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# Common fixed point theorems for nonlinear contractions in a Menger space

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**Abstract**

The main purpose of this paper is to introduce a new class of Jungck-type contraction and to present some common fixed point theorems for this mapping. Several examples are given to show that our result is a proper extension of many known results.

**MSC:** 47H10

**Keywords:** Menger space; common fixed point theorem

## 1 Introduction

Probabilistic metric space has been introduced and studied in 1942 by Menger in USA [1], and since then the theory of probabilistic metric spaces has developed in many directions [2–6]. The idea of Menger was to use distribution functions instead of nonnegative real numbers as values of the metric. The notion of a probabilistic metric space corresponds to the situation when we do not know exactly the distance between two points, we know only probabilities of possible values of this distance. Such a probabilistic generalization of metric spaces appears to be well adapted for the investigation of physiological thresholds and physical quantities, particularly in connection with both string and  $E$ -infinity which were introduced and studied by a well-known scientific hero El Naschie [7–9].

It is observed by many authors that the contraction condition in a metric space may be exactly translated into a probabilistic metric space endowed with min norms. Sehgal and Bharucha-Reid [10] obtained a generalization of the Banach contraction principle on a complete Menger space, which is a milestone in developing fixed point theorems in a Menger space.

Jungck's fixed point theorem [11] has many applications in nonlinear analysis. This theorem is extended by several authors; see [12–16] and the references therein.

In this paper, we introduce a new class of Jungck-type contraction and present some common fixed point theorems for this mapping. Several examples are given to show that our result is a proper extension of many known results.

## 2 Preliminaries

Throughout this paper we denote by  $N$  the set of all positive integers, by  $Q$  the set of all rational numbers, by  $Z^+$  the set of all nonnegative integers, by  $R$  the set of all real numbers and by  $R^+$  the set of all nonnegative real numbers. We shall recall some definitions and lemmas related to a Menger space.

**Definition 2.1** A mapping  $F : R \rightarrow R^+$  is called a distribution if it is nondecreasing left continuous with  $\inf\{F(t) : t \in R\} = 0$  and  $\sup\{F(t) : t \in R\} = 1$ . We shall denote by  $L$  the set of all distribution functions. The specific distribution function  $H : R \rightarrow R^+$  is defined by

$$H(t) = \begin{cases} 0, & t \leq 0, \\ 1, & t > 0. \end{cases}$$

**Definition 2.2** ([13]) Probabilistic metric space (PM-space) is an ordered pair  $(X, F)$ , where  $X$  is an abstract set of elements and  $F : X \times X \rightarrow L$  is defined by  $(p, q) \rightarrow F_{p,q}$ , where  $\{F_{p,q} : p, q \in X\} \subseteq L$ , where the functions  $F_{p,q}$  satisfy the following:

- (a)  $F_{p,q}(x) = 1$  for all  $x > 0$  if and only if  $p = q$ ;
- (b)  $F_{p,q}(0) = 0$ ;
- (c)  $F_{p,q} = F_{q,p}$ ;
- (d)  $F_{p,q}(x) = 1$  and  $F_{q,r}(y) = 1$ , then  $F_{p,r}(x + y) = 1$ .

**Definition 2.3** A mapping  $t : [0, 1] \times [0, 1] \rightarrow [0, 1]$  is called a  $t$ -norm if

- (e)  $t(0, 0) = 0$  and  $t(a, 1) = a$  for all  $a \in [0, 1]$ ;
- (f)  $t(a, b) = t(b, a)$  for all  $a, b \in [0, 1]$ ;
- (g)  $t(a, b) \leq t(c, d)$  for all  $a, b, c, d \in [0, 1]$  with  $a \leq c$  and  $b \leq d$ ;
- (h)  $t(t(a, b), c) = t(a, t(b, c))$  for all  $a, b, c \in [0, 1]$ .

**Definition 2.4** A Menger space is a triplet  $(X, F, t)$ , where  $(X, F)$  is a PM-space and  $t$  is a  $t$ -norm such that for all  $p, q, r \in X$  and all  $x, y \geq 0$ ,

$$F_{p,r}(x + y) \geq t(F_{p,q}(x), F_{q,r}(y)).$$

**Definition 2.5** ([13]) Let  $(X, F, t)$  be a Menger space and  $f : X \rightarrow X$ .

- (1) A sequence  $\{p_n\}$  in  $X$  is said to converge to a point  $p$  in  $X$  (written as  $p_n \rightarrow p$ ) if for every  $\varepsilon > 0$  and  $\lambda > 0$ , there exists a positive integer  $M(\varepsilon, \lambda)$  such that  $F_{p_n,p}(\varepsilon) > 1 - \lambda$  for all  $n \geq M(\varepsilon, \lambda)$ .
- (2) A sequence  $\{p_n\}$  in  $X$  is said to be Cauchy if for each  $\varepsilon > 0$  and  $\lambda > 0$ , there is a positive integer  $M(\varepsilon, \lambda)$  such that  $F_{p_n,p_m}(\varepsilon) \geq 1 - \lambda$  for all  $n, m \in N$  with  $n, m \geq M(\varepsilon, \lambda)$ .
- (3) A Menger space  $(X, F, t)$  is said to be complete if every Cauchy sequence in  $X$  converges to a point of it.
- (4)  $f$  is said to be continuous at a point  $p$  in  $X$  if for every sequence  $\{p_n\}$  in  $X$ , which converges to  $p$ , the sequence  $\{f(p_n)\}$  in  $X$  converges to  $f(p)$ .
- (5)  $f$  is said to be continuous on  $X$  if  $f$  is continuous at every point in  $X$ .

**Definition 2.6** ([4]) A  $t$ -norm  $t$  is said to be of  $H$ -type if a family of functions  $\{t^n(a)\}_{n=1}^\infty$  is equicontinuous at  $a = 1$ , that is, for any  $\varepsilon \in (0, 1)$ , there exists  $\delta \in (0, 1)$  such that  $a > 1 - \delta$  and  $n \geq 1$  imply  $t^n(a) > 1 - \varepsilon$ . The  $t$ -norm  $t = \min$  is a trivial example of a  $t$ -norm of  $H$ -type, but there are  $t$ -norms of  $H$ -type with  $t$ -norm  $\neq \min$  (see, e.g., Hadzic [5]).

From Definition 2.1-Definition 2.5, we can prove easily the following lemmas.

**Lemma 2.7** ([10]) *If  $(X, d)$  is a metric, then the metric induces a mapping  $X \times X \rightarrow L$ , defined by  $F_{p,q}(x) = H(x - d(p, q))$ ,  $p, q \in X$  and  $x \in R$ . Further, if the  $t$ -norm  $t : [0, 1] \times$*

$[0, 1] \rightarrow [0, 1]$  is defined by  $t(a, b) = \min\{a, b\}$  for all  $a, b \in [0, 1]$ , then  $(X, F, t)$  is a Menger space. It is complete if  $(X, d)$  is complete.

**Lemma 2.8** In a Menger space  $(X, F, t)$ , if  $t(x, x) \geq x$  for all  $x \in [0, 1]$ , then  $t(a, b) = \min\{a, b\}$  for all  $a, b \in [0, 1]$ .

### 3 Jungck-type fixed point theorems

In 1976, Jungck proved the following theorem.

**Theorem A** (Jungck [11], 1976) Let  $f$  be a continuous mapping of a complete metric space  $(X, d)$  into itself and let  $g : X \rightarrow X$  be a map that satisfies the following conditions:

- (a)  $g(X) \subseteq f(X)$ ;
- (b)  $g$  commutes with  $f$ ; (\*1)
- (c)  $d(g(x), g(y)) \leq kd(f(x), f(y))$

for all  $x, y \in X$  and for some  $0 < k < 1$ . Then  $f$  and  $g$  have a unique common fixed point.

**Definition 3.1** Let  $(X, F, t)$  be a Menger space with  $t(x, x) \geq x$  for all  $x \in [0, 1]$  and let  $f, g : X \rightarrow X$  be two self-mappings of  $X$ . We will say that  $f$  and  $g$  are Jungck-type generalized contraction if

$$F_{g(p),g(q)}(\varphi(x)) \geq F_{f(p),f(q)}(x) \tag{*2}$$

for all  $p, q \in X$  and  $x > 0$ , where  $\varphi : [0, \infty) \rightarrow [0, \infty)$  is a mapping such that  $\varphi(x) < x$  for all  $x > 0$ , and for all  $p, q \in X$  and  $x \in R$ ,  $F_{p,q}(x)$  is the same as in Definition 2.2.

**Remark 3.2**

- (1) It is clear that (\*1) implies (\*2) if  $F_{p,q}(x) = H(x - d(p, q))$  for all  $p, q \in X$ ,  $x \in R$ , and  $\varphi(x) = kx$  for all  $x \in R^+$ , where  $0 < k < 1$ .
- (2) In Example 3.10, we shall show that the condition (\*2) is satisfied, but the condition (\*1) is not satisfied.

**Definition 3.3** Let  $\varphi : [0, \infty) \rightarrow [0, \infty)$  be a mapping such that  $\varphi(x) < x$  for all  $x > 0$ . We say that  $\varphi$  is the  $U$ -generalized contraction if

$$\varphi\left((x - \varphi(x))\left(\frac{\varphi(x)}{x}\right)^r\right) \leq (x - \varphi(x))\left(\frac{\varphi(x)}{x}\right)^{r+1} \tag{*3}$$

for all  $x > 0$  and  $r \in Z^+$ .

**Lemma 3.4** Let  $k \in (0, 1)$  be as in (c) of Theorem A and let  $\varphi : [0, \infty) \rightarrow [0, \infty)$  be defined by

$$\varphi(x) = \begin{cases} (\frac{1+k}{2})x + (\frac{1-k}{2})x^2, & 0 \leq x \leq k, \\ (\frac{1}{2} + k - \frac{k^2}{2})x, & k < x. \end{cases}$$

Then  $\varphi$  is an  $U$ -generalized contraction.

*Proof* It follows from hypotheses that for all  $x > 0$ ,

$$\varphi(x) < x. \tag{*4}$$

Now we shall show that condition (\*3) is satisfied. Since

$$(x - \varphi(x)) \left( \frac{\varphi(x)}{x} \right)^r \leq k \quad \text{for all } x \in (0, k] \text{ and } r \in \mathbb{Z}^+,$$

there are three cases which need to be considered.

Case 1. Let  $x \in (0, k]$  and  $r \in \mathbb{Z}^+$ . Then, since

$$\left( 1 - \frac{\varphi(x)}{x} \right) \left( \frac{\varphi(x)}{x} \right)^r \leq 1,$$

(\*3) is satisfied.

Case 2. Let  $x \in (k, \infty)$  and  $r \in \mathbb{Z}^+$  with  $(x - \varphi(x)) \left( \frac{\varphi(x)}{x} \right)^r \leq k$ . Then, since

$$\left( \frac{1+k}{2} \right) + \left( \frac{1-k}{2} \right) (x - \varphi(x)) \left( \frac{\varphi(x)}{x} \right)^r \leq \frac{1}{2} + k - \frac{k^2}{2},$$

(\*3) is satisfied.

Case 3. Let  $x \in (k, \infty)$  and  $r \in \mathbb{Z}^+$  with  $k < (x - \varphi(x)) \left( \frac{\varphi(x)}{x} \right)^r$ . Then, since

$$\frac{1}{2} + k - \frac{k^2}{2} = \frac{\varphi(x)}{x},$$

(\*3) is satisfied. From (\*4), Case 1, Case 2 and Case 3,  $\varphi$  is  $U$ -generalized contraction. □

The following example shows that  $f$  and  $g$  do not have a common fixed point even though  $f, g$  and  $\varphi$  satisfy (\*2) and (\*3).

**Example 3.5** Let  $k \in (0, 1)$  and  $\varphi : [0, \infty) \rightarrow [0, \infty)$  be as in Lemma 3.4. Let  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x + 1$  and  $g(x) = \frac{k}{2}x$ . Define  $F_{p,q} : \mathbb{R} \rightarrow \mathbb{R}^+$  by

$$F_{p,q}(x) = H(x - |p - q|) \quad \text{for all } p, q \in \mathbb{R} \text{ and } x \in \mathbb{R},$$

where  $F_{p,g}$  and  $H$  are the same as in Definition 2.1 and Definition 2.2. Let  $t : [0, 1] \times [0, 1] \rightarrow [0, 1]$  be defined by  $t(a, b) = \min\{a, b\}$  for all  $a, b \in [0, 1]$ . Then, by Lemma 3.4 and simple calculations, (\*2) and (\*3) are satisfied. But  $f$  and  $g$  do not have a common fixed point.

**Remark 3.6** It follows from Example 3.5 that  $f$  and  $g$  must satisfy (\*2) and (\*3), and other conditions additionally in order to have a common fixed point of  $f$  and  $g$ .

The following is Jungck-type common fixed point theorem which is a generalization of Jungck's common fixed point theorem [11].

**Theorem 3.7** *Let  $(X, F, t)$  be a complete Menger space with continuous  $t$ -norm and  $t(x, x) \geq x$  for all  $x \in [0, 1]$ , let  $f$  be a continuous self-mapping on  $X$  and let  $\varphi : R^+ \rightarrow R^+$  be a mapping that satisfies the following conditions:*

- (i)  $g(X) \subseteq f(X)$ ;
- (ii)  $g$  commutes with  $f$ ;
- (iii)  $f, g$  and  $\varphi$  satisfy  $(\ast 2)$  and  $(\ast 3)$ ;
- (iv)  $\varphi$  is a strictly increasing and bijective;
- (v)  $\lim_{n \rightarrow \infty} \varphi^{-n}(x) = \infty$  for each  $x > 0$ , where  $\varphi^{-n}$  is  $n$ -times repeated composition of  $\varphi^{-1}$  with itself.

*Then  $f$  and  $g$  have a unique common fixed point.*

*Proof*

It is easy to see that the self-mapping  $g$  on  $X$  in Theorem 3.7 is continuous on  $X$ . (3.1)

Let  $x_0 \in X$ . By (i), there exists a sequence  $\{x_n\}_{n=0}^\infty$  in  $X$  such that

$$f(x_n) = g(x_{n-1}) \quad \text{for all } n \in N. \tag{3.2}$$

From (iii) and (3.2), we have

$$F_{g(x_{n-1}), g(x_n)}(\varphi(x)) \geq F_{f(x_{n-1}), f(x_n)}(x) \quad \text{for all } n \in N \text{ and } x > 0. \tag{3.3}$$

By virtue of (iv), (3.2) and (3.3), we obtain

$$F_{f(x_n), f(x_{n+1})}(x) = F_{g(x_{n-1}), g(x_n)}(x) \geq F_{f(x_{n-1}), f(x_n)}(\varphi^{-1}(x)) \tag{3.4}$$

for all  $n \in N$  and  $x > 0$ . In view of (3.4), we have

$$F_{f(x_n), f(x_{n+1})}(x) \geq F_{f(x_0), f(x_1)}(\varphi^{-n}(x)) \tag{3.5}$$

for all  $n \in N$  and  $x > 0$ . By repeated application of (3.5), we have

$$F_{f(x_{n+j}), f(x_{n+1+j})}(x) \geq F_{f(x_n), f(x_{n+1})}(\varphi^{-j}(x)) \tag{3.6}$$

for all  $n, j \in N$  and  $x > 0$ . From (iii), we have

$$0 < \frac{\varphi(x)}{x} < 1 \quad \text{for all } x > 0. \tag{3.7}$$

On account of (3.7), we obtain that

$$\sum_{k=0}^{\infty} \left[ \frac{\varphi(x)}{x} \right]^k = \frac{1}{1 - \left(\frac{\varphi(x)}{x}\right)} \quad \text{for all } x > 0. \tag{3.8}$$

In terms of (3.8), we get that

$$x = (x - \varphi(x)) \sum_{k=0}^{\infty} \left(\frac{\varphi(x)}{x}\right)^k \quad \text{for all } x > 0. \tag{3.9}$$

Now we shall show that  $\{f(x_n)\}$  is a Cauchy sequence.

$$\text{Let } n, m \in N \text{ be such that } n < m. \tag{3.10}$$

From (iii), (iv), (3.5)-(3.10) and Definition 2.4, we deduce that

$$\begin{aligned} & F_{f(x_n)f(x_m)}(x) \\ &= F_{f(x_n)f(x_m)}\left((x - \varphi(x)) \sum_{k=0}^{\infty} \left(\frac{\varphi(x)}{x}\right)^k\right) \\ &\geq F_{f(x_n)f(x_m)}\left((x - \varphi(x)) \sum_{k=0}^{m-n-1} \left(\frac{\varphi(x)}{x}\right)^k\right) \\ &\geq \min\left\{F_{f(x_n)f(x_{n+1})}((x - \varphi(x))), \right. \\ &\quad F_{f(x_{n+1})f(x_{n+2})}\left((x - \varphi(x))\left(\frac{\varphi(x)}{x}\right)\right), \dots, \\ &\quad \left.F_{f(x_{m-1})f(x_m)}\left((x - \varphi(x))\left(\frac{\varphi(x)}{x}\right)^{m-n-1}\right)\right\} \\ &\geq \min\left\{F_{f(x_n)f(x_{n+1})}((x - \varphi(x))), \right. \\ &\quad \left.F_{f(x_n)f(x_{n+1})}\left(\varphi^{-1}\left((x - \varphi(x))\left(\frac{\varphi(x)}{x}\right)\right)\right), \dots, \right. \\ &\quad \left.F_{f(x_n)f(x_{n+1})}\left(\varphi^{-(m-n-1)}\left((x - \varphi(x))\left(\frac{\varphi(x)}{x}\right)^{m-n-1}\right)\right)\right\} \\ &\geq F_{f(x_n)f(x_{n+1})}((x - \varphi(x))) \\ &\geq F_{f(x_0)f(x_1)}(\varphi^{-n}(x - \varphi(x))) \end{aligned} \tag{3.11}$$

for all  $x > 0$  and  $n, m \in N$  with  $n < m$ . In terms of (iii), (v) and Definition 2.2, we have

$$\lim_{n \rightarrow \infty} F_{f(x_0)f(x_1)}(\varphi^{-n}(x - \varphi(x))) = 1 \quad \text{for all } x > 0. \tag{3.12}$$

By (3.11), (3.12) and Definition 2.2,  $\{f(x_n)\}$  is a Cauchy sequence in  $X$ . Since  $X$  is complete and  $\{f(x_n)\}$  is a Cauchy sequence in  $X$ , there exists  $z \in X$  such that

$$\lim_{n \rightarrow \infty} f(x_n) = z. \tag{3.13}$$

On account of (3.2) and (3.13), we have

$$\lim_{n \rightarrow \infty} g(x_n) = z. \tag{3.14}$$

By (ii), (3.1), (3.13), (3.14) and hypotheses,

$$f(g(x_n)) = g(f(x_n)) \quad \text{for all } n \in N, \tag{3.15}$$

$$\lim_{n \rightarrow \infty} f(g(x_n)) = f(z)$$

and

$$\lim_{n \rightarrow \infty} g(f(x_n)) = g(z).$$

From (3.15), we get that

$$f(z) = g(z). \tag{3.16}$$

In view of (ii), (3.16) and (\*2), we have

$$F_{g(z),g(g(z))}(\varphi(x)) \geq F_{f(z),f(g(z))}(x) \geq F_{g(z),g(f(z))}(x) \geq F_{g(z),g(g(z))}(x) \tag{3.17}$$

for all  $x > 0$ .

By (iv) and (3.17),

$$F_{g(z),g(g(z))}(x) \geq F_{g(z),g(g(z))}(\varphi^{-n}(x)) \tag{3.18}$$

for all  $n \in N$  and  $x > 0$ . Due to (v), (3.18), Definition 2.1 and Definition 2.2, we get that

$$g(z) = g(g(z)). \tag{3.19}$$

From (ii), (3.16) and (3.19), we have

$$g(z) = g(g(z)) = g(f(z)) = f(g(z)). \tag{3.20}$$

By (3.20),  $g(z)$  is a common fixed point of  $f$  and  $g$ . To prove the uniqueness of a common fixed point of  $f$  and  $g$ , let  $u$  and  $w$  be common fixed points of  $f$  and  $g$ . Then  $f(u) = g(u) = u$  and  $f(w) = g(w) = w$ . Putting  $p = u$  and  $q = w$  in (\*2), we get

$$F_{g(u),g(w)}(\varphi(x)) = F_{u,w}(\varphi(x)) \geq F_{f(u),f(w)}(x) = F_{u,w}(x) \tag{3.21}$$

for all  $x > 0$ , which gives  $u = w$ . Thus  $g(z)$  is a unique common fixed point of  $f$  and  $g$ .  $\square$

Now we give an example to support Theorem 3.7.

**Example 3.8** Let  $X = R$  be the set of reals with the usual metric and let  $f, g : X \rightarrow X$  and  $\varphi : R^+ \rightarrow R^+$  be mappings defined as follows:

$$f(x) = 3x, \quad g(x) = 2x \quad \text{and} \quad \varphi(x) = \begin{cases} \frac{2}{3}x + \frac{1}{3}x^2, & 0 \leq x \leq \frac{2}{3}, \\ \frac{8}{9}x, & \frac{2}{3} < x. \end{cases} \tag{3.22}$$

Let the mappings  $F_{p,q}$ ,  $H$  and  $t$  be as in Example 3.5. Then from Lemma 2.7,  $(X, F, t)$  is a complete Menger space. By the same method as in Lemma 3.4 and simple calculations, the conditions of Theorem 3.7 are satisfied. Thus  $f$  and  $g$  have a unique common fixed point 0.

From Theorem 3.7, we have the following corollary.

**Corollary 3.9** *Let  $(X, F, t)$  be a complete Menger space with continuous  $t$ -norm and  $t(x, x) \geq x$  for all  $x \in [0, 1]$ . Let  $f, g : X \rightarrow X$  be maps that satisfy the following conditions:*

- (a)  $g(X) \subseteq f(X)$ ;
- (b)  $f$  is continuous;
- (c)  $g$  commutes with  $f$ ;
- (d)  $F_{g(p),g(q)}(kx) \geq F_{f(p),f(q)}(x)$  for all  $p, q \in X$ ,  $x > 0$  and for some  $0 < k < 1$ . Then  $f$  and  $g$  have a unique common fixed point.

*Proof* Let  $\varphi : R^+ \rightarrow R^+$  be defined by

$$\varphi(x) = kx, \quad 0 < k < 1. \tag{3.23}$$

From (b) and (d), we deduce that  $g$  is continuous. Thus, by (3.23), the same method as in Lemma 3.4 and simple calculations, the conditions of Theorem 3.7 are satisfied. Therefore  $f$  and  $g$  have a unique common fixed point.

In the next example, we shall show that all the conditions of Theorem 3.7 are satisfied, but condition (d) in Corollary 3.9 and condition (\*1) in Theorem A are not satisfied.  $\square$

**Example 3.10** Let  $k \in (0, 1)$  be as in (c) of Theorem A and let  $X = R$  be the set of reals with usual metric. Suppose that  $f, g : X \rightarrow X$  and  $\varphi : R^+ \rightarrow R^+$  are mappings defined as follows:

$$f(x) = kx, \quad g(x) = \left(\frac{k + k^2}{2}\right)x$$

and

$$\varphi(x) = \begin{cases} \left(\frac{1+k}{2}\right)x + \left(\frac{1-k}{2}\right)x^2, & 0 \leq x \leq k, \\ \left(\frac{1}{2} + k - \frac{k^2}{2}\right)x, & k < x. \end{cases}$$

Let the mappings  $F_{p,q}$ ,  $H$  and  $t$  be the same as in Example 3.5. Then, from Lemma 2.7 and Lemma 3.4,  $(X, F, t)$  is a complete Menger space and  $\varphi$  satisfies (\*3). Since

$$|g(p) - g(q)| \leq \varphi(|f(p) - f(q)|)$$

for all  $p, q \in X$ , we deduce that

$$F_{g(p),g(q)}(\varphi(x)) \geq F_{f(p),f(q)}(x)$$

for all  $p, q \in X$  and  $x > 0$ , which implies (\*2). By simple calculations, conditions (i), (ii), (iv) and (v) of Theorem 3.7 are satisfied. Thus all the conditions of Theorem 3.7 are satisfied.



Hence  $f$  and  $g$  have a unique common fixed point 0. By hypotheses, there exist  $p_1 = -k \in R$ ,  $q_1 = 0 \in R$  and  $x_1 = \frac{3}{4}k^2 + \frac{1}{4}k > 0$  such that

$$F_{g(p_1),g(q_1)}(kx_1) < F_{f(p_1),f(q_1)}(x_1),$$

which implies that condition (d) of Corollary 3.9 is not satisfied. By hypotheses, there exist  $p_2 = -k^2 \in R$  and  $q_2 = 0 \in R$  such that

$$|g(p_2) - g(q_2)| > k|f(p_2) - f(q_2)|,$$

which implies that condition (\*1) in Theorem A is not satisfied. Therefore Theorem 3.7 is a proper extension of Theorem A and Corollary 3.9.

A natural question arises from Example 3.5.

**Question** Would Theorem 3.7 remain true if (i)-(v) in Theorem 3.7 were substituted by some suitable conditions?

#### Competing interests

The author declares that he has no competing interests.

#### Authors' contributions

The author completed the paper himself. The author read and approved the final manuscript.

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