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Fixed point theorems for weakly C-contractive mappings in partial metric spaces

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Abstract

In this work, we establish some fixed point theorems for weakly C-contractive mappings in partial metric spaces. Presented theorems extend and generalize some existence results in the literature. Also, an example is given to support our results.

MSC: 47H10; 54H25

Keywords: fixed point; common fixed point; partial metric space; weakly C-contraction

1 Introduction and preliminaries

Fixed point theory has fascinated many mathematicians since 1922 with the celebrated Banach's fixed point theorem. Fixed point theory plays a major role within as well as outside mathematics, so the attraction of fixed point theory to large numbers of researchers is understandable, and the problem of fixed point has been studied in several directions; see for example, [1–4]. The study of metric fixed point theory has been researched extensively in the past decades. Recently, some generalizations of the notion of a metric space have been proposed by some authors. In 1992, Matthews introduced a new notion of generalized metric space called partial metric space (for short PMS) [5, 6], in which the distance of a point from itself may not be zero. After the appearance of partial metric spaces, some authors started to generalize Banach contraction mapping theorem to partial metric spaces and focus on fixed point theory on partial metric spaces (see, *e.g.*, [7–24]). A new category of fixed point problems was addressed by Khan *et al.* [25]. In this study, they introduced the concept of altering distance function. In [26], Choudhury introduced the concept of weakly C-contractive mapping as follows.

Definition 1.1 [26] Let (X,d) be a metric space and $T : X \to X$ be a mapping. Then T is said to be weakly C-contractive (or a weakly C-contraction) if for all $x,y \in X$, the following inequality holds:

$$d(Tx,Ty) \leq \frac{1}{2}\big(d(x,Ty)+d(Tx,y)\big) - \phi\big(d(x,Ty),d(Tx,y)\big),$$

where $\phi : [0,+\infty) \times [0,+\infty) \to [0,+\infty)$ is a continuous function such that $\phi(x,y) = 0$ if and only if $x = y = 0$.

Shatanawi [27] investigated some fixed point theorems and coupled fixed point theorems for weakly C-contractive mapping by using an altering distance function in metric and partially ordered metric spaces.

Recently, Haghi *et al.* [28] pointed that many fixed point generalizations to partial metric spaces can be obtained from the corresponding results in metric spaces and considered some cases to demonstrate this fact. The aim of this paper is to research fixed point and common fixed point theorems for weakly C-contractive type mappings in partial metric spaces. Our results extend and generalize some results of [27] to partial metric spaces; all of our results cannot be obtained from the corresponding results in metric spaces. Moreover, even in metric spaces, our results are the generalizations of some results of [27]. Also, we give an example to illustrate our results.

Throughout this paper, the letters N and N^+ denote the set of all nonnegative integer numbers and the set of all positive integer numbers, respectively. Let us recall some definitions and properties of partial metric spaces.

Definition 1.2 [6] Let X be a nonempty set. The mapping $p : X \times X \to [0,+\infty)$ is said to be a partial metric on X if the following conditions hold:

(P_1) $x = y \Leftrightarrow p(x,y) = p(x,x) = p(y,y)$,
(P_2) $p(x,x) \leq p(x,y)$,
(P_3) $p(x,y) = p(y,x)$,
(P_4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$,

for any $x,y,z \in X$. The pair (X,p) is then called a partial metric space.

It is clear that, if $p(x,y) = 0$, then from (P_1) and (P_2), $x = y$. But if $x = y$, $p(x,y)$ may not be 0.

For a partial metric p on X, the function $d_p : X \times X \to [0,+\infty)$ given by

$$d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y)$$

is a (usual) metric on X. Each partial metric p on X generates a T_0-topology τ_p on X with a base of the family of open p-balls $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

Let (X,p) be a partial metric space. Then:

A sequence $\{x_n\}$ in a partial metric space (X,p) converges to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to+\infty} p(x,x_n)$.

A sequence $\{x_n\}$ in a partial metric space (X,p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to+\infty} p(x_m,x_n)$.

A partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to+\infty} p(x_m,x_n)$.

The following lemmas play a major role in proving our main results.

Lemma 1.1 [29] *Let (X,p) be a partial metric space.*

(A) *A sequence $\{x_n\}$ is a Cauchy sequence in (X,p) if and only if $\{x_n\}$ is a Cauchy sequence in (X,d_p).*
(B) *(X,p) is complete if and only if (X,d_p) is complete. Moreover,*

$$\lim_{n\to+\infty} d_p(x_n,x) = 0 \quad \Leftrightarrow \quad p(x,x) = \lim_{n\to+\infty} p(x_n,x) = \lim_{n,m\to+\infty} p(x_n,x_m). \tag{1.1}$$

Lemma 1.2 [29, 30] *Assume that $x_n \to z$ as $n \to +\infty$ in a PMS (X, p) such that $p(z, z) = 0$. Then $\lim_{n\to+\infty} p(x_n, y) = p(z, y)$ for every $y \in X$.*

Lemma 1.3 [31] *Let (X, p) be a partial metric space and let $\{x_n\}$ be a sequence in X such that*

$$\lim_{n\to+\infty} p(x_{n+1}, x_n) = 0.$$

If $\{x_{2n}\}$ is not a Cauchy sequence in (X, p), then there exist $\varepsilon > 0$ and two sequences $\{m(k)\}$ and $\{n(k)\}$ of positive integers such that $n(k) > m(k) > k$ and the following four sequences tend to ε when $k \to +\infty$:

$$\begin{aligned} &p(x_{2m(k)}, x_{2n(k)}), \qquad p(x_{2m(k)}, x_{2n(k)+1}), \\ &p(x_{2m(k)-1}, x_{2n(k)}), \qquad p(x_{2m(k)-1}, x_{2n(k)+1}). \end{aligned} \tag{1.2}$$

2 Main results

We start this section with the following definition, which can be seen in [9, 16, 17, 30].

Definition 2.1 Let (x, P) be a partial metric space. A mapping $T : X \to X$ is said to be continuous at $x_0 \in X$ if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $T(B_p(x_0, \delta)) \subset B_p(Tx_0, \varepsilon)$.

Definition 2.2 [25] The function $\varphi : [0, +\infty) \to [0, +\infty)$ is called an altering distance function, if the following properties are satisfied:

(1) φ is continuous and nondecreasing;

(2) $\varphi(t) = 0$ if and only if $t = 0$.

Lemma 2.1 [31] *Let (X, p) be a partial metric space, $T : X \to X$ be a given mapping. Suppose that T is continuous at $x_0 \in X$. Then, for each sequence $\{x_n\}$ in X, $x_n \to x_0$ in $\tau_p \Rightarrow Tx_n \to Tx_0$ in τ_p holds.*

Theorem 2.1 *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a partial metric p on X such that (X, p) is complete. Let $f : X \to X$ be a continuous nondecreasing mapping. Suppose that for comparable $x, y \in X$, we have*

$$\psi\big(p(fx, fy)\big) \le \varphi\left(\frac{p(x, fy) + p(fx, y)}{2}\right) - \phi\big(p(x, fy), p(fx, y)\big), \tag{2.1}$$

where ψ and φ are altering distance functions with

$$\psi(t) - \varphi(t) \ge 0 \tag{2.2}$$

for all $t \ge 0$, and $\phi : [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ is a continuous function with $\phi(x, y) = 0$ if and only if $x = y = 0$. If there exists $x_0 \in X$ such that $x_0 \preceq fx_0$, then f has a fixed point.

Proof If $x_0 = fx_0$, then x_0 is a fixed point of f. Suppose that $x_0 \prec fx_0$, we can choose $x_1 \in X$ such that $fx_0 = x_1$. Since f is a nondecreasing function, we have

$$x_0 \prec x_1 = fx_0 \preceq x_2 = fx_1 \preceq x_3 = fx_2.$$

Continuing this process, we can construct a sequence $\{x_n\}$ in X such that $x_{n+1} = fx_n$ with

$$x_0 \prec x_1 \preceq x_2 \preceq \cdots \preceq x_n \preceq x_{n+1} \preceq \cdots.$$

It is clear that if $p(x_n, x_{n+1}) = 0$ for some $n_0 \in N$, then f has a fixed point. Taking $p(x_n, x_{n+1}) > 0$ for all $n \in N$, now let us prove the following inequality:

$$p(x_n, x_{n+1}) \leq p(x_{n-1}, x_n), \quad n \in N^+. \tag{2.3}$$

Suppose this is not true, then $p(x_n, x_{n+1}) > p(x_{n-1}, x_n)$ for some n_0, that is,

$$p(x_{n_0}, x_{n_0+1}) > p(x_{n_0-1}, x_{n_0}). \tag{2.4}$$

From (2.1) and (2.4), we obtain that

$$\begin{aligned}
&\psi\big(p(x_{n_0}, x_{n_0+1})\big) \\
&\quad = \psi\big(p(fx_{n_0-1}, fx_{n_0})\big) \\
&\quad \leq \varphi\left(\frac{p(x_{n_0-1}, fx_{n_0}) + p(fx_{n_0-1}, x_{n_0})}{2}\right) - \phi\big(p(x_{n_0-1}, fx_{n_0}), p(fx_{n_0-1}, x_{n_0})\big) \\
&\quad = \varphi\left(\frac{p(x_{n_0-1}, x_{n_0+1}) + p(x_{n_0}, x_{n_0})}{2}\right) - \phi\big(p(x_{n_0-1}, x_{n_0+1}), p(x_{n_0}, x_{n_0})\big) \\
&\quad \leq \varphi\left(\frac{p(x_{n_0-1}, x_{n_0}) + p(x_{n_0}, x_{n_0+1})}{2}\right) - \phi\big(p(x_{n_0-1}, x_{n_0+1}), p(x_{n_0}, x_{n_0})\big) \\
&\quad \leq \varphi\big(\max\{p(x_{n_0-1}, x_{n_0}), p(x_{n_0}, x_{n_0+1})\}\big) - \phi\big(p(x_{n_0-1}, x_{n_0+1}), p(x_{n_0}, x_{n_0})\big) \\
&\quad = \varphi\big(p(x_{n_0}, x_{n_0+1})\big) - \phi\big(p(x_{n_0-1}, x_{n_0+1}), p(x_{n_0}, x_{n_0})\big),
\end{aligned}$$

this together with (2.2) shows that

$$\phi\big(p(x_{n_0-1}, x_{n_0+1}), p(x_{n_0}, x_{n_0})\big) = 0.$$

Using the property of ϕ, we have

$$p(x_{n_0-1}, x_{n_0+1}) = 0, \qquad p(x_{n_0}, x_{n_0}) = 0. \tag{2.5}$$

Since

$$\begin{aligned}
&\psi\big(p(x_{n_0}, x_{n_0+1})\big) \\
&\quad = \psi\big(p(fx_{n_0-1}, fx_{n_0})\big) \\
&\quad \leq \varphi\left(\frac{p(x_{n_0-1}, fx_{n_0}) + p(fx_{n_0-1}, x_{n_0})}{2}\right) - \phi\big(p(x_{n_0-1}, fx_{n_0}), p(fx_{n_0-1}, x_{n_0})\big) \\
&\quad = \varphi\left(\frac{p(x_{n_0-1}, x_{n_0+1}) + p(x_{n_0}, x_{n_0})}{2}\right) - \phi\big(p(x_{n_0-1}, x_{n_0+1}), p(x_{n_0}, x_{n_0})\big),
\end{aligned}$$

applying (2.5), we get

$$\psi\big(p(x_{n_0}, x_{n_0+1})\big) = 0. \tag{2.6}$$

From the property of ψ, we have $p(x_{n_0}, x_{n_0+1}) = 0$, which contradicts with $p(x_n, x_{n+1}) > 0$ for all $n \in N$; hence (2.3) holds. Therefore, $\{p(x_n, x_{n+1})\}$ is a nonincreasing sequence, and thus there exists $r \geq 0$ such that

$$\lim_{n\to+\infty} p(x_n, x_{n+1}) = r.$$

Using (2.1), we obtain

$$\begin{aligned}
&\psi\big(p(x_{n+1}, x_{n+2})\big) \\
&\quad = \psi\big(p(fx_n, fx_{n+1})\big) \\
&\quad \leq \varphi\left(\frac{p(x_n, fx_{n+1}) + p(fx_n, x_{n+1})}{2}\right) - \phi\big(p(x_n, fx_{n+1}), p(fx_n, x_{n+1})\big) \\
&\quad = \varphi\left(\frac{p(x_n, x_{n+2}) + p(x_{n+1}, x_{n+1})}{2}\right) - \phi\big(p(x_n, x_{n+2}), p(x_{n+1}, x_{n+1})\big) \\
&\quad \leq \varphi\left(\frac{p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2})}{2}\right) - \phi\big(p(x_n, x_{n+2}), p(x_{n+1}, x_{n+1})\big),
\end{aligned} \tag{2.7}$$

it means that

$$\phi\big(p(x_n, x_{n+2}), p(x_{n+1}, x_{n+1})\big) \leq \varphi\left(\frac{p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2})}{2}\right) - \psi\big(p(x_{n+1}, x_{n+2})\big).$$

Letting $n \to +\infty$ in the above inequality, we get

$$\liminf_{n\to+\infty} \phi\big(p(x_n, x_{n+2}), p(x_{n+1}, x_{n+1})\big) = 0,$$

the continuity of ϕ guarantees that

$$\phi\Big(\liminf_{n\to+\infty} p(x_n, x_{n+2}), \liminf_{n\to+\infty} p(x_{n+1}, x_{n+1})\Big) = 0,$$

and the property of ϕ gives that

$$\liminf_{n\to+\infty} p(x_n, x_{n+2}) = 0, \qquad \liminf_{n\to+\infty} p(x_{n+1}, x_{n+1}) = 0. \tag{2.8}$$

Since

$$\begin{aligned}
&\psi\big(p(x_{n+1}, x_{n+2})\big) \\
&\quad = \psi\big(p(fx_n, fx_{n+1})\big) \\
&\quad \leq \varphi\left(\frac{p(x_n, fx_{n+1}) + p(fx_n, x_{n+1})}{2}\right) - \phi\big(p(x_n, fx_{n+1}), p(fx_n, x_{n+1})\big) \\
&\quad = \varphi\left(\frac{p(x_n, x_{n+2}) + p(x_{n+1}, x_{n+1})}{2}\right) - \phi\big(p(x_n, x_{n+2}), p(x_{n+1}, x_{n+1})\big),
\end{aligned}$$

on taking inferior limit in the above inequalities and using (2.8), we obtain that $\psi(r) = 0$ and so $r = 0$, therefore,

$$\lim_{n\to+\infty} p(x_n, x_{n+1}) = 0,$$

moreover, we have

$$\lim_{n\to+\infty} p(x_n,x_n) = 0.$$

Now, we claim that $\{x_n\}$ is a Cauchy sequence in the metric space (X,d_p) (and so also in the space (X,p) by Lemma 1.1). For this, it is sufficient to show that $\{x_{2n}\}$ is a Cauchy sequence in (X,d_p). Suppose that this is not the case, then using Lemma 1.1 we have that $\{x_{2n}\}$ is not a Cauchy sequence in (X,p). By Lemma 1.3, we obtain that there exist $\varepsilon > 0$ and two sequences $\{m(k)\}$ and $\{n(k)\}$ of positive integers such that $n(k) > m(k) > k$ and sequences in (1.2) tend to ε when $k \to +\infty$. For two comparable elements $y = x_{2n(k)+1}$ and $x = x_{2m(k)}$, we can obtain, from (2.1), that

$$\begin{aligned}
&\psi\bigl(p(x_{2n(k)+1},x_{2m(k)})\bigr)\\
&\quad= \psi\bigl(p(fx_{2n(k)},fx_{2m(k)-1})\bigr)\\
&\quad\le \varphi\biggl(\frac{p(x_{2n(k)},fx_{2m(k)-1}) + p(fx_{2n(k)},x_{2m(k)-1})}{2}\biggr)\\
&\qquad- \phi\bigl(p(x_{2n(k)},fx_{2m(k)-1}),p(fx_{2n(k)},x_{2m(k)-1})\bigr)\\
&\quad= \varphi\biggl(\frac{p(x_{2n(k)},x_{2m(k)}) + p(x_{2n(k)+1},x_{2m(k)-1})}{2}\biggr)\\
&\qquad- \phi\bigl(p(x_{2n(k)},x_{2m(k)}),p(x_{2n(k)+1},x_{2m(k)-1})\bigr).
\end{aligned}\tag{2.9}$$

Taking $k \to +\infty$ in (2.9), we get

$$\psi(\varepsilon) \le \varphi(\varepsilon) - \phi(\varepsilon,\varepsilon),$$

which implies that $\phi(\varepsilon,\varepsilon) = 0$, hence $\varepsilon = 0$, a contradiction. Thus, $\{x_{2n}\}$ is a Cauchy sequence in (X,d_p) and so $\{x_n\}$ is a Cauchy sequence both in (X,d_p) and in (X,p). Since (X,p) is complete then the sequence $\{x_n\}$ converges to some $z \in X$, that is

$$p(z,z) = \lim_{n\to+\infty} p(x_n,z) = \lim_{n,m\to+\infty} p(x_n,x_m).\tag{2.10}$$

Moreover, since $\{x_n\}$ is a Cauchy sequence in (X,d_p), we have $\lim_{n\to+\infty} d_p(x_n,x_m) = 0$. By $d_p(x_n,x_m) = 2p(x_n,x_m) - p(x_n,x_n) - p(x_m,x_m)$ and $\lim_{n\to+\infty} p(x_n,x_n) = 0$, we have $\lim_{n\to+\infty} p(x_n,x_m) = 0$. Then (2.10) yields that

$$p(z,z) = \lim_{n\to+\infty} p(x_n,z) = 0.\tag{2.11}$$

Applying the triangular inequality, we have

$$p(z,fz) \le p(z,x_n) + p(x_n,fz) - p(x_n,x_n) \le p(z,x_n) + p(x_n,fz) = p(z,x_n) + p(fx_{n-1},fz),$$

taking $n \to +\infty$ in the above inequalities, then the continuity of f and Lemma 2.1 give that

$$p(z,fz) \le p(fz,fz),$$

hence

$$p(z,fz) = p(fz,fz). \tag{2.12}$$

By combining (2.1) and (2.12), we have

$$\begin{aligned}\psi\bigl(p(z,fz)\bigr) &= \psi\bigl(p(fz,fz)\bigr)\\ &\le \varphi\left(\frac{p(z,fz)+p(fz,z)}{2}\right) - \phi\bigl(p(z,fz),p(fz,z)\bigr)\\ &= \varphi\bigl(p(z,fz)\bigr) - \phi\bigl(p(z,fz),p(fz,z)\bigr),\end{aligned}$$

which yields that $\phi(p(z,fz),p(fz,z)) = 0$, and thus $p(z,fz) = 0$, that is $z = fz$. Therefore, z is a fixed point of f. □

Theorem 2.2 *Suppose that X, f, ψ, φ, and ϕ are the same as in Theorem* 2.1 *except the continuity of f. Suppose that for a nondecreasing sequence $\{x_n\}$ in X with $x_n \to x \in X$, we have $x_n \preceq x$ for all $n \in N$. If there exists $x_0 \in X$ such that $x_0 \preceq fx_0$, then f has a fixed point.*

Proof As in the proof of Theorem 2.1, we have a Cauchy sequence $\{x_n\}$ in X. Since (X,p) is complete, there exists $z \in X$ such that $x_n \to z$, that is,

$$p(z,z) = \lim_{n\to+\infty} p(x_n,z) = \lim_{n,m\to+\infty} p(x_n,x_m),$$

due to the hypothesis, we get $x_n \preceq z$. Similar to the proof of Theorem 2.1, we have that

$$p(z,z) = \lim_{n\to+\infty} p(x_n,z) = 0.$$

From (2.1), we obtain that

$$\begin{aligned}\psi\bigl(p(x_n,fz)\bigr) &= \psi\bigl(p(fx_{n-1},fz)\bigr)\\ &\le \varphi\left(\frac{p(x_{n-1},fz)+p(fx_{n-1},z)}{2}\right) - \phi\bigl(p(x_{n-1},fz),p(x_n,z)\bigr)\\ &= \varphi\left(\frac{p(x_{n-1},fz)+p(x_n,z)}{2}\right) - \phi\bigl(p(x_{n-1},fz),p(x_n,z)\bigr).\end{aligned}$$

Letting $n \to +\infty$ in the above inequalities, and by Lemma 1.2, we have

$$\psi\bigl(p(z,fz)\bigr) \le \varphi\bigl(p(z,fz)\bigr) - \phi\bigl(p(z,fz),0\bigr),$$

which implies, from (2.2), that $\phi(p(z,fz),0) = 0$, hence $p(z,fz) = 0$, and thus $z = fz$. Therefore, f has a fixed point. □

Theorem 2.3 *Let (X,p) be a complete partial metric space, f and g be self-mappings on X. Suppose that for all $x,y \in X$*

$$\psi\bigl(p(fx,gy)\bigr) \le \varphi\left(\frac{p(x,gy)+p(fx,y)}{2}\right) - \phi\bigl(p(x,gy),p(fx,y)\bigr), \tag{2.13}$$

where ψ and φ are altering distance functions with

$$\psi(t) - \varphi(t) \geq 0 \tag{2.14}$$

for all $t \geq 0$, and $\phi : [0,+\infty) \times [0,+\infty) \to [0,+\infty)$ is a continuous function with $\phi(x,y) = 0$ if and only if $x = y = 0$.

Then f and g have a unique common fixed point.

Proof Let x_0 be an arbitrary point in X. One can choose $x_1 \in X$ such that $fx_0 = x_1$. Also, one can choose $x_2 \in X$ such that $gx_1 = x_2$. Continuing this process, one can construct a sequence $\{x_n\}$ in X such that

$$x_{2n+1} = fx_{2n}, \qquad x_{2n+2} = gx_{2n+1}, \quad n \in N. \tag{2.15}$$

Now, we discuss the following two cases.

Case 1. If $p(x_n, x_{n+1}) = 0$ for some $n_0 \in N$, then f and g have at least one common fixed point. In fact, if $p(x_n, x_{n+1}) = 0$ for some $n_0 \in N$, that is $p(x_{n_0}, x_{n_0+1}) = 0$, which implies that $x_{n_0} = x_{n_0+1}$. If $n_0 = 2k$ ($k \in N$), then $x_{2k} = x_{2k+1}$. Using (2.13), we have

$$\begin{aligned}
&\psi\bigl(p(x_{2k+1}, x_{2k+2})\bigr)\\
&\quad = \psi\bigl(p(fx_{2k}, gx_{2k+1})\bigr)\\
&\quad \leq \varphi\left(\frac{p(x_{2k}, gx_{2k+1}) + p(fx_{2k}, x_{2k+1})}{2}\right) - \phi\bigl(p(x_{2k}, gx_{2k+1}), p(fx_{2k}, x_{2k+1})\bigr)\\
&\quad = \varphi\left(\frac{p(x_{2k}, x_{2k+2}) + p(x_{2k+1}, x_{2k+1})}{2}\right) - \phi\bigl(p(x_{2k}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})\bigr)\\
&\quad \leq \varphi\left(\frac{p(x_{2k}, x_{2k+1}) + p(x_{2k+1}, x_{2k+2})}{2}\right) - \phi\bigl(p(x_{2k}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})\bigr)\\
&\quad \leq \varphi\bigl(\max\bigl\{p(x_{2k}, x_{2k+1}), p(x_{2k+1}, x_{2k+2})\bigr\}\bigr) - \phi\bigl(p(x_{2k}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})\bigr)\\
&\quad = \varphi\bigl(\max\bigl\{p(x_{2k+1}, x_{2k+1}), p(x_{2k+1}, x_{2k+2})\bigr\}\bigr) - \phi\bigl(p(x_{2k+1}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})\bigr)\\
&\quad = \varphi\bigl(p(x_{2k+1}, x_{2k+2})\bigr) - \phi\bigl(p(x_{2k+1}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})\bigr).
\end{aligned} \tag{2.16}$$

With the help of (2.14) and (2.16), we conclude that $\phi(p(x_{2k+1}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})) = 0$, hence, using the property of ϕ, we get $p(x_{2k+1}, x_{2k+2}) = 0$, that is $x_{2k+1} = x_{2k+2}$. By similar arguments, we obtain $x_{2k+2} = x_{2k+3}$, $x_{2k+3} = x_{2k+4}$ and so on. Thus, $\{x_n\}$ becomes a constant from $n = 2k$, that is,

$$x_{2k} = x_{2k+1} = x_{2k+2} = \cdots. \tag{2.17}$$

Equations (2.15) and (2.17) yield that

$$x_{2k} = gx_{2k} = fx_{2k}, \tag{2.18}$$

which implies that x_{2k} is the common fixed point of f and g. Similarly, one can show that if $n_0 = 2k + 1$ ($k \in N$), then f and g have at least one common fixed point. Therefore, we have proved that if $p(x_n, x_{n+1}) = 0$ for some $n_0 \in N$, then f and g have at least one common fixed point.

Case 2. If $p(x_n, x_{n+2}) = 0$ for some $n_0 \in N$, then f and g have at least one common fixed point. Indeed, if $n_0 = 2k$ ($k \in N$), then $p(x_{2k}, x_{2k+2}) = 0$. Hence, $x_{2k} = x_{2k+2}$, due to (2.13), we have

$$\begin{aligned}
&\psi\bigl(p(x_{2k+1}, x_{2k+2})\bigr)\\
&\quad= \psi\bigl(p(fx_{2k}, gx_{2k+1})\bigr)\\
&\quad\le \varphi\left(\frac{p(x_{2k}, gx_{2k+1}) + p(fx_{2k}, x_{2k+1})}{2}\right) - \phi\bigl(p(x_{2k}, gx_{2k+1}), p(fx_{2k}, x_{2k+1})\bigr)\\
&\quad= \varphi\left(\frac{p(x_{2k}, x_{2k+2}) + p(x_{2k+1}, x_{2k+1})}{2}\right) - \phi\bigl(p(x_{2k}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})\bigr)\\
&\quad\le \varphi\left(\frac{p(x_{2k}, x_{2k+1}) + p(x_{2k+1}, x_{2k+2})}{2}\right) - \phi\bigl(p(x_{2k}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})\bigr)\\
&\quad= \varphi\left(\frac{p(x_{2k+2}, x_{2k+1}) + p(x_{2k+1}, x_{2k+2})}{2}\right) - \phi\bigl(p(x_{2k}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})\bigr)\\
&\quad= \varphi\bigl(p(x_{2k+2}, x_{2k+1})\bigr) - \phi\bigl(p(x_{2k}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})\bigr).
\end{aligned} \tag{2.19}$$

Applying (2.14) and (2.19), we obtain $\phi(p(x_{2k}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})) = 0$. Using the property of ϕ, we have

$$p(x_{2k+1}, x_{2k+1}) = 0. \tag{2.20}$$

From (2.20) and using $p(x_{2k}, x_{2k+2}) = 0$, we get that

$$\begin{aligned}
&\psi\bigl(p(x_{2k+1}, x_{2k+2})\bigr)\\
&\quad= \psi\bigl(p(fx_{2k}, gx_{2k+1})\bigr)\\
&\quad\le \varphi\left(\frac{p(x_{2k}, gx_{2k+1}) + p(fx_{2k}, x_{2k+1})}{2}\right) - \phi\bigl(p(x_{2k}, gx_{2k+1}), p(fx_{2k}, x_{2k+1})\bigr)\\
&\quad= \varphi\left(\frac{p(x_{2k}, x_{2k+2}) + p(x_{2k+1}, x_{2k+1})}{2}\right) - \phi\bigl(p(x_{2k}, x_{2k+2}), p(x_{2k+1}, x_{2k+1})\bigr)\\
&\quad= \varphi(0) - \phi(0,0)\\
&\quad= 0,
\end{aligned} \tag{2.21}$$

which implies that $\psi(p(x_{2k+1}, x_{2k+2})) = 0$, and thus $p(x_{2k+1}, x_{2k+2}) = 0$. Hence we obtain that f and g have at least one common fixed point from case 1. Similarly, it is easy to show that if $p(x_n, x_{n+2}) = 0$ for some $n = 2k + 1$ ($k \in N$), then f and g have at least one common fixed point, this completes the proof of case 2.

Taking $p(x_n, x_{n+1}) > 0$ and $p(x_n, x_{n+2}) > 0$ for all $n \in N$. Now we prove that for every $k \in N$, we have

$$p(x_{2k+2}, x_{2k+1}) \le p(x_{2k+1}, x_{2k}). \tag{2.22}$$

Suppose this is not true, then $p(x_{2k+2}, x_{2k+1}) > p(x_{2k+1}, x_{2k})$ for some $k = k_0$, that is,

$$p(x_{2k_0+2}, x_{2k_0+1}) > p(x_{2k_0+1}, x_{2k_0}).$$

Using (2.13) and (2.15), we obtain that

$$\begin{aligned}
&\psi\bigl(p(x_{2k_0+1}, x_{2k_0+2})\bigr) \\
&\quad= \psi\bigl(p(fx_{2k_0}, gx_{2k_0+1})\bigr) \\
&\quad\le \varphi\biggl(\frac{p(x_{2k_0}, gx_{2k_0+1}) + p(fx_{2k_0}, x_{2k_0+1})}{2}\biggr) - \phi\bigl(p(x_{2k_0}, gx_{2k_0+1}), p(fx_{2k_0}, x_{2k_0+1})\bigr) \\
&\quad= \varphi\biggl(\frac{p(x_{2k_0}, x_{2k_0+2}) + p(x_{2k_0+1}, x_{2k_0+1})}{2}\biggr) - \phi\bigl(p(x_{2k_0}, x_{2k_0+2}), p(x_{2k_0+1}, x_{2k_0+1})\bigr) \\
&\quad\le \varphi\biggl(\frac{p(x_{2k_0}, x_{2k_0+1}) + p(x_{2k_0+1}, x_{2k_0+2})}{2}\biggr) - \phi\bigl(p(x_{2k_0}, x_{2k_0+2}), p(x_{2k_0+1}, x_{2k_0+1})\bigr) \\
&\quad\le \varphi\bigl(\max\bigl\{p(x_{2k_0}, x_{2k_0+1}), p(x_{2k_0+1}, x_{2k_0+2})\bigr\}\bigr) - \phi\bigl(p(x_{2k_0}, x_{2k_0+2}), p(x_{2k_0+1}, x_{2k_0+1})\bigr) \\
&\quad= \varphi\bigl(p(x_{2k_0+1}, x_{2k_0+2})\bigr) - \phi\bigl(p(x_{2k_0}, x_{2k_0+2}), p(x_{2k_0+1}, x_{2k_0+1})\bigr).
\end{aligned} \tag{2.23}$$

Equations (2.14) and (2.23) give that $\phi(p(x_{2k_0}, x_{2k_0+2}), p(x_{2k_0+1}, x_{2k_0+1})) = 0$. Using the property of ϕ, we get $p(x_{2k_0}, x_{2k_0+2}) = 0$, which contradicts with $p(x_n, x_{n+2}) > 0$ for $n \in N$, hence (2.22) holds.

Similarly, one can show that for every $k \in N^+$, the following inequality holds.

$$p(x_{2k+1}, x_{2k}) \le p(x_{2k}, x_{2k-1}). \tag{2.24}$$

Equations (2.22) and (2.24) imply that the sequence $\{p(x_n, x_{n+1})\}$ is nonincreasing, and consequently there exists some $r \ge 0$ such that

$$\lim_{n\to+\infty} p(x_n, x_{n+1}) = r. \tag{2.25}$$

By (2.25) and the following inequalities,

$$\begin{aligned}
p(x_{2n}, x_{2n+2}) &\le p(x_{2n}, x_{2n+1}) + p(x_{2n+1}, x_{2n+2}) - p(x_{2n+1}, x_{2n+1}) \\
&\le p(x_{2n}, x_{2n+1}) + p(x_{2n+1}, x_{2n+2}),
\end{aligned}$$

we get that $\{p(x_{2n}, x_{2n+2})\}$ is bounded, and hence it has some subsequence $\{p(x_{2n(k)}, x_{2n(k)+2})\}$ converging to some r_0, that is,

$$\lim_{k\to+\infty} p(x_{2n(k)}, x_{2n(k)+2}) = r_0. \tag{2.26}$$

Taking (P_2) into account, we have

$$p(x_{2n(k)+1}, x_{2n(k)+1}) \le p(x_{2n(k)+1}, x_{2n(k)}),$$

which combining with (2.25) shows that $p(x_{2n(k)+1}, x_{2n(k)+1})$ is bounded, and hence there exists subsequence $p(x_{2n(k_i)+1}, x_{2n(k_i)+1})$ of $p(x_{2n(k)+1}, x_{2n(k)+1})$ such that $p(x_{2n(k_i)+1}, x_{2n(k_i)+1})$ converges to some r_1, that is,

$$\lim_{i\to+\infty} p(x_{2n(k_i)+1}, x_{2n(k_i)+1}) = r_1. \tag{2.27}$$

By (2.13), we have

$$\begin{aligned}
\psi\bigl(p(x_{2n(k_i)+1}, x_{2n(k_i)+2})\bigr) &= \psi\bigl(p(fx_{2n(k_i)}, gx_{2n(k_i)+1})\bigr) \\
&\le \varphi\left(\frac{p(x_{2n(k_i)}, gx_{2n(k_i)+1}) + p(fx_{2n(k_i)}, x_{2n(k_i)+1})}{2}\right) \\
&\quad - \phi\bigl(p(x_{2n(k_i)}, gx_{2n(k_i)+1}), p(fx_{2n(k_i)}, x_{2n(k_i)+1})\bigr) \\
&= \varphi\left(\frac{p(x_{2n(k_i)}, x_{2n(k_i)+2}) + p(x_{2n(k_i)+1}, x_{2n(k_i)+1})}{2}\right) \\
&\quad - \phi\bigl(p(x_{2n(k_i)}, x_{2n(k_i)+2}), p(x_{2n(k_i)+1}, x_{2n(k_i)+1})\bigr) \\
&\le \varphi\left(\frac{p(x_{2n(k_i)}, x_{2n(k_i)+1}) + p(x_{2n(k_i)+1}, x_{2n(k_i)+2})}{2}\right) \\
&\quad - \phi\bigl(p(x_{2n(k_i)}, x_{2n(k_i)+2}), p(x_{2n(k_i)+1}, x_{2n(k_i)+1})\bigr).
\end{aligned} \tag{2.28}$$

Letting $i \to +\infty$ in (2.28), and using (2.25)-(2.27), we obtain that

$$\psi(r) \le \varphi(r) - \phi(r_0, r_1), \tag{2.29}$$

which means that $\phi(r_0, r_1) = 0$, hence $r_0 = 0$ and $r_1 = 0$.

Since

$$\begin{aligned}
\psi\bigl(p(x_{2n(k_i)+1}, x_{2n(k_i)+2})\bigr) &= \psi\bigl(p(fx_{2n(k_i)}, gx_{2n(k_i)+1})\bigr) \\
&\le \varphi\left(\frac{p(x_{2n(k_i)}, gx_{2n(k_i)+1}) + p(fx_{2n(k_i)}, x_{2n(k_i)+1})}{2}\right) \\
&\quad - \phi\bigl(p(x_{2n(k_i)}, gx_{2n(k_i)+1}), p(fx_{2n(k_i)}, x_{2n(k_i)+1})\bigr) \\
&= \varphi\left(\frac{p(x_{2n(k_i)}, x_{2n(k_i)+2}) + p(x_{2n(k_i)+1}, x_{2n(k_i)+1})}{2}\right) \\
&\quad - \phi\bigl(p(x_{2n(k_i)}, x_{2n(k_i)+2}), p(x_{2n(k_i)+1}, x_{2n(k_i)+1})\bigr),
\end{aligned}$$

taking the limit as $i \to +\infty$, we have $\psi(r) = 0$, which implies that $r = 0$, that is,

$$\lim_{n\to+\infty} p(x_n, x_{n+1}) = 0. \tag{2.30}$$

Now, we claim that $\{x_n\}$ is a Cauchy sequence in the metric space (X, d_p) (and so also in the space (X, p) by Lemma 1.1). For this, it is sufficient to show that $\{x_{2n}\}$ is a Cauchy sequence in (X, d_p). Suppose that this is not the case, then using Lemma 1.1, we have that $\{x_{2n}\}$ is not a Cauchy sequence in (X, p). By Lemma 1.3, we obtain that there exist $\varepsilon > 0$ and two sequences $\{m(k)\}$ and $\{n(k)\}$ of positive integers such that $n(k) > m(k) > k$ and sequences in (1.2) tend to ε when $k \to +\infty$.

From (2.13), we get that

$$\begin{aligned}\psi\big(p(x_{2n(k)+1},x_{2m(k)})\big) &= \psi\big(p(fx_{2n(k)},gx_{2m(k)-1})\big)\\ &\le \varphi\left(\frac{p(x_{2n(k)},gx_{2m(k)-1})+p(fx_{2n(k)},x_{2m(k)-1})}{2}\right)\\ &\quad - \phi\big(p(x_{2n(k)},gx_{2m(k)-1}),p(fx_{2n(k)},x_{2m(k)-1})\big)\\ &= \varphi\left(\frac{p(x_{2n(k)},x_{2m(k)})+p(x_{2n(k)+1},x_{2m(k)-1})}{2}\right)\\ &\quad - \phi\big(p(x_{2n(k)},x_{2m(k)}),p(x_{2n(k)+1},x_{2m(k)-1})\big).\end{aligned}$$

Letting $k\to+\infty$ in the above inequalities and using the continuity of ψ, φ and ϕ, we get that

$$\psi(\varepsilon)\le\varphi(\varepsilon)-\phi(\varepsilon,\varepsilon),$$

therefore, we get that $\phi(\varepsilon,\varepsilon)=0$. Hence, $\varepsilon=0$ which is a contradiction. Thus, $\{x_n\}$ is a Cauchy sequence in (X,d_p), and $\{x_n\}$ is also a Cauchy sequence in (X,p). Since (X,p) is complete, then the sequence $\{x_n\}$ converges to some $z\in X$, that is,

$$p(z,z)=\lim_{n\to+\infty}p(x_n,z)=\lim_{n,m\to+\infty}p(x_n,x_m).$$

Moreover, the sequence $\{x_{2n}\}$ and $\{x_{2n+1}\}$ converge to $z\in X$, that is,

$$p(z,z)=\lim_{n\to+\infty}p(x_{2n},z)=\lim_{n,m\to+\infty}p(x_{2n},x_{2m})$$

and

$$p(z,z)=\lim_{n\to+\infty}p(x_{2n+1},z)=\lim_{n,m\to+\infty}p(x_{2n+1},x_{2m+1}).$$

Using the fact that $\{x_n\}$ is a Cauchy sequence in (X,d_p), we have $\lim_{n\to+\infty}d_p(x_n,x_m)=0$, which together with $d_p(x_n,x_m)=2p(x_n,x_m)-p(x_n,x_n)-p(x_m,x_m)$ yields that $\lim_{n\to+\infty}p(x_n,x_m)=0$. Hence, we have

$$p(z,z)=\lim_{n\to+\infty}p(x_n,z)=\lim_{n\to+\infty}p(x_{2n},z)=\lim_{n\to+\infty}p(x_{2n+1},z)=0.$$

By substituting $x=x_{2n(k)+1}$, $y=z$ in (2.13), we get that

$$\begin{aligned}\psi\big(p(x_{2n(k)+1},gz)\big) &= \psi\big(p(fx_{2n(k)},gz)\big)\\ &\le \varphi\left(\frac{p(x_{2n(k)},gz)+p(fx_{2n(k)},z)}{2}\right)\\ &\quad - \phi\big(p(x_{2n(k)},gz),p(fx_{2n(k)},z)\big)\\ &= \varphi\left(\frac{p(x_{2n(k)},gz)+p(x_{2n(k)+1},z)}{2}\right)\\ &\quad - \phi\big(p(x_{2n(k)},gz),p(x_{2n(k)+1},z)\big),\end{aligned}$$

letting $k \to +\infty$ and applying Lemma 1.2, we conclude that

$$\begin{aligned}\psi\big(p(z,gz)\big) &\le \varphi\left(\frac{p(z,gz)+p(z,z)}{2}\right) - \phi\big(p(z,gz),p(z,z)\big)\\ &\le \varphi\big(p(z,gz)\big) - \phi\big(p(z,gz),0\big),\end{aligned}$$

which yields that $\phi(p(z,gz),0) = 0$; hence, $p(z,gz) = 0$, and thus $z = gz$. Similarly, one can easily show that $z = fz$, therefore, z is the common fixed point of f and g.

Now we prove the uniqueness of common fixed point. Let us suppose that u is also the common fixed point of f and g. Since

$$\begin{aligned}\psi\big(p(u,z)\big) &= \psi\big(p(fu,gz)\big)\\ &\le \varphi\left(\frac{p(u,gz)+p(fu,z)}{2}\right) - \phi\big(p(u,gz),p(fu,z)\big)\\ &= \varphi\big(p(u,z)\big) - \phi\big(p(u,z),p(u,z)\big),\end{aligned}$$

which means that $\phi(p(u,z),p(u,z)) = 0$; hence, $p(u,z) = 0$, and so $u = z$. Thus, the uniqueness of the common fixed point is proved. □

By taking $\varphi = \psi$ in Theorems 2.1-2.3, respectively, we have the following results.

Corollary 2.1 *Let $(X,\preceq)$ be a partially ordered set and suppose that there exists a partial metric p on X such that (X,p) is complete. Let $f : X \to X$ be a continuous nondecreasing mapping. Suppose that for comparable $x, y \in X$, we have*

$$\psi\big(p(fx,fy)\big) \le \psi\left(\frac{p(x,fy)+p(fx,y)}{2}\right) - \phi\big(p(x,fy),p(fx,y)\big),$$

where ψ is an altering distance function and $\phi : [0,+\infty)\times[0,+\infty) \to [0,+\infty)$ is a continuous function with $\phi(x,y) = 0$ if and only if $x = y = 0$. If there exists $x_0 \in X$ such that $x_0 \preceq fx_0$, then f has a fixed point.

Corollary 2.2 *Suppose that X, f, ψ, and ϕ are the same as in Corollary* 2.1 *except the continuity of f. Suppose that for a nondecreasing sequence $\{x_n\}$ in X with $x_n \to x \in X$, we have $x_n \preceq x$ for all $n \in N$. If there exists $x_0 \in X$ such that $x_0 \preceq fx_0$, then f has a fixed point.*

Corollary 2.3 *Let (X,p) be a complete partial metric space, f and g be self-mappings on X. Suppose that there exist functions ψ and ϕ such that for all $x, y \in X$*

$$\psi\big(p(fx,gy)\big) \le \psi\left(\frac{p(x,gy)+p(fx,y)}{2}\right) - \phi\big(p(x,gy),p(fx,y)\big),$$

where ψ is an altering distance function and $\phi : [0,+\infty)\times[0,+\infty) \to [0,+\infty)$ is a continuous function with $\phi(x,y) = 0$ if and only if $x = y = 0$.

Then f and g have a unique common fixed point.

Remark 2.1 *If we replace the partial metric p by* (*usual*) *metric d in Corollaries* 2.1-2.3, *then we get Theorems* 2.1-2.3 *of* [27].

Now, we introduce an example to support the usability of our results.

Example 2.1 Let $X = [0,1]$ be endowed with the usual partial metric $p : X \times X \to [0,+\infty)$ defined by $p(x,y) = \max\{x,y\}$. It is easy to show that the partial metric space (X,p) is complete. Also, define the mappings $f,g : X \to X$ by $fx = \frac{x^2}{4}$ and $gx = \frac{x^2}{5}$, respectively. Let us take $\psi,\varphi : [0,+\infty) \to [0,+\infty)$ such that $\psi(t) = t^2$ and $\varphi(t) = \frac{t^2}{2}$, respectively, and take $\phi : [0,+\infty) \times [0,+\infty) \to [0,+\infty)$ such that $\phi(t,s) = \frac{(t+s)^2}{16}$. If $x \geq y$, then

$$p(fx,gy) = \max\left\{\frac{x^2}{4},\frac{y^2}{5}\right\} = \frac{x^2}{4},$$

and

$$p(x,gy) + p(fx,y) = p\left(x,\frac{y^2}{5}\right) + p\left(\frac{x^2}{4},y\right) = \max\left\{x,\frac{y^2}{5}\right\} + p\left(\frac{x^2}{4},y\right) = x + p\left(\frac{x^2}{4},y\right).$$

So, we have

$$\begin{aligned}\psi\bigl(p(fx,gy)\bigr) &= \frac{x^4}{16} \leq \frac{x^2}{16}\\ &\leq \frac{(x + p(\frac{x^2}{4},y))^2}{16}\\ &= \frac{(x + p(\frac{x^2}{4},y))^2}{8} - \frac{(x + p(\frac{x^2}{4},y))^2}{16}\\ &= \varphi\left(\frac{p(x,gy) + p(fx,y)}{2}\right) - \phi\bigl(p(x,gy),p(fx,y)\bigr).\end{aligned}$$

If $x \leq y$, then

$$p(fx,gy) = \max\left\{\frac{x^2}{4},\frac{y^2}{5}\right\} \leq \frac{y^2}{4}$$

and

$$p(x,gy) + p(fx,y) = p\left(x,\frac{y^2}{5}\right) + p\left(\frac{x^2}{4},y\right) = p\left(x,\frac{y^2}{5}\right) + \max\left\{\frac{x^2}{4},y\right\} = p\left(x,\frac{y^2}{5}\right) + y.$$

So, we have

$$\begin{aligned}\psi\bigl(p(fx,gy)\bigr) &= \frac{y^4}{16} \leq \frac{y^2}{16}\\ &\leq \frac{(y + p(x,\frac{y^2}{5}))^2}{16}\\ &= \frac{(y + p(x,\frac{y^2}{5}))^2}{8} - \frac{(y + p(x,\frac{y^2}{5}))^2}{16}\\ &= \varphi\left(\frac{p(x,gy) + p(fx,y)}{2}\right) - \phi\bigl(p(x,gy),p(fx,y)\bigr).\end{aligned}$$

From the above arguments, we conclude that (2.13) holds; hence, all the required hypotheses of Theorem 2.3 are satisfied. Thus, we deduce the existence and uniqueness of a common fixed point of f and g. Here, 0 is the unique common fixed point.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the work. All authors read and approved the final manuscript.

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