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Lie group analysis and similarity solutions for hydro-magnetic Maxwell fluid through a porous medium

Khaled Saad Mekheimer[1*], Mostafa Fatouh El-Sabbagh[2] and Rabea Elshennawy Abo-Elkhair[1]

* Correspondence: kh_Mekheimer@yahoo.com
[1]Mathematics Department, Faculty of Science, Al-Azhar University, Nasr City 11448, Cairo, Egypt
Full list of author information is available at the end of the article

Abstract

The equations of two dimensional incompressible fluid flow for hydro-magnetic Maxwell fluid through a porous medium have been studied. Lie group analysis has been employed and the group invariant solutions are obtained. Solutions corresponding to translational and rotational symmetries are obtained. A boundary value problem for the translational symmetry is investigated and the results are also sketched graphically. The effects of physical parameters have been noticed.

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1 Introduction

Non-Newtonian fluid behavior, which is characterized by a nonlinear viscosity dependence on the strain, can be observed in many complex fluids, for example, polymers, dense colloidal dispersions, surfactant solutions, micellar solutions chemical, and petroleum industries [1]. In addition to shear-thinning and shear-thickening behavior, a dynamic or even chaotic response can be found in some fluids subjected to a steady shear flow. Because of the difficulty to suggest a single model which exhibits all properties of non-Newtonian fluids, they cannot be described as simply as Newtonian fluids. Due to this fact many models of constitutive equations have been proposed and most of them are empirical or semi empirical [2]. Amongst these the differential type fluid model gained considerable attention of many researchers. The flows of non-Newtonian fluids are not only important because of their technological significance but also in the interesting mathematical features presented by the equations governing the flow. However on the other hand there are much controversies on these models as well. Such fluids are also inadequate to describe the relaxation phenomena. For a complete and detailed discussion of the relevant issues for differential type fluids, we refer the readers to Dunn and Rajagopal [3] and Aksel [4].

The non-Newtonian fluids are mainly classified into three types namely differential, rate and integral. The simplest subclass of the rate type fluids is the Maxwell model [5]. This fluid model can very well describe the relaxation time effects. Specifically the Maxwell fluid model has been used for the viscoelastic flows where the dimensionless

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relaxation time is small. However in some more concentrated polymeric fluids the Maxwell model is also useful for large dimensionless relaxation time. Some recent investigations dealing with the flows of Maxwell fluids are given in the references [6-9].

Modified Darcy's law for a Maxwell fluid including the Hall current has been used for the modeling. In fact, the Hall effect is important when the Hall parameter, which is the ratio between the electron-cyclotron frequency and the electronatom-collision frequency, is high. This happens, when the magnetic field is high or when the collision frequency is low. In most cases, the Hall term has been ignored in applying Ohm's law as it has no marked effects for small and moderate values of the magnetic field. However, the current trend in the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable. Under these conditions, the Hall current is important and it has marked effects on the magnitude and direction of the current density and consequently on the magnetic-force term. Therefore, it is of interest to study the influence of the Hall current on the flow.

In the Earth there are a large number of problems that can be described by the interaction of a low viscosity fluid (water, oil, gas, magma) in a permeable (possibly deformable) matrix. Darcy's Law is the classic, empirically derived equation for the flux of a low viscosity fluid in a permeable matrix. This equation assumes that flow in the pores or cracks of the medium is essentially laminar and provides the average flux through a representative area that is larger than the pore scale and smaller than the scale of significant permeability variation (if such a scale exists). Various approaches have been used to justify this rule from first principles (see e.g., Dagan [10]) but it generally seems to work.

In this article, we apply the so-called symmetry methods for a particular problem of fluid mechanics. The main advantage of such methods is that they can successfully be applied to nonlinear differential equations [11-13]. The similarity solutions are quite popular because they result in the reduction of the independent variables of the problem. The symmetry transformations method transform the given family of equations of n independent variables, say, to another family of equations of n - 1 independent variables, which can further be solved [14,15]. The fundamental concepts of this approach can be found in [16-19]. In our case, the problem under investigation is (2 + 1)-nonlinear partial differential equations (PDEs). Hence, any similarity solution will transform the system of (n + 1)-nonlinear PDEs into a system of (n)-nonlinear PDEs and any similarity solution will transform the system of (2)-nonlinear PDEs into a system of ordinary differential equations (ODEs).

Many authors used Lie group analysis method to obtain the exact solutions for some problems in fluid mechanics. Yurusoy and Pakdemirli [20] investigated the boundary layer equations of a non-Newtonian fluid model in which the shear stress is an arbitrary function of the velocity gradient. Yurusoy et al. [21] have obtained the solution for the creeping flow of the second grade fluid. Also the two-dimensional equations of motions for the slowly flowing and heat transfer in second grade fluid in cartesian coordinates neglecting the inertial terms are considered by Yürüsoy [22]. Shahzad et al. [23] found the analytical solution of a micropolar fluid by using the Lie group analysis. Recently, Mekheimer et al. studied the Lie group analysis and similarity solutions for a couple stress fluid with heat transfer [24], Lie point symmetries and similarity solutions for an electrically conducting Jeffrey fluid [25] and Lie point symmetries and similarity solution for a micro-polar fluid through a porous medium [26].

From discussion above, we attend to find the analytical (similarity) solutions for the flow problem of an incompressible hydro-magnetic Maxwell fluid through a porous medium using Lie group analysis. The problem is presented as follows, in Section 2, the equations governing two-dimensional motion of an incompressible, MHD Maxwell fluid are introduced. In Section 3, the basic idea of the Lie group analysis method are given and used to find the isovector field of our equations. The similarity solutions corresponding to translational and rotational symmetry are obtained in Sections 3.1 and 3.2. Also a boundary value problem for the similarity solutions corresponding to translational symmetry are obtained in Section 4. The graphs for a boundary value problem (magma flow) are plotted and discussed in Section 5. Finally a concluding remarks are pointed in Section 6.

2 Equations of motion

The continuity and momentum equations governing the two-dimensional motion of an incompressible hydro-magnetic Maxwell fluid through a porous medium can be written as:

$$
\begin{aligned}
E_1 &= \frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0, \\
E_2 &= \left(1 + \tilde{\lambda}\frac{\partial}{\partial t}\right)\left[\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}}\right] + \left(1 + \tilde{\lambda}\frac{\partial}{\partial \tilde{t}}\right)\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial \tilde{x}} - \frac{\mu}{\rho}\nabla^2 \tilde{u} \\
&\quad + \left(1 + \tilde{\lambda}\frac{\partial}{\partial \tilde{t}}\right)\left[\frac{\sigma B_0^2 \theta}{\rho}(\tilde{u} - m\tilde{v})\right] + \frac{\mu\varphi}{\rho\tilde{k}}\tilde{u} = 0, \\
E_3 &= \left(1 + \tilde{\lambda}\frac{\partial}{\partial t}\right)\left[\frac{\partial \tilde{v}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}}\right] + \left(1 + \tilde{\lambda}\frac{\partial}{\partial \tilde{t}}\right)\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial \tilde{y}} - \frac{\mu}{\rho}\nabla^2 \tilde{v} \\
&\quad + \left(1 + \tilde{\lambda}\frac{\partial}{\partial \tilde{t}}\right)\left[\frac{\partial B_0^2 \theta}{\rho}(\tilde{v} + m\tilde{u})\right] + \frac{\mu\varphi}{\rho\tilde{k}}\tilde{v} = 0,
\end{aligned}
\tag{1}
$$

where $\nabla^2 = \frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{y}^2}$, $\theta = \frac{1}{1+m^2}$, $m = \frac{\partial B_0}{e\, n_e}$ are the fluid velocities in the $\tilde{x}, \tilde{y}$ directions, $\tilde{p}$ is the pressure, and $\tilde{t}$ is the time. Here $\tilde{\lambda}$, ρ, μ, ϕ, k, e, n_e, σ, B_0, and m are the relaxation time, density, coefficient of viscosity, porosity of the porous medium, permeability, electric charge, the number density of electrons, electrical conductivity of the fluid, magnetic field and Hall parameter respectively.

Using the following dimensionless parameters

$$
u = \frac{\tilde{u}}{U}, \quad v = \frac{\tilde{v}}{U}, \quad x = \frac{\tilde{x}}{L}, \quad y = \frac{\tilde{y}}{L}, \quad t = \frac{U}{L}\tilde{t}, \quad p = \frac{\tilde{p}}{\rho U^2}, \quad \lambda = \frac{U}{L}\tilde{\lambda}, \quad k = \frac{\tilde{k}}{\varphi L^2}, \tag{2}
$$

the system (1) becomes

$$
\begin{aligned}
E_1 &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \\
E_2 &= \left(1 + \lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] + \left(1 + \lambda\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial x} - \frac{1}{R}\nabla^2 u \\
&\quad + \left(1 + \lambda\frac{\partial}{\partial t}\right)\left[M\theta(u - mv)\right] + \frac{1}{Rk}u = 0, \\
E_3 &= \left(1 + \lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] + \left(1 + \lambda\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial y} - \frac{1}{R}\nabla^2 v \\
&\quad + \left(1 + \lambda\frac{\partial}{\partial t}\right)\left[M\theta(v + mu)\right] + \frac{1}{Rk}v = 0,
\end{aligned}
\tag{3}
$$

where $R = \dfrac{\rho LU}{\mu}$ is the Reynolds number, $M = \dfrac{\sigma B_0^2 L}{\rho U}$ is the Hartmann number and L, U are the dimensionless length and velocity, respectively.

3 Lie group analysis and isovector fields

In order to obtain the analytical solution, we apply the Lie group analysis theory to system (3). For this we write

$$\begin{cases} x_i^* = x_i + \varepsilon \xi_i(x_j, u_\alpha) + o(\varepsilon^2), \\ u_\alpha^* = u_\alpha + \varepsilon \eta_\alpha(x_j, u_\beta) + o(\varepsilon^2), \end{cases} \quad i, j = 1, 2, 3, \quad \alpha, \beta = 1, 2, 3, \tag{4}$$

as the infinitesimal Lie point transformations. We have assumed that system (3) is invariant under the transformations given in Eq. (4). The corresponding infinitesimal generator of Lie groups (symmetries) is given by

$$X = \xi_i \frac{\partial}{\partial x_i} + \eta_\alpha \frac{\partial}{\partial u_\alpha}, \tag{5}$$

with summation convention over the repeated index and $x_1 = x$, $x_2 = y$, $x_3 = t$, $u_1 = u$, $u_2 = v$, $u_3 = p$. The coefficients ξ_1, ξ_2, ξ_3, η_1, η_2, and η_3 are the functions of all independent and dependent variables. There coefficients are the components of the infinitesimals symmetries corresponding to x, y, t, u, v, and p, respectively to be determined from the invariance conditions:

$$\mathrm{Pr}^{(2)} X(E_a)\Big|_{E_{\alpha=0}} = 0, \quad a = 1, 2, 3, \tag{6}$$

where $E_a = 0$, $i = 1, 2, 3$ represent the system of Eq. (3) and $\mathrm{Pr}^{(2)}$ is the second prolongation of the isovector field X. Since the system (3) is of order two, then our prolongation will be in the form

$$\begin{aligned} \mathrm{Pr}^{(1)} X &= X + \eta_{\alpha i} \frac{\partial}{\partial u_{\alpha,i}}, \\ \mathrm{Pr}^{(2)} X &= \mathrm{Pr}^{(1)} X + \eta_{\alpha ij} \frac{\partial}{\partial u_{\alpha,ij}}. \end{aligned} \tag{7}$$

where

$$\begin{aligned} \eta_{\alpha i} &= D_i \left[\eta_\alpha - \xi_j u_{\alpha,j} \right] + \xi_j u_{\alpha,ji}, \\ \eta_{\alpha ij} &= D_{ij} \left[\eta_\alpha - \xi_k u_{\alpha,k} \right] + \xi_k u_{\alpha,kij.} \end{aligned} \tag{8}$$

and the operator $D_{i_1 i_2 \ldots i_s}$ is called the *total derivative* (*Hash operator*) and has the following form:

$$D_i = \frac{\partial}{\partial x_i} + u_{\alpha,i} \frac{\partial}{\partial u_\alpha} + u_{\alpha,ij} \frac{\partial}{\partial u_{\alpha,j}} + u_{\alpha,ijk} \frac{\partial}{\partial u_{\alpha,jk}}, \tag{9}$$

where $D_{ij} = D_i(D_j) = D_j(D_i) = D_{ji}$ and $u_{\alpha,i} = \dfrac{\partial u_\alpha}{\partial x_i}$.

Expanding the system of Eq. (6) with the aid of *Mathematica programm*, along with the original system of Eq. (3) to eliminate u_x, p_{xt}, p_{yt} and setting the coefficients involving u_y, u_{yy}, v_x, v_y, v_{xx}, v_{xy}, v_{yy} and various products to zero give rise the essential set of over-determined equations. Solving these set of determining equations we obtain the required components of isovector field as follows:

$$\begin{aligned}&\xi_1 = a_2 - a_1 y,\quad \xi_2 = a_3 + a_1 x,\quad \xi_3 = a_4,\\&\eta_1 = -\alpha_1 v,\ \eta_2 = a_1 u,\quad \eta_3 = \delta(t),\end{aligned} \tag{10}$$

where a_i, $i = 1,..., 5$ are arbitrary constants, $\delta(t)$ is arbitrary function of the variable t only.

3.1 Translational symmetry

In this case we take $a_1 = 0$. The characteristic equations corresponding to the translational symmetry are:

$$\frac{dx}{a_2} = \frac{dy}{a_3} = \frac{dt}{a_4} = \frac{du}{0} = \frac{dv}{0} = \frac{dp}{\delta(t)}. \tag{11}$$

By solving the ODEs (11), we can obtain the similarity variables and similarity functions as follows:

$$\phi = x - m_1 t, \qquad \psi = y - m_2 t, \tag{12}$$

$$u(x,y,t) = \hat{u}(\phi,\psi),\quad v(x,y,t) = \hat{v}(\phi,\psi),\quad p(x,y,t) = \hat{p}(\phi,\psi) + \delta_1(t), \tag{13}$$

where $m_1 = \dfrac{a_2}{a_4}, m_2 = \dfrac{a_3}{a_4}$ are arbitrary constants and $\delta_1(t) = \int \delta(t)\ dt$ is an arbitrary function. Substituting the transformations (12), (13) in the Eq. (3) lead to the following system of PDEs:

$$\left\{\begin{aligned}&E_1 = \hat{u}_\phi + \hat{v}_\psi = 0,\\&E_2 = \hat{u}\left[\frac{1}{Rk} + M\theta + \hat{u}_\phi - \lambda(m_2\hat{u}_{\phi\psi} + m_1\hat{u}_{\phi\phi})\right] - \frac{1}{R}(\hat{u}_{\phi\phi} + \hat{u}_{\psi\psi}) + \lambda m_2^2\hat{u}_{\phi\phi} + \hat{p}_\phi\\&\quad -\hat{v}\left[mM\theta - \hat{u}_\psi + \lambda\left(m_2\hat{u}_{\psi\psi} + m_1\hat{u}_{\phi\psi}\right)\right] - m_2\left[\hat{u}_\psi\left(1 + M\lambda\theta + \lambda\hat{v}_\psi + \lambda\hat{u}_\phi\right)\right.\\&\quad \left.+\lambda\left(-mM\theta\hat{v}_\psi - 2m_1\hat{u}_{\phi\psi} + \hat{p}_{\phi\psi}\right)\right] - m_1\left[\hat{u}_\phi\left(1 + M\lambda\theta + \lambda\hat{u}_\phi\right)\right.\\&\quad \left.+\lambda\left(\hat{u}_\psi\hat{v}_\phi - mM\theta\hat{v}_\phi - m_1\hat{u}_{\phi\phi} + \hat{p}_{\phi\phi}\right)\right] = 0,\\&E_3 = \hat{v}\left[\frac{1}{Rk} + M\theta + \hat{v}_\psi - \lambda\left(m_2\hat{v}_{\psi\psi} + m_1\hat{v}_{\phi\psi}\right)\right] - \frac{1}{R}(\hat{v}_{\phi\phi} + \hat{v}_{\psi\psi}) - \lambda m_2^2\hat{v}_{\psi\psi} - \hat{p}_\psi\\&\quad +\hat{u}\left[mM\theta - \hat{v}_\phi - \lambda\left(m_2\hat{v}_{\phi\psi} + m_1\hat{v}_{\phi\phi}\right)\right] + m_2\left[\hat{v}_\psi\left(1 + M\lambda\theta + \lambda\hat{v}_\psi\right)\right.\\&\quad \left.+\lambda\left(\hat{u}_\psi\left(mM\theta + \hat{v}_\phi\right) - 2m_1\hat{v}_{\phi\psi} + \hat{p}_{\psi\psi}\right)\right] + m_1\left[\hat{v}_\phi\left(1 + M\lambda\theta + \lambda\hat{v}_\psi\right)\right.\\&\quad \left.+\lambda\left(\hat{u}_\phi\hat{v}_\phi + mM\theta\hat{u}_\phi + m_1\hat{v}_{\phi\phi} + \hat{p}_{\phi\psi}\right)\right] = 0,\end{aligned}\right. \tag{14}$$

To transform Eq. (14) to an (ODEs), we use the Lie group analysis again and obtain the infinitesimal generator corresponding to system of equation (14) in the following form:

$$\xi_{11} = b_1,\quad \xi_{21} = b_2,\quad \eta_{11} = 0,\quad \eta_{21} = 0,\quad \eta_{31} = \beta(\phi,\psi), \tag{15}$$

where b_i, $i = 1, 2$ are arbitrary constants and $\beta(\varphi, \psi)$ an arbitrary function that satisfy two conditions:

$$\left\{\begin{aligned}&\beta_\phi - \lambda\left(m_2\beta_{\phi\psi} + m_1\beta_{\phi\phi}\right) = 0,\\&\beta_\psi - \lambda\left(m_2\beta_{\psi\psi} + m_1\beta_{\phi\psi}\right) = 0,\end{aligned}\right. \tag{16}$$

By solving the characteristic equations,

$$\frac{d\phi}{b_1} = \frac{d\psi}{b_2} = \frac{d\hat{u}}{0} = \frac{d\hat{v}}{0} = \frac{d\hat{p}}{\beta(\phi,\psi)}, \tag{17}$$

if we take $\beta(\varphi, \psi) = 0$ then we can obtain the similarity variable and similarity functions as the following:

$$\chi = \psi - m_0\phi, \quad \hat{u}(\phi,\psi) = f(\chi), \quad \hat{v}(\phi,\psi) = g(\chi), \quad \hat{p}(\phi,\psi) = h(\chi), \tag{18}$$

where $m_0 = \dfrac{b_2}{b_1}$ an arbitrary constant. Substituting the transformations (18) in Eq. (14) lead to the following system of ODEs:

$$\begin{cases} g' - m_0 f' = 0 \\ f\left[\frac{1}{Rk} + M\theta - m_0 f' - \lambda m_0 (m_0 m_1 - m_2) f''\right] - \frac{1}{R}\left(1 + m_0^2\right) f'' - g\left[mM\theta - f'\right. \\ \quad \left. -\lambda (m_0 m_1 - m_2) f''\right] - m_2\left[\left(1 + M\lambda\theta + \lambda g'\right) f' - \lambda\left(mM\theta g' + m_2 f''\right)\right] \\ \quad + m_0 m_1\left[\left(1 + M\lambda\theta + \lambda g'\right) f' - \lambda\left(mM\theta g' + 2m_2 f''\right)\right] + \lambda m_0\left[-(m_0 m_1 - m_2)(f')^2\right. \\ \quad \left. + m_0 m_1^2 f''\right] + \lambda m_0 (m_0 m_1 - m_2) h'' - m_0 h = 0, \\ g\left[\frac{1}{Rk} + M\theta + g' + \lambda (m_0 m_1 - m_2) g''\right] - \frac{1}{R}(1 + m_0^2) g'' - f\left[mM\theta - m_0 g'\right. \\ \quad \left. - m_0\lambda (m_0 m_1 - m_2) g''\right] - m_2\left[\left(1 + M\lambda\theta + \lambda g'\right) g' + \lambda\left(mM\theta f' - m_2 g''\right)\right] \\ \quad + m_0 m_1\left[\left(1 + M\lambda\theta + \lambda g'\right) g' + \lambda\left(mM\theta f' - 2m_2 g''\right)\right] + \lambda m_0\left[-(m_0 m_1 - m_2) f' g'\right. \\ \quad \left. + m_0 m_1^2 g''\right] + \lambda (m_0 m_1 - m_2) h'' + h = 0. \end{cases} \tag{19}$$

Integrating the first equation in (19) yields

$$g = m_0 f + c_1, \tag{20}$$

where c_1 is an arbitrary constant. Eliminating $h(\xi)$ from the second and third equations in (19) along with Eq. (20) we get the following equation:

$$Af'' + Bf' + Cf + D = 0. \tag{21}$$

where

$$\begin{aligned} A &= -\frac{1}{R}\left(1 + m_0^2\right) + \lambda\left(m_0^2 m_1^2 + m_2^2 - 2m_0 m_1 m_2 + c_1 (m_0 m_1 - m_2)\right), \\ B &= c_1 + (1 + M\lambda\theta)(m_0 m_1 - m_2), \\ C &= \frac{1}{Rk} + M\theta, \\ D &= \frac{c_1}{\left(1 + m_0^2\right)}\left[m_0\left(\frac{1}{Rk} + M\theta\right) - mM\theta\right]. \end{aligned} \tag{22}$$

By solving equation (21) we get

$$f(\chi) = c_2 \exp[\alpha_1 \chi] + c_3 \exp[\alpha_2 \chi] - \tilde{D}, \tag{23}$$

where $\tilde{D} = \dfrac{D}{C}$ and c_2, c_3 are arbitrary constants and α_1, α_2 are roots of the following equation:

$$A\gamma^2 + B\gamma + C = 0, \tag{24}$$

From equations (20) and (23) the expression of the function $g(\chi)$ becomes

$$g(\chi) = m_0 \left(c_2 \exp[\alpha_1 \chi] + c_3 \exp[\alpha_2 \chi] - \tilde{D} + c_1, \right. \tag{25}$$

and from the second and third equations in (19) we get

$$\begin{aligned} h(\chi) = {} & \left[\tilde{D} m M\theta - \frac{c_1}{(1+m_0^2)} \left(\frac{1}{Rk} + M\theta + mM\theta m_0 \right) \right] \chi - mM\theta \left(\frac{c_2}{\alpha_1} \exp[\alpha_1 \chi] \right. \\ & \left. + \frac{c_3}{\alpha_2} \exp[\alpha_2 \chi] \right) - \lambda c_4 \left(m_0 m_1 - m_2 \right) \exp\left[- \frac{\chi}{\lambda(m_0 m_1 - m_2)} \right] + c_5, \end{aligned} \tag{26}$$

where c_4 and c_5 are arbitrary constants.

In the form of the original variables, our exact solutions can be written as follows:

$$\begin{cases} u(x, y, t) = c_2 \exp\left[\alpha_1 \left(y - m_0 x + (m_0 m_1 - m_2)\, t \right) \right] \\ \qquad + c_3 \exp\left[\alpha_2 \left(y - m_0 x + (m_0 m_1 - m_2)\, t \right) \right] - \tilde{D}, \\ v(x, y, t) = m_0 \left(c_2 \exp\left[\alpha_1 \left(y - m_0 x + (m_0 m_1 - m_2)\, t \right) \right] \right. \\ \qquad \left. + c_3 \exp\left[\alpha_2 \left(y - m_0 x + (m_0 m_1 - m_2)\, t \right) \right] - \tilde{D} \right) + c_1, \\ p(x, y, t) = \left[\tilde{D} mM\theta - \frac{c_1}{(1+m_0^2)} \left(\frac{1}{Rk} + M\theta + mM\theta m_0 \right) \right] \left(y - m_0 x + (m_0 m_1 - m_2) \right) \\ \qquad - mM\theta \left(\frac{c_2}{\alpha_1} \exp\left[\alpha_1 \left(y - m_0 x + (m_0 m_1 - m_2)\, t \right) \right] \right. \\ \qquad \left. + \frac{c_3}{\alpha_2} \exp\left[\alpha_2 \left(y - m_0 x + (m_0 m_1 - m_2)\, t \right) \right] \right) \lambda \\ \qquad - c_4 \left(m_0 m_1 - m_2 \right) \exp\left[- \frac{\left(y - m_0 x + (m_0 m_1 - m_2)\, t \right)}{\lambda(m_0 m_1 - m_2)} \right] + c_5 + \delta_1(t). \end{cases} \tag{27}$$

3.2 Rotational symmetry

In this section, the parameter a_1 is taken to be an arbitrary non-zero constant. The characteristic equations corresponding to the rotational symmetry are:

$$\frac{dx}{-a_1 y + a_2} = \frac{dy}{a_1 x + a_3} = \frac{dt}{a_4} = \frac{du}{-a_1 v} = \frac{dv}{a_1 u} = \frac{dp}{\delta(t)}. \tag{28}$$

Integrating equations (28) using the Lie group analysis method, we get the rotationally invariant solutions for our problem in the following form:

$$\begin{cases} \psi = \sqrt{(x+\beta_1)^2 + (y+\beta_2)^2}, \\ \phi = \tan^{-1}\left[\frac{y+\beta_2}{x+\beta_1} \right], \\ u = G_1(\psi, t) \cos[\phi] + G_2(\psi, t) \sin[\phi], \\ v = G_1(\psi, t) \sin[\phi] - G_2(\psi, t) \cos[\phi], \\ p = G_3(\psi, t) + \delta_1(t), \end{cases} \tag{29}$$

where $\beta_1 = \frac{a_3}{a_1}$, $\beta_2 = \frac{-a_2}{a_1}$, and G_3 are functions of ψ and t.

Substituting the new (similarity) variables (φ, ψ) and functions (G_1, G_2, G_3) into the original system (3) yields the following set of equations:

$$\begin{cases} \frac{G_1}{\psi} + \frac{\partial G_1}{\partial \psi} = 0, \\ G_1 \left[k + \psi^2 \left(1 + RkM\theta \left(\frac{\partial G_1}{\partial \psi} + \lambda \frac{\partial^2 G_1}{\partial \psi\, \partial t} \right) \right) \right] + k\psi \left[RG_2 \left(mM\theta\psi - G_2 - 2\lambda \frac{\partial G_2}{\partial t} \right) - \frac{\partial G_1}{\partial \psi} \right] \\ \quad -k\psi^2 \frac{\partial^2 G_1}{\partial \psi^2} + Rk\psi^2 \left[\left(1 + M\lambda\theta + \lambda \frac{\partial G_1}{\partial \psi} \right) \frac{\partial G_1}{\partial t} + \frac{\partial G_3}{\partial \psi} \right. \\ \quad \left. +\lambda \left(mM\theta \frac{\partial G_2}{\partial t} + \frac{\partial^2 G_1}{\partial t^2} + \frac{\partial^2 G_3}{\partial \psi\, \partial t} \right) \right] = 0, \\ G_2 \left[k + \psi^2 (1 + RkM\theta) + Rk\lambda\psi \frac{\partial G_1}{\partial t} \right] + k\psi \left[RG_1 \left(G_2 + \lambda \frac{\partial G_2}{\partial t} \right) - \frac{\partial G_2}{\partial \psi} - \psi \frac{\partial^2 G_2}{\partial \psi^2} \right] \\ \quad +Rk\psi^2 \left[(1 + M\lambda\theta) \frac{\partial G_2}{\partial t} + \lambda \left(\frac{\partial^2 G_2}{\partial t^2} + \frac{\partial G_1}{\partial t} \frac{\partial G_2}{\partial \psi} + G_1 \frac{\partial^2 G_2}{\partial \psi \partial t} \right) \right. \\ \quad \left. -mM\theta G_1 - mM\lambda\theta \frac{\partial G_1}{\partial t} + G_1 \frac{\partial G_2}{\partial \psi} \right] = 0 \end{cases} \tag{30}$$

To transform Eq. (30) to (ODEs), we use the Lie group analysis again and obtain the infinitesimal generator corresponding to system of equations (30) in the following form:

$$\xi_{12} = 0, \quad \xi_{22} = d_1, \quad \eta_{11} = 0, \quad \eta_{21} = 0, \quad \eta_{32} = \eta(\psi, t, G_1), \tag{31}$$

where d_1 is an arbitrary constant and $\eta(\psi,t,G_1)$ an arbitrary function that satisfy the following condition:

$$\psi \frac{\partial \eta}{\partial \psi} - G_1 \frac{\partial \eta}{\partial G_1} = 0. \tag{32}$$

If we take $\eta(\psi,t,G_1) = 0$ is a simple solution of Eq. (32) then the characteristic equations are:

$$\frac{d\psi}{0} = \frac{dt}{d_1} = \frac{dG_1}{0} = \frac{dG_2}{0} = \frac{dG_3}{0}. \tag{33}$$

The similarity variables and resulting functions are

$$\zeta = \psi, \quad G_1 = F_1(\zeta), \quad G_2 = F_2(\zeta), \quad G_3 = F_3(\zeta). \tag{34}$$

One now substitutes the similarity variable and the functions into the equations (30) and obtains

$$\begin{cases} F'_1 + \frac{F_1}{\zeta} = 0, \\ F_1 \left[k + \zeta^2 \left(1 + RkM\theta + RkF'_1 \right) \right] + k\zeta \left[RF_2 \left(mM\theta\zeta - F_2 \right) - F'_1 + R\zeta F'_3 - \zeta F''_1 \right] = 0, \\ F_2 \left[k + \zeta^2 (1 + RkM\theta) \right] + Rk\zeta F_1 \left[F_2 + \zeta \left(F'_2 - mM\theta \right) \right] - k\zeta F'_2 + \zeta F''_2 = 0. \end{cases} \tag{35}$$

Integrating the first equation in the system (35) we get:

$$F_1 = \frac{g_1}{\zeta}, \tag{36}$$

where g_1 is an arbitrary constant. From the second and the third equations in (35) along with (36) we get:

$$F'_3 - \frac{F_2^2}{\zeta} + mM\theta F_2 - \frac{g_1^2}{\zeta^3} + \frac{(1 + RkM\theta)\, g_1}{Rk\zeta} = 0, \tag{37}$$

$$F_2 \left[k + \zeta^2 (1 + RkM\theta) \right] + Rkg_1 \left[F_2 + \zeta \left(F'_2 - mM\theta \right) \right] - k\zeta \left[F'_2 + \zeta F''_2 \right] = 0, \tag{38}$$

by integrating equation (38) we obtain

$$\begin{aligned} F_2 = {} & \frac{1}{k} 2^{-\left(1+\frac{Rg_1}{2}\right)} \zeta^{\frac{Rg_1}{2}} \left(-i\zeta\sqrt{\tfrac{1}{k} + RM\theta}\right)^{-\left(1+\frac{Rg_1}{2}\right)} \\ & \left[2^{2+R_{g_1}} kg_3 \csc\left[\frac{1}{2}\pi Rg_1\right] {}_0F_1\ Regularized \left(-\frac{Rg_1}{2}, \frac{\zeta^2(1+RkM\theta)}{4k}\right)\right. \\ & - \zeta^2 \left(-i\zeta\sqrt{\frac{1}{k} + RM\theta}\right)(1+RkM\theta)\left[g_2 + g_3 \cot\left[\frac{1}{2}\pi Rg_1\right]\right] \\ & \left. {}_0F_1\ Regularized \left(2 + \frac{Rg_1}{2}, \frac{\zeta^2(1+RkM\theta)}{4k}\right)\right] \\ & - \frac{RkmM\theta g_1}{\zeta(1+RkM\theta)}\left[-1 + {}_0F_1\left(-\frac{Rg_1}{2}, \frac{\zeta^2(1+RkM\theta)}{4k}\right)\right], \end{aligned} \tag{39}$$

and

$$F_3 = \int \left(\frac{F_2^2}{\zeta} - mM\theta F_2 + \frac{g_1^2}{\zeta^3} - \frac{(1+RkM\theta)g_1}{Rk\zeta}\right) d\zeta, \tag{40}$$

where g_2 and g_3 are arbitrary constants, and the function ${}_pF_q$ and ${}_pF_q$ *Regularized* are defined as:

$$\begin{aligned} {}_pF_q\left(\vec{a};\vec{b};z\right) &= \text{HypergeometricPFQ}\left(\{a_1, \ldots, a_p\}; \{b_1, \ldots, b_q\}; z\right) \\ &= \sum_{k=0}^{\infty} \left[\frac{(a_1)k \ldots (a_p)k}{(b_1)k \ldots (b_q)k}\right] \frac{z^k}{k!}, \end{aligned} \tag{41}$$

$${}_pF_q\text{Regularized} = \frac{{}_pF_q}{\Gamma(b_1)\ldots\Gamma(b_q)}. \tag{42}$$

Then the solution of our problem in the original variables using symbolic computations is:

$$\begin{aligned} u(x,y,t) = {} & \frac{g_1}{\psi}\cos[\phi] + \left\{ \frac{1}{k} 2^{-(1+\frac{Rg_1}{2})} \psi^{\frac{Rg_1}{2}} \left(-i\psi\sqrt{\frac{1}{k} + RM\theta}\right)^{-(1+\frac{Rg_1}{2})} \right. \\ & \left[2^{2+R_{g_1}} kg_3 \csc[\frac{1}{2}\pi Rg_1]\ {}_0F_1\ Regularized\left(-\frac{Rg_1}{2}, \frac{\psi^2(1+RkM\theta)}{4k}\right) - \right. \\ & \psi^2 \left(-i\psi\sqrt{\frac{1}{k} + RM\theta}\right)(1+RkM\theta)\left[g_2 + g_3\cot[\frac{1}{2}\pi Rg_1]\right] \\ & \left. {}_0F_1\ Regularized\left(2 + \frac{Rg_1}{2}, \frac{\psi^2(1+RkM\theta)}{4k}\right)\right] - \\ & \left. \frac{RkmM\theta g_1}{\psi(1+RkM\theta)}\left[-1 + {}_0F_1\left(-\frac{Rg_1}{2}, \frac{\psi^2(1+RkM\theta)}{4k}\right)\right]\right\} \sin[\phi] \\ v(x,y,t) = {} & \frac{g_1}{\psi}\sin[\phi] - \left\{ \frac{1}{k} 2^{-(1+\frac{Rg_1}{2})} \psi^{\frac{Rg_1}{2}} \left(-i\psi\sqrt{\frac{1}{k} + RM\theta}\right)^{-(1+\frac{Rg_1}{2})} \right. \\ & \left[2^{2+R_{g_1}} kg_3 \csc[\frac{1}{2}\pi Rg_1]\ {}_0F_1\ Regularized\left(-\frac{Rg_1}{2}, \frac{\psi^2(1+RkM\theta)}{4k}\right) - \right. \\ & - \psi^2 \left(-i\psi\sqrt{\frac{1}{k} + RM\theta}\right)(1+RkM\theta)\left[g_2 + g_3\cot\left[\frac{1}{2}\pi Rg_1\right]\right] \\ & \left. {}_0F_1\ Regularized\left(2 + \frac{Rg_1}{2}, \frac{\psi^2(1+RkM\theta)}{4k}\right)\right] - \\ & \left. \frac{RkmM\theta g_1}{\psi(1+RkM\theta)}\left[-1 + {}_0F_1(-\frac{Rg_1}{2}, \frac{\psi^2(1+RkM\theta)}{4k})\right]\right\} \cos[\phi], \\ p(x,y,t) = {} & \int \left(\frac{F_2^2}{\psi} - mM\theta F_2 + \frac{g_1^2}{\psi^3} - \frac{(1+RkM\theta)g_1}{Rk\psi}\right) d\psi + \delta_1(t), \end{aligned} \tag{43}$$

where ψ and φ are the same in (29).

4 Solutions for hydro-magnetic Maxwell fluid through a porous medium: (magmatic fluid) problem

One of the important applications in geology is the magmatic fluid. Consider a magmatic fluid as an incompressible hydro-magnetic Maxwell fluid through a porous medium and a plate over it. The plate occupies the position $y = 0$, where the positive y goes deep into the fluid beneath the plate. The relevant boundary conditions are of the form:

$$\begin{aligned} &u(x,0,0) = U_0, \quad u(x,\infty,t) = 0, \quad \frac{\partial u}{\partial y}(x,0,0) = 0, \\ &v(x,0,0) = -V_0, \quad p(x,\infty,0) = P_0, \quad p(x,0,0) = P_a, \end{aligned} \tag{44}$$

where U_0 is the velocity of the plate, V_0 is the magmatic fluid velocity penetrating into the plate, P_0 is the pressure deep in the magmatic fluid and P_a is the atmosphere pressure. The expressions (27) for the translational symmetry case solution after using conditions (44) give

$$\begin{cases} u(x,y,t) = \dfrac{U_0}{\alpha_2 - \alpha_1}\left[\alpha_2 \exp\left[\alpha_1(y + W\,t)\right] - \alpha_1 \exp\left[\alpha_2(y + W\,t)\right]\right], \\ v(x,y,t) = \dfrac{V_0}{\alpha_2 - \alpha_1}\left[\alpha_2 \exp\left[\alpha_1(y + W\,t)\right] - \alpha_1 \exp\left[\alpha_2(y + W\,t)\right]\right], \\ p(x,y,t) = \dfrac{mM\theta U_0}{\alpha_1\alpha_2(\alpha_2 - \alpha_1)}\left[-\alpha_1^2 \exp\left[\alpha_2(y + W\,t)\right] + \alpha_2^2 \exp\left[\alpha_1(y + W\,t)\right]\right] \\ \qquad - \left[p_0 - p_a + \dfrac{mM\theta U_0}{\alpha_1\alpha_2}(\alpha_1 + \alpha_2)\right] \exp\left[-\dfrac{(y + W\,t)}{C\lambda}\right] + P_0, \end{cases} \tag{45}$$

where $W = m_0m_1 - m_2$, α_1 and α_2 are the negative roots of Eq. (24).

5 Discussion of the magmatic fluid problem

This section deals with the graphics on the magmatic fluid. So, the interpretation of the relaxation time λ, Reynolds number R, Hartmann number M, Hall parameter m, the time parameter t, and the permeability parameter k have been studied on the pressure p, and the x and y components of the velocity distributions u and v.

Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, and 14 describe the variations of the velocity components u and v with the time t at $y = 0$ for different values of

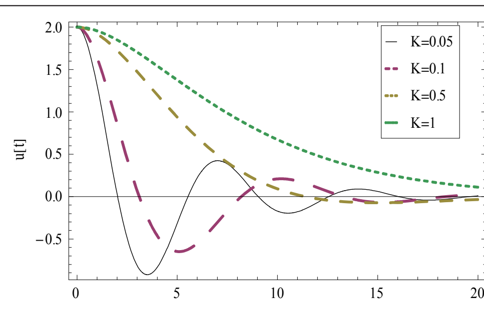

Figure 1 Variation of the dimensionless velocity distribution along the *x*-axis with *t* for different values of permeability parameter $k(U_0 = V_0 = 2; m_0 = m_1 = m_2 = 2; y = 0; m = 0.5; M = 0.5; R = 0.5; \lambda = 50)$.

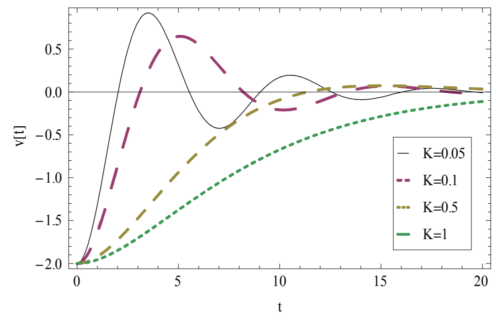

Figure 2 Variation of the dimensionless velocity distribution along the *y*-axis with *t* for different values of permeability parameter $k(U_0 = V_0 = 2; m_0 = m_1 = m_2 = 2; y = 0; m = 0.5; M = 0.5; R = 0.5; \lambda = 50)$.

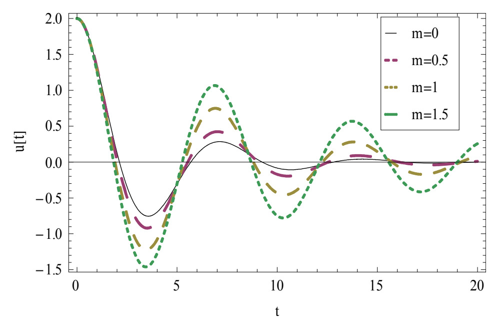

Figure 3 Variation of the dimensionless velocity distribution along the *x*-axis with *t* for different values of Hall parameter $m(U_0 = V_0 = 2; y = 0; m_0 = m_1 = m_2 = 2; k = 0.05; M = 0.5; R = 0.5; \lambda = 50)$.

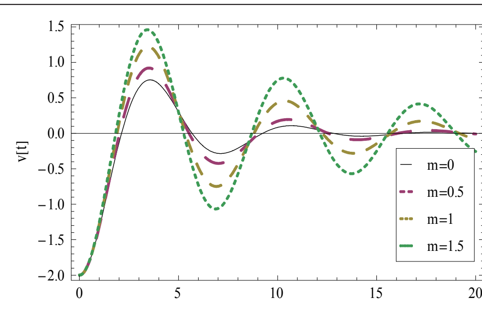

Figure 4 Variation of the dimensionless velocity distribution along the *y*-axis with *t* for different values of Hall parameter $m(U_0 = V_0 = 2; y = 0; m_0 = m_1 = m_2 = 2; k = 0.05; M = 0.5; R = 0.5; \lambda = 50)$.

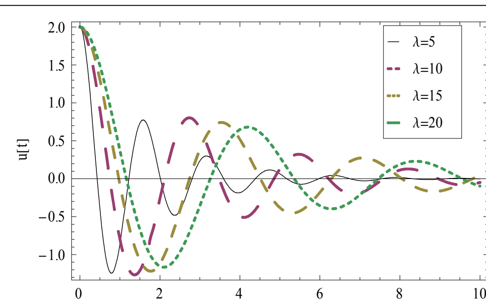

Figure 5 Variation of the dimensionless velocity distribution along the *x*-axis with *t* for different values of relaxation time $\lambda(U_0 = V_0 = 2; y = 0; m_0 = m_1 = m_2 = 2; k = 0.05; M = 0.5; R = 0.5; m = 0.5)$.

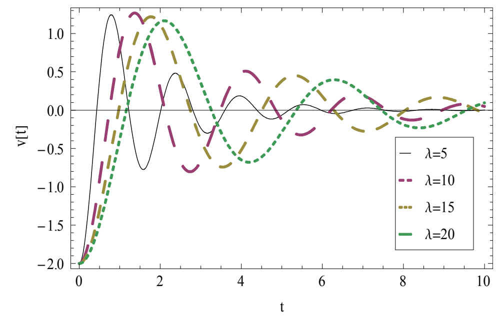

Figure 6 Variation of the dimensionless velocity distribution along the *y*-axis with *t* for different values of relaxation time $\lambda(U_0 = V_0 = 2; y = 0; m_0 = m_1 = m_2 = 2; k = 0.05; M = 0.5; R = 0.5; m = 0.5)$.

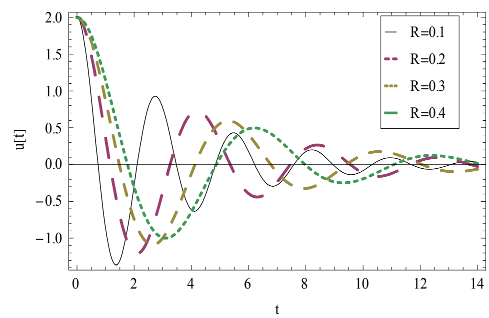

Figure 7 Variation of the dimensionless velocity distribution along the *x*-axis with *t* for different values of Reynolds number $R(U_0 = V_0 = 2; y = 0; m = 0.5; m_0 = m_1 = m_2 = 2; k = 0.05; M = 0.5; \lambda = 50)$.

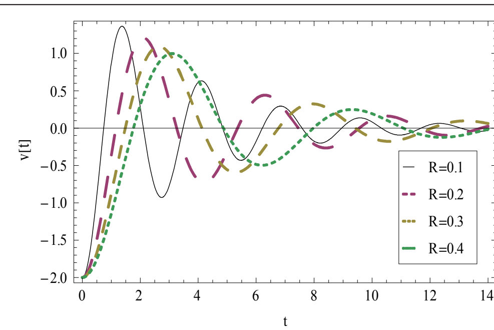

Figure 8 Variation of the dimensionless velocity distribution along the *y*-axis with *t* for different values of Reynolds number $R(U_0 = V_0 = 2; y = 0; m = 0.5; m_0 = m_1 = m_2 = 2; k = 0.05; M = 0.5; \lambda = 50)$.

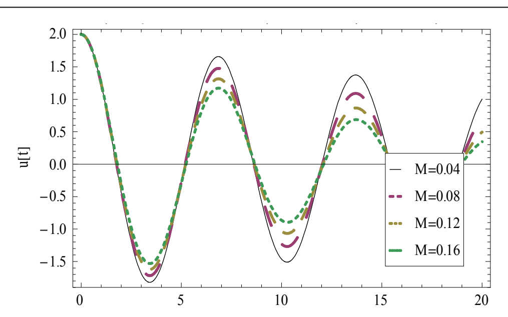

Figure 9 Variation of the dimensionless velocity distribution along the *x*-axis with *t* for different values of Hartmann number $M(U_0 = V_0 = 2; m_0 = m_1 = m_2 = 2; y = 0; k = 0.05; R = 0.5; \lambda = 50; m = 0.5)$.

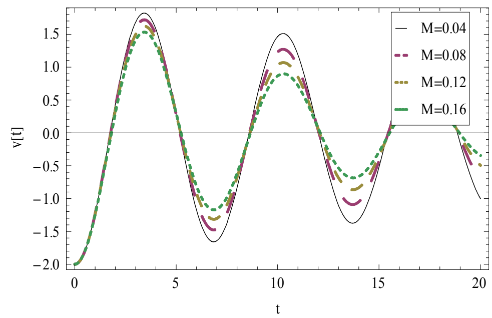

Figure 10 **Variation of the dimensionless velocity distribution along the *y*-axis with *t* for different values of Hartmann number *M*($U_0 = V_0 = 2$; $m_0 = m_1 = m_2 = 2$; $y = 0$; $k = 0.05$; $R = 0.5$; $\lambda = 50$; $m = 0.5$).**

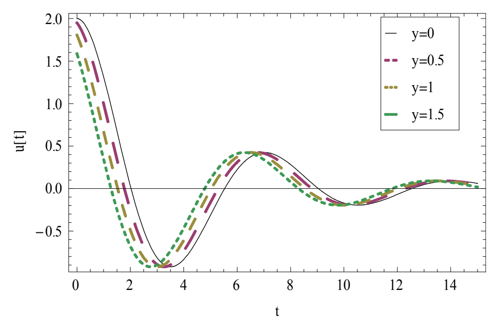

Figure 11 **Variation of the dimensionless velocity distribution along the *x*-axis with *t* for different values *y*($U_0 = V_0 = 2$; $m_0 = m_1 = m_2 = 2$; $M = 0.5$; $k = 0.05$; $R = 0.5$; $\lambda = 50$; $m = 0.5$).**

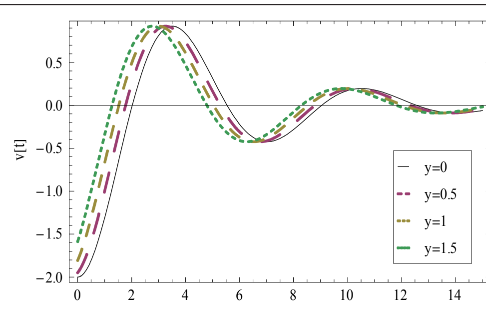

Figure 12 **Variation of the dimensionless velocity distribution along the *y*-axis with *t* for different values *y*($U_0 = V_0 = 2$; $m_0 = m_1 = m_2 = 2$; $M = 2$; $k = 0.05$; $R = 50$; $\lambda = 50$; $m = 0.5$).**

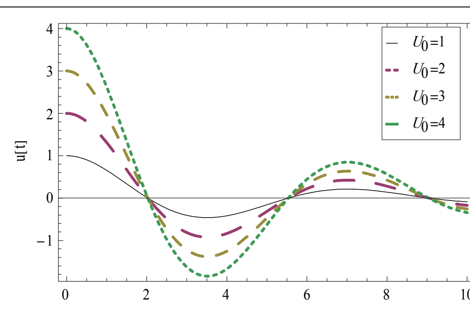

Figure 13 **Variation of the dimensionless velocity distribution along the *x*-axis with *t* for different values of the velocity U_0($V_0 = 2$; $y = 0$; $m_0 = m_1 = m_2 = 2$; $k = 0.05$; $R = 0.5$; $\lambda = 50$; $m = 0.5$; $M = 0.5$).**

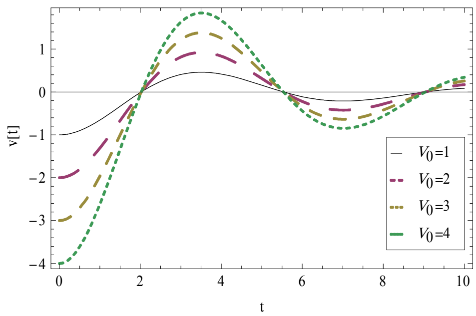

Figure 14 Variation of the dimensionless velocity distribution along the *y*-axis with *t* for different values of magmatic velocity V_0(U_0 = 2; *y* = 0; $m_0 = m_1 = m_2 = 2$; *k* = 0.05; *R* = 0.5; λ = 50; *m* = 0.5; *M* = 0.5).

permeability parameter *k*, Hall parameter *m*, the relaxation time λ, Reynolds number R, and Hartmann number *M*. For all of these figures at $y = 0$ we note that as the time *t* increases the variation of each component of the velocity decreases and vanishes for large values of *t*. This is expected, where for small values of *t* and at the magma plate ($y = 0$), the variation of the velocity components is obvious. Also, the gap between the curves for small values of *t* at the magma plate increases than those as *t* increases.

Figures 1 and 2 show that as the permeability parameter *k* increases the horizontal velocity component *u* increases, while the vertical velocity component *v* decreases. Figures 3 and 4 illustrate the variation of the velocity components *u* and *v* with the Hall parameter *m*, which indicate that for small values of *t* (or at initial values of *t*) the curves are the same with no obvious different which for $t > 2$, the gap between the curves appears. Also, we can see that curves with small values of *m* ($m = 0, 0.5$) are vanishing rabidly than those for ($m = 1,1.5$) i.e., as the Hall parameter *m* increases the disturbance of the velocity components increase. (decreasing the number of density electrons or the electronic charges).

Figures 5 and 6 illustrate the variations of *u* and *v* with *t* for different values of the relaxation time λ, which show that for small values λ the disturbance in *u* and *v* will vanish rapidly than those as λ increases. Also, the figures show that the disturbance in *u* and *v* for a Newtonian fluid less than those for a Non-Newtonian fluid in the case of magma flow.

Figures 7 and 8 show that the variation with the Reynolds number *R*. As *R* increases the velocity components *u* and *v* increase. Figures 9 and 10 show that as the Hartmann number *M* increases the velocity components *u* and *v* decrease, i.e., the fluid moves as a block and takes a constant value for large values of *M*. Figures 11 and 12 illustrate the variation of *u* and *v* with *t* for different values of the *y* axis, which show that the velocity components take the initial values of the magma plate at $y = 0$ and the velocity components decreases as *y* increases. Figures 13 and 14 describe the variations of *u* and *v* with *t* for different values of U_0 and V_0 (velocities of the magma plate), the figures show that the gab between the curves decreases with time and finally vanishes and for certain values of *t* the velocity components *u* and *v* equal to zero. Also, the magnitudes of *u* and *v* increase with increasing U_0 and V_0.

Figures 15, 16, 17, 18, 19, and 20 illustrate the variation of the pressure *p* with *y* for different values of *k*, *m*, λ, *R*, *M*, and *t*. We can see that the pressure decreases as the

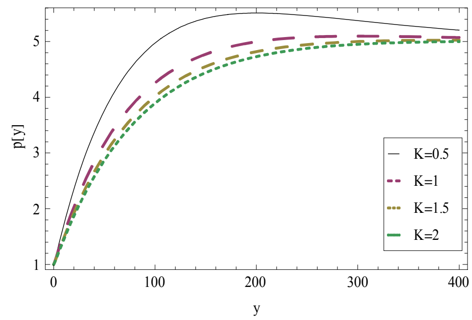

Figure 15 Variation of the dimensionless pressure distribution with *y* for different values of permeability parameter $k(U_0 = V_0 = 2; p_0 = 5; pa = 1; m_0 = m_1 = m_2 = 2; t = 0; m = 0.5; M = 2; R = 50; \lambda = 50)$.

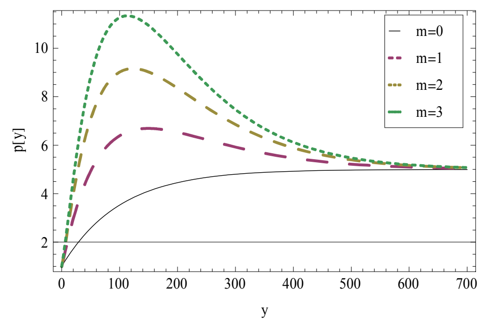

Figure 16 Variation of the dimensionless pressure distribution with *y* for different values of Hall parameter $m(U_0 = V_0 = 2; m_0 = m_1 = m_2 = 2; t = 0; k = 0.5; M = 2; R = 50; \lambda = 50; p_0 = 5; p_a = 1)$.

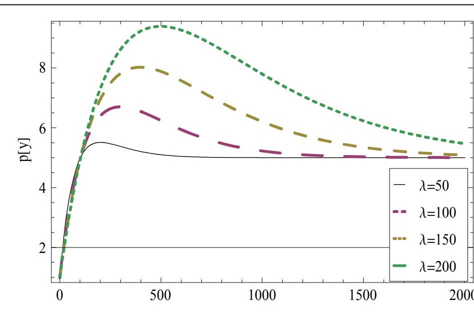

Figure 17 Variation of the dimensionless pressure distribution with *y* for different values of relaxation time $\lambda(U_0 = V_0 = 2; m_0 = m_1 = m_2 = 2; t = 0; k = 0.5; M = 2; R = 50; m = 0.5; p_0 = 5; p_a = 1)$.

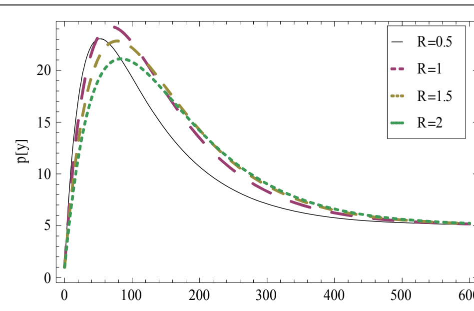

Figure 18 Variation of the dimensionless pressure distribution with *y* for different values of Reynolds number $R(U_0 = V_0 = 2; m_0 = m_1 = m_2 = 2; t = 0; k = 0.5; M = 2; \lambda = 50; m = 0.5; p_0 = 5; p_a = 1)$.

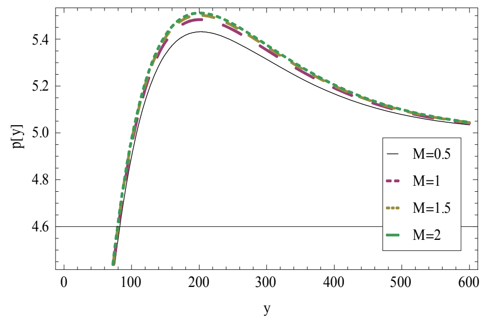

Figure 19 Variation of the dimensionless pressure distribution with *y* for different values of Hartmann number *M*($U_0 = V_0 = 2$; $m_0 = m_1 = m_2 = 2$; $t = 0$; $k = 0.5$; $R = 50$; $\lambda = 50$; $m = 0.5$; $p_0 = 5$; $p_a = 1$).

permeability parameter k increases and take a constant value for large values of k. However an inverse effective behavior for p with the Hartmann number M is shown in Figure 19. Figures 16 and 17 describe the variation of p with the Hall parameter m and the relaxation time λ, which shows that as y increases the pressure increases and the gabs between the curves are more obvious near to the magma plate. Also, the pressure takes the same values of the pressure deep in the magmatic fluid P_0 for the large values of y (as we move deep into the fluid) and the same effect is shown with λ. The pressure decreases as the Reynolds number R increases as shown in Figure 18. Figure 20 shows the variation of the pressure with y for different values of t. We can see that the pressure increases as t increases for small values of y.

Other cases of symmetry will be considered for other boundary value problems else where for other applications.

6 Concluding remarks

The significant features of Lie group analysis for hydro-magnetic Maxwell fluid through a porous medium have been presented. Similarities solutions are obtained and applied to an important phenomena in geology, which is the magmatic fluid. The main points have been summarized as follows:

- As the Hall parameter m increases the disturbances of the velocity components are increase.

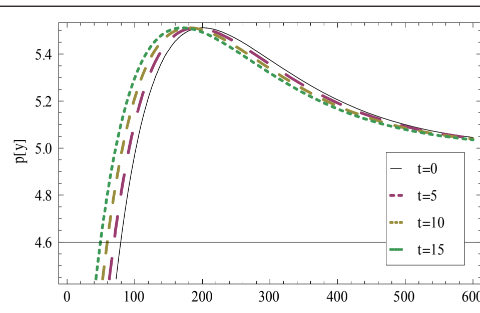

Figure 20 Variation of the dimensionless pressure distribution with *y* for different values of time *t* ($U_0 = V_0 = 2$; $m_0 = m_1 = m_2 = 2$; $M = 2$; $k = 0.5$; $R = 50$; $\lambda = 50$; $m = 0.5$; $p_0 = 5$; $p_a = 1$; $y = 0$).

•The disturbances in the fluid velocity components for a Newtonian fluid are less than those for a non-Newtonian fluid (magmatic fluid)
•The magmatic fluid moves as a block for large values of the Hartmann number M.
•The pressure near to the magma plate is higher for a magneto-magma flow than that for a magma flow without a magnetic field. Also, this pressure for a porous medium is less than that for a medium with high permeability.
•The pressure increase near to the magma plate and take the constant value of the pressure deep in the magmatic fluid for large values of y.

Author details
[1]Mathematics Department, Faculty of Science, Al-Azhar University, Nasr City 11448, Cairo, Egypt [2]Mathematics Department, Faculty of Science, Minia University, Minia 61111, Egypt

Authors' contributions
Khaled S. Mekheimer, Mostafa F El-Sabbagh and and Rabea E. Abo-Elkhair contributed to each part of this work equally.
All the authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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