\partial y}\ln\sqrt{x^2+y^2}, \\ F^{-1}\left[\frac{\xi_1}{(\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2}\right] &= \frac{iA}{2\alpha_y}\frac{\partial}{\partial x}H\left(x - \frac{\alpha_x}{\alpha_y}y - \frac{A}{\alpha_y}|y|\right), \\ F^{-1}\left[\frac{\xi_2}{(\frac{\xi_1\alpha_x+\xi_2\alpha_y}{A})^2 - \xi_1^2}\right] &= \frac{iA}{2\alpha_y}\frac{\partial}{\partial y}H\left(x - \frac{\alpha_x}{\alpha_y}y - \frac{A}{\alpha_y}|y|\right) \end{aligned}$$

(where $H(x) = \begin{cases} 1, & x \geq 0, \\ 0, & x < 0 \end{cases}$ is Heaviside's function), we get

$$\begin{aligned} v_x^{(2)} &= \frac{a_{21}}{2\pi}\frac{x}{x^2+y^2} + \frac{b_{21}}{2\pi}\frac{y}{x^2+y^2} \\ &\quad - \frac{Ac_{21}}{2\alpha_y}\frac{\partial}{\partial x}H\left(x - \frac{\alpha_x}{\alpha_y}y - \frac{A}{\alpha_y}|y|\right) - \frac{Ad_{21}}{2\alpha_y}\frac{\partial}{\partial y}H\left(x - \frac{\alpha_x}{\alpha_y}y - \frac{A}{\alpha_y}|y|\right), \end{aligned} \tag{15}$$

$$\begin{aligned} v_y^{(2)} &= -\frac{a_{30}}{2\pi}\frac{x}{x^2+y^2} - \frac{b_{30}}{2\pi}\frac{y}{x^2+y^2} \\ &\quad + \frac{Ac_{30}}{2\alpha_y}\frac{\partial}{\partial x}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right) + \frac{Ad_{30}}{2\alpha_y}\frac{\partial}{\partial y}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right), \end{aligned} \tag{16}$$

$$b_x^{(2)} = \alpha_x v_x^{(2)} + \alpha_y v_y^{(2)} - \frac{A}{2}\frac{\partial}{\partial x}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right), \tag{17}$$

$$p_x^{(2)} = -v_x^{(2)} - \frac{\alpha_x}{2A}\frac{\partial}{\partial x}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right) - \frac{\alpha_y}{2A}\frac{\partial}{\partial y}H\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right). \tag{18}$$

Integral representations

In thin airfoil theory, the linearized boundary conditions are usually imposed on the segment $[-1,1]$ and the functions we are looking for are defined on $\mathbf{R}^2\setminus[-1,1]$. Since v_x, v_y, b_x, b_y and p are integrable functions, they may be regarded as regular distributions. Taking into account the boundary conditions ([2], Chapter 5)

$$[\mathbf{b}](x) = 0, \qquad v_y(x,\pm 0) = h'_{\pm}(x), \quad x \in [-1,1], \tag{19}$$

we obtain the following linearized system for the distributions v_x, v_y, b_x, b_y and p:

$$\begin{aligned} &\frac{\partial v_x}{\partial x} + \frac{\partial p}{\partial x} + \frac{\alpha_y}{A}\left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right) = 0 = f_1, \\ &\frac{\partial v_y}{\partial x} + \frac{\partial p}{\partial y} - \frac{\alpha_x}{A}\left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right) = [p]\delta_{[-1,1]} = f_2, \\ &\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = [h']\delta_{[-1,1]} = f_3, \\ &\frac{\partial b_x}{\partial x} + \frac{\partial b_y}{\partial y} = 0 = f_4, \\ &b_y + \alpha_y v_x - \alpha_x v_y = 0, \end{aligned} \tag{20}$$

where $[\mathbf{b}](x)$ and $[p](x)$ represent the jumps of $\mathbf{b}$ and p over the segment $[-1,1]$ and $[p]\delta_{[-1,1]}$, $[h']\delta_{[-1,1]}$ are simple layer distributions. We may easily verify that

$$\begin{aligned} &v_x = \sum_{j=1}^{4} v_x^{(j)} \star f_j, \qquad v_y = \sum_{j=1}^{4} v_y^{(j)} \star f_j, \\ &b_x = \sum_{j=1}^{4} b_x^{(j)} \star f_j, \qquad p = \sum_{j=1}^{4} p^{(j)} \star f_j, \end{aligned} \tag{21}$$

where $\star$ stands for the convolution product. We shall consider, for the sake of simplicity, the case of zero thickness wing, *i.e.*, $h_+(x) = h_-(x)$. Hence $f_1 = f_3 = f_4 = 0$ and

$$\begin{aligned} &v_x = v_x^{(2)} \star [p]\delta_{[-1,1]}, \qquad v_y = v_y^{(2)} \star [p]\delta_{[-1,1]}, \\ &b_x = b_x^{(2)} \star [p]\delta_{[-1,1]}, \qquad p = p^{(2)} \star [p]\delta_{[-1,1]}. \end{aligned}$$

We calculate the convolutions and obtain the following representation for v_x:

$$\begin{aligned}
v_x(x,y) &= \frac{a_{21}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{x-\xi}{(x-\xi)^2+y^2}\,d\xi + \frac{b_{21}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{y}{(x-\xi)^2+y^2}\,d\xi \\
&\quad - \frac{Ac_{21}}{2\alpha_y}\frac{\partial}{\partial x}\int_{-1}^{1}[p](\xi)H\left(x-\xi-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right)d\xi \\
&\quad - \frac{Ad_{21}}{2\alpha_y}\frac{\partial}{\partial y}\int_{-1}^{1}\bigl[p(\xi)\bigr]H\left(x-\xi-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right)d\xi \\
&= \frac{a_{21}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{x-\xi}{(x-\xi)^2+y^2}\,d\xi + \frac{b_{21}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{y}{(x-\xi)^2+y^2}\,d\xi \\
&\quad - \frac{A}{2\alpha_y^2}\left(c_{21}\alpha_y-d_{21}\alpha_x-d_{21}A\frac{y}{|y|}\right) \\
&\quad \times [p]\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right)\times H\left(1-\left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|\right).
\end{aligned}\tag{22}$$

Similarly, we get

$$\begin{aligned}
v_y(x,y) &= -\frac{a_{30}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{x-\xi}{(x-\xi)^2+y^2}\,d\xi - \frac{b_{30}}{2\pi}\int_{-1}^{1}[p](\xi)\frac{y}{(x-\xi)^2+y^2}\,d\xi \\
&\quad + \frac{A}{2\alpha_y^2}\left(c_{30}\alpha_y-d_{30}\alpha_x-d_{30}A\frac{y}{|y|}\right) \\
&\quad \times [p]\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right)\times H\left(1-\left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|\right),
\end{aligned}\tag{23}$$

$$\begin{aligned}
b_x(x,y) &= \alpha_x v_x(x,y)+\alpha_y v_y(x,y) \\
&\quad - \frac{A}{2}[p]\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right)\times H\left(1-\left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|\right),
\end{aligned}\tag{24}$$

$$\begin{aligned}
p(x,y) &= -v_x(x,y) \\
&\quad + \frac{y}{2|y|}[p]\left(x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right)\times H\left(1-\left|x-\frac{\alpha_x}{\alpha_y}y-\frac{A}{\alpha_y}|y|\right|\right).
\end{aligned}\tag{25}$$

The integral equation for the jump of the pressure

Using the Plemelj formulas and linearized boundary conditions (19), we get from (23) the integral equation

$$\begin{aligned}
h'(x) &= v_y(x,\pm 0) \\
&= -\frac{a_{30}}{2\pi}p.v.\int_{-1}^{1}\frac{[p(\xi)]}{x-\xi}\,d\xi + \frac{A(c_{30}\alpha_y-d_{30}\alpha_x)}{2\alpha_y^2}\bigl[p(x)\bigr], \quad x\in(-1,1),
\end{aligned}\tag{26}$$

which is equivalent to

$$-\beta[p](x)+\frac{k}{\pi}p.v.\int_{-1}^{1}\frac{[p](\xi)}{\xi-x}\,d\xi = \chi h'(x),\tag{27}$$

where $k = A^2 + 1 - 2\alpha_x^2$, $\beta = \alpha_y(1 + A^2)A^{-1}$ and $\chi = 2(4\alpha_x^2 + A^{-2} - A^2)$. As it is shown in [3, 5] and [2], Chapter 5, the solution of equation (27) is

$$[p](x) = \frac{-\beta\chi}{\beta^2 + k^2}h'(x) - \frac{k\chi}{\pi(\beta^2 + k^2)}\left(\frac{1-x}{1+x}\right)^\theta p.v.\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^\theta \frac{h'(\xi)}{\xi - x}\,d\xi$$
$$+ \frac{2C\sin\theta\pi}{(1-x)^{1-\theta}(1+x)^\theta}, \quad \tan\theta\pi = \frac{k}{\beta}, 0 \le \theta < 1. \qquad (28)$$

We may take $C = 0$ if we impose Kutta-Joukovsky's condition. Other choices of the constant C were considered by Stewartson [6].

The flat plate

In this case, $h(x) = -\varepsilon x$, $h'(x) = -\varepsilon$. Taking into account that

$$p.v.\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^\theta \frac{1}{\xi - x}\,d\xi = \frac{\pi}{\sin\theta\pi}\left\{1 - \left(\frac{1+x}{1-x}\right)^\theta \cos\theta\pi\right\},$$

we get from (28)

$$[p](x) = \frac{\varepsilon\chi}{\sqrt{\beta^2 + k^2}}\left(\frac{1-x}{1+x}\right)^\theta.$$

The lift is ([2], 5.2.6)

$$L = -\int_{-1}^{1}[p](x)\,dx = 4\varepsilon\pi\frac{(A^2-1)^2 + 4A^2\alpha_y^2}{A^2(A^2 - 1 + 2\alpha_y^2)}\theta.$$

In order to obtain graphic representations for the velocity (Figure 1), magnetic induction (Figure 2) and lift (Figure 3), we use the quadrature formulas

$$I_1(x,y) = \int_{-1}^{1}\left(\frac{1-\xi}{1+\xi}\right)^\theta \frac{x-\xi}{(x-\xi)^2 + y^2} = \sum_{k=1}^{N} W_k \frac{x - t_k}{(x-t_k)^2 + y^2},$$
$$I_2 = \int_{-1}^{1}\left(\frac{1-\xi}{1+\xi}\right)^\theta \frac{y}{(x-\xi)^2 + y^2} = \sum_{k=1}^{N} W_k \frac{y}{(x-t_k)^2 + y^2},$$

where

$$W_k = -\frac{4\Gamma(N+\theta+1)\Gamma(N-\theta+1)}{[(N+1)!]^2 P_{N-1}^{(\theta+1,-\theta+1)}(t_k)P_{N+1}^{(\theta,-\theta)}(t_k)}.$$

$P_{N-1}^{(\theta+1,-\theta+1)}(t_k)$, $P_{N+1}^{(\theta,-\theta)}(t_k)$ are the Jacobi polynomials and t_k, $k = 1,\dots,N$, are the roots of $P_N^{(\theta,-\theta)}(t_k)$. We considered $\varepsilon = 0.2$.

Conclusions

We have calculated the velocity, the pressure and the magnetic induction for the steady flow of an incompressible perfectly conducting fluid past a thin airfoil. The integral rep-

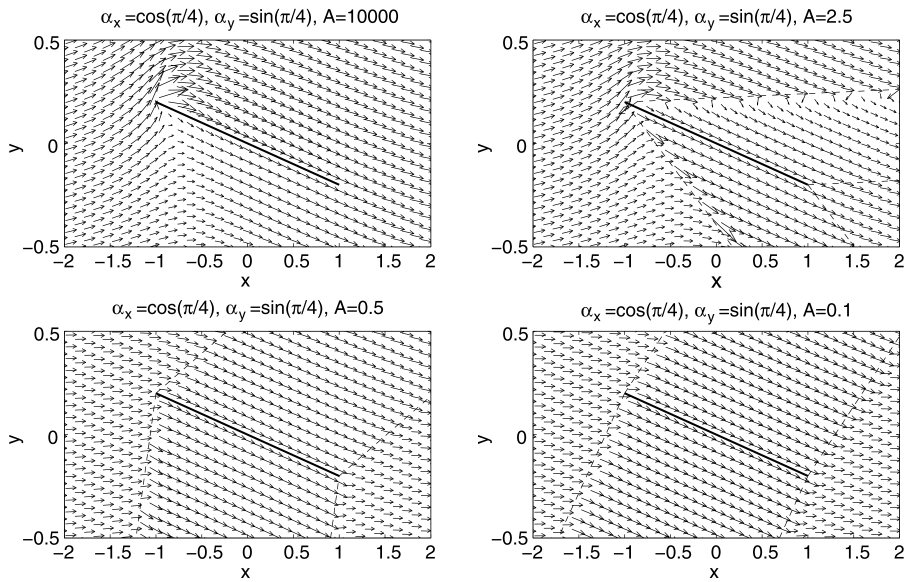

Figure 1 Velocity field. The velocity is represented by arrows in the grid points.

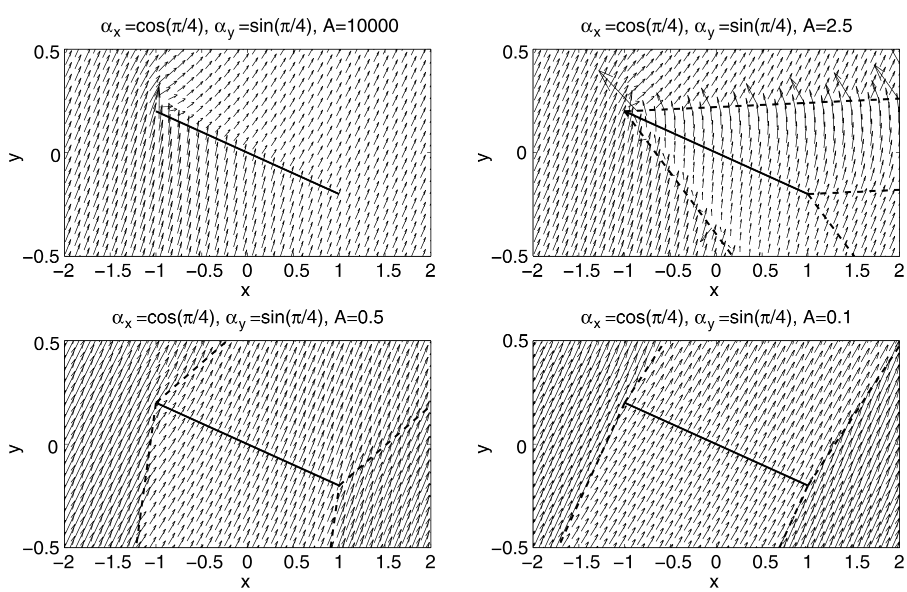

Figure 2 Magnetic induction field. The magnetic induction is represented by arrows in the grid points.

resentations of the velocity, pressure and magnetic induction contain respectively an elliptic part and a hyperbolic one. Some calculations were performed for the flat plate. In the graphic representation of the velocity and magnetic induction from Figures 1 and 2, we may observe the simple waves which are determined by the influence of the magnetic field. In Figure 3 we represent the lift coefficient against Alfvén's number A. We notice that the lift coefficient increases when A decreases, *i.e.*, when the value of the magnetic induction at infinity increases.

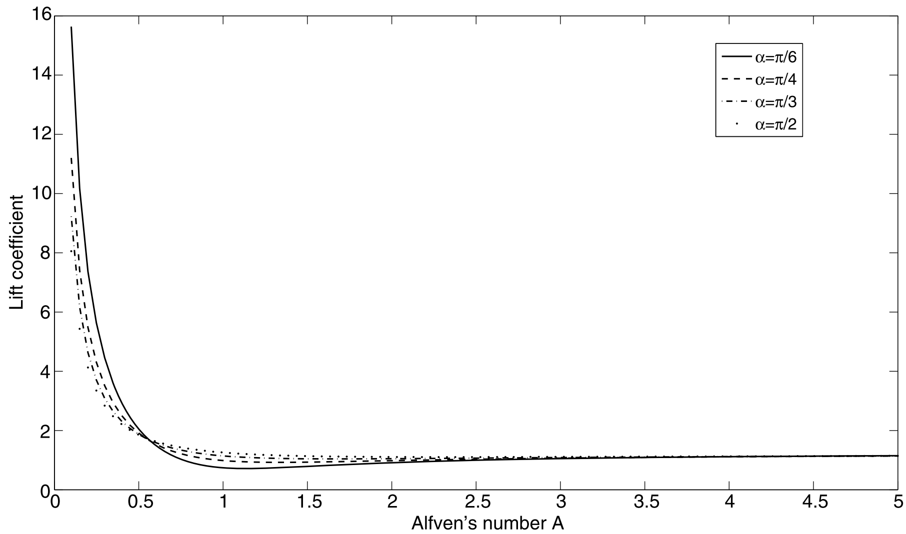

Figure 3 Lift coefficient versus Alfvén's number. The lift coefficient is calculated for various values of the angle α.

Competing interests
The author declares that he has no competing interests.

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References

1. Cork, TC, Post, ML, Orlov, DM: Single dielectric barrier discharge plasma enhanced aerodynamics: physics, modeling and applications. Exp. Fluids **46**, 1-26 (2009)
2. Dragoş, L: Magnetofluid Dynamics. Ed. Academiei Române, Bucharest (1975)
3. Carabineanu, A: Fundamental solutions for the plane-parallel steady flow of incompressible perfectly conducting fluids past thin airfoils. Rev. Roum. Math. Pures Appl. **40**, 279-288 (1995)
4. Dragoş, L: Mathematical Methods in Aerodynamics. Kluwer Academic, Dordrecht (2003)
5. Homentcovschi, D: Sur la résolution explicite du problème de Hilbert. Application au calcul de la portance d'un profil mince dans un fluide électroconducteur. Rev. Roum. Math. Pures Appl. **14**, 203-223 (1969)
6. Stewartson, K: Magneto-fluid dynamics of thin bodies in oblique fields. Z. Angew. Math. Phys. **12**, 261-271 (1961)

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