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Lacunary Δ-statistical convergence in intuitionistic fuzzy n-normed space

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Abstract

The concept of lacunary statistical convergence was introduced in intuitionistic fuzzy n-normed spaces in Sen and Debnath (Math. Comput. Model. 54:2978-2985, 2011). In this article, we introduce the notion of lacunary Δ-statistically convergent and lacunary Δ-statistically Cauchy sequences in an intuitionistic fuzzy n-normed space. Also, we give their properties using lacunary density and prove relation between these notions.

MSC: 47H10; 54H25

Keywords: statistical convergence; lacunary sequence; difference sequence; intuitionistic fuzzy n-normed space

1 Introduction

Fuzzy set theory was introduced by Zadeh [1] in 1965. This theory has been applied not only in different branches of engineering such as in nonlinear dynamic systems [2], in the population dynamics [3], in the quantum physics [4], but also in many fields of mathematics such as in metric and topological spaces [5–7], in the theory of functions [8, 9], in the approximation theory [10]. 2-normed and n-normed linear spaces were initially introduced by Gähler [11, 12] and further studied by Kim and Cho [13], Malceski [14] and Gunawan and Mashadi [15]. Vijayabalaji and Narayanan [16] defined fuzzy n-normed linear space. After Saadati and Park [17] introduced the concept of intuitionistic fuzzy normed space, Vijayabalaji *et al.* [18] defined the notion of intuitionistic fuzzy n-normed space. The notion of statistical convergence was investigated by Steinhaus [19] and Fast [20]. Then a lot of authors applied this concept to probabilistic normed spaces [21, 22], random 2-normed spaces [23] and finally intuitionistic fuzzy normed spaces [24, 25]. Fridy and Orhan [26] introduced the idea of lacunary statistical convergence. Using this idea, Mursaleen and Mohiuddine [27], Sen and Debnath [28] investigated lacunary statistical convergence in intuitionistic fuzzy normed spaces and intuitionistic fuzzy n-normed spaces, respectively. The idea of difference sequences was introduced by Kızmaz [29] where $\Delta x = (\Delta x_k) = x_k - x_{k+1}$. Başarır [30] introduced the Δ-statistical convergence of sequences. Bilgin [31] introduced the definition of lacunary strongly Δ-convergence of fuzzy numbers. Hazarika [32] gave the definition of lacunary generalized difference statistical convergence in random 2-normed spaces. Also, the generalized difference sequence spaces were studied by various authors [33–35]. In this article, we shall introduce lacunary Δ-statistical convergence and lacunary Δ-statistically Cauchy sequences in IFnNLS.

2 Preliminaries, background and notation

In this section, we give the basic definitions.

Definition 2.1 ([27]) A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative,
(ii) $*$ is continuous,
(iii) $a * 1 = a$ for all $a \in [0,1]$,
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for each $a, b, c, d \in [0,1]$.

Definition 2.2 ([27]) A binary operation $\circ : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-conorm if it satisfies the following conditions:

(i) $\circ$ is associative and commutative,
(ii) $\circ$ is continuous,
(iii) $a \circ 0 = a$ for all $a \in [0,1]$,
(iv) $a \circ b \le c \circ d$ whenever $a \le c$ and $b \le d$ for each $a, b, c, d \in [0,1]$.

Definition 2.3 ([27]) Let $n \in \mathbb{N}$ and X be a real vector space of dimension $d \ge n$ (here we allow it to be infinite). A real-valued function $\|\bullet, \dots, \bullet\|$ on $X \times \cdots \times X = X^n$ satisfying the following four properties:

(i) $\|x_1, x_2, \dots, x_n\| = 0$ if and only if $x_1, x_2, \dots, x_n$ are linearly dependent,
(ii) $x_1, x_2, \dots, x_n$ are invariant under any permutation,
(iii) $\|x_1, x_2, \dots, \alpha x_n\| = |\alpha| \|x_1, x_2, \dots, x_n\|$ for any $\alpha \in \mathbb{R}$,
(iv) $\|x_1, x_2, \dots, x_{n-1}, y + z\| \le \|x_1, x_2, \dots, x_{n-1}, y\| + \|x_1, x_2, \dots, x_{n-1}, z\|$,

is called an n-norm on X and the pair is called an n-normed space.

Definition 2.4 ([28]) An IFnNLS is the five-tuple $(X, \mu, \upsilon, *, \circ)$ where X is a linear space over a field F, $*$ is a continuous t-norm, $\circ$ is a continuous t-conorm, μ, υ are fuzzy sets on $X^n \times (0, \infty)$, μ denotes the degree of membership and υ denotes the degree of non-membership of $(x_1, x_2, \dots, x_n, t) \in X^n \times (0, \infty)$ satisfying the following conditions for every $(x_1, x_2, \dots, x_n) \in X^n$ and $s, t > 0$:

(i) $\mu(x_1, x_2, \dots, x_n, t) + \upsilon(x_1, x_2, \dots, x_n, t) \le 1$,
(ii) $\mu(x_1, x_2, \dots, x_n, t) > 0$,
(iii) $\mu(x_1, x_2, \dots, x_n, t) = 1$ if and only if $x_1, x_2, \dots, x_n$ are linearly dependent,
(iv) $\mu(x_1, x_2, \dots, x_n, t)$ is invariant under any permutation of $x_1, x_2, \dots, x_n$,
(v) $\mu(x_1, x_2, \dots, cx_n, t) = \mu(x_1, x_2, \dots, x_n, \frac{t}{|c|})$ for all $c \ne 0$, $c \in F$,
(vi) $\mu(x_1, x_2, \dots, x_n, s) * \mu(x_1, x_2, \dots, x_n', t) \le \mu(x_1, x_2, \dots, x_n + x_n', s + t)$,
(vii) $\mu(x_1, x_2, \dots, x_n, t) : (0, \infty) \to [0,1]$ is continuous in t,
(viii) $\lim_{t\to\infty} \mu(x_1, x_2, \dots, x_n, t) = 1$ and $\lim_{t\to 0} \mu(x_1, x_2, \dots, x_n, t) = 0$,
(ix) $\upsilon(x_1, x_2, \dots, x_n, t) < 1$,
(x) $\upsilon(x_1, x_2, \dots, x_n, t) = 0$ if and only if $x_1, x_2, \dots, x_n$ are linearly dependent,
(xi) $\upsilon(x_1, x_2, \dots, x_n, t)$ is invariant under any permutation of $x_1, x_2, \dots, x_n$,
(xii) $\upsilon(x_1, x_2, \dots, cx_n, t) = \upsilon(x_1, x_2, \dots, x_n, \frac{t}{|c|})$ for all $c \ne 0$, $c \in F$,
(xiii) $\upsilon(x_1, x_2, \dots, x_n, s) \circ \upsilon(x_1, x_2, \dots, x_n', t) \ge \upsilon(x_1, x_2, \dots, x_n + x_n', s + t)$
(xiv) $\upsilon(x_1, x_2, \dots, x_n, t) : (0, \infty) \to [0,1]$ is continuous in t,
(xv) $\lim_{t\to\infty} \upsilon(x_1, x_2, \dots, x_n, t) = 0$ and $\lim_{t\to 0} \upsilon(x_1, x_2, \dots, x_n, t) = 1$.

Example 2.1 ([28]) Let $(X, \|\bullet, \dots, \bullet\|)$ be an n-normed linear space. Also let $a * b = ab$ and $a \circ b = \min\{a + b, 1\}$ for all $a, b \in [0, 1]$,

$$\mu(x_1, x_2, \dots, x_n, t) = \frac{t}{t + \|x_1, x_2, \dots, x_n\|} \quad \text{and} \quad \upsilon(x_1, x_2, \dots, x_n, t) = \frac{\|x_1, x_2, \dots, x_n\|}{t + \|x_1, x_2, \dots, x_n\|}.$$

Then $(X, \mu, \upsilon, *, \circ)$ is an IFnNLS.

Definition 2.5 ([26]) A lacunary sequence is an increasing integer sequence $\theta = \{k_r\}$ such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$ and the ratio $\frac{k_r}{k_{r-1}}$ will be abbreviated as q_r. Let $K \subseteq \mathbb{N}$. The number

$$\delta_\theta(K) = \lim_r \frac{1}{h_r} \bigl|\{k \in I_r : k \in K\}\bigr|$$

is said to be the θ-density of K, provided the limit exists.

Definition 2.6 ([28]) Let θ be a lacunary sequence. A sequence $x = \{x_k\}$ of numbers is said to be lacunary statistically convergent (or S_θ-convergent) to the number L if for every $\varepsilon > 0$, the set $K(\varepsilon)$ has θ-density zero, where

$$K(\varepsilon) = \bigl\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\bigr\}.$$

In this case, we write $S_\theta\text{-}\lim x = L$.

3 Δ-Convergence and lacunary Δ-statistical convergence in IFnNLS

In this section, we define Δ-convergence and lacunary Δ-statistical convergence in intuitionistic fuzzy n-normed spaces.

Definition 3.1 Let $(X, \mu, \upsilon, *, \circ)$ be an IFnNLS. A sequence $x = \{x_k\}$ in X is said to be Δ-convergent to $L \in X$ with respect to the intuitionistic fuzzy n-norm $(\mu, \upsilon)^n$ if, for every $\varepsilon > 0$, $t > 0$ and $y_1, y_2, \dots, y_{n-1} \in X$, there exists $k_0 \in \mathbb{N}$ such that $\mu(y_1, y_2, \dots, y_{n-1}, \Delta x_k - L, t) > 1 - \varepsilon$ and $\upsilon(y_1, y_2, \dots, y_{n-1}, \Delta x_k - L, t) < \varepsilon$ for all $k \geq k_0$, where $k \in \mathbb{N}$ and $\Delta x_k = (x_k - x_{k+1})$. It is denoted by $(\mu, \upsilon)^n\text{-}\lim \Delta x = L$ or $\Delta x_k \to L$ as $k \to \infty$.

Definition 3.2 Let $(X, \mu, \upsilon, *, \circ)$ be an IFnNLS. A sequence $x = \{x_k\}$ in X is said to be lacunary Δ-statistically convergent or $S_\theta(\Delta)$-convergent to $L \in X$ with respect to the intuitionistic fuzzy n-norm $(\mu, \upsilon)^n$ provided that for every $\varepsilon > 0$, $t > 0$ and $y_1, y_2, \dots, y_{n-1} \in \mathbb{X}$,

$$\begin{aligned}&\delta_\theta(\Delta)\bigl(\bigl\{k \in \mathbb{N} : \mu(y_1, y_2, \dots, y_{n-1}, \Delta x_k - L, t) \leq 1 - \varepsilon \\ &\quad \text{or } \upsilon(y_1, y_2, \dots, y_{n-1}, \Delta x_k - L, t) \geq \varepsilon\bigr\}\bigr) = 0,\end{aligned}$$

or, equivalently,

$$\begin{aligned}&\delta_\theta(\Delta)\bigl(\bigl\{k \in \mathbb{N} : \mu(y_1, y_2, \dots, y_{n-1}, \Delta x_k - L, t) > 1 - \varepsilon \\ &\quad \text{and } \upsilon(y_1, y_2, \dots, y_{n-1}, \Delta x_k - L, t) < \varepsilon\bigr\}\bigr) = 1.\end{aligned}$$

It is denoted by ${S_\theta}^{(\mu,\upsilon)^n}(\Delta)\text{-}\lim x = L$ or $x_k \to L(S_\theta(\Delta))$. Using Definition 3.2 and properties of the θ-density, we can easily obtain the following lemma.

Lemma 3.1 *Let $(X,\mu,\upsilon,*,\circ)$ be an IFnNLS and θ be a lacunary sequence. Then, for every $\varepsilon > 0$, $t > 0$ and $y_1, y_2, \ldots, y_{n-1} \in \mathbb{X}$, the following statements are equivalent*:

(i) ${S_\theta}^{(\mu,\upsilon)^n}(\Delta)\text{-}\lim x = L$,

(ii) $\delta_\theta(\Delta)(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \le 1 - \varepsilon\}) = \delta_\theta(\Delta)(\{k \in \mathbb{N} : \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \ge \varepsilon\}) = 0$,

(iii) $\delta_\theta(\Delta)(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) > 1 - \varepsilon$ *and* $\upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) < \varepsilon\}) = 1$,

(iv) $\delta_\theta(\Delta)(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) > 1 - \varepsilon\}) = \delta_\theta(\Delta)(\{k \in \mathbb{N} : \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) < \varepsilon\}) = 1$,

(v) $S_\theta\text{-}\lim \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) = 1$ *and* $S_\theta\text{-}\lim \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) = 0$.

Proceeding exactly in a similar way as in [36], the following theorem can be proved.

Theorem 3.1 *Let $(X,\mu,\upsilon,*,\circ)$ be an IFnNLS and θ be a lacunary sequence. If a sequence $x = \{x_k\}$ in X is lacunary Δ-statistically convergent or $S_\theta(\Delta)$-convergent to $L \in X$ with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$, ${S_\theta}^{(\mu,\upsilon)^n}(\Delta)\text{-}\lim x$ is unique.*

Theorem 3.2 *Let $(X,\mu,\upsilon,*,\circ)$ be an IFnNLS and θ be a lacunary sequence. If $(\mu,\upsilon)^n$-$\lim \Delta x = L$, then ${S_\theta}^{(\mu,\upsilon)^n}(\Delta)\text{-}\lim x = L$.*

Proof Let $(\mu,\upsilon)^n$-$\lim \Delta x = L$. Then, for every $\varepsilon > 0$, $t > 0$ and $y_1, y_2, \ldots, y_{n-1} \in \mathbb{X}$, there exists $k_0 \in \mathbb{N}$ such that $\mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) > 1 - \varepsilon$ and $\upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) < \varepsilon$ for all $k \ge k_0$. Hence the set

$$\begin{aligned}&\bigl\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \le 1 - \varepsilon \\ &\quad \text{or } \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \ge \varepsilon\bigr\}\end{aligned}$$

has a finite number of terms. Since every finite subset of $\mathbb{N}$ has lacunary density zero,

$$\begin{aligned}&\delta_\theta(\Delta)\bigl(\bigl\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \le 1 - \varepsilon \\ &\quad \text{or } \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \ge \varepsilon\bigr\}\bigr) = 0,\end{aligned}$$

that is, ${S_\theta}^{(\mu,\upsilon)^n}(\Delta)\text{-}\lim x = L$.

It follows from the following example that the converse of Theorem 3.2 is not true in general.

Example 3.1 Consider $X = \mathbb{R}^n$ with

$$\|x_1, x_2, \ldots, x_n\| = abs\left(\begin{vmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nn} \end{vmatrix}\right),$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \in \mathbb{R}^n$ for each $i = 1, 2, \ldots, n$, and let $a * b = ab$, $a \circ b = \min\{a + b, 1\}$ for all $a, b \in [0,1]$. Now, for all $y_1, y_2, \ldots, y_{n-1}, x \in \mathbb{R}^n$ and $t > 0$, $\mu(y_1, y_2, \ldots, y_{n-1}, x, t) = \frac{t}{t + \|y_1, y_2, \ldots, y_{n-1}, x\|}$ and $\upsilon(y_1, y_2, \ldots, y_{n-1}, x, t) = \frac{\|y_1, y_2, \ldots, y_{n-1}, x\|}{t + \|y_1, y_2, \ldots, y_{n-1}, x\|}$. Then $(\mathbb{R}^n, \mu, \upsilon, *, \circ)$ is an IFnNLS. Let I_r and h_r be as defined in Definition 2.5. Define a sequence $x = \{x_k\}$ whose terms are given by

$$x_k = \begin{cases} (\frac{(n-[\sqrt{h_r}]+1)(-n+[\sqrt{h_r}])}{2}, 0, \ldots, 0) \in \mathbb{R}^n & \text{if } 1 \le k \le n - [\sqrt{h_r}], \\ (-\frac{1}{2}k^2 + \frac{1}{2}k, 0, \ldots, 0) \in \mathbb{R}^n & \text{if } n - [\sqrt{h_r}] + 1 \le k \le n, \\ (-\frac{1}{2}n^2 - \frac{1}{2}n, 0, \ldots, 0) \in \mathbb{R}^n & \text{if } k > n \end{cases}$$

such that

$$\Delta x_k = \begin{cases} (k, 0, \ldots, 0) \in \mathbb{N} & \text{if } n - [\sqrt{h_r}] + 1 \le k \le n, \\ (0, 0, \ldots, 0) \in \mathbb{N} & \text{otherwise.} \end{cases}$$

For every $0 < \varepsilon < 1$ and for any $y_1, y_2, \ldots, y_{n-1} \in X$, $t > 0$, let

$$K(\varepsilon, t) = \bigl\{k \in I_r : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \le 1 - \varepsilon \\ \text{or } \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \ge \varepsilon\bigr\}.$$

Now,

$$K(\varepsilon, t) = \left\{k \in I_r : \|y_1, y_2, \ldots, y_{n-1}, \Delta x_k\| \ge \frac{\varepsilon t}{1 - \varepsilon} > 0\right\} \\ \subseteq \bigl\{k \in I_r : \Delta x_k = (k, 0, \ldots, 0) \in \mathbb{R}^n\bigr\}.$$

Thus we have $\frac{1}{h_r}|\{k \in I_r : k \in K(\varepsilon, t)\}| \le \frac{[\sqrt{h_r}]}{h_r} \to 0$ as $r \to \infty$. Hence $S_\theta{}^{(\mu,\upsilon)^n}(\Delta)$-$\lim x = 0$.

On the other hand, $x = \{x_k\}$ in X is not Δ-convergent to 0 with respect to the intuitionistic fuzzy n-norm since

$$\mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k, t) = \frac{t}{t + \|y_1, y_2, \ldots, y_{n-1}, \Delta x_k\|} \\ = \begin{cases} \frac{t}{t + \|y_1, y_2, \ldots, y_{n-1}, \Delta x_k\|} & \text{if } n - [\sqrt{h_r}] + 1 \le k \le n, \\ 1, & \text{otherwise,} \end{cases} \\ \le 1$$

and

$$\upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k, t) = \frac{\|y_1, y_2, \ldots, y_{n-1}, \Delta x_k\|}{t + \|y_1, y_2, \ldots, y_{n-1}, \Delta x_k\|} \\ = \begin{cases} \frac{\|y_1, y_2, \ldots, y_{n-1}, \Delta x_k\|}{t + \|y_1, y_2, \ldots, y_{n-1}, \Delta x_k\|} & \text{if } n - [\sqrt{h_r}] + 1 \le k \le n, \\ 0, & \text{otherwise} \end{cases} \\ \ge 0.$$

This completes the proof of the theorem. □

Theorem 3.3 *Let $(X,\mu,\upsilon,*,\circ)$ be an IFnNLS. Then $S_\theta{}^{(\mu,\upsilon)^n}(\Delta)$-$\lim x = L$ if and only if there exists an increasing sequence $K = \{k_n\}$ of the natural numbers such that $\delta_\theta(\Delta)(K) = 1$ and $(\mu,\upsilon)^n$-$\lim_{k\in K}\Delta x_k = L$.*

Proof Necessity. Suppose that $S_\theta{}^{(\mu,\upsilon)^n}(\Delta)$-$\lim x = L$. Then, for every $y_1,y_2,\ldots,y_{n-1}\in\mathbb{X}$, $t>0$ and $j=1,2,\ldots,$

$$\begin{aligned} K(j,t) = \biggl\{ & k\in\mathbb{N} : \mu(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t) > 1-\frac{1}{j} \\ & \text{and } \upsilon(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t) < \frac{1}{j}\biggr\} \quad \text{and} \\ M(j,t) = \biggl\{ & k\in\mathbb{N} : \mu(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t) \le 1-\frac{1}{j} \\ & \text{or } \upsilon(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t) \ge \frac{1}{j}\biggr\}. \end{aligned}$$

Then $\delta_\theta(\Delta)(M(j,t)) = 0$ since

$$K(j,t)\supset K(j+1,t) \tag{3.1}$$

and

$$\delta_\theta(\Delta)\bigl(K(j,t)\bigr) = 1 \tag{3.2}$$

for $t>0$ and $j=1,2,\ldots$. Now we have to show that for $k\in K(j,t)$ suppose that for some $k\in K(j,t)$, $x=\{x_k\}$ not Δ-convergent to L with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$. Therefore there is $\alpha>0$ and a positive integer k_0 such that $\mu(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t)\le 1-\alpha$ or $\upsilon(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t)\ge\alpha$ for all $k\ge k_0$. Let $\alpha>\frac{1}{j}$ and

$$\begin{aligned} K(\alpha,t) = \bigl\{ & k\in\mathbb{N} : \mu(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t) > 1-\alpha \\ & \text{and } \upsilon(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t) < \alpha\bigr\}. \end{aligned}$$

Then $\delta_\theta(\Delta)(K(\alpha,t)) = 0$. Since $\alpha>\frac{1}{j}$, by (3.1) we have $\delta_\theta(\Delta)(K(j,t)) = 0$, which contradicts by equation (3.2).

Sufficiency. Suppose that there exists an increasing sequence $K=\{k_n\}$ of the natural numbers such that $\delta_\theta(\Delta)(K) = 1$ and $(\mu,\upsilon)^n$-$\lim_{k\in K}\Delta x_k = L$, *i.e.*, for every $y_1,y_2,\ldots,y_{n-1}\in\mathbb{X}$, $\varepsilon>0$ and $t>0$, there exists $n_0\in\mathbb{N}$ such that $\mu(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t) > 1-\varepsilon$ and $\upsilon(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t) < \varepsilon$.

Let

$$\begin{aligned} M(\varepsilon,t) := \bigl\{ & k\in\mathbb{N} : \mu(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t) \le 1-\varepsilon \\ & \text{or } \upsilon(y_1,y_2,\ldots,y_{n-1},\Delta x_k - L,t) \ge \varepsilon\bigr\} \\ \subseteq & -\{k_{n_0+1},k_{n_0+2},\ldots\} \end{aligned}$$

and consequently $\delta_\theta(\Delta)(M(\varepsilon,t)) \leq 1-1=0$. Hence $S_\theta^{(\mu,\upsilon)^n}(\Delta)$-$\lim x = L$. This completes proof of the theorem. □

Theorem 3.4 *Let $(X,\mu,\upsilon,*,\circ)$ be an IFnNLS. Then $S_\theta^{(\mu,\upsilon)^n}(\Delta)$-$\lim x = L$ if and only if there exist a convergent sequence $y=\{y_k\}$ and a lacunary Δ-statistically null sequence $z=\{z_k\}$ with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$ such that $(\mu,\upsilon)^n$-$\lim y = L$, $\Delta x = y+\Delta z$ and $\delta_\theta(\Delta)(\{k\in\mathbb{N}:\Delta z_k=0\})=1$.*

Proof Necessity. Suppose that $S_\theta^{(\mu,\upsilon)^n}(\Delta)$-$\lim x = L$ and

$$K(j,t)=\left\{k\in\mathbb{N}:\mu(y_1,y_2,\ldots,y_{n-1},\Delta x_k-L,t)>1-\frac{1}{j}\right.$$
$$\left.\text{and } \upsilon(y_1,y_2,\ldots,y_{n-1},\Delta x_k-L,t)<\frac{1}{j}\right\}.$$

Using Theorem 3.3 for any $y_1,y_2,\ldots,y_{n-1}\in X$, $t>0$ and $j\in\mathbb{N}$, we can construct an increasing index sequence $\{r_j\}$ of the natural numbers such that $r_j\in K(j,t)$, $\delta_\theta(\Delta)(K(j,t))=1$, and so we can conclude that for all $r>r_j$ ($j\in\mathbb{N}$),

$$\frac{1}{h_r}\left|\left\{k\in I_r:\mu(y_1,y_2,\ldots,y_{n-1},\Delta x_k-L,t)>1-\frac{1}{j}\right.\right.$$
$$\left.\left.\text{and } \upsilon(y_1,y_2,\ldots,y_{n-1},\Delta x_k-L,t)<\frac{1}{j}\right\}\right|>\frac{j-1}{j}.$$

We define $y=\{y_k\}$ and $z=\{z_k\}$ as follows. If $1<k<r_1$, we set $y_k=\Delta x_k$ and $z_k=0$. Now suppose that $j\geq 1$ and $r_j<k\leq r_{j+1}$. If $k\in K(j,t)$, *i.e.*, $\mu(y_1,y_2,\ldots,y_{n-1},\Delta x_k-L,t)>1-\frac{1}{j}$ and $\upsilon(y_1,y_2,\ldots,y_{n-1},\Delta x_k-L,t)<\frac{1}{j}$, we set $y_k=\Delta x_k$ and $\Delta z_k=0$. Otherwise $y_k=L$ and $\Delta z_k=\Delta x_k-L$. Hence it is clear that $\Delta x=y+\Delta z$.

We claim that $(\mu,\upsilon)^n$-$\lim y=L$. Let $\varepsilon>\frac{1}{j}$. If $k\in K(j,t)$ for all $k>r_j$, $\mu(y_1,y_2,\ldots,y_{n-1},y_k-L,t)>1-\varepsilon$ and $\upsilon(y_1,y_2,\ldots,y_{n-1},y_k-L,t)<\varepsilon$. Since ε was arbitrary, we have proved the claim.

Next we claim that $z=\{z_k\}$ is a lacunary Δ-statistically null sequence with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$, *i.e.*, $S_\theta^{(\mu,\upsilon)^n}(\Delta)$-$\lim z=0$. It suffices to see that $\delta_\theta(\Delta)(\{k\in\mathbb{N}:\Delta z_k=0\})=1$ to prove the claim. This follows from observing that

$$\left|\{k\in I_r:\Delta z_k=0\}\right|$$
$$\leq\left|\left\{k\in I_r:\mu(y_1,y_2,\ldots,y_{n-1},\Delta z_k,t)>1-\varepsilon \text{ and } \upsilon(y_1,y_2,\ldots,y_{n-1},\Delta z_k,t)<\varepsilon\right\}\right|$$

for any $r\in\mathbb{N}$ and $\varepsilon>0$.

We show that if $\delta>0$ and $j\in\mathbb{N}$ such that $\frac{1}{j}<\delta$, then

$$\frac{1}{h_r}\left|\{k\in I_r:\Delta z_k=0\}\right|>1-\delta$$

for all $r>r_j$. Recall from the construction that if $k\in K(j,t)$, then $\Delta z_k=0$ for $r_j<k\leq r_{j+1}$.

Now, for $t > 0$ and $s \in \mathbb{N}$, let

$$K(s,t) = \left\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) > 1 - \frac{1}{s} \right. \\ \left. \text{and } \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) < \frac{1}{s}\right\}.$$

For $s > j$ and $r_s < k \leq r_{s+1}$ by (3.2),

$$K(s,t) = \left\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) > 1 - \frac{1}{s} \right. \\ \left. \text{and } \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) < \frac{1}{s}\right\} \\ \subset \{k \in \mathbb{N} : \Delta z_k = 0\}.$$

Consequently, if $r_s < k \leq r_{s+1}$ and $s > j$, then

$$\begin{aligned} &\frac{1}{h_r}\left|\{k \in I_r : \Delta z_k = 0\}\right| \\ &\quad \geq \frac{1}{h_r}\left|\left\{k \in I_r : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) > 1 - \frac{1}{s}\right.\right. \\ &\qquad \left.\left. \text{and } \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) < \frac{1}{s}\right\}\right| \\ &\quad > 1 - \frac{1}{s} > 1 - \frac{1}{j} > 1 - \delta. \end{aligned}$$

Hence we get $\delta_\theta(\Delta)(\{k \in \mathbb{N} : \Delta z_k = 0\}) = 1$, which establishes the claim.

Sufficiency. Let x, y and z be sequences such that $(\mu, \upsilon)^n$-$\lim y = L$, $\Delta x = y + \Delta z$ and $\delta_\theta(\Delta)(\{k \in \mathbb{N} : \Delta z_k = 0\}) = 1$. Then, for any $y_1, y_2, \ldots, y_{n-1} \in X$, $\varepsilon > 0$ and $t > 0$, we have

$$\begin{aligned} &\left\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \leq 1 - \varepsilon \text{ or } \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \geq \varepsilon\right\} \\ &\quad \subseteq \left\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \leq 1 - \varepsilon \text{ or } \upsilon(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \geq \varepsilon\right\} \\ &\qquad \cup \{k \in \mathbb{N} : \Delta z_k \neq 0\}. \end{aligned}$$

Therefore

$$\begin{aligned} &\delta_\theta(\Delta)\left(\left\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \leq 1 - \varepsilon\right.\right. \\ &\qquad \left.\left. \text{or } \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t) \geq \varepsilon\right\}\right) \\ &\quad \leq \delta_\theta\left(\left\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \leq 1 - \varepsilon \text{ or } \upsilon(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \geq \varepsilon\right\}\right) \\ &\qquad + \delta_\theta(\Delta)\left(\{k \in \mathbb{N} : \Delta z_k \neq 0\}\right). \end{aligned}$$

Since $(\mu, \upsilon)^n$-$\lim y = L$, the set

$$\left\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \leq 1 - \varepsilon \text{ or } \upsilon(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \geq \varepsilon\right\}$$

contains at most finitely many terms and thus

$$\delta_\theta\bigl(\bigl\{k\in\mathbb{N}:\mu(y_1,y_2,\dots,y_{n-1},y_k-L,t)\le 1-\varepsilon \text{ or } \upsilon(y_1,y_2,\dots,y_{n-1},y_k-L,t)\ge\varepsilon\bigr\}\bigr).$$

Also by hypothesis, $\delta_\theta(\Delta)(\{k\in\mathbb{N}:\Delta z_k\neq 0\})$. Hence,

$$\delta_\theta(\Delta)\bigl(\bigl\{k\in\mathbb{N}:\mu(y_1,y_2,\dots,y_{n-1},\Delta x_k-L,t)\le 1-\varepsilon$$
$$\text{or } \upsilon(y_1,y_2,\dots,y_{n-1},\Delta x_k-L,t)\ge\varepsilon\bigr\}\bigr)=0,$$

and consequently $S_\theta{}^{(\mu,\upsilon)^n}(\Delta)\text{-}\lim x=L$. □

4 Δ-Cauchy and lacunary Δ-statistically Cauchy sequences in IFnNLS

In this section, we introduce the notion of Cauchy sequences and lacunary statistically Cauchy sequences in IFnNLS.

Definition 4.1 Let $(X,\mu,\upsilon,*,\circ)$ be an IFnNLS. A sequence $x=\{x_k\}$ in X is said to be Δ-Cauchy with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$ if, for every $\varepsilon>0$, $t>0$ and $y_1,y_2,\dots,y_{n-1}\in\mathbb{X}$, there exists $k_0\in\mathbb{N}$ such that $\mu(y_1,y_2,\dots,y_{n-1},\Delta x_k-\Delta x_m,t)>1-\varepsilon$ and $\upsilon(y_1,y_2,\dots,y_{n-1},\Delta x_k-\Delta x_m,t)<\varepsilon$ for all $k,m\ge k_0$.

Definition 4.2 Let $(X,\mu,\upsilon,*,\circ)$ be an IFnNLS. A sequence $x=\{x_k\}$ in X is said to be lacunary Δ-statistically Cauchy or $S_\theta(\Delta)$-Cauchy with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$ if, for every $\varepsilon>0$, $t>0$ and $y_1,y_2,\dots,y_{n-1}\in\mathbb{X}$, there exists a number $m\in\mathbb{N}$ satisfying

$$\delta_\theta(\Delta)\bigl(\bigl\{k\in\mathbb{N}:\mu(y_1,y_2,\dots,y_{n-1},\Delta x_k-\Delta x_m,t)\le 1-\varepsilon$$
$$\text{or } \upsilon(y_1,y_2,\dots,y_{n-1},\Delta x_k-\Delta x_m,t)\ge\varepsilon\bigr\}\bigr)=0.$$

Theorem 4.1 *Let $(X,\mu,\upsilon,*,\circ)$ be an IFnNLS. If a sequence $x=\{x_k\}$ in X is lacunary Δ-statistically convergent with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$ if and only if it is lacunary Δ-statistically Cauchy with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$.*

Proof Let $x=\{x_k\}$ be a lacunary Δ-statistically convergent sequence which converges to L. For a given $\varepsilon>0$, choose $s>0$ such that $(1-\varepsilon)*(1-\varepsilon)>1-s$ and $\varepsilon\circ\varepsilon<s$. Let

$$A(\varepsilon,t)=\bigl\{k\in\mathbb{N}:\mu(y_1,y_2,\dots,y_{n-1},\Delta x_k-L,t/2)\le 1-\varepsilon$$
$$\text{or } \upsilon(y_1,y_2,\dots,y_{n-1},\Delta x_k-L,t/2)\ge\varepsilon\bigr\}.$$

Then, for any $t>0$ and $y_1,y_2,\dots,y_{n-1}\in\mathbb{X}$,

$$\delta_\theta(\Delta)\bigl(A(\varepsilon,t)\bigr)=0, \tag{4.1}$$

which implies that $\delta_\theta(\Delta)(A^c(\varepsilon,t))=1$.

Let $q\in A^c(\varepsilon,t)$. Then

$$\mu(y_1,y_2,\dots,y_{n-1},\Delta x_q-L,t/2)>1-\varepsilon$$

and

$$\upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t/2) < \varepsilon.$$

Now, let

$$B(s,t) = \{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - \Delta x_q, t) \leq 1 - s \\ \text{or } \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - \Delta x_q, t) \geq s\}.$$

We need to show that $B(s,t) \subset A(\varepsilon, t)$. Let $k \in B(s,t) \cap A^c(\varepsilon, t)$. Hence $\mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - \Delta x_q, t) \leq 1 - s$ and $\mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t/2) > 1 - \varepsilon$, in particular, $\mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_q - L, t/2) > 1 - \varepsilon$. Then

$$\begin{aligned} 1 - s &\geq \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - \Delta x_q, t) \\ &\geq \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t/2) * \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_q - L, t/2) \\ &> (1 - \varepsilon) * (1 - \varepsilon) > 1 - s, \end{aligned}$$

which is not possible. On the other hand, $\upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - \Delta x_q, t) \geq s$ and $\upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t/2) < \varepsilon$, in particular, $\upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_q - L, t/2) < \varepsilon$. Hence,

$$\begin{aligned} s &\leq \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - \Delta x_q, t) \\ &\leq \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t/2) \circ \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_q - L, t/2) \\ &< \varepsilon \circ \varepsilon < s, \end{aligned}$$

which is not possible. Hence $B(s,t) \subset A(\varepsilon, t)$ and by (4.1) $\delta_\theta(\Delta)(B(\varepsilon, t)) = 0$. This proves that x is lacunary Δ-statistically Cauchy with respect to the intuitionistic fuzzy n-norm $(\mu, \upsilon)^n$.

Conversely, let $x = \{x_k\}$ be lacunary Δ-statistically Cauchy but not lacunary Δ-statistically convergent with respect to the intuitionistic fuzzy n-norm $(\mu, \upsilon)^n$. For a given $\varepsilon > 0$, choose $s > 0$ such that $(1 - \varepsilon) * (1 - \varepsilon) > 1 - s$ and $\varepsilon \circ \varepsilon < s$. Since x is not lacunary Δ-convergent

$$\begin{aligned} &\mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - \Delta x_m, t) \\ &\quad \geq \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t/2) * \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_q - L, t/2) \\ &\quad > (1 - \varepsilon) * (1 - \varepsilon) > 1 - s, \\ &\upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - \Delta x_m, t) \\ &\quad \leq \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - L, t/2) \circ \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_q - L, t/2) \\ &\quad < \varepsilon \circ \varepsilon < s. \end{aligned}$$

Therefore $\delta_\theta(\Delta)(E^c(s,t)) = 0$, where

$$B(s,t) = \{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - \Delta x_q, t) \leq 1 - s \\ \text{or } \upsilon(y_1, y_2, \ldots, y_{n-1}, \Delta x_k - \Delta x_q, t) \geq s\}$$

and so $\delta_\theta(\Delta)(E(s,t)) = 1$, which is a contradiction, since x was lacunary Δ-statistically Cauchy with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$. So, x must be lacunary Δ-statistically convergent with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$. □

Corollary 4.1 *Let $(X,\mu,\upsilon,*,\circ)$ be an IFnNLS and θ be a lacunary sequence. Then, for any sequence $x = \{x_k\}$ in X, the following conditions are equivalent:*

(i) *x is $S_\theta(\Delta)$-convergent with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$.*

(ii) *x is $S_\theta(\Delta)$-Cauchy with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$.*

(iii) *There exists an increasing sequence $K = \{k_n\}$ of the natural numbers such that $\delta_\theta(\Delta)(K) = 1$ and the subsequence $\{x_{k_n}\}$ is $S_\theta(\Delta)$-Cauchy with respect to the intuitionistic fuzzy n-norm $(\mu,\upsilon)^n$.*

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
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