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On some power series with algebraic coefficients and Liouville numbers

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Abstract

In this work, we consider some power series with algebraic coefficients from a certain algebraic number field K of degree m and investigate transcendence of the values of the given series for some Liouville number arguments.

1 Introduction

The theory of transcendental numbers has a long history and was originated back to Liouville in his famous paper [1] in which he produced the first explicit examples of transcendental numbers at a time where their existence was not yet known. Later, Cantor [2] gave another proof of the existence of transcendental numbers by establishing the denumerability of the set of algebraic numbers. It follows from this that almost all real numbers are transcendental. Further, the theory of transcendental numbers is closely related to the study of Diophantine approximation. Recent advances in Diophantine approximation can be found in the excellent surveys of Moshchevitin [3] and Waldschimidt [4].

Mahler [5] introduced a classification of the set of all transcendental numbers into three disjoint classes, termed S, T and U and this classification has proved to be of considerable value in the general development of the subject. The first classification of this kind was outlined by Maillet in [6], and others were described by Perna in [7] and Morduchai-Boltovskoj [8] but to Mahler's classification attaches by for the most interest. Mahler described this classification in the following way.

Let $P(x) = a_n x^n + \cdots + a_1 x + a_0$ be a polynomial with integral coefficients. The height $H(P)$ of P is defined by $H(P) = \max(|a_n|, \ldots, |a_0|)$ and the degree of P is denoted by $\deg(P)$. Given an arbitrary complex number ξ, Mahler puts

$$\omega_n(H,\xi) = \min\left\{\left|P(\xi)\right| : \deg(P) \le n, H(P) \le H, P(\xi) \ne 0\right\},$$

where n and H are positive integers. Next Mahler puts

$$\omega_n(\xi) = \overline{\lim_{H\to\infty}} \frac{-\log \omega_n(H,\xi)}{\log H} \quad \text{and} \quad \omega(\xi) = \overline{\lim_{n\to\infty}} \frac{\omega_n(\xi)}{n}.$$

The inequalities $0 \le \omega_n(\xi) \le \infty$ and $0 \le \omega(\xi) \le \infty$ hold. From $\omega_{n+1}(H,\xi) \le \omega_n(H,\xi)$, we get $\omega_{n+1}(\xi) \ge \omega_n(\xi)$. If for an index $\omega_n(\xi) = \infty$, the $\mu(\xi)$ is defined as the smallest of them, otherwise $\mu(\xi) = \infty$. Thus, $\mu(\xi)$ is uniquely determined. Furthermore, the two quantities $\mu(\xi)$ and $\omega(\xi)$ are never finite simultaneously, for the finiteness of $\mu(\xi)$ implies that there is an $n < \infty$ such that $\omega_n = \infty$, whence $\omega = \infty$. Therefore, there are the following four possibilities for ξ, ξ is called

an A - number if $\omega(\xi) = 0, \mu(\xi) = \infty$,

an S - number if $0 < \omega(\xi) < \infty, \mu(\xi) = \infty$,

a T - number if $\omega(\xi) = \infty, \mu(\xi) = \infty$,

a U - number if $\omega(\xi) = \infty, \mu(\xi) < \infty$.

In [9], Koksma introduced an analogous classification of complex numbers. He divided the complex numbers into four classes A^*, S^*, T^* and U^* in the following way.

Let α be an arbitrary algebraic number. If we denote its minimal defining polynomial by $P(x)$, then the height $H(\alpha)$ of α is defined by $H(\alpha) = H(P)$ and the degree $\deg(\alpha)$ of α is defined by $\deg(\alpha) = \deg(P)$. Given an arbitrary complex number ξ and positive integers n, H, let α be an algebraic number with degree at most n and height at most H such that $|\xi - \alpha|$ takes the smallest positive value; Koksma defines $\omega_n^*(H,\xi)$ by the following equation:

$$\omega_n^*(H,\xi) = \min\left\{|\xi - \alpha| : \alpha \text{ is algebraic}, \deg(\alpha) \leq n, H(\alpha) \leq H, \alpha \neq \xi\right\}.$$

Next, Koksma puts

$$\omega_n^*(\xi) = \overline{\lim_{H\to\infty}} \frac{-\log(H\omega_n^*(H,\xi))}{\log H} \quad \text{and} \quad \omega^*(\xi) = \overline{\lim_{n\to\infty}} \frac{\omega_n^*(\xi)}{n}.$$

The inequalities $0 \leq \omega_n^*(\xi) \leq \infty$ and $0 \leq \omega^*(\xi) \leq \infty$ hold. If for an index $\omega_n^*(\xi) = \infty$, the $\mu^*(\xi)$ is defined as the smallest of them, otherwise $\mu^*(\xi) = \infty$. Thus, $\mu^*(\xi)$ is uniquely determined. Furthermore, the two quantities $\mu^*(\xi)$ and $\omega^*(\xi)$ are never finite simultaneously. Therefore, there are the following four possibilities for ξ, ξ is called:

an A^* - number if $\omega^*(\xi) = 0, \mu^*(\xi) = \infty$,

an S^* - number if $0 < \omega^*(\xi) < \infty, \mu^*(\xi) = \infty$,

a T^* - number if $\omega^*(\xi) = \infty, \mu^*(\xi) = \infty$,

a U^* - number if $\omega^*(\xi) = \infty, \mu^*(\xi) < \infty$.

Wirsing [10] proved that both classifications are equivalent. Namely, A, S, T and U numbers are the same as A^*, S^*, T^* and U^* numbers. The class A is precisely the set of algebraic numbers. ξ is called a U-number of degree m if $\mu(\xi) = m$. The set of U-numbers of degree m is denoted by U_m. It is obvious that for any $m \geq 1$, the U_m is a subclass of U and U is the union of all disjoint sets U_m. Leveque [11] proved that U_m is not empty for any $m \geq 1$.

In [12], Oryan considered a class of power series with algebraic coefficients and proved that under certain conditions these series take values in the subclass U_m for algebraic arguments. Later in [13], similar relations are investigated for Liouville number arguments, and it is proved that these series take values in the set of Mahler's U-numbers. In [14], Saradha and Tijdeman considered certain convergent sums and showed that they are either rational or transcendental. Later in [15], Yuan and Li obtained further results for some convergent sums. In [16], Nyblom employed a variation on the proof used to established Liouville's theorem concerning the rational approximation of algebraic numbers, to deduce explicit growth conditions for a certain series to converge to a transcendental number. Later, Nyblom [17] derived a sufficiency condition for a series of positive rational terms to converge to a transcendental number. Further, Duverney [18] proved a theorem that gives a criterion for the sums of infinite series to be transcendental. The terms of these series consist of the rational numbers and converge regularly and very quickly to zero. In [19], Hančl introduced the concept of transcendental sequences and proved a criterion for sequences to be transcendental. Later, a new concept of a Liouville sequence was introduced in [20] by means of the related Liouville series. Some recent results for the transcendence of infinite series can also be found in Borwein and Coons [21], Hančl and Rucki [22], Hančl and Štěpnička [23], Murty and Weatherby [24], Weatherby [25].

In the present work, we considered certain power series with algebraic coefficients from a certain algebraic number field K of degree m and showed that under certain conditions these series take values belonging to either the algebraic number field K or $\bigcup_{i=1}^{m} U_i$ in Mahler's classification of the complex numbers for some Liouville number arguments.

2 Preliminaries

In this paper, $|x|$ means the absolute value of x and the least common multiple of $x_1, x_2, \ldots, x_n$ is denoted by $[x_1, x_2, \ldots, x_n]$.

Definition 1 A real number ξ is called a Liouville number if and only if for every positive integer n there exists integers p_n, q_n ($q_n > 1$) with

$$0 < \left|\xi - \frac{p_n}{q_n}\right| < \frac{1}{q_n^n}.$$

The set of all Liouville numbers is identical with the U_1 subclass. More information about Liouville numbers may be found in [26–28]. Now, in order to prove our main theorem we need the following lemmas.

Lemma 2 [29] *Let $\alpha_1, \ldots, \alpha_k$ ($k \geq 1$) be algebraic numbers which belong to an algebraic number field K of degree m, and let $F(y, x_1, \ldots, x_k)$ be a polynomial with rational integral coefficients and with degree at least* 1 *in y. If η is any algebraic number such that $F(\eta, \alpha_1, \ldots, \alpha_k) = 0$, then $\deg(\eta) \leq dm$ and*

$$H(\eta) \leq 3^{2dm+(l_1+\cdots+l_k)m} H^m H(\alpha_1)^{l_1 m} \cdots H(\alpha_k)^{l_k m},$$

where H is the height of the polynomial F, d is the degree of F in y and l_i is the degree of F in x_i for $i = 1, \ldots, k$.

Lemma 3 [11] *Let α be an algebraic number of degree m, and let $\alpha^{(1)} = \alpha, \ldots, \alpha^{(m)}$ be its conjugates. Then $\overline{|\alpha|} \leq 2H(\alpha)$, where $\overline{|\alpha|} = \max(|\alpha^{(1)}|, \ldots, |\alpha^{(m)}|)$.*

Lemma 4 [30] *Let α be an algebraic number of degree m, then $H(\alpha) \leq (2\overline{|\alpha|})^m$, where $\overline{|\alpha|} = \max(|\alpha^{(1)}|, \ldots, |\alpha^{(m)}|)$.*

3 Main result

Theorem 5 *Let K be an algebraic number field of degree m, and let*

$$g(x) = \sum_{n=0}^{\infty} \frac{\alpha_n}{e_n} x^n$$

be a power series such that $\alpha_n \in K$ are non-zero algebraic numbers and $e_n > 1$ are rational integers satisfying the following conditions:

$$\underline{\lim}_{n\to\infty} \frac{\log e_{n+1}}{\log e_n} = \eta > 1, \tag{1}$$

$$\overline{\lim}_{n\to\infty} \frac{\log e_{n+1}}{\log e_n} = \infty, \tag{2}$$

$$\overline{\lim}_{n\to\infty} \frac{\log H(\alpha_n)}{\log e_n} = \mu < 1. \tag{3}$$

Further, let ξ be a Liouville number satisfying the following two properties:

1. *ξ has an approximation with rational numbers $\frac{p_n}{q_n}$ $(q_n > 1)$ so that the following inequality holds for sufficiently large n*

$$\left|\xi - \frac{p_n}{q_n}\right| < \frac{1}{q_n^{ns_n}} \quad \Bigl(\lim_{n\to\infty} s_n = +\infty\Bigr). \tag{4}$$

2. *There exist two positive real constants γ_1 and γ_2 with $\frac{\eta}{\eta-1} < \gamma_1 < \gamma_2$ and*

$$e_n^{\gamma_1} \leq q_n^n \leq e_n^{\gamma_2} \tag{5}$$

for sufficiently large n.

Then $g(\xi)$ belongs to either the algebraic number field K or $\bigcup_{i=1}^{m} U_i$.

Proof It follows from (1) that

$$\log e_{n+1} > \eta_1 \log e_n \tag{6}$$

for sufficiently large n, where $\eta_1 = \eta - \varepsilon_1$ and ε_1 is to be chosen as $0 < \varepsilon_1 < \eta - \frac{\gamma_1}{\gamma_1 - 1}$. It follows from (6) that the sequence $\{e_n\}$ is strictly increasing, thus we have

$$\lim_{n\to\infty} e_n = \infty, \tag{7}$$

$$\lim_{n\to\infty} \frac{\log e_n}{n} = \infty \quad \text{and} \quad \lim_{n\to\infty} \frac{\log e_n}{n^2} = \infty. \tag{8}$$

Furthermore, from (6) we get

$$e_n < e_{n+1}^{\frac{1}{\eta_1}} \tag{9}$$

for sufficiently large n.

Let $E_n = [e_0, e_1, \dots, e_n]$. Then by using (7) and (9), we obtain for sufficiently large n

$$e_n \le E_n \le e_n^{\varepsilon_2 + \frac{\eta_1}{\eta_1 - 1}}, \tag{10}$$

where ε_2 is to be chosen as $0 < \varepsilon_2 < \gamma_1 - \frac{\eta_1}{\eta_1 - 1}$. We can easily deduce from (3) and $\mu < \frac{\mu+1}{2}$ that

$$\frac{\log H(\alpha_n)}{\log e_n} < \frac{\mu + 1}{2} \tag{11}$$

for sufficiently large n. Since (11) holds, there is a natural number N_0 such that

$$H(\alpha_n) < e_n^{(\frac{\mu+1}{2})} \tag{12}$$

for every $n > N_0$. On the other hand, we get from Lemma 3 that

$$\overline{|\alpha_n|} \le 2H(\alpha_n) \tag{13}$$

since α_n are algebraic numbers. From here and (12), we obtain

$$|\alpha_n| \le \overline{|\alpha_n|} \le 2e_n^{(\frac{\mu+1}{2})} \tag{14}$$

for every $n > N_0$. Now, we shall define the algebraic numbers

$$\beta_n = \sum_{\gamma=0}^{n} \frac{\alpha_\gamma}{e_\gamma} \left(\frac{p_n}{q_n}\right)^\gamma$$

for $n = 0, 1, 2, \dots$. Since $\beta_n \in K$, $\deg(\beta_n) \le m$ for $n = 0, 1, 2, \dots$. Let us determine an upper bound for the heights of the algebraic numbers β_n. By multiplying both sides of this equality by $E_n q_n^n$ and putting $l_i = \frac{E_n}{e_i}$ for $i = 0, 1, 2, \dots$, we obtain the equality

$$E_n q_n^n \beta_n = (l_0 q_n^n)\alpha_0 + (l_1 q_n^{n-1} p_n)\alpha_1 + \cdots + (l_n p_n^n)\alpha_n.$$

Since ξ is a Liouville number, we can assume that $p_n \neq 0$ for $n = 0, 1, 2, \dots$. Then we get a polynomial

$$G(y, x_0, x_1, \dots, x_n) = E_n q_n^n y - \sum_{\gamma=0}^{n} (l_\gamma q_n^{n-\gamma} p_n^\gamma) x_\gamma$$

with rational integral coefficients such that $G(\beta_n, \alpha_0, \dots, \alpha_n) = 0$. Further, this polynomial is of degree 1 in each $y, x_0, x_1, \dots, x_n$. Thus, we deduce from Lemma 2 that

$$H(\beta_n) \le 3^{2m+(n+1)m} H^m H(\alpha_0)^m \cdots H(\alpha_n)^m, \tag{15}$$

where H is the height of the polynomial $G(y, x_0, x_1, \dots, x_n)$. By using (4), we obtain $|\frac{p_n}{q_n}|^i \le k_0^n$ for $i = 0, 1, 2, \dots$, where $k_0 = |\xi| + 1 > 1$ is a real constant. From here, we can easily get

$$H \le E_n q_n^n k_0^n. \tag{16}$$

It follows from (15) and (16) that

$$H(\beta_n) \le k_1^{mn} E_n^m q_n^{mn} H(\alpha_0)^m \cdots H(\alpha_n)^m, \tag{17}$$

where $k_1 = 3^4 k_0 > 1$ is a real constant. Moreover, we get

$$H(\beta_i) \le \left(2\overline{|\beta_i|}\right)^m \quad (i = 0, 1, 2, \ldots) \tag{18}$$

from Lemma 4. Then we deduce from (17) and (18) that

$$H(\beta_n) \le 2^{m^2(n+1)} k_1^{mn} E_n^m q_n^{mn} \left(\overline{|\beta_0|} \cdots \overline{|\beta_n|}\right)^{m^2}. \tag{19}$$

By using (10), (13) and (14), we obtain

$$\left(\overline{|\beta_0|} \cdots \overline{|\beta_n|}\right)^{m^2} \le 2^{m^2(n+1)} k_2 e_n^{\gamma_1 m^2 (\frac{\mu+1}{2})},$$

where $k_2 = \max(1, (H(\alpha_0) \cdots H(\alpha_{N_0}))^{m^2}) \ge 1$ is a real constant. It follows from here, (5) and (10) that

$$H(\beta_n) \le k_3^n e_n^{k_4}, \tag{20}$$

where $k_3 = 2^{4m^2} k_1^m k_2 > 1$ and $k_4 = \gamma_1 m + \gamma_1 m^2 (\frac{\mu+1}{2}) + \gamma_2 m > 1$ are real constants.

Now, we consider the following polynomials:

$$g_n(x) = \sum_{\nu=0}^{n} \frac{\alpha_\nu}{e_\nu} x^\nu$$

for $n = 1, 2, \ldots$. Since $g_n(x)$ are continuous and differentiable for all real numbers, at least one real number c_n exists between ξ and $\frac{p_n}{q_n}$ such that for every n

$$g_n(\xi) - \beta_n = g_n(\xi) - g_n\left(\frac{p_n}{q_n}\right) = \left(\xi - \frac{p_n}{q_n}\right) g_n'(c_n). \tag{21}$$

It is obvious that $|c_n| \le \max(|\xi|, |\frac{p_n}{q_n}|)$. Since $|\frac{p_n}{q_n}| < |\xi| + 1$, we obtain $|c_n| \le |\xi| + 1$ for sufficiently large n. Furthermore, from here and (4) and (21), we get

$$\left| g_n(\xi) - g_n\left(\frac{p_n}{q_n}\right) \right| \le \frac{1}{q_n^{ns_n}} \sum_{\nu=1}^{n} \frac{|\alpha_\nu|}{e_\nu} \nu \left(|\xi| + 1\right)^{\nu-1} \tag{22}$$

for sufficiently large n.

Define $\sigma_n = \max(|\alpha_0|, \ldots, |\alpha_n|)$. Then we obtain

$$\sum_{\nu=1}^{n} \frac{|\alpha_\nu|}{e_\nu} \nu \left(|\xi| + 1\right)^{\nu-1} \le n^2 \sigma_n \left(|\xi| + 1\right)^{n-1} \tag{23}$$

for sufficiently large n. It follows from (14) that $\sigma_n \leq 2e_n^{(\frac{1+\mu}{2})}$ for sufficiently large n. We get from here and (22), (23)

$$\left|g_n(\xi) - \beta_n\right| \leq \frac{2n^2(|\xi| + 1)^{n-1} e_n^{(\frac{1+\mu}{2})}}{q_n^{ns_n}}.$$

By using (5), we obtain from here that

$$\left|g_n(\xi) - \beta_n\right| \leq \frac{2n^2(|\xi| + 1)^{n-1}}{e_n^{\gamma_1 s_n - (\frac{1+\mu}{2})}}. \tag{24}$$

From (8) and $\lim_{n\to\infty} s_n = \infty$, it is possible to find a sequence $\{\omega_n'\}$ with $\lim_{n\to\infty}\omega_n' = \infty$ such that

$$\frac{2n^2(|\xi| + 1)^{n-1}}{e_n^{\gamma_1 s_n - (\frac{1+\mu}{2})}} \leq \frac{1}{2(k_3^n e_n^{k_4})^{\omega_n'}}. \tag{25}$$

Therefore, we get from (20), (24) and (25)

$$\left|g_n(\xi) - \beta_n\right| \leq \frac{1}{2H(\beta_n)^{\omega_n'}} \tag{26}$$

for sufficiently large n. Moreover, the following inequality holds:

$$\left|g(\xi) - g_n(\xi)\right| \leq \sum_{i=1}^{\infty} \frac{|\alpha_{n+i}|}{e_{n+i}} |\xi|^{n+i}.$$

We get from here and (14)

$$\left|g(\xi) - g_n(\xi)\right| \leq \frac{2|\xi|^{n+1}}{e_{n+1}^{(\frac{1-\mu}{2})}} \left[1 + \left(\frac{e_{n+1}}{e_{n+2}}\right)^{\frac{1-\mu}{2}} |\xi| + \left(\frac{e_{n+1}}{e_{n+3}}\right)^{\frac{1-\mu}{2}} |\xi|^2 + \cdots\right].$$

Thus, we deduce from (9) that

$$0 < \frac{e_{n+1}}{e_{n+2}} < \frac{1}{e_{n+2}^{(1-\frac{1}{\eta_1})}}.$$

Since (7) holds, then we obtain $\lim_{n\to\infty} \frac{e_{n+1}}{e_{n+2}} = 0$. Similarly, since $\frac{1-\mu}{2} > 0$, we have

$$\left(\frac{e_{n+1}}{e_{n+1+k}}\right)^{\frac{1-\mu}{2}} |\xi|^k < \left(\frac{1}{2}\right)^k$$

for $k = 1, 2, 3, \ldots$ and, therefore,

$$\left|g(\xi) - g_n(\xi)\right| \leq \frac{4|\xi|^{n+1}}{e_{n+1}^{(\frac{1-\mu}{2})}}.$$

On the other hand from (8), we get $4|\xi|^{n+1} \leq e_{n+1}^{(\frac{1-\mu}{4})}$ for sufficiently large n. From here, we obtain

$$\left|g(\xi) - g_n(\xi)\right| \leq \frac{1}{e_{n+1}^{(\frac{1-\mu}{4})}}$$

for sufficiently large n. Now if we define $r_n = \frac{\log e_{n+1}}{\log e_n}$, then we have

$$\left|g(\xi) - g_n(\xi)\right| \leq \frac{1}{e_n^{r_n(\frac{1-\mu}{4})}}.$$

Using (2), then it follows that there exists a subsequence $\{r_{n_k}\}$ of $\{r_n\}$ such that $\lim_{k\to\infty} r_{n_k} = \infty$. Therefore, we get

$$\left|g(\xi) - g_{n_k}(\xi)\right| \leq \frac{1}{e_{n_k}^{r_{n_k}(\frac{1-\mu}{4})}} \tag{27}$$

for sufficiently large n_k. From (8) and $\lim_{k\to\infty} r_{n_k} = \infty$, there exists a suitable sequence $\{r'_{n_k}\}$ with $\lim_{k\to\infty} r'_{n_k} = \infty$ such that

$$\frac{1}{e_{n_k}^{r_{n_k}(\frac{1-\mu}{4})}} \leq \frac{1}{2(k_3^n e_n^{k_4})^{r'_{n_k}}}$$

and, therefore, from (20) and (27), we obtain

$$\left|g(\xi) - g_{n_k}(\xi)\right| \leq \frac{1}{2H(\beta_{n_k})^{r'_{n_k}}} \tag{28}$$

for sufficiently large n_k. On the other hand by using (26), we deduce that

$$\left|g_{n_k}(\xi) - \beta_{n_k}\right| \leq \frac{1}{2H(\beta_{n_k})^{\omega'_{n_k}}} \tag{29}$$

for sufficiently large n_k. Let $\omega_{n_k} = \min(r'_{n_k}, \omega'_{n_k})$. It follows from (28) and (29) that

$$\left|g(\xi) - \beta_{n_k}\right| \leq \frac{1}{H(\beta_{n_k})^{\omega_{n_k}}},$$

where $\lim_{k\to\infty} \omega_{n_k} = \infty$. It follows from here that $\lim_{k\to\infty} \beta_{n_k} = g(\xi)$. Thus, if the sequence $\{\beta_{n_k}\}$ is constant, then $g(\xi)$ is an algebraic number in K. Otherwise $g(\xi) \in \bigcup_{i=1}^m U_i$. □

4 Conclusion

In this work, the series with algebraic coefficients are treated and it is shown that under certain conditions the values of these series are either algebraic numbers or U-numbers for Liouville number arguments. The similar results can be proved for the power series, which are defined in the p-adic field Q_p and in the field of formal Laurent series.

Competing interests
The author declares that she has no competing interests.

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