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Some fixed point results in dislocated quasi metric (dq-metric) spaces

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Abstract

The aim of this paper is to investigate some fixed point results in dislocated quasi metric (dq-metric) spaces. Fixed point results for different types of contractive conditions are established, which generalize, modify and unify some existing fixed point theorems in the literature. Appropriate examples for the usability of the established results are also given. We notice that by using our results some fixed point results in the context of dislocated quasi metric spaces can be deduced.

MSC: Primary 47H10; secondary 54H25

Keywords: complete dq-metric space; contraction mapping; self-mapping; Cauchy sequence; fixed point

1 Introduction

Fixed point theory is one of the most dynamic research subjects in nonlinear analysis. In this area, the first important and significant result was proved by Banach in 1922 for a contraction mapping in a complete metric space. The well-known Banach contraction theorem may be stated as follows: 'Every contraction mapping of a complete metric space X into itself has a unique fixed point' (Bonsall 1962).

Dass and Gupta [1] generalized the Banach contraction principle in a metric space for some rational type contractive conditions.

The role of topology in logic programming has come to be recognized (see [2–6] and the references cited therein). Particularly, topological methods are applied to obtain fixed point semantics for logic programs. Such considerations motivated the concept of dislocated metric spaces. This idea was not new and it had been studied in the context of domain theory [4] where the dislocated metrics were known as metric domains.

Hitzler and Seda [3] investigated the useful applications of dislocated topology in the context of logic programming semantics. In order to obtain a unique supported model for these programs, they introduced the notation of dislocated metric space and generalized the Banach contraction principle in such spaces.

Furthermore, Zeyada *et al.* [7] generalized the results of Hitzler and Seda [3] and introduced the concept of complete dislocated quasi metric space. Aage and Salunke [8, 9] derived some fixed point theorems in dislocated quasi metric spaces. Similarly, Isufati [10] proved some fixed point results for continuous contractive condition with rational type expression in the context of a dislocated quasi metric space. Kohli *et al.* [11] investigated a fixed point theorem which generalized the result of Isufati. In [12] Zoto gave some new results in dislocated and dislocated quasi metric spaces. For a continuous self-mapping, a fixed point theorem in dislocated quasi metric spaces was investigated by Madhu Shrivastava *et al.* [13]. In 2013, Patel and Patel [14] constructed some new fixed point results in a dislocated quasi metric space.

In the current manuscript, we establish some fixed point results for single and a pair of continuous self-mappings in the context of dislocated quasi metric spaces which generalize, modify and unify the results of Aage and Salunke [8, 9], Manvi Kohli [11], Patel and Patel [14], Madhu Shrivastava *et al.* [13] and Zeyada *et al.* [7]. Throughout the paper $\mathbb{R}^+$ represents the set of non-negative real numbers.

2 Preliminaries

Definition 2.1 ([7]) Let X be a non-empty set, and let $d : X \times X \to \mathbb{R}^+$ be a function satisfying the following conditions:

(d_1) $d(x,x) = 0$;
(d_2) $d(x,y) = d(y,x) = 0$ implies that $x = y$;
(d_3) $d(x,y) = d(y,x)$ for all $x,y,z \in X$;
(d_4) $d(x,y) \leq d(x,z) + d(z,y)$ for all $x,y,z \in X$.

If d satisfies the conditions from d_1 to d_4, then it is called a metric on X, if d satisfies conditions d_2 to d_4, then it is called a dislocated metric (d-metric) on X, and if d satisfies conditions d_2 and d_4, only then it is called a dislocated quasi metric (dq-metric) on X.

It is evident that every metric on X is a dislocated metric on X, but the converse is not necessarily true as is clear from the following example.

Example 2.1 Let $X = \mathbb{R}^+$ define the distance function $d : X \times X \to \mathbb{R}^+$ by

$$d(x,y) = \max\{x,y\}.$$

Furthermore, from the following example one can say that a dislocated quasi metric on X needs not be a dislocated metric on X.

Example 2.2 Let $X = [0,1]$, we define the function $d : X \times X \to \mathbb{R}^+$ as

$$d(x,y) = |x - y| + |x| \quad \text{for all } x,y \in X.$$

In our main work we will use the following definitions which can be found in [7].

Definition 2.2 A sequence $\{x_n\}$ in a dq-metric space is called a Cauchy sequence if for $\epsilon > 0$ there exists a positive integer N such that for $m,n \geq N$, we have $d(x_m,x_n) < \epsilon$.

Definition 2.3 A sequence $\{x_n\}$ is called dq-convergent in X if for $n \geq N$, we have $d(x_n,x) < \epsilon$, where x is called the dq-limit of the sequence $\{x_n\}$.

Definition 2.4 A dq-metric space (X,d) is said to be complete if every Cauchy sequence in X converges to a point of X.

Definition 2.5 Let (X, d) be a dq-metric space, a mapping $T : X \to X$ is called a contraction if there exists $0 \le \alpha < 1$ such that

$$d(Tx, Ty) \le \alpha d(x, y) \quad \text{for all } x, y \in X \text{ and } \alpha \in [0, 1).$$

The following statement is well known (see [7]).

Lemma 1 *Limit in a dq-metric space is unique.*

In [15] Kannan defined a contraction of the following type.

Definition 2.6 Let (X, d) be a metric space, and let $T : X \to X$ be a self-mapping. Then T is called a Kannan mapping if

$$d(Tx, Ty) \le \alpha\bigl[d(x, Tx) + d(y, Ty)\bigr] \quad \text{for all } x, y \in X \text{ and } \alpha \in [0, 1/2). \tag{1}$$

Kannan [15] established a unique fixed point theorem for a mapping which satisfies condition (1) in metric spaces.

Definition 2.7 ([16]) Let (X, d) be a metric space, a self-mapping $T : X \to X$ is called a generalized contraction if and only for all $x, y \in X$, there exist c_1, c_2, c_3, c_4 such that $\sup\{c_1 + c_2 + c_3 + 2c_4\} < 1$ and

$$d(Tx, Ty) \le c_1 \cdot d(x, y) + c_2 \cdot d(x, Tx) + c_3 \cdot d(y, Ty) + c_4 \cdot \bigl[d(x, Ty) + d(y, Tx)\bigr]. \tag{2}$$

Ciric [16] investigated a unique fixed point theorem for a mapping which satisfies condition (2) in the context of metric spaces.

In the following theorem, Zeyada *et al.* [7] generalized the Banach contraction principle in dislocated quasi metric spaces.

Theorem 2.1 *Let (X, d) be a complete dq-metric space, $T : X \to X$ be a continuous contraction, then T has a unique fixed point in X.*

Aage and Salunke [8] established the following results for single and a pair of continuous mappings in dislocated quasi metric spaces.

Theorem 2.2 *Let (X, d) be a complete dq-metric space and $T : X \to X$ be a continuous self-mapping satisfying the following condition*:

$$d(Tx, Ty) \le a \cdot d(x, y) + b \cdot d(x, Tx) + c \cdot d(y, Ty),$$

where $a, b, c \ge 0$ with $a + b + c < 1$ and for all $x, y \in X$. Then T has a unique fixed point.

Theorem 2.3 *Let (X, d) be a complete dq-metric space and $S, T : X \to X$ be continuous self-mappings satisfying the following condition*:

$$d(Sx, Ty) \le a \cdot d(x, y) + b \cdot d(x, Sx) + c \cdot d(y, Ty),$$

where $a, b, c \geq 0$ *with* $a + b + c < 1$ *and for all* $x, y \in X$. *Then* S *and* T *have a unique common fixed point.*

Furthermore, Aage and Salunke [9] derived the following fixed point theorems with a Kannan-type contraction and a generalized contraction in the setting of dislocated quasi metric spaces, respectively.

Theorem 2.4 *Let* (X, d) *be a complete dq-metric space and* $T : X \to X$ *be a continuous self-mapping satisfying the following condition*:

$$d(Tx, Ty) \leq a \cdot \left[d(x, Tx) + d(y, Ty)\right],$$

where $a \geq 0$ *with* $a < \frac{1}{2}$ *and for all* $x, y \in X$. *Then* T *has a unique fixed point.*

Theorem 2.5 *Let* (X, d) *be a complete dq-metric space and* $T : X \to X$ *be a continuous self-mapping satisfying the following condition*:

$$d(Tx, Ty) \leq a \cdot d(x, y) + b \cdot d(x, Tx) + c \cdot d(y, Ty) + e \cdot \left[d(x, Ty) + d(y, Tx)\right],$$

where $a, b, c, e \geq 0$ *with* $a + b + c + 2e < 1$ *and for all* $x, y \in X$. *Then* T *has a unique fixed point.*

Isufati [10] derived the following two results, where the first one generalized the result of Dass and Gupta [1] in dislocated quasi metric spaces.

Theorem 2.6 *Let* (X, d) *be a complete dq-metric space and* $T : X \to X$ *be a continuous self-mapping satisfying the following condition*:

$$d(Tx, Ty) \leq a \cdot \frac{d(y, Ty)[1 + d(x, Tx)]}{1 + d(x, y)} + b \cdot d(x, y),$$

where $a, b > 0$ *with* $a + b < 1$ *and for all* $x, y \in X$. *Then* T *has a unique fixed point.*

Theorem 2.7 *Let* (X, d) *be a complete dq-metric space and* $T : X \to X$ *be a continuous self-mapping satisfying the following condition*:

$$d(Tx, Ty) \leq a \cdot d(x, y) + b \cdot d(y, Tx) + c \cdot d(x, Ty),$$

where $a, b, c > 0$ *with* $\sup\{a + 2b + 2c\} < 1$ *and for all* $x, y \in X$. *Then* T *has a unique fixed point.*

In [11] Kohli, Shrivastava and Sharma proved the following theorem in the context of dislocated quasi metric spaces which generalized Theorem 2.6.

Theorem 2.8 *Let* (X, d) *be a complete dq-metric space and* $T : X \to X$ *be a continuous self-mapping satisfying the following condition*:

$$d(Tx, Ty) \leq a \cdot \frac{d(y, Ty)[1 + d(x, Tx)]}{1 + d(x, y)} + b \cdot d(x, y) + c \cdot d(y, Ty),$$

where $a, b, c > 0$ *with* $a + b + c < 1$ *and for all* $x, y \in X$. *Then* T *has a unique fixed point.*

For rational type contraction conditions Madhu Shrivastava *et al.* [13] proved the following theorem in a dislocated quasi metric space.

Theorem 2.9 *Let (X,d) be a complete dq-metric space and $T : X \to X$ be a continuous self-mapping satisfying the following condition*:

$$d(Tx,Ty) \leq a \cdot \frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)} + b \cdot d(x,y) + c \cdot \frac{d(y,Ty)+d(y,Tx)}{1+d(y,Ty)d(y,Tx)},$$

where $a,b,c > 0$ with $a+b+c < 1$ and for all $x,y \in X$. Then T has a unique fixed point.

In 2013, Patel and Patel [14] derived the following result in dislocated quasi metric spaces.

Theorem 2.10 *Let (X,d) be a complete dq-metric space, and let $T : X \to X$ be a continuous self-mapping satisfying the following condition*:

$$\begin{aligned} d(Tx,Ty) &\leq c_1 \cdot d(x,y) + c_2 \cdot d(x,Tx) + c_3 \cdot d(y,Ty) \\ &\quad + c_4 \cdot d\big[(x,Tx) + d(y,Ty)\big] + c_5 \cdot d\big[d(x,Ty) + d(y,Tx)\big], \end{aligned}$$

where $c_1,c_2,c_3,c_4,c_5 \geq 0$ with $c_1+c_2+c_3+2(c_4+c_5) < 1$ and for all $x,y \in X$. Then T has a unique fixed point.

3 Main results

In this section we derive some fixed point theorems with examples for single and a pair of continuous self-mappings in the context of dislocated quasi metric spaces.

Theorem 3.1 *Let (X,d) be a complete dq-metric space, and let $T : X \to X$ be a continuous self-mapping satisfying the following condition*:

$$\begin{aligned} d(Tx,Ty) &\leq a_1 \cdot d(x,y) + a_2 \cdot d(x,Ty) + a_3 \cdot d(y,Tx) + a_4 \cdot d(y,Ty) \\ &\quad + a_5 \cdot \frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)} + a_6 \cdot \frac{d(y,Ty)+d(y,Tx)}{1+d(y,Ty)d(y,Tx)} \\ &\quad + a_7 \cdot \frac{d(x,Tx)[1+d(y,Tx)]}{1+d(x,y)+d(y,Ty)}, \end{aligned} \tag{3}$$

where $a_1,a_2,a_3,a_4,a_5,a_6,a_7 \geq 0$ with $a_1+2(a_2+a_3)+a_4+a_5+3a_6+a_7 < 1$ and for all $x,y \in X$. Then T has a unique fixed point in X.

Proof Let x_0 be arbitrary in X, we define a sequence $\{x_n\}$ by the rule

$$x_0, x_1 = Tx_0, x_2 = Tx_1, \ldots, x_{n+1} = Tx_n \quad \text{for all } n \in \mathrm{N}.$$

Now we show that $\{x_n\}$ is a Cauchy sequence in X. Suppose

$$d(x_n,x_{n+1}) = d(Tx_{n-1},Tx_n).$$

By using condition (3) we have

$$\begin{aligned} d(x_n,x_{n+1}) &\le a_1\cdot d(x_{n-1},x_n)+a_2\cdot d(x_{n-1},Tx_n)+a_3\cdot d(x_n,Tx_{n-1})+a_4\cdot d(x_n,Tx_n)\\ &\quad +a_5\cdot\frac{d(x_n,Tx_n)[1+d(x_{n-1},Tx_{n-1})]}{1+d(x_{n-1},x_n)}+a_6\cdot\frac{d(x_n,Tx_n)+d(x_n,Tx_{n-1})}{1+d(x_n,Tx_n)d(x_n,Tx_{n-1})}\\ &\quad +a_7\cdot\frac{d(x_{n-1},Tx_{n-1})[1+d(x_n,Tx_{n-1})]}{1+d(x_{n-1},x_n)+d(x_n,Tx_n)}\\ &\le a_1\cdot d(x_{n-1},x_n)+a_2\cdot d(x_{n-1},x_{n+1})+a_3\cdot d(x_n,x_n)+a_4\cdot d(x_n,x_{n+1})\\ &\quad +a_5\cdot d(x_n,x_{n+1})+a_6\cdot d(x_n,x_{n+1})+a_6\cdot d(x_n,x_n)+a_7\cdot d(x_{n-1},x_n),\\ d(x_n,x_{n+1}) &\le \frac{a_1+a_2+a_3+a_6+a_7}{1-(a_2+a_3+a_4+a_5+2a_6)}\cdot d(x_{n-1},x_n). \end{aligned}$$

Let

$$h=\frac{a_1+a_2+a_3+a_5+a_6+a_7}{1-(a_2+a_3+a_4+2a_6)}.$$

Clearly, $h<1$ because $a_1+2a_2+2a_3+a_4+a_5+3a_6+a_7<1$.

So,

$$d(x_n,x_{n+1})\le h\cdot d(x_{n-1},x_n).$$

Similarly,

$$d(x_{n-1},x_n)\le h\cdot d(x_{n-2},x_{n-1}).$$

Thus

$$d(x_n,x_{n+1})\le h^2\cdot d(x_{n-2},x_{n-1}).$$

Continuing the same procedure, we have

$$d(x_n,x_{n+1})\le h^n d(x_0,x_1).$$

But $0\le h<1$ so $h^n\to 0$ as $n\to\infty$, which shows that $\{x_n\}$ is a Cauchy sequence in a complete dq-metric space. So there exists $z\in X$ such that $x_n\to z$ as $n\to\infty$.

Now we show that z is a fixed point of T. Since $x_n\to z$ as $n\to\infty$, using the continuity of T, we have

$$\lim_{n\to\infty} Tx_n = Tz,$$

which implies that

$$\lim_{n\to\infty} x_{n+1} = Tz.$$

Thus $Tz=z$. Hence z is a fixed point of T.

Uniqueness. Suppose that T has two fixed points z and w for $z \neq w$. Consider

$$\begin{aligned} d(z,w) = d(Tz,Tw) &\leq a_1 \cdot d(z,w) + a_2 \cdot d(z,Tw) + a_3 \cdot d(w,Tz) + a_4 \cdot d(w,Tw) \\ &+ a_5 \cdot \frac{d(w,Tw)[1 + d(z,Tz)]}{1 + d(z,w)} + a_6 \cdot \frac{d(w,Tw) + d(w,Tz)}{1 + d(w,Tw)d(w,Tz)} \\ &+ a_7 \cdot \frac{d(z,Tz)[1 + d(w,Tz)]}{1 + d(z,w) + d(w,Tw)}. \end{aligned} \tag{4}$$

Since z and w are fixed points of T, therefore condition (3) implies that $d(z,z) = 0$ and $0 = d(w,w)$. Finally, from (4) we get

$$d(z,w) \leq (a_1 + a_2) \cdot d(z,w) + (a_3 + a_6) \cdot d(w,z). \tag{5}$$

Similarly, we have

$$d(w,z) \leq (a_1 + a_2) \cdot d(w,z) + (a_3 + a_6) \cdot d(z,w). \tag{6}$$

Subtracting (6) from (5) we have

$$\big|d(z,w) - d(w,z)\big| \leq \big|(a_1 + a_2) - (a_3 + a_6)\big| \cdot \big|d(z,w) - d(w,z)\big|. \tag{7}$$

Since $|(a_1 + a_2) - (a_3 + a_6)| < 1$, so the above inequality (7) is possible if

$$d(z,w) - d(w,z) = 0. \tag{8}$$

Taking equations (5), (6) and (8) into account, we have $d(z,w) = 0$ and $d(w,z) = 0$. Thus by (d_2) $z = w$. Hence T has a unique fixed point in X. □

Example 3.1 Let $X = [0,1]$ with a complete dq-metric defined by

$$d(x,y) = |x| \quad \text{for all } x, y \in X,$$

and define the continuous self-mapping T by $Tx = \frac{x}{2}$ with $a_1 = 1/8$, $a_2 = 1/10$, $a_3 = 1/12$, $a_4 = 1/15$, $a_5 = 1/20$, $a_6 = 1/24$, $a_7 = 1/30$. Then T satisfies all the conditions of Theorem 3.1, and $x = 0$ is the unique fixed point of T in X.

Remarks In the above Theorem 3.1:

- If $a_4 = a_5 = a_6 = a_7 = 0$, then we get the result of Isufati [10].
- If $a_2 = a_3 = a_4 = a_7 = 0$, then we get the result of Madhu Shrivastava *et al.* [13].
- If $a_2 = a_3 = a_4 = a_6 = a_7 = 0$, then we get the result of Isufati [10].
- if $a_2 = a_3 = a_6 = a_7 = 0$, then we get the result of Manvi Kohli [11].

Theorem 3.2 *Let* (X,d) *be a complete dq-metric space, and let* $S, T : X \to X$ *be two continuous self-mappings satisfying the following condition*:

$$\begin{aligned} d(Sx,Ty) &\leq c_1 \cdot d(x,y) + c_2 \cdot d(x,Sx) + c_3 \cdot d(y,Ty) \\ &+ c_4 \cdot d\big[(x,Sx) + d(y,Ty)\big] + c_5 \cdot d\big[d(x,Ty) + d(y,Sx)\big], \end{aligned} \tag{9}$$

where $c_1, c_2, c_3, c_4, c_5 \geq 0$ *with* $c_1 + c_2 + c_3 + 2c_4 + 4c_5 < 1$ *and for all* $x, y \in X$. *Then* S *and* T *have a unique common fixed point in* X.

Proof Let x_0 be arbitrary in X, we define a sequence $\{x_n\}$ by the rule $x_0, x_1 = Sx_0, \ldots, x_{2n+1} = Sx_{2n}$ and $x_2 = Tx_1, \ldots, x_{2n} = Tx_{2n-1}$ for all $n \in \mathrm{N}$. We claim that $\{x_n\}$ is a Cauchy sequence in X. For this consider

$$d(x_{2n+1}, x_{2n+2}) = d(Sx_{2n}, Tx_{2n+1}).$$

By using condition (9) we have

$$\begin{aligned} &d(x_{2n+1}, x_{2n+2}) \\ &\quad \leq c_1 \cdot d(x_{2n}, x_{2n+1}) + c_2 \cdot d(x_{2n}, Sx_{2n}) + c_3 \cdot d(x_{2n+1}, Tx_{2n+1}) \\ &\qquad + c_4 \cdot \bigl[d(x_{2n}, Sx_{2n}) + d(x_{2n+1}, Tx_{2n+1})\bigr] + c_5 \cdot \bigl[d(x_{2n}, Tx_{2n+1}) + d(x_{2n+1}, Tx_{2n})\bigr] \\ &\quad \leq (c_1 + c_2 + c_4 + 2c_5) \cdot d(x_{2n}, x_{2n+1}) + (c_3 + c_4 + 2c_5) \cdot d(x_{2n+1}, x_{2n+2}). \end{aligned}$$

Therefore, finally we have

$$d(x_{2n+1}, x_{2n+2}) \leq \frac{(c_1 + c_2 + c_4 + 2c_5)}{1 - (c_3 + c_4 + 2c_5)} \cdot d(x_{2n}, x_{2n+1}).$$

Let

$$h = \frac{(c_1 + c_2 + c_4 + 2c_5)}{1 - (c_3 + c_4 + 2c_5)}.$$

Then $h < 1$ as $c_1 + c_2 + c_3 + 2c_4 + 4c_5 < 1$. Thus

$$d(x_{2n+1}, x_{2n+2}) \leq hd(x_{2n}, x_{2n+1}) \quad \text{for } n \geq 0$$

and

$$d(x_{2n}, x_{2n+1}) \leq h \cdot d(x_{2n-1}, x_{2n}).$$

So

$$d(x_{2n+1}, x_{2n+2}) \leq h^2 d(x_{2n-1}, x_{2n}).$$

Similarly, we proceed to get

$$d(x_n, x_{n+1}) \leq h^n d(x_0, x_1).$$

Since $0 \leq h < 1$ and $n \to \infty$ implies that $h^n \to 0$, which proved that $\{x_n\}$ is a Cauchy sequence in a complete dq-metric space. Therefore there exists z in X such that $x_n \to z$ as $n \to \infty$. Also the sub-sequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ converge to z. Since T is a continuous mapping, therefore

$$\lim_{n\to\infty} x_{2n+1} = z \quad \Rightarrow \quad \lim_{n\to\infty} Tx_{2n+1} = Tz \quad \Rightarrow \quad \lim_{n\to\infty} x_{2n+2} = Tz.$$

Hence

$$Tz = z.$$

Similarly, taking the continuity of S, we can show that $Sz = z$.

Hence z is the common fixed point of S and T.

Uniqueness. Suppose that S and T have two common fixed points z and w for $z \neq w$. Consider

$$\begin{aligned} d(z,w) = d(Sz,Tw) \leq c_1 \cdot d(z,w) + c_2, d(z,Sz) + c_3 \cdot d(w,Tw) \\ + c_4 \cdot \bigl[d(z,Sz) + d(w,Tw)\bigr] + c_5 \cdot \bigl[d(z,Tw) + d(w,Tz)\bigr]. \end{aligned} \tag{10}$$

Since z and w are common fixed points of T and S, condition (9) implies that $d(z,z) = 0$ and $d(w,w)$. Thus equation (10) becomes

$$d(z,w) \leq (c_1 + c_5) \cdot d(z,w) + c_5 \cdot d(w,z). \tag{11}$$

Similarly,

$$d(w,z) \leq (c_1 + c_5) \cdot d(w,z) + c_5 \cdot d(z,w). \tag{12}$$

Subtracting (12) from (11) we get

$$\bigl|d(z,w) - d(w,z)\bigr| \leq |c_1| \cdot \bigl|d(z,w) - d(w,z)\bigr|.$$

Since $c_1 < 1$, so the above inequality is possible if

$$d(z,w) - d(w,z) = 0. \tag{13}$$

By combining equations (11), (12) and (13), one can get $d(z,w) = 0$ and $d(w,z) = 0$. Using (d_2) we have $z = w$. Hence S and T have a unique common fixed point in X. □

Example 3.2 Let $X = [0,1]$ and complete dq-metric is defined by

$$d(x,y) = |x|,$$

where the continuous self-mappings S and T are defined by $Sx = 0$ and $Tx = x/5$ for all $x \in X$. Suppose $c_1 = 1/5$, $c_2 = 1/6$, $c_3 = 1/8$, $c_4 = 1/10$, $c_5 = 1/12$.

Then S and T satisfy all the conditions of Theorem 3.2, so $x = 0$ is the unique common fixed point of S and T in X.

Theorem 3.2 yields the following corollaries.

Corollary 3.1 *If $S = T$ and all other conditions of Theorem* 3.2 *are satisfied, then T has a unique fixed point.*

Corollary 3.2 *Let $c_4 = c_5 = 0$, and let $S, T : X \to X$ be two self-continuous mappings satisfying all other conditions of Theorem* 3.2. *Then S and T have a unique common fixed point in X.*

Corollary 3.3 *Let $c_4 = c_5 = 0$, and $S = T$ and all other conditions of Theorem* 3.2 *be satisfied, then again T has a unique fixed point.*

Corollary 3.4 *Suppose $c_1 = c_2 = c_3 = c_4 = 0$. Let $S, T : X \to X$ be two self-continuous mappings satisfying all other conditions of Theorem* 3.2. *Then S and T have a unique fixed point in X.*

Corollary 3.5 *Suppose $c_1 = c_2 = c_3 = c_5 = 0$ and $S = T$, and all other conditions of Theorem* 3.2 *are satisfied. Then T has a unique fixed point in X.*

Remarks

- Corollary 3.1 is the result of Patel and Patel [14].
- Corollaries 3.2 and 3.3 are the results of Aage and Salunke [8].
- Corollary 3.5 is the result of Aage and Salunke [9].

Conclusion

The derived results extend some theorems of [7–11, 13, 14] in the setting of dislocated quasi metric spaces.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the final version.

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