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G-β-ψ-contractive type mappings in G-metric spaces

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Abstract

In this paper, we introduce G-β-ψ-contractive mappings which are generalizations of α-ψ-contractive mappings in the context of G-metric spaces. Additionally, we prove existence and uniqueness of fixed points of such contractive mappings. Our results generalize, extend and improve the existing results in the literature. We state some examples to illustrate our results.

1 Introduction and preliminaries

In the last few decades, fixed point theory has been one of the most interesting research fields in nonlinear functional analysis. In addition to many branches of applied and pure mathematics, fixed point theory results have wide application areas in many disciplines such as economics, computer science, engineering *etc.* The most remarkable results in this direction were given by Banach [1] in 1922. He proved that each contraction in a complete metric space has a unique fixed point. Due to application potential of the theory, many authors have directed their attention to this field and have generalized the Banach fixed point theorem in various ways (see, *e.g.*, [1–50]). Very recently, Samet *et al.* [38] introduced the notion of α-ψ-contractive mappings and proved the related fixed point theorems. The authors [38] showed that Banach fixed point theorems and some other theorems in the literature became direct consequences of their results. On the other hand, in 2004, Mustafa and Sims [24] defined the notion of a G-metric space and characterized the Banach fixed point theorem in the context of a G-metric space. Following these results, many authors have discussed fixed point theorems in the framework of G-metric spaces; see, *e.g.*, [9, 10, 14–16, 18–20, 23–29, 40–43, 50]. In this paper, we combine these two notions by introducing G-β-ψ-contractive mappings, which are a characterization of α-ψ-contractive mappings in the context of G-metric spaces. Our main results generalize, extend and improve the existing results on the topic in the literature.

Throughout this paper, $\mathbb{N}$ denotes the set of nonnegative integers, and $\mathbb{R}^+$ denotes the set of nonnegative reals.

Let Ψ be a family of functions $\psi : [0,\infty) \to [0,\infty)$ satisfying the following conditions:

(i) ψ is nondecreasing;

(ii) there exist $k_0 \in \mathbb{N}$ and $a \in (0,1)$ and a convergent series of nonnegative terms $\sum_{k=1}^{\infty} v_k$ such that

$$\psi^{k+1}(t) \le a\psi^k(t) + v_k$$

for $k \ge k_0$ and any $t \in \mathbb{R}^+$.

These functions are known in the literature as (c)-comparison functions.

Lemma 1 (See [5]) *If $\psi \in \Psi$, then the following hold*:

(i) $(\psi^n(t))_{n\in\mathbb{N}}$ *converges to* 0 *as* $n \to \infty$ *for all* $t \in \mathbb{R}^+$;
(ii) $\psi(t) < t$ *for any* $t \in (0,\infty)$;
(iii) ψ *is continuous at* 0;
(iv) *the series* $\sum_{k=1}^{\infty} \psi^k(t)$ *converges for any* $t \in \mathbb{R}^+$.

Remark 2 In some sources, (c)-comparison functions are called Bianchini-Grandolfi gauge functions (see, *e.g.*, [34–36]).

Very recently, Samet *et al.* [38] introduced the following concepts.

Definition 3 Let (X,d) be a metric space and let $T : X \to X$ be a given mapping. We say that T is an α-ψ-contractive mapping if there exist two functions $\alpha : X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that

$$\alpha(x,y)d(Tx,Ty) \le \psi\bigl(d(x,y)\bigr),$$

for all $x,y \in X$.

Clearly, any contractive mapping, that is, a mapping satisfying Banach contraction, is an α-ψ-contractive mapping with $\alpha(x,y) = 1$ for all $x,y \in X$ and $\psi(t) = kt$, for all $t \ge 0$ and some $k \in [0,1)$.

Definition 4 Let $T : X \to X$ and $\alpha : X \times X \to [0,\infty)$. We say that T is α-admissible if for all $x,y \in X$, we have

$$\alpha(x,y) \ge 1 \quad \Longrightarrow \quad \alpha(Tx,Ty) \ge 1.$$

Various examples of such mappings are presented in [38]. The main results in [38] are the following fixed point theorems.

Theorem 5 *Let (X,d) be a complete metric space and $T : X \to X$ be an α-ψ-contractive mapping. Suppose that*

(i) T *is* α*-admissible*;
(ii) *there exists* $x_0 \in X$ *such that* $\alpha(x_0,Tx_0) \ge 1$;
(iii) T *is continuous*.

Then there exists $u \in X$ such that $Tu = u$.

Theorem 6 *Let (X,d) be a complete metric space and $T : X \to X$ be an α-ψ-contractive mapping. Suppose that*

(i) T *is* α*-admissible*;
(ii) *there exists* $x_0 \in X$ *such that* $\alpha(x_0,Tx_0) \ge 1$;
(iii) *if* $\{x_n\}$ *is a sequence in* X *such that* $\alpha(x_n,x_{n+1}) \ge 1$ *for all* n *and* $x_n \to x \in X$ *as* $n \to \infty$, *then* $\alpha(x_n,x) \ge 1$ *for all* n.

Then there exists $u \in X$ such that $Tu = u$.

Theorem 7 *Adding to the hypotheses of Theorem* 5 (*resp. Theorem* 6) *the condition*: *For all* $x, y \in X$, *there exists* $z \in X$ *such that* $\alpha(x,z) \geq 1$ *and* $\alpha(y,z) \geq 1$, *we obtain the uniqueness of a fixed point of* T.

Mustafa and Sims [24] introduced the concept of G-metric spaces as follows.

Definition 8 ([24]) Let X be a non-empty set and $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying the following properties:

(G1) $G(x,y,z) = 0$ if $x = y = z$;

(G2) $0 < G(x,x,y)$ for all $x, y \in X$ with $x \neq y$;

(G3) $G(x,x,y) \leq G(x,y,z)$ for all $x,y,z \in X$ with $y \neq z$;

(G4) $G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots$ (symmetry in all three variables);

(G5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for all $x,y,z,a \in X$ (rectangle inequality).

Then the function G is called a generalized metric, or, more specifically, a G-metric on X, and the pair (X,G) is called a G-metric space.

Every G-metric on X defines a metric d_G on X by

$$d_G(x,y) = G(x,y,y) + G(y,x,x) \quad \text{for all } x, y \in X.$$

Example 9 Let (X,d) be a metric space. The function $G : X \times X \times X \to \mathbb{R}^+$, defined as either

$$G(x,y,z) = \max\{d(x,y), d(y,z), d(z,x)\}$$

or

$$G(x,y,z) = d(x,y) + d(y,z) + d(z,x)$$

for all $x,y,z \in X$, is a G-metric on X.

Definition 10 ([24]) Let (X,G) be a G-metric space, and let $\{x_n\}$ be a sequence of points of X. We say that $\{x_n\}$ is G-convergent to $x \in X$ if

$$\lim_{n,m\to\infty} G(x,x_n,x_m) = 0,$$

that is, for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x,x_n,x_m) < \varepsilon$ for all $n,m \geq N$. We call x the limit of the sequence and write $x_n \to x$ or $\lim_{n\to\infty} x_n = x$.

Proposition 11 ([24]) *Let* (X,G) *be a G-metric space. The following are equivalent*:

(1) $G(x_n,x_n,x) \to 0$ *as* $n \to \infty$;

(2) $G(x_n,x,x) \to 0$ *as* $n \to \infty$;

(3) $G(x_n,x_m,x) \to 0$ *as* $n,m \to \infty$.

Definition 12 ([24]) Let (X,G) be a G-metric space. A sequence $\{x_n\}$ is called a G-Cauchy sequence if for any $\varepsilon > 0$, there is $N \in \mathbb{N}$ such that $G(x_n,x_m,x_l) < \varepsilon$ for all $n,m,l \geq N$, that is, $G(x_n,x_m,x_l) \to 0$ as $n,m,l \to \infty$.

Proposition 13 ([24]) *Let (X,G) be a G-metric space. Then the following are equivalent*:

(1) *the sequence $\{x_n\}$ is G-Cauchy*;
(2) *for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$ for all $n, m \geq N$.*

Definition 14 ([24]) A G-metric space (X,G) is called G-complete if every G-Cauchy sequence is G-convergent in (X,G).

Lemma 15 ([24]) *Let (X,G) be a G-metric space. Then, for any $x,y,z,a \in X$, it follows that*:

(i) *if $G(x,y,z) = 0$, then $x = y = z$*;
(ii) $G(x,y,z) \leq G(x,x,y) + G(x,x,z)$;
(iii) $G(x,y,y) \leq 2G(y,x,x)$;
(iv) $G(x,y,z) \leq G(x,a,z) + G(a,y,z)$;
(v) $G(x,y,z) \leq \frac{2}{3}[G(x,y,a) + G(x,a,z) + G(a,y,z)]$;
(vi) $G(x,y,z) \leq G(x,a,a) + G(y,a,a) + G(z,a,a)$.

Definition 16 (See [24]) Let (X,G) be a G-metric space. A mapping $T : X \to X$ is said to be G-continuous if $\{T(x_n)\}$ is G-convergent to $T(x)$, where $\{x_n\}$ is a G-convergent sequence converging to x.

In [23], Mustafa characterized the well-known Banach contraction principle mapping in the context of G-metric spaces in the following ways.

Theorem 17 (See [23]) *Let (X,G) be a complete G-metric space and let $T : X \to X$ be a mapping satisfying the following condition for all $x,y,z \in X$*:

$$G(Tx, Ty, Tz) \leq kG(x,y,z), \tag{1}$$

where $k \in [0,1)$. Then T has a unique fixed point.

Theorem 18 (See [23]) *Let (X,G) be a complete G-metric space and let $T : X \to X$ be a mapping satisfying the following condition for all $x,y \in X$*:

$$G(Tx, Ty, Ty) \leq kG(x,y,y), \tag{2}$$

where $k \in [0,1)$. Then T has a unique fixed point.

Remark 19 The condition (1) implies the condition (2). The converse is true only if $k \in [0, \frac{1}{2})$. For details, see [23].

From [23, 24], each G-metric G on X generates a topology τ_G on X whose base is a family of open G-balls $\{B_G(x,\varepsilon) : x \in X, \varepsilon > 0\}$, where $B_G(x,\varepsilon) = \{y \in X : G(x,y,y) < \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$. A nonempty set A in the G-metric space (X,G) is G-closed if $\overline{A} = A$. Moreover,

$$x \in \overline{A} \quad \Leftrightarrow \quad B_G(x,\varepsilon) \cap A \neq \emptyset \quad \text{for all } \varepsilon > 0.$$

Proposition 20 *Let (X,G) be a G-metric space and let A be a nonempty subset of X. The set A is G-closed if for any G-convergent sequence $\{x_n\}$ in A with limit x, one has $x \in A$.*

2 Main results

We introduce the concept of generalized α-ψ-contractive mappings as follows.

Definition 21 Let (X,G) be a G-metric space and let $T:X\to X$ be a given mapping. We say that T is a G-β-ψ-contractive mapping of type I if there exist two functions $\beta: X\times X\times X\to[0,\infty)$ and $\psi\in\Psi$ such that for all $x,y,z\in X$, we have

$$\beta(x,y,z)G(Tx,Ty,Tz)\le\psi\bigl(G(x,y,z)\bigr). \tag{3}$$

Definition 22 Let (X,G) be a G-metric space and let $T:X\to X$ be a given mapping. We say that T is a G-β-ψ-contractive mapping of type II if there exist two functions $\beta: X\times X\times X\to[0,\infty)$ and $\psi\in\Psi$ such that for all $x,y\in X$, we have

$$\beta(x,y,y)G(Tx,Ty,Ty)\le\psi\bigl(G(x,y,y)\bigr). \tag{4}$$

Definition 23 Let (X,G) be a G-metric space and let $T:X\to X$ be a given mapping. We say that T is a G-β-ψ-contractive mapping of type A if there exist two functions $\beta: X\times X\times X\to[0,\infty)$ and $\psi\in\Psi$ such that for all $x,y\in X$, we have

$$\beta(x,y,Tx)G\bigl(Tx,Ty,T^2x\bigr)\le\psi\bigl(G\bigl(x,y,T^2x\bigr)\bigr). \tag{5}$$

Remark 24 Clearly, any contractive mapping, that is, a mapping satisfying (1), is a G-β-ψ-contractive mapping of type I with $\beta(x,y,z)=1$ for all $x,y,z\in X$ and $\psi(t)=kt$, $k\in(0,1)$. Analogously, a mapping satisfying (2), is a G-β-ψ-contractive mapping of type II with $\beta(x,y,y)=1$ for all $x,y\in X$ and $\psi(t)=kt$, $k\in(0,1)$.

Definition 25 Let $T:X\to X$ and $\beta:X\times X\times X\to[0,\infty)$. We say that T is β-admissible if for all $x,y,z\in X$, we have

$$\beta(x,y,z)\ge 1 \quad\Longrightarrow\quad \beta(Tx,Ty,Tz)\ge 1.$$

Example 26 Let $X=[0,\infty)$ and $T:X\to X$. Define $\beta(x,y,z):X\times X\times X\to[0,\infty)$ by

$$Tx=\begin{cases}2\ln x & \text{if } x\neq 0,\\ e & \text{otherwise,}\end{cases}\quad\text{and}\quad \beta(x,y,z)=\begin{cases}e & \text{if } x\ge y\ge z,\\ 0 & \text{otherwise.}\end{cases}$$

Then T is β-admissible.

Our first result is the following.

Theorem 27 *Let (X,G) be a complete G-metric space. Suppose that $T:X\to X$ is a G-β-ψ-contractive mapping of type A and satisfies the following conditions*:

(i) *T is β-admissible*;
(ii) *there exists $x_0\in X$ such that $\beta(x_0,Tx_0,Tx_0)\ge 1$*;
(iii) *T is G-continuous.*

Then there exists $u\in X$ such that $Tu=u$.

Proof Let $x_0 \in X$ such that $\beta(x_0, Tx_0, Tx_0) \geq 1$ (such a point exists from the condition (ii)). Define the sequence $\{x_n\}$ in X by $x_{n+1} = Tx_n$ for all $n \geq 0$. If $x_{n_0} = x_{n_0+1}$ for some n_0, then $u = x_{n_0}$ is a fixed point of T. So, we can assume that $x_n \neq x_{n+1}$ for all n. Since T is β-admissible, we have

$$\beta(x_0, x_1, x_1) = \beta(x_0, Tx_0, Tx_0) \geq 1 \quad \Longrightarrow \quad \beta(Tx_0, Tx_1, Tx_1) = \beta(x_1, x_2, x_2) \geq 1.$$

Inductively, we have

$$\beta(x_n, x_{n+1}, x_{n+1}) \geq 1 \quad \text{for all } n = 0, 1, \ldots. \tag{6}$$

From (5) and (6), it follows that for all $n \geq 1$, we have

$$\begin{aligned} G(x_n, x_{n+1}, x_{n+1}) &= G(Tx_{n-1}, Tx_n, Tx_n) \\ &= G\bigl(Tx_{n-1}, Tx_n, T^2 x_{n-1}\bigr) \\ &\leq \beta(x_{n-1}, x_n, Tx_{n-1}) G\bigl(Tx_{n-1}, Tx_n, T^2 x_{n-1}\bigr) \\ &\leq \psi\bigl(G(x_{n-1}, x_n, Tx_{n-1})\bigr) \\ &\leq \psi\bigl(G(x_{n-1}, x_n, x_n)\bigr). \end{aligned}$$

Since ψ is nondecreasing, by induction, we have

$$G(x_n, x_{n+1}, x_{n+1}) \leq \psi^n\bigl(G(x_0, x_1, x_1)\bigr) \quad \text{for all } n \geq 1. \tag{7}$$

Using (G5) and (7), we have

$$\begin{aligned} G(x_n, x_m, x_m) &\leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2}) \\ &\quad + G(x_{n+2}, x_{n+3}, x_{n+3}) + \cdots + G(x_{m-1}, x_m, x_m) \\ &\leq \sum_{k=n}^{m-1} G(x_k, x_{k+1}, x_{k+1}) \\ &\leq \sum_{k=n}^{m-1} \psi^k\bigl(G(x_0, x_1, x_1)\bigr). \end{aligned}$$

Since $\psi \in \Psi$ and $G(x_0, x_1, x_1) > 0$, by Lemma 1, we get

$$\sum_{k=0}^{\infty} \psi^k\bigl(G(x_0, x_1, x_1)\bigr) < \infty.$$

Thus, we have

$$\lim_{n,m \to 0} G(x_n, x_m, x_m) = 0.$$

By Proposition 13, this implies that $\{x_n\}$ is a G-Cauchy sequence in the G-metric space (X, G). Since (X, G) is complete, there exists $u \in X$ such that $\{x_n\}$ is G-convergent to u.

Since T is G-continuous, it follows that $\{Tx_n\}$ is G-convergent to Tu. By the uniqueness of the limit, we get $u = Tu$, that is, u is a fixed point of T. □

The next theorem does not require continuity.

Theorem 28 *Let (X, G) be a complete G-metric space. Suppose that $T : X \to X$ is a G-β-ψ-contractive mapping of type A and satisfies the following conditions*:

(i) *T is β-admissible*;
(ii) *there exists $x_0 \in X$ such that $\beta(x_0, Tx_0, Tx_0) \geq 1$*;
(iii) *if $\{x_n\}$ is a sequence in X such that $\beta(x_n, x_{n+1}, x_{n+1}) \geq 1$ for all n and $\{x_n\}$ is a G-convergent to $x \in X$, then $\beta(x_n, x, x_{n+1}) \geq 1$ for all n.*

Then there exists $u \in X$ such that $Tu = u$.

Proof Following the proof of Theorem 29, we know that the sequence $\{x_n\}$ defined by $x_{n+1} = Tx_n$ for all $n \geq 0$ is a G-Cauchy sequence in the complete G-metric space (X, G) that is G-convergent to $u \in X$. From (6) and (iii), we have

$$\beta(x_n, u, u) \geq 1 \quad \text{for all } n \geq 0. \tag{8}$$

Using the basic properties of G-metric together with (5) and (8), we have

$$\begin{aligned} G(x_{n+1}, Tu, x_{n+2}) &\leq G\big(Tx_n, Tu, T^2x_n\big) \\ &\leq \beta(x_n, u, x_{n+1})G\big(Tx_n, Tu, T^2x_n\big) \\ &\leq \psi\big(G(x_n, u, x_{n+1})\big). \end{aligned}$$

Letting $n \to \infty$, using Proposition 11 and since ψ is continuous at $t = 0$, it follows that

$$G(u, Tu, u) = 0.$$

By Lemma 15, we obtain $u = Tu$. □

The following theorem can be derived easily from Theorems 27 and 28.

Theorem 29 *Let (X, G) be a complete G-metric space. Suppose that $T : X \to X$ is a G-β-ψ-contractive mapping of type A and satisfies the following conditions*:

(i) *T is β-admissible*;
(ii) *there exists $x_0 \in X$ such that $\beta(x_0, Tx_0, Tx_0) \geq 1$*;
(iii) *T is G-continuous.*

Then there exists $u \in X$ such that $Tu = u$.

Theorem 30 *Let (X, G) be a complete G-metric space. Suppose that $T : X \to X$ is a G-β-ψ-contractive mapping of type II and satisfies the following conditions*:

(i) *T is β-admissible*;
(ii) *there exists $x_0 \in X$ such that $\beta(x_0, Tx_0, Tx_0) \geq 1$*;
(iii) *if $\{x_n\}$ is a sequence in X such that $\beta(x_n, x_{n+1}, x_{n+1}) \geq 1$ for all n and $\{x_n\}$ is a G-convergent to $x \in X$, then $\beta(x_n, x, x_{n+1}) \geq 1$ for all n.*

Then there exists $u \in X$ such that $Tu = u$.

Remark 31 We notice that some fixed point theorems in the context of G-metric space can be derived from usual fixed point results via certain substitutions (see, *e.g.*, [21, 39]). On the other hand, our main result cannot be obtained via a substitution technique because the expressions in our statements do not allow one to achieve a metric by writing a simple substitution.

With the following example, we show that the hypotheses in Theorems 27-30 do not guarantee uniqueness.

Example 32 Let $X = [0,\infty)$ be the G-metric space, where

$$G(x,y,z) = |x-y| + |y-z| + |z-x|$$

for all $x, y \in X$. Consider the self-mapping $T : X \to X$ given by

$$Tx = \begin{cases} 2x - \frac{7}{4} & \text{if } x > 1, \\ \frac{x}{4} & \text{if } 0 \leq x \leq 1. \end{cases}$$

Notice that Theorem 18 in [23], a characterization of the Banach fixed point theorem, cannot be applied in this case because $G(T1, T2, T2) = 4 > 2 = G(1, 2, 2)$.

Define $\beta : X \times X \times X \to [0,\infty)$ as

$$\beta(x,y,z) = \begin{cases} 1 & \text{if } x,y,z \in [0,1], \\ 0 & \text{otherwise.} \end{cases}$$

Let $\psi(t) = \frac{t}{2}$ for $t \geq 0$. Then we conclude that T is a G-β-ψ-contractive mapping. In fact, for all $x, y \in X$, we have

$$\beta(x,y,y)G(Tx,Ty,Ty) \leq \frac{1}{2}G(x,y,y).$$

On the other hand, there exists $x_0 \in X$ such that $\beta(x_0, Tx_0, Tx_0) \geq 1$. Indeed, for $x_0 = 1$, we have $\beta(1, T1, T1) = \beta(1, \frac{1}{4}, \frac{1}{4}) = 1$.

Notice also that T is continuous. To show that T satisfies all the hypotheses of Theorem 29, it is sufficient to observe that T is β-admissible. For this purpose, let $x, y \in X$ such that $\beta(x,y,y) \geq 1$, which is equivalent to saying that $x, y \in [0,1]$. Due to the definitions of β and T, we have

$$Tx = \frac{x}{4} \in [0,1], \qquad Ty = \frac{y}{4} \in [0,1].$$

Hence, $\beta(Tx,Ty,Ty) \geq 1$. As a result, all the conditions of Theorem 29 are satisfied. Note that Theorem 29 guarantees the existence of a fixed point but not the uniqueness. In this example, 0 and $\frac{7}{4}$ are two fixed points of T.

In the following example, T is not continuous.

Example 33 Let X, G and β be defined as in Example 32. Let $T : X \to X$ be a map given by

$$Tx = \begin{cases} 2x - \frac{7}{4} & \text{if } x > 1, \\ \frac{x}{3} & \text{if } 0 \leq x \leq 1. \end{cases}$$

Let $\psi(t) = \frac{t}{3}$ for $t \geq 0$. Then we conclude that T is a G-β-ψ-contractive mapping. In fact, for all $x, y \in X$, we have

$$\beta(x,y,y)G(Tx,Ty,Ty) \leq \frac{1}{2}G(x,y,y).$$

Furthermore, there exists $x_0 \in X$ such that $\beta(x_0, Tx_0, Tx_0) \geq 1$. For $x_0 = 1$, we have $\beta(1, T1, T1) = \beta(1, \frac{1}{3}, \frac{1}{3}) = 1$.

Let $\{x_n\}$ be a sequence such that $\beta(x_n, x_{n+1}, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and $x_n \to x$ as $n \to \infty$. By the definition of β, we have $\beta(x_n, x_{n+1}, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$. Then we see that $x_n \in [0,1]$. Thus, $\beta(x_n, x, x) \geq 1$.

To show that T satisfies all of the hypotheses of Theorem 30, it is sufficient to observe that T is β-admissible. For this purpose, let $x, y \in X$ such that $\beta(x,y,y) \geq 1$. It is equivalent to saying that $x, y \in [0,1]$. Due to the definitions of β and T, we have

$$Tx = \frac{x}{3} \in [0,1], \qquad Ty = \frac{y}{3} \in [0,1].$$

Hence, $\beta(Tx, Ty, Ty) \geq 1$.

As a result, all the conditions of Theorem 30 are satisfied. Note that Theorem 30 guarantees the existence of a fixed point but not uniqueness. In this example, 0 and $\frac{7}{4}$ are two fixed points of T.

Theorem 34 *Adding the following condition to the hypotheses of Theorem* 27 (*resp. Theorem* 29-*Theorem* 30) *we obtain the uniqueness of a fixed point of T.*

(iv) *For all $x, y \in X$, there exists $z \in X$ such that $\beta(x,z,z) \geq 1$ and $\beta(y,z,z) \geq 1$.*

Proof Let u, $u^* \in X$ be two fixed points of T. By (iv), there exists $z \in X$ such that

$$\beta(u,u,z) \geq 1 \quad \text{and} \quad \beta\bigl(u^*, u^*, z\bigr) \geq 1.$$

Since T is β-admissible, we get by induction that

$$\beta\bigl(u,u,T^n z\bigr) \geq 1 \quad \text{and} \quad \beta\bigl(u^*, u^*, T^n z\bigr) \geq 1 \quad \text{for all } n = 1, 2, \ldots. \tag{9}$$

From (9) and (5), we have

$$\begin{aligned} G\bigl(u, T^n z, u\bigr) &= G\bigl(Tu, T\bigl(T^{n-1}\bigr), T^2 u\bigr) \\ &\leq \beta\bigl(u, T^{n-1}z, Tu\bigr) G\bigl(Tu, T\bigl(T^{n-1}z\bigr), T^2 u\bigr) \\ &\leq \psi\bigl(G\bigl(u, T^{n-1}z, Tu\bigr)\bigr) = \psi\bigl(G\bigl(u, T^{n-1}z, u\bigr)\bigr). \end{aligned}$$

Thus, we get by induction that

$$G(u, T^n z, u) \le \psi^n\big(G(u,z,u)\big) \quad \text{for all } n = 1, 2, \dots.$$

By (G4), we get

$$G(u, u, T^n z) \le \psi^n\big(G(u,u,z)\big) \quad \text{for all } n = 1, 2, \dots.$$

Letting $n \to \infty$, and since $\psi \in \Psi$, we have

$$G(u, u, T^n z,) \to 0.$$

This implies that $\{T^n z\}$ is G-convergent to u. Similarly, we get $\{T^n z\}$ is G-convergent to u^*. By the uniqueness of the limit, we get $u = u^*$, that is, the fixed point of T is unique. □

3 Consequences

3.1 Cyclic contraction

Now, we prove our results for cyclic contractive mappings in a G-metric space.

Theorem 35 *Let A, B be a non-empty G-closed subset of a complete G-metric (X,G) space, let $Y = A \cup B$, and let $T : Y \to Y$ be a given self-mapping satisfying*

$$T(A) \subset B \quad \text{and} \quad T(B) \subset A. \tag{10}$$

If there exists a function $\psi \in \Psi$ such that

$$G(Tx, Ty, Ty) \le \psi\big(G(x,y,y)\big) \quad \text{for all } x \in A, y \in B, \tag{11}$$

then T has a unique fixed point $u \in A \cap B$, that is, $Tu = u$.

Proof Notice that (Y,G) is a complete G-metric space because A, B are closed subsets of a complete G-metric space (X,G). We define $\beta : X \times X \times X \to [0,\infty)$ in the following way:

$$\beta(x,y,y) = \begin{cases} 1 & \text{if } (x,y) \in (A \times B) \cup (B \times A), \\ 0 & \text{otherwise.} \end{cases}$$

Due to the definition of β and assumption (11), we have

$$\beta(x,y,y)G(Tx,Ty,Ty) \le \psi\big(G(x,y,y)\big), \quad \forall x, y \in Y. \tag{12}$$

Hence, T is a G-β-ψ-contractive mapping.

Let $(x,y) \in Y \times Y$ such that $\beta(x,y,y) \ge 1$. If $(x,y) \in A \times B$, then by assumption (10), $(Tx,Ty) \in B \times A$, which yields that $\beta(Tx,Ty,Ty) \ge 1$. If $(x,y) \in B \times A$, we get again $\beta(Tx,Ty,Ty) \ge 1$ by analogy. Thus, in any case we have $\beta(Tx,Ty,Ty) \ge 1$, that is, T is β-admissible. Notice also that for any $z \in A$, we have $(z,Tz) \in A \times B$, which yields that $\beta(z,Tz,Tz) \ge 1$.

Take a sequence $\{x_n\}$ in X such that $\beta(x_n, x_{n+1}, x_{n+1}) \geq 1$ for all n and $x_n \to z \in X$ as $n \to \infty$. Regarding the definition of β, we derive that

$$(x_n, x_{n+1}) \in (A \times B) \cup (B \times A) \quad \text{for all } n. \tag{13}$$

By the assumption, A, B and $(A \times B) \cup (B \times A)$ are closed sets. Hence we get that $(z,z) \in (A \times B) \cup (B \times A)$, which implies that $z \in A \cap B$. We conclude, by the definition of β, that $\beta(x_n, z, z) \geq 1$ for all n.

Now all the hypotheses of Theorem 30 are satisfied, and we conclude that T has a fixed point. Next, we show the uniqueness of a fixed point z of T. Suppose that $w = Tw$, where $w \in A \cap B$. Since $(z,w)(A \times B) \cup (B \times A)$, we have $\beta(y,z,z) \geq 1$ and $\beta(z,y,y) \geq 1$. Thus the condition (iv) of Theorem 34 is satisfied. □

3.2 Coupled fixed point theorems

For the rest of the paper, we suppose that all G-metric spaces (X, G) are symmetric, that is, $G(x,y,y) = G(x,x,y)$ for all $x, y \in X$.

In 1987, Guo and Lakshmikantham [8] introduced the notion of a coupled fixed point. The concept of a coupled fixed point was reconsidered by Gnana-Bhaskar and Lakshmikantham [7] in 2006. In this paper, they proved the existence and uniqueness of a coupled fixed point of an operator $F : X \times X \to X$ on a partially ordered metric space under a condition called the mixed monotone property.

Definition 36 ([7]) Let $(X, \preceq)$ be a partially ordered set and $F : X \times X \to X$. The mapping F is said to have the mixed monotone property if $F(x,y)$ is monotone non-decreasing in x and monotone non-increasing in y, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, \quad x_1 \preceq x_2 \quad \Rightarrow \quad F(x_1, y) \preceq F(x_2, y)$$

and

$$y_1, y_2 \in X, \quad y_1 \preceq y_2 \quad \Rightarrow \quad F(x, y_1) \succeq F(x, y_2).$$

Definition 37 ([7]) An element $(x,y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X \to X$ if

$$x = F(x,y) \quad \text{and} \quad y = F(y,x).$$

Lemma 38 (See [38]) *Let $F : X \times X \to X$ be a given mapping. Define the mapping $T_F : X \times X \to X \times X$ by $T_F(x,y) = (F(x,y), F(y,x))$ for all $(x,y) \in X \times X$. Then (x,y) is a fixed point of T_F if and only if (x,y) is a coupled fixed point of F.*

Definition 39 Let (X, G) be a G-metric space. A mapping $F : X \times X \to X$ is said to be continuous if for any two G-convergent sequences $\{x_n\}$ and $\{y_n\}$ converging to x and y, respectively, $\{F(x_n, y_n)\}$ is G-convergent to $F(x,y)$.

Theorem 40 *Let (X,G) be a complete G-metric space and let $F : X \times X \to X$ be a given mapping. Suppose there exist $\psi \in \Psi$ and a function $\beta : X^2 \times X^2 \times X^2 \to [0,\infty)$ such that*

$$\beta\big((x,y),(u,v),(u,v)\big)G\big(F(x,y),F(u,v),F(u,v)\big) \le \frac{1}{2}\psi\big(G(x,u,u)+G(y,v,v)\big) \tag{14}$$

for all $(x,y),(u,v) \in X \times X$. Suppose also that

(a) *for all $(x,y),(u,v) \in X \times X$, we have*

$$\begin{aligned}&\beta\big((x,y),(u,v),(u,v)\big) \ge 1\\ &\quad\Rightarrow\quad \beta\big(\big(F(x,y),F(y,x)\big),\big(F(u,v),F(v,u)\big),\big(F(u,v),F(v,u)\big)\big) \ge 1;\end{aligned}$$

(b) *there exists (x_0,y_0) such that*

$$\begin{aligned}&\beta\big((x_0,y_0),\big(F(x_0,y_0),F(y_0,x_0)\big),\big(F(x_0,y_0),F(y_0,x_0)\big)\big) \ge 1, \quad \textit{and}\\ &\beta\big(\big(F(y_0,x_0),F(x_0,y_0)\big),\big(F(y_0,x_0),F(x_0,y_0)\big),(y_0,x_0)\big) \ge 1;\end{aligned}$$

(c) *F is continuous.*

Then F has a coupled fixed point, that is, there exists $(x^*,y^*) \in X \times X$ such that $F(x^*,y^*) = x^*$ and $F(y^*,x^*) = y^*$.

Proof Let (Y,δ) be a complete G-metric space with $Y = X \times X$ and

$$\delta\big((x,y),(u,v),(s,t)\big) = G(x,u,s) + G(y,v,t)$$

for all $(x,y),(u,v),(s,t) \in Y$. By using (14) and (G4), we get

$$\beta\big((x,y),(u,v),(u,v)\big)G\big(F(x,y),F(u,v),F(u,v)\big) \le \frac{1}{2}\psi\big(\delta\big((x,y),(u,v),(u,v)\big)\big), \tag{15}$$

and

$$\begin{aligned}&\beta\big((v,u),(v,u),(y,x)\big)G\big(F(v,u),F(v,u),F(y,x)\big)\\ &\quad\le \frac{1}{2}\psi\big(G(v,v,y)+G(u,u,x)\big)\\ &\quad= \frac{1}{2}\psi\big(G(x,u,u)+G(y,v,v)\big)\\ &\quad= \frac{1}{2}\psi\big(\delta\big((x,y),(u,v),(u,v)\big)\big).\end{aligned} \tag{16}$$

Combining (15) and (16), we have

$$\gamma(\zeta,\eta,\eta)\delta(T\zeta,T\eta,T\eta) \le \psi\big(\delta(\zeta,\eta,\eta)\big)$$

for all $\zeta = (x,y), \eta = (u,v) \in Y$, where $T : Y \to Y$ is defined by

$$T(x,y) = \big(F(x,y),F(y,x)\big) \quad \text{for all } \zeta = (x,y) \in Y,$$

and $\gamma : Y \times Y \times Y \to [0,\infty)$ is given by

$$\gamma\big((x,y),(u,v),(u,v)\big) = \min\big\{\beta\big((x,y),(u,v),(u,v)\big), \beta\big((v,u),(v,u),(y,x)\big)\big\}.$$

It follows that T is a G-continuous and G-γ-ψ-contractive mapping of type II.

Suppose that $\gamma(\zeta,\eta,\eta) \geq 1$ for $\zeta = (x,y), \eta = (u,v) \in Y$. Then, by the condition (a), we have $\gamma(T\zeta,T\eta,T\eta) \geq 1$. Therefore, T is γ-admissible.

From the condition (b), there exists (x_0,y_0) such that

$$\gamma\big((x_0,y_0),T(x_0,y_0),T(x_0,y_0)\big) \geq 1.$$

Since all the hypotheses of Theorem 29 are satisfied, it follows that T has a fixed point, and by Lemma 38, F has a coupled fixed point. □

Theorem 41 *Let (X,G) be a complete G-metric space and let $F : X \times X \to X$ be a given mapping. Suppose there exist $\psi \in \Psi$ and a function $\beta : X^2 \times X^2 \times X^2 \to [0,\infty)$ such that*

$$\beta\big((x,y),(u,v),(u,v)\big)G\big(F(x,y),F(u,v),F(u,v)\big) \leq \frac{1}{2}\psi\big(G(x,u,u) + G(y,v,v)\big)$$

for all $(x,y),(u,v) \in X \times X$. Suppose also that

(a) *for all $(x,y),(u,v) \in X \times X$, we have*

$$\beta\big((x,y),(u,v),(u,v)\big) \geq 1 \\ \Rightarrow \quad \beta\big(\big(F(x,y),F(y,x)\big),\big(F(u,v),F(v,u)\big),\big(F(u,v),F(v,u)\big)\big) \geq 1;$$

(b) *there exists (x_0,y_0) such that*

$$\beta((x_0,y_0),\big(F(x_0,y_0),F(y_0,x_0)\big),\big(F(x_0,y_0),F(y_0,x_0)\big) \geq 1, \quad \text{and}$$
$$\beta\big(\big(F(y_0,x_0),F(x_0,y_0)\big),\big(F(y_0,x_0),F(x_0,y_0)\big),(y_0,x_0)\big) \geq 1;$$

(c) *if $\{x_n\}$ and $\{y_n\}$ are sequences in X such that*

$$\beta\big((x_n,y_n),(x_{n+1},y_{n+1}),(x_{n+1},y_{n+1})\big) \geq 1$$

and

$$\beta\big((y_{n+1},x_{n+1}),(y_{n+1},x_{n+1}),(y_n,x_n)\big) \geq 1,$$

$\{x_n\}$ and $\{y_n\}$ are G-convergent to x and y, respectively, then

$$\beta\big((x_n,y_n),(x,y),(x,y)\big) \geq 1$$

and

$$\beta\big((y,x),(y,x),(y_n,x_n)\big) \geq 1$$

for all n.

Then F has a coupled fixed point, that is, there exists $(x^*, y^*) \in X \times X$ such that $F(x^*, y^*) = x^*$ and $F(y^*, x^*) = y^*$.

Proof Let $\{(x_n, y_n)\}$ be a sequence in Y such that

$$\gamma\big((x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+1}, y_{n+1})\big) \geq 1$$

and (x_n, y_n) is G-convergent to (x, y). From the condition (c), we get

$$\gamma\big((x_n, y_n), (x, y), (x, y)\big) \geq 1.$$

This implies that all the hypotheses of Theorem 30 are satisfied. It follows that T has a fixed point, and by Lemma 38, the mapping F has a coupled fixed point. □

Theorem 42 *Adding the following condition to the hypotheses of Theorem* 40 (*resp. Theorem* 41), *we obtain the uniqueness of a coupled fixed point of F.*

(d) *For all $(x, y), (u, v) \in X \times X$, there exists $(z_1, z_2) \in X \times X$ such that*

$$\beta\big((x, y), (z_1, z_2), (z_1, z_2)\big) \geq 1, \qquad \beta\big((z_2, z_1), (z_2, z_1), (y, x)\big) \geq 1$$

and

$$\beta\big((u, v), (z_1, z_2), (z_1, z_2)\big) \geq 1, \qquad \beta\big((z_2, z_1), (z_2, z_1), (v, u)\big) \geq 1.$$

Proof With the condition (d), T and γ satisfy the condition (iv) of Theorem 34. From Theorem 34 and Lemma 38, the result follows. □

3.3 Choudhury and Maity's coupled fixed point results in a *G*-metric space

Definition 43 Let $(X, \preceq)$ be a partially ordered set, and let (X, G) be a G-metric space. A partially ordered G-metric space, $(X, G, \preceq)$, is called ordered complete if for each convergent sequence $\{x_n\}_{n=0}^{\infty} \subset X$, the following conditions hold:

(OC_1) if $\{x_n\}$ is a non-increasing sequence in X such that $x_n \to x^*$, then $x^* \preceq x_n$ $\forall n \in \mathbb{N}$;
(OC_2) if $\{y_n\}$ is a non-decreasing sequence in X such that $y_n \to y^*$, then $y^* \succeq y_n$ $\forall n \in \mathbb{N}$.

Choudhury and Maity [10] proved the following coupled fixed point theorems on ordered G-metric spaces.

Theorem 44 *Let $(X, \preceq)$ be a partially ordered set and let G be a G-metric on X such that (X, G) is a complete G-metric space. Let $F : X \times X \to X$ be a G-continuous mapping having the mixed monotone property on X. Suppose that there exists a $k \in [0, 1)$ such that*

$$G\big(F(x, y), F(u, v), F(w, z)\big) \leq \frac{k}{2}\big[G(x, u, w) + G(y, v, z)\big] \tag{17}$$

for all $x, y, u, v, w, z \in X$ with $x \succeq u \succeq w$ and $y \preceq v \preceq z$, where either $u \neq w$ or $v \neq z$. If there exist $x_0, y_0 \in X$ such that $x_0 \preceq F(x_0, y_0)$ and $F(y_0, x_0) \preceq y_0$, then F has a coupled fixed point, that is, there exists $(x, y) \in X \times X$ such that $x = F(x, y)$ and $y = F(y, x)$.

Proof Let $Y = X^2$. Suppose that $\beta, \gamma : Y \times Y \times Y \to [0,\infty)$ such that

$$\gamma\big((x,y),(u,v),(u,v)\big) = \begin{cases} 1 & \text{if } x \succeq u \text{ and } y \preceq v, \\ 0 & \text{otherwise,} \end{cases} \tag{18}$$

where

$$\gamma\big((x,y),(u,v),(u,v)\big) = \min\big\{\beta\big((x,y),(u,v),(u,v)\big), \beta\big((v,u),(v,u),(y,x)\big)\big\}.$$

From (17), for all $(x,y),(u,v) \in X \times X$, we have

$$\beta\big((x,y),(u,v),(u,v)\big)G\big(F(x,y),F(u,v),F(u,v)\big) \le \frac{k}{2}\big[G(x,u,u) + G(y,v,v)\big]$$

and

$$\beta\big((v,u),(v,u),(y,x)\big)G\big(F(v,u),F(v,u),F(y,x)\big) \le \frac{k}{2}\big[G(v,v,y) + G(u,u,x)\big].$$

It follows that T is a G-γ-ψ-contractive mapping of type II with $\psi(t) = kt$, $t \ge 0$. Let $(x,y),(u,v) \in X \times X$ such that

$$\gamma\big((x,y),(u,v),(u,v)\big) \ge 1.$$

By the definition of γ, we get $x \succeq u$ and $y \preceq v$. This implies that

$$F(x,y) \succeq F(u,v) \quad \text{and} \quad F(y,x) \preceq F(v,u),$$

since F has the mixed monotone property. Thus,

$$\gamma\big(\big(F(x,y),F(y,x)\big),\big(F(u,v),F(v,u)\big),\big(F(u,v),F(v,u)\big)\big) \ge 1.$$

By the assumption, there exist $x_0, y_0 \in X$ such that $x_0 \preceq F(x_0,y_0)$ and $F(y_0,x_0) \preceq y_0$. By the definition of γ, it implies that

$$\beta\big((s,t),\big(F(s,t),F(t,s)\big),\big(F(s,t),F(t,s)\big)\big) \ge 1, \quad \text{and}$$
$$\beta\big(\big(F(t,s),F(s,t)\big),\big(F(t,s),F(s,t)\big),(t,s)\big) \ge 1,$$

where $s = x_0$ and $t = y_0$. From Theorem 40, F has a coupled fixed point. □

Theorem 45 *If, instead of G-continuity of F in the theorem above, we assume that X is ordered complete, then F has a coupled fixed point.*

Proof It is sufficient to prove that the condition (c) of Theorem 41 is satisfied under the setting of (18). For this purpose, we take two sequences $\{s_n\}$ and $\{t_n\}$ in X such that $s_n \to s \in X$ and $t_n \to t \in X$ as $n \to \infty$. Assume that $\beta((s_n,t_n),(s_{n+1},t_{n+1}),(s_{n+1},t_{n+1})) \ge 1$ and $\beta((t_{n+1},s_{n+1}),(t_{n+1},s_{n+1}),(t_n,s_n)) \ge 1$. Due to the definition of β, the sequences $\{s_n\}$ and

$\{t_n\}$ are nonincreasing and nondecreasing, respectively. Regarding (i) and (ii), we derive that

$$s_n \succeq s \quad \text{and} \quad t_n \preceq t,$$

which yields that

$$\beta\bigl((s_n,t_n),(s,t),(s,t)\bigr) = 1 \quad \text{and}$$
$$\beta\bigl((t,s),(t,s),(t_n,s_n)\bigr) = 1.$$

Then, the assumption (c) of Theorem 41 holds. Hence, F has a coupled fixed point. □

Remark 46 Notice that analogs of all of the theorems proved in Section 2 can be derived by replacing type I and type II with type A.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the final manuscript.

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