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Difference inequality for stability of impulsive difference equations with distributed delays

Dingshi Li[1*], Shujun Long[2] and Xiaohu Wang[1*]

* Correspondence:
lidingshi2006@163.com;
xiaohuwang1111@163.com
[1]Yangtze Center of Mathematics,
Sichuan University, Chengdu
610064, P. R. China
Full list of author information is
available at the end of the article

Abstract

In this paper, we consider a class of impulsive difference equations with distributed delays. By establishing an impulsive delay difference inequality and using the properties of "ρ-cone" and eigenspace of the spectral radius of non-negative matrices, some new sufficient conditions for global exponential stability of the impulsive difference equations with distributed delays are obtained. An example is given to demonstrate the effectiveness of the theory.

Keywords: Difference equations, Impulsive, Distributed delays, Difference inequality, Global exponential stability

1 Introduction

Difference equations usually appear in the investigation of systems with discrete time or in the numerical solution of systems with continuous time [1]. In recent years, the stability investigation of difference equations has been interesting to many investigators, and various advanced results on this problem have been reported [2,3]. However, almost all available results have been focused on systems with discrete delays. In reality, difference systems with distributed delays become important because it is essential to formulate the discrete-time analogue of the continuous-time system with distributed delays when one wants to simulate or compute the continuous-time one after obtaining its dynamical characteristics. Fortunately, such an issue has been addressed in [4-7].

However, besides the delay effect, an impulsive effect likewise exists in a wide variety of evolutionary processes in which states are changed abruptly at certain moments of time, involving such fields as medicine, biology, economics, mechanics, electronics, and telecommunications. Recently, the asymptotic behaviors of impulsive difference equations have attracted considerable attention. Many interesting results on impulsive effect have been obtained [8-11].

It is well known that distributed delay differential equations with impulses or without impulses have been considered by many authors (see, for instance [12-14]). But, to the best of our knowledge, there is no concerning on the stability of impulsive difference equations with distributed delays in literature. Motivated by the above discussion, we here make a first attempt to arrive at results on the global exponential stability of impulsive difference equations with distributed delays.

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2 Model description and preliminaries

Let $R^n(R^n_+)$ be the space of n-dimensional (non-negative) real column vectors and $R^{m \times n}(R^{m \times n}_+)$ denotes the set of $m \times n$ (non-negative) real matrices. Usually, E denotes an $n \times n$ unit matrix. For $A, B \in R^{m \times n}$ or $A, B \in R^n$, the notation $A \geq B$ ($A > B$) means that each pair of corresponding elements of A and B satisfies the inequality " $\geq$ (>)". Especially, $A \in R^{m \times n}$ is called a nonnegative matrix if $A \geq 0$, and $z \in R^n$ is called a positive vector if $z > 0$. Z denotes the integer set, $Z_\infty = \{j \in Z \mid -\infty < j \leq 0\}$ and $Z^+_\infty = \{j \in Z \mid 0 \leq j < \infty\}$. C denotes the set of all bounded functions $\phi(j) \in R^n, j \in Z_\infty$.

For $x \in R^n$, $A \in R^{n \times n}$, $\phi \in C$, we define

$$[x]^+ = (|x_1|, \ldots, |x_n|)^T, \quad [A]^+ = (|a_{ij}|)_{n \times n},$$
$$[\varphi(m)]_\infty = ([\varphi_1(m)]_\infty, \ldots, [\varphi_n(m)]_\infty)^T, \quad [\varphi(m)]^+_\infty = [[\varphi(m)]^+]_\infty,$$

where $[\varphi_i(m)]_\infty = \sup_{s \in Z_\infty} \{\varphi_i(m+s)\}$, and introduce the corresponding norm for them as follows:

$$\|x\| = \max_{1 \leq i \leq n} \{|x_i|\}, \quad \|A\| = \max_{1 \leq i \leq n} \sum_{j=1}^n |a_{ij}|, \quad \|\varphi\| = \max_{1 \leq i \leq n} \left\{[\varphi_i(m)]^+_\infty\right\}.$$

In this paper, we mainly consider the following impulsive difference equations with distributed delays

$$\begin{cases} x_i(m+1) = a_i x_i(m) + \sum_{j=1}^n b_{ij} f_j(x_j(m)) + \sum_{j=1}^n c_{ij} \sum_{k=1}^\infty \mu_{ij}(k) g_j(x_j(m-k)), & m \in Z^+_\infty, \ m \neq m_k, \\ x_i(m+1) = H_{im}(x_1(m), \ldots, x_m(m)), & m = m_k, \\ x_i(m) = \varphi_i(m), & m \in Z_\infty, \end{cases} \quad (1)$$

where $0 < i \leq n$ and a_i, b_{ij}, c_{ij} are constants. The fixed moments of time $m_k \in Z$, and satisfy $0 < m_1 < m_2 < \cdots$, $\lim_{k \to \infty} m_k = \infty$. The constants $\mu_{ij}(k)$ satisfy the following convergence conditions:

$$(H): \sum_{k=1}^\infty e^{\lambda_0 k} |\mu_{ij}(k)| < \infty, \quad i, j = 1, 2, \ldots,$$

where λ_0 is a positive constant.

For convenience, we shall rewrite (1) in the vector form:

$$\begin{cases} x(m+1) = Ax(m) + Bf(x(m)) + C \sum_{k=1}^\infty \mu(k) g(x(m-k)), & m \in Z^+_\infty, \ m \neq m_k, \\ x(m+1) = H_m(x(m)), & m = m_k, \\ x(m) = \varphi(m), & m \in Z_\infty, \end{cases} \quad (2)$$

where $x(m) = (x_1(m), \ldots, x_n(m))^T$, $A = \text{diag}\{a_1, \ldots, a_n\}$, $B = \{b_{ij}\}_{n \times n}$, $C = \{c_{ij}\}_{n \times n}$, $f(x) = (f_1(x_1), \ldots, f_n(x_n))^T$, $g(x) = (g_1(x_1), \ldots, g_n(x_n))^T$, $\mu(k) = (\mu_{ij}(k))_{n \times n}$, $H_m(x(m)) = (H_{1m}(x(m)), \ldots, H_{nm}(x(m)))^T$, $\phi \in C$, and $f(x), g(x), H_m(x) \in C[R^n, R^n]$.

We will assume that there exists one solution of system (2) which is denoted by $x(m, 0, \phi)$, or, $x(m)$, if no confusion occurs. We will also assume that $g(0) = 0, f(0) = 0$ and $H_m(0) = 0, m = m_k$, for the stability purpose of this paper. Then system (2) admits an equilibrium solution $x(m) \equiv 0$.

Definition 2.1. The zero solution of Equation 2 is called globally exponentially stable if there are positive constants λ and $M \geq 1$ such that for any initial condition $\phi \in C$,

$$\|x(m,\ 0,\ \varphi)\| \leq M\,\|\varphi\|e^{-\lambda m}, \quad m \geq 0.$$

Here λ is called the exponential convergence rate.

For $A \in R_+^{n \times n}$, the spectral radius $\rho(A)$ is an eigenvalue of A and its eigenspace is denoted by

$$W_\rho(A) \triangleq \{z \in R^n | Az = \rho(A)z\},$$

which includes all positive eigenvectors of A provided that the non-negative matrix A has at least one positive eigenvector(see [15]).

Lemma 2.1. [16] Suppose that $M \in R_+^{n \times n}$ and $\rho(M) < 1$, then there exists a positive vector z such that

$$(E - M)z\ > 0.$$

For $M \in R_+^{n \times n}$ and $\rho(M) < 1$, we denote

$$\Omega_\rho(M) = \{z \in R^n | (E - M)z > 0,\ z > 0\},$$

which is a nonempty set by Lemma 2.1, and satisfying that $k_1 z_1 + k_2 z_2 \in \Omega_\rho(M)$ for any scalars $k_1 > 0$, $k_2 > 0$ and vectors $z_1, z_2 \in \Omega_\rho(M)$. So $\Omega_\rho(M)$ is a cone without vertex in R^n, we call it a "ρ-cone."

Lemma 2.2. Suppose $P \in R_+^{n \times n}$ and $Q(k) = (q_{ij}(k))_{n \times n}$, where $q_{ij}(k) \geq 0$ and satisfy

$$\sum_{k=1}^{\infty} e^{\lambda_1 k} q_{ij}(k) < \infty, \quad i,\ j = 1, 2,\ \ldots,\ n,$$

where λ_1 is a positive constant. Denote $Q = (q_{ij})_{n \times n} \triangleq (\sum_{k=1}^{\infty} q_{ij}(k))_{n \times n} \in R_+^{n \times n}$ and let $\rho(P + Q) < 1$ and $u(m) = (u_1(m), \ldots, u_n(m))^T \in R_+^n$ be a solution of the following inequality with the initial condition $u(m_0 + m) \in C$, $m \in Z_\infty$,

$$u(m + 1) \leq Pu(m) + \sum_{k=1}^{\infty} Q(k)u(m - k), \quad m \geq m_0. \tag{3}$$

Then

$$u(m) \leq z e^{-\lambda(m - m_0)}, \quad m \geq m_0, \tag{4}$$

provided that the initial conditions satisfy

$$u(s) \leq z e^{-\lambda(s - m_0)}, \quad -\infty < s \leq m_0, \tag{5}$$

where $z = (z_1, z_2, \ldots, z_n)^T \in \Omega_\rho(P + Q)$, $m_0 \in Z$ and the positive number $\lambda \leq \lambda_1$ is determined by the following inequality

$$\left(e^\lambda \left(P + \sum_{k=1}^{\infty} Q(k)e^{\lambda k}\right) - E\right)z < 0. \tag{6}$$

Proof. Since $\rho(P + Q) < 1$ and $P + Q \in R_+^{n \times n}$, then, by Lemma 2.1, there exists a positive vector $z \in \Omega_\rho(P + Q)$ such that $(E - (P + Q))z > 0$. Using continuity, there must be a sufficiently small constant $\lambda > 0$ such that $\left(e^\lambda \left(P + \sum_{k=1}^{\infty} Q(k)e^{\lambda k}\right) - E\right)z < 0$, i.e., inequality (6) has at least one positive solution $\lambda \leq \lambda_1$.

Let

$$\gamma(m) = u(m)e^{\lambda(m-m_0)} \quad \text{or} \quad u(m) = \gamma(m)e^{-\lambda(m-m_0)}.$$

Then, from (5), we have

$$\gamma(s) \leq z, \quad -\infty < s \leq m_0. \tag{7}$$

By (3), we have

$$\gamma(m+1) = u(m+1)e^{\lambda(m+1-m_0)} \leq \left(Pu(m) + \sum_{k=1}^{\infty} Q(k)u(m-k)\right)e^{\lambda(m+1-m_0)}, \quad m \geq m_0. \tag{8}$$

Since $P + Q \in R_+^{n \times n}$, we derive that

$$\gamma(m+1) \leq \left(P\gamma(m)e^{-\lambda(m-m_0)} + \sum_{k=1}^{\infty} Q(k)\gamma(m-k)e^{-\lambda(m-k-m_0)}\right)e^{\lambda(m+1-m_0)} \tag{9}$$

$$\leq \left(P\gamma(m) + \sum_{k=1}^{\infty} Q(k)\gamma(m-k)e^{\lambda k}\right)e^{\lambda}.$$

We next show for any $m \geq m_0$

$$\gamma(m) \leq z. \tag{10}$$

If this is not true, then there must be a positive constant $m^* \geq m_0$ and some integer i such that

$$\gamma_i(m^*+1) > z \quad \text{and} \quad \gamma(m) \leq z, \quad -\infty < m \leq m^*. \tag{11}$$

By (6), (9), and the second inequality of (11), we obtain that

$$\gamma(m^*+1) \leq \left(P\gamma(m^*) + \sum_{k=1}^{\infty} Q(k)\gamma(m^*-k)e^{\lambda k}\right)e^{\lambda}$$

$$\leq \left(P + \sum_{k=1}^{\infty} Q(k)e^{\lambda k}\right)e^{\lambda}z \leq z,$$

which contradicts the first inequality of (11). Thus (10) holds for all $m \geq m_0$. Therefore, we have

$$u(m) \leq ze^{-\lambda(m-m_0)}, \quad m \geq m_0,$$

and the proof is completed.

3 Main results

To obtain the global exponential stability of the zero solution of system (2), we introduce the following assumptions.

(A_1) For any $x \in R^n$, there exist non-negative diagonal matrices U and V such that

$$[f(x)]^+ \leq U[x]^+, \quad [g(x)]^+ \leq V[x]^+.$$

(A_2) For any $x \in R^n$, there exist non-negative matrices R_k such that

$$[H_{m_k}(x)]^+ \leq R_k[x]^+, \quad k = 1, 2, \ldots.$$

(A_3) Let $P = [A]^+ + [B]^+U$, $Q = [C]^+ \sum_{k=1}^{\infty} [\mu(k)]^+V$, and $\rho(P + Q) < 1$.

(A_4) The set $\Omega = \bigcap_{k=1}^{\infty} [W_\rho(R_k)] \bigcap \Omega_\rho(P + Q)$ is nonempty.

(A_5) Let

$$\gamma k \geq \max\{1, \ \rho(R_k)\}, \tag{12}$$

and there exists a constant γ such that

$$\frac{\ln \gamma_k}{m_k - m_{k-1}} \leq \gamma < \lambda, \quad k = 1, 2, \ldots, \tag{13}$$

where the positive number $\lambda \leq \lambda_0$ is determined by the following inequality

$$\left(e^\lambda \left(P + \sum_{k=1}^{\infty} Q(k) e^{\lambda k} \right) - E \right) z < 0, \quad \text{for a given} \quad z \in \Omega. \tag{14}$$

Theorem 3.1. Assume that the hypothesis (H) and Conditions (A_1)-(A_5) hold. Then the zero solution of (2) is globally exponentially stable and the exponential convergent rate equals $\lambda - \gamma$.

Proof. Since $\rho(P + Q) < 1$ and $P + Q \in R_+^{n \times n}$, then, by Lemma 2.1, there exists a positive vector $z \in \Omega_\rho(P + Q)$ such that $(E - (P + Q))z > 0$. Using continuity and hypothesis (H), there must be a sufficiently small constant $\lambda > 0$ such that $(e^\lambda(P + \sum_{k=1}^{\infty} Q(k) e^{\lambda k}) - E)z < 0$, i.e., inequality (14) has at least one positive solution $\lambda \leq \lambda_0$.

From (2), Conditions (A_1) and (A_3), we have

$$
\begin{aligned}
[x(m + 1)]^+ \ &\leq [Ax(m)]^+ + [Bf(x(m))]^+ + \left[C \sum_{k=1}^{\infty} \mu(k) g(x(m - k)) \right]^+ \\
&\leq [A]^+[x(m)]^+ + [B]^+ U[(x(m))]^+ + [C]^+ V \sum_{k=1}^{\infty} [\mu(k)]^+ [x(m - k)]^+ \\
&= P[x(m)]^+ + [C]^+ V \sum_{k=1}^{\infty} [\mu(k)]^+ [x(m - k)]^+, \quad m_{k-1} \leq m \leq m_k, \quad k = 1, 2, 3 \ldots,
\end{aligned}
\tag{15}
$$

where $m_0 = 0$.

For the initial conditions: $x(s) = \phi(s)$, $-\infty < s \leq 0$, where $\phi \in C$, we can get

$$[x(m)]^+ \leq d||\varphi|| e^{-\lambda(m - m_0)}, \quad -\infty < m \leq 0, \tag{16}$$

where

$$d = \frac{1}{\min\limits_{1 \leq i \leq n} z_i} z, \quad z \in \Omega.$$

By the property of "ρ-cone" and $z \in \Omega \subseteq \Omega_\rho(P + Q)$, we have $d \ ||\phi|| \in \Omega_\rho(P + Q)$. Then, all the conditions of Lemma 2.2 are satisfied by (15), (16), and Condition (A_3), we derive that

$$[x(m)]^+ \leq d||\varphi|| e^{-\lambda(m - m_0)}, \quad m_0 \leq m \leq m_1. \tag{17}$$

Suppose for all $q = 1, \ldots, k$, the inequalities

$$[x(m)]^+ \leq \gamma_0 \cdots \gamma_{q-1} d||\varphi|| e^{-\lambda(m - m_0)}, \quad m_{q-1} \leq m \leq m_q, \tag{18}$$

hold, where $\gamma_0 = 1$. Then, from Condition (A_2) and (18), we have

$$
\begin{aligned}
[x(m_q + 1)]^+ = [H_{m_q}(x(m_q))]^+ \\
\leq R_q[x(m_q)]^+ \\
\leq R_q d\gamma_0 \cdots \gamma_{q-1} ||\varphi|| e^{-\lambda(m-m_0)}.
\end{aligned}
\tag{19}
$$

Since $d \in \Omega \subseteq W_\rho(R_q)$, we have $R_q d = \rho(R_q)d$. Therefore, from (12) and (19), we obtain

$$
[x(m_q + 1)]^+ \leq \gamma_0 \cdots \gamma_{q-1} \gamma_q d ||\varphi|| e^{-\lambda(m-m_0)}.
\tag{20}
$$

This, together with (18), leads to

$$
[x(m)]^+ \leq \gamma_0 \cdots \gamma_{k-1} \gamma_k d ||\varphi|| e^{-\lambda(m-m_0)}, \quad -\infty < m \leq m_k + 1.
\tag{21}
$$

By the property of "ρ-cone" again, the vector $\gamma_0 \ldots \gamma_{k-1}\gamma_k d \in \Omega_\rho(P + Q)$. It follows from (21) and Lemma 2.2 that

$$
[x(m)]^+ \leq \gamma_0 \cdots \gamma_{k-1} \gamma_k d ||\varphi|| e^{-\lambda(m-m_0)}, \quad m_k + 1 \leq m \leq m_{k+1}.
$$

yielding, together with (18), that

$$
[x(m)]^+ \leq \gamma_0 \cdots \gamma_{k-1} \gamma_k d ||\varphi|| e^{-\lambda(m-m_0)}, \quad m_k \leq m \leq m_{k+1}.
$$

By mathematical induction, we can conclude that

$$
[x(m)]^+ \leq \gamma_0 \cdots \gamma_{k-1} d ||\varphi|| e^{-\lambda(m-m_0)}, \quad m_{k-1} \leq m \leq m_k, \quad k = 1, 2, \ldots.
\tag{22}
$$

Noticing that $\gamma_k \leq e^{\gamma(m_k - m_{k-1})}$ by (13), we can use (22) to conclude that

$$
\begin{aligned}
[x(m)]^+ &\leq e^{\gamma(m_1 - m_0)} \cdots e^{\gamma(m_{k-1} - m_{k-2})} d ||\varphi|| e^{-\lambda(m-m_0)} \\
&\leq d ||\varphi|| e^{\gamma(m-m_0)} e^{-\lambda(m-m_0)} \\
&= d ||\varphi|| e^{-(\lambda-\gamma)(m-m_0)}, \quad m_{k-1} \leq m \leq m_k, \quad k = 1, 2, \ldots,
\end{aligned}
$$

which implies that the conclusions of the theorem hold.

Remark 3.1. In Theorem 3.1, we may properly choose the matrix R_k in the condition (A_2) such that $\Omega \equiv \varnothing$ Especially, when $R_k = \alpha_k E$ (α_k are non-negative constants), Ω is certainly nonempty. So, by using Theorem 3.1, we can easily obtain the following corollary.

Remark 3.2. The conditions (A_1)-(A_5) is conservative. For example, we get the absolute value of all coefficients of (2). Recently, the delay-fractioning or delay-partitioning approach [17,18] is widely used that has shown the potential of reducing conservatism. We will combine delay-partitioning approach with difference inequality approach in our future work to reduce the conservatism.

Corollary 3.1. Assume that (H), (A_1), (A_3), and (A_5) hold. For any $x \in R^n$, there exist non-negative constants α_k such that

$$
[H_{m_k}(x)]^+ \leq \alpha_k[x]^+, \quad k = 1, 2, \ldots.
\tag{23}
$$

And let $\gamma_k \geq \{1, \alpha_k\}$, where the scalar $0 < \lambda < \lambda_0$ is determined by (14). Then the zero solution of (2) is globally exponentially stable and the exponential convergent rate equals $\lambda - \gamma$.

Proof. Noticing that (23) is a special case of Condition (A_2). Since $\rho(R_k) = \alpha_k$, then $W_\rho(R_k) = R^n$. So, we have $\Omega = \bigcap_{k=1}^{\infty} [W_\rho(R_k)] \bigcap \Omega_\rho(P + Q) = \Omega_\rho(P + Q)$. Since the

"ρ-cone" $\Omega_\rho(P + Q)$ is nonempty by Lemma 2.1, (A_4) obviously holds. Thus we can deduce the conclusion in terms of Theorem 3.1.

Remark 3.3. If $H_k(x) = x$, then Equation 2 becomes difference equations with distributed delays without impulses in vector form

$$x(m + 1) = Ax(m) + Bf(x(m)) + C\sum_{k=1}^{\infty} \mu_k g\ (x(m - k)), \qquad (24)$$

which contains many popular models such as discrete-time Hopfield neural networks, discrete-time cellular neural networks, and discrete-time recurrent neural networks, and so on.

Corollary 3.2. Assume that (H), (A_1), and (A_3) hold. Then Equation 24 has exactly one equilibrium point, which is globally exponentially stable.

4 An illustrate example

In this section, we will give an example to illustrate the global exponential stability of Equation 1 further.

Example. Consider the following difference equation with distributed delays:

$$\begin{aligned}
x_1(m + 1) &= \frac{1}{4}x_1(m) + \frac{1}{5}\sin(x_1(m)) + \frac{1}{10}x_2(m) \\
&\quad - \frac{1}{6}\sum_{k=1}^{\infty} e^{-k}|x_1(m - k)| + \frac{1}{8}\sum_{k=1}^{\infty} e^{-k}|x_2(m - k)|, \quad m \neq m_k, \\
x_2(m + 1) &= \frac{1}{5}x_1(m) + \frac{1}{6}\sin(x_1(m)) + \frac{1}{8}x_2(m) \\
&\quad - \frac{1}{3}\sum_{k=1}^{\infty} e^{-k}|x_1(m - k)| + \frac{1}{10}\sum_{k=1}^{\infty} e^{-k}|x_2(m - k)|
\end{aligned} \qquad (25)$$

with

$$\begin{aligned}
x_1(m_k + 1) &= H_{1m_k}(x_1(m_k),\ x_2(m_k)), \\
x_2(m_k + 1) &= H_{2m_k}(x_1(m_k),\ x_2(m_k))
\end{aligned} \qquad (26)$$

and $m_1 = 4$, $m_k = m_{k-1} + k$ for $k = 2, 3,....$ One can check that all the properties given in (H) are satisfied provided that $0 < \lambda_0 < 1$.

Case 1. If $H_{im_k}(x_1,\ x_2) = x_i$ for $i = 1, 2$ and $k = 1, 2,...$, then Equation 25 becomes difference equation with distributed delays without impulses. The parameters of Conditions (A_1) and (A_3) are as follows:

$$A = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{5} \end{pmatrix}, \quad B = \begin{pmatrix} \frac{1}{5} & \frac{1}{10} \\ \frac{1}{6} & \frac{1}{8} \end{pmatrix}, \quad C = \begin{pmatrix} -\frac{1}{6} & \frac{1}{8} \\ -\frac{1}{3} & \frac{1}{10} \end{pmatrix}, \quad U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mu_k = \begin{pmatrix} e^{-k} & 0 \\ 0 & e^{-k} \end{pmatrix}, \quad P = \begin{pmatrix} \frac{9}{20} & \frac{1}{10} \end{pmatrix},$$

$$Q = \begin{pmatrix} \frac{1}{6(e-1)} & \frac{1}{8(e-1)} \\ \frac{1}{3(e-1)} & \frac{1}{10(e-1)} \end{pmatrix}, \quad P + Q = \begin{pmatrix} \frac{1}{6(e-1)} + \frac{9}{20} & \frac{1}{8(e-1)} + \frac{1}{10} \\ \frac{1}{3(e-1)} + \frac{1}{6} & \frac{1}{10(e-1)} + \frac{13}{40} \end{pmatrix},$$

where $P = A + [B]^+U$, $Q = [C]^+ \sum_{k=1}^{m} |\mu(k)|V$. We can easily observe that $\rho(P + Q) = 0.8345 < 1$. By Corollary 3.2, Equation 25 has exactly one globally exponentially stable equilibrium $(0, 0)^T$.

Case 2. Next we consider the case where

$$H_{1m_k} = e^{0.04k}x_1, \quad H_{2m_k} = e^{0.04k}x_2.$$

We can verify that point $(0, 0)^T$ is also an equilibrium point of the impulsive difference equation with distributed delays (25)-(26) and the parameters of Conditions (A_2) and (A_4) as follows:

$$R_k = e^{0.04k}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \rho(R_k) = e^{0.04k}, \quad W_\rho(R_k) = \left\{(z_1, z_2)^T | z_1, z_2 \in R\right\},$$

$$\Omega_\rho(P + Q) = \left\{(z_1, z_2)^T > 0 \left| \frac{40(e-1) + 40}{81(e-1) - 12}z_1 < z_2 < \frac{30(e-1) - 10}{6(e-1) + 30}z_1 \right.\right\}.$$

So $\Omega = \{(z_1, z_2)^T > 0 | z_2 = z_1\}$ is not empty. Let $z = (1, 1)^T \in \Omega$ and $\lambda = 0.05$ which satisfies the inequality $((e^\lambda(P + [C]^+V\sum_{k=1}^m |\mu_k|e^{\lambda k})) - E)z < 0$. We can obtain that for $k = 1, 2, ...$

$$\gamma_k = e^{0.04k} \geq \max\{1, e^{0.04k}\}, \quad \frac{\ln \gamma_k}{m_k - m_{k-1}} \leq \frac{\ln e^{0.04k}}{k} = 0.04 < \lambda.$$

Clearly, all conditions of Theorem 3.1 are satisfied, so the equilibrium $(0, 0)^T$ is globally exponentially stable and the exponential convergent rate is equal to 0.01.

5 Conclusion

In this paper, we consider a class of impulsive difference equations with distributed delays. By establishing an impulsive delay difference inequality and using the properties of "ρ-cone" and eigenspace of the spectral radius of non-negative matrices, some new sufficient conditions for global exponential stability of the impulsive difference equations with distributed delays are obtained. The conditions (A_1)-(A_5) are conservative. For example, we get the absolute value of all coefficients of (2). We will combine delay-partitioning approach with difference inequality approach in our future work to reduce the conservatism.

Author details
[1]Yangtze Center of Mathematics, Sichuan University, Chengdu 610064, P. R. China [2]College of Mathematics and Information Science, Leshan Teachers College, Leshan 614004, P.R. China

Authors' contributions
Dingshi Li carried out the main proof of the theorems in this paper. Shujun Long carried out the expample. Xiaohu Wang provided the main idea of this paper. All authors read and approve the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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