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On some trace inequalities for positive definite Hermitian matrices

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Abstract

Let $\mathbf{A}$ be a positive definite Hermitian matrix, we investigate the trace inequalities of $\mathbf{A}$. By using the equivalence of the deformed matrix, according to some properties of positive definite Hermitian matrices and some elementary inequalities, we extend some previous works on the trace inequalities for positive definite Hermitian matrices, and we obtain some valuable theory.

MSC: Primary 15A45; 15A57

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1 Introduction

In mathematics, a Hermitian matrix (or self-adjoint matrix) is a square matrix with complex entries that is equal to its own conjugate transpose. That is, the elements in the ith row and jth column are equal to the complex conjugates of the elements in the jth row and ith column. In other words, the matrix $\mathbf{A}$ is Hermitian if and only if $\mathbf{A} = \overline{\mathbf{A}}^T$, where $\overline{\mathbf{A}}^T$ denotes the conjugate transpose of matrix $\mathbf{A}$.

Hermitian matrices play an important role in statistical mechanics [1], engineering; in cases such as communication, to describe n-dimensional signal cross-correlation properties, like conjugate symmetry, we can use Hermitian matrices.

The earliest study of matrix inequality work in the literature was [2]. In 1980, Bellman [3] proved some trace inequalities for positive definite Hermitian matrices:

(1) $\operatorname{tr}(\mathbf{AB})^2 \le \operatorname{tr}(\mathbf{A}^2\mathbf{B}^2)$;

(2) $2\operatorname{tr}(\mathbf{AB}) \le \operatorname{tr}\mathbf{A}^2 + \operatorname{tr}\mathbf{B}^2$;

(3) $\operatorname{tr}(\mathbf{AB}) \le (\operatorname{tr}\mathbf{A}^2)^{\frac{1}{2}}(\operatorname{tr}\mathbf{B}^2)^{\frac{1}{2}}$.

Since then, the problems of the trace inequality for positive definite (semidefinite) Hermitian matrices have caught the attention of scholars, getting a lot of interesting results. There exists a vast literature that studies the trace (see [4–8]). In the paper, using the identical deformation of matrix, and combined with some elementary inequalities, our purpose is to derive some new results on the trace inequality for positive definite Hermitian matrices.

The rest of this paper is organized as follows. In Section 2, we will give the relevant definitions and properties of Hermitian matrices. In Section 3, we will quote some lemmas; in Section 4, which is the main part of the paper, using the properties of Hermitian matrices, we investigate the trace inequalities for positive definite Hermitian matrices.

2 Preliminaries

By $\mathbf{M}_{n,m}(\mathbf{F})$ we denote the n-by-m matrices over a field $\mathbf{F}$, usually the real numbers $\mathbf{R}$ or the complex numbers $\mathbf{C}$. Most often, the facts discussed are valid in the setting of the complex-entried matrices, in which case $\mathbf{M}_{n,m}(\mathbf{C})$ is abbreviated as $\mathbf{M}_{n,m}$. In case of square matrices we replace $\mathbf{M}_{n,n}$ by $\mathbf{M}_n$.

Let $\mathbf{A} = (a_{ij}) \in \mathbf{M}_n$; we may denote the eigenvalues of $\mathbf{A}$ by $\lambda_1, \lambda_2, \ldots, \lambda_n$, without loss of generality, where we let $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. Let $\sigma(\mathbf{A})$ denote the singular value, and $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n$. It is well known that if $\mathbf{A}$ is Hermitian, then all eigenvalues of $\mathbf{A}$ are real numbers and if $\mathbf{A}$ is unitary, then every eigenvalue of $\mathbf{A}$ has modulus 1. The sum of two Hermitian matrices of the same size is Hermitian. If $\mathbf{A}$ is Hermitian, then $\mathbf{A}^k$ is Hermitian for all $k = 1, 2, \ldots$. If $\mathbf{A}$ is invertible as well, then $\mathbf{A}^{-1}$ is Hermitian.

A Hermitian matrix $\mathbf{A} \in \mathbf{M}_n$ is said to be positive semidefinite, denoted by $\mathbf{A} \geq 0$, if $(\mathbf{A}x, x) \geq 0$ for all $x \in \mathbf{C}^n$, and it is called positive definite, denoted by $\mathbf{A} > 0$, if $(\mathbf{A}x, x) > 0$ for all nonzero $x \in \mathbf{C}^n$ ($\mathbf{C}^n$ denotes complex vector spaces), where $(\cdot)$ denotes the Euclidean inner product on $\mathbf{C}^n$.

Remark 2.1

(1) The sum of any two positive definite matrices of the same size is positive definite.
(2) Each eigenvalue of a positive definite matrix is a non-negative (positive) real number.
(3) The trace and the determinant of a positive definite matrix are non-negative (positive) real numbers.
(4) Any principal submatrix of a positive definite matrix is positive definite.

A Hermitian matrix is positive definite if and only if all of its eigenvalues are non-negative (positive) real numbers.

We will use this fact several times.

Let $\mathbf{A} \in \mathbf{M}_n$. Then the trace of $\mathbf{A}$ is given by $\operatorname{tr}\mathbf{A} = \sum_{i=1}^n a_{ii}$. The trace function has the following properties.

Let $\mathbf{A}, \mathbf{B} \in \mathbf{M}_n$, $\alpha \in \mathbf{C}$. Then

(1) $\operatorname{tr}(\mathbf{A} + \mathbf{B}) = \operatorname{tr}\mathbf{A} + \operatorname{tr}\mathbf{B}$;
(2) $\operatorname{tr}\alpha\mathbf{A} = \alpha \operatorname{tr}\mathbf{A}$;
(3) $\operatorname{tr}(\mathbf{AB}) = \operatorname{tr}(\mathbf{BA})$.

3 Some lemmas

The following lemmas play a fundamental role in this paper.

Lemma 3.1 [9] *Let* $\mathbf{A}_i \in \mathbf{M}_n$ $(i = 1, 2, \ldots, m)$. *Then*

$$\left|\operatorname{tr}(\mathbf{A}_1\mathbf{A}_2\cdots\mathbf{A}_m)\right| \leq \sum_{i=1}^n \sigma_i(\mathbf{A}_1\mathbf{A}_2\cdots\mathbf{A}_m) \leq \sum_{i=1}^n \sigma_i(\mathbf{A}_1)\sigma_i(\mathbf{A}_2)\cdots\sigma_i(\mathbf{A}_m),$$

where $\sigma_1(\mathbf{A}_i) \geq \sigma_2(\mathbf{A}_i) \geq \cdots \geq \sigma_n(\mathbf{A}_i)$.

Lemma 3.2 [10] *Let* $\alpha_i > 0$ $(i = 1, 2, \ldots, n)$, *and* $\sum_{i=1}^n \alpha_i \geq 1$. *Let* $a_{ij} > 0$ $(j = 1, 2, \ldots, m)$. *Then*

$$\sum_{j=1}^m a_{1j}^{\alpha_1} a_{2j}^{\alpha_2} \cdots a_{nj}^{\alpha_n} \leq \left(\sum_{j=1}^m a_{1j}\right)^{\alpha_1} \left(\sum_{j=1}^m a_{2j}\right)^{\alpha_2} \cdots \left(\sum_{j=1}^m a_{nj}\right)^{\alpha_n}.$$

Lemma 3.3 [11] *Let* $\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_n$ *be same size positive definite matrices, and* $\alpha_1, \alpha_2, \ldots, \alpha_n$ *are positive real numbers, and* $\sum_{i=1}^{n} \alpha_i = 1$. *Then*

$$\operatorname{tr} \prod_{i=1}^{n} \mathbf{A}_i^{\alpha_i} \le \sum_{i=1}^{n} \alpha_i \operatorname{tr} \mathbf{A}_i.$$

4 Main results

Now, we prove the following theorem.

Theorem 4.1 *Let* $\mathbf{A}_i$, $\mathbf{B}_i$ $(i = 1, 2, \ldots, m)$ *be same size positive definite matrices,* $p > 1$, *and* $\frac{1}{p} + \frac{1}{q} = 1$. *Then*

$$\operatorname{tr} \sum_{i=1}^{m} (\mathbf{A}_i \mathbf{B}_i) \le \left(\sum_{i=1}^{m} \operatorname{tr} \mathbf{A}_i^p \right)^{\frac{1}{p}} \left(\sum_{i=1}^{m} \operatorname{tr} \mathbf{B}_i^q \right)^{\frac{1}{q}}.$$

Proof Since the eigenvalues and traces of positive definite matrices are all positive real numbers, the eigenvalues are equal to the singular values. Then, according to Lemma 3.1 and the spectral mapping theorem, we have

$$\begin{aligned}
&\sum_{i=1}^{m} \operatorname{tr} \mathbf{A}_i^{\frac{1}{p}} \mathbf{B}_i^{\frac{1}{q}} \\
&\quad \le \sum_{i=1}^{m} \sum_{j=1}^{n} \sigma_j \bigl(\mathbf{A}_i^{\frac{1}{p}} \mathbf{B}_i^{\frac{1}{q}} \bigr) \\
&\quad \le \sum_{i=1}^{m} \sum_{j=1}^{n} \sigma_j \bigl(\mathbf{A}_i^{\frac{1}{p}} \bigr) \sigma_j \bigl(\mathbf{B}_i^{\frac{1}{q}} \bigr) \\
&\quad = \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_j \bigl(\mathbf{A}_i^{\frac{1}{p}} \bigr) \lambda_j \bigl(\mathbf{B}_i^{\frac{1}{q}} \bigr) \\
&\quad = \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_j^{\frac{1}{p}} (\mathbf{A}_i) \lambda_j^{\frac{1}{q}} (\mathbf{B}_i).
\end{aligned} \tag{4.1}$$

By using Lemma 3.2, we get

$$\begin{aligned}
&\sum_{i=1}^{m} \operatorname{tr} \mathbf{A}_i^{\frac{1}{p}} \mathbf{B}_i^{\frac{1}{q}} \\
&\quad \le \sum_{i=1}^{m} \left(\sum_{j=1}^{n} \lambda_j (\mathbf{A}_i) \right)^{\frac{1}{p}} \left(\sum_{j=1}^{n} \lambda_j (\mathbf{B}_i) \right)^{\frac{1}{q}} \\
&\quad \le \sum_{i=1}^{m} (\operatorname{tr} \mathbf{A}_i)^{\frac{1}{p}} (\operatorname{tr} \mathbf{B}_i)^{\frac{1}{q}} \\
&\quad \le \left(\sum_{i=1}^{m} \operatorname{tr} \mathbf{A}_i \right)^{\frac{1}{p}} \left(\sum_{i=1}^{m} \operatorname{tr} \mathbf{B}_i \right)^{\frac{1}{q}}.
\end{aligned} \tag{4.2}$$

Let $\mathbf{A}_i = \mathbf{A}_i^p$, $\mathbf{B}_i = \mathbf{B}_i^q$. Then we obtain

$$\sum_{i=1}^{m} \operatorname{tr}(\mathbf{A}_i\mathbf{B}_i) \le \left(\sum_{i=1}^{m} \operatorname{tr}\mathbf{A}_i^p\right)^{\frac{1}{p}} \left(\sum_{i=1}^{m} \operatorname{tr}\mathbf{B}_i^q\right)^{\frac{1}{q}}.$$

By $\operatorname{tr}(\mathbf{A}+\mathbf{B}) = \operatorname{tr}\mathbf{A} + \operatorname{tr}\mathbf{B}$, we have

$$\operatorname{tr}\sum_{i=1}^{m} (\mathbf{A}_i\mathbf{B}_i) \le \left(\sum_{i=1}^{m} \operatorname{tr}\mathbf{A}_i^p\right)^{\frac{1}{p}} \left(\sum_{i=1}^{m} \operatorname{tr}\mathbf{B}_i^q\right)^{\frac{1}{q}}.$$

Thus, the proof is completed. □

Next, we give a trace inequality for positive definite matrices.

Theorem 4.2 *Let* $\alpha_i > 0$ $(i = 1, 2, \ldots, n)$ *and* $\sum_{i=1}^{n} \alpha_i = 1$. $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{C}_i$ $(i = 1, 2, \ldots, n)$ *are same size positive definite matrices. Then*

$$\operatorname{tr}\left[\prod_{i=1}^{n} \mathbf{A}_i^{\alpha_i} + \prod_{i=1}^{n} \mathbf{B}_i^{\alpha_i} + \prod_{i=1}^{n} \mathbf{C}_i^{\alpha_i}\right] \le \prod_{i=1}^{n} \left[\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)\right]^{\alpha_i}.$$

Proof Since the trace of a matrix is a linear operation, by using Lemma 3.3, it follows that

$$\begin{aligned}
&\frac{\operatorname{tr}(\prod_{i=1}^{n} \mathbf{A}_i^{\alpha_i} + \prod_{i=1}^{n} \mathbf{B}_i^{\alpha_i} + \prod_{i=1}^{n} \mathbf{C}_i^{\alpha_i})}{\prod_{i=1}^{n} [\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)]^{\alpha_i}} \\
&\quad = \operatorname{tr}\prod_{i=1}^{n}\left[\frac{\mathbf{A}_i^{\alpha_i}}{\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)^{\alpha_i}} + \frac{\mathbf{B}_i^{\alpha_i}}{\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)^{\alpha_i}} + \frac{\mathbf{C}_i^{\alpha_i}}{\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)^{\alpha_i}}\right] \\
&\quad = \operatorname{tr}\prod_{i=1}^{n}\left[\frac{\mathbf{A}_i}{\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)}\right]^{\alpha_i} + \operatorname{tr}\prod_{i=1}^{n}\left[\frac{\mathbf{B}_i}{\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)}\right]^{\alpha_i} \\
&\qquad + \operatorname{tr}\prod_{i=1}^{n}\left[\frac{\mathbf{C}_i}{\operatorname{tr}(\mathbf{A}_i + B_i + \mathbf{C}_i)}\right]^{\alpha_i} \\
&\quad \le \sum_{i=1}^{n} \frac{\alpha_i \operatorname{tr}\mathbf{A}_i}{\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)} + \sum_{i=1}^{n} \frac{\alpha_i \operatorname{tr}\mathbf{B}_i}{\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)} + \sum_{i=1}^{n} \frac{\alpha_i \operatorname{tr}\mathbf{C}_i}{\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)} \\
&\quad = \sum_{i=1}^{n} \frac{\alpha_i(\operatorname{tr}\mathbf{A}_i + \operatorname{tr}\mathbf{B}_i + \operatorname{tr}\mathbf{C}_i)}{\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)} \\
&\quad = \sum_{i=1}^{n} \alpha_i = 1.
\end{aligned} \tag{4.3}$$

Then we have

$$\operatorname{tr}\left[\prod_{i=1}^{n} \mathbf{A}_i^{\alpha_i} + \prod_{i=1}^{n} \mathbf{B}_i^{\alpha_i} + \prod_{i=1}^{n} \mathbf{C}_i^{\alpha_i}\right] \le \prod_{i=1}^{n} \left[\operatorname{tr}(\mathbf{A}_i + \mathbf{B}_i + \mathbf{C}_i)\right]^{\alpha_i}.$$ □

Now we use mathematical induction to deduce our third result.

Theorem 4.3 *Let* $\mathbf{A}_i$ $(i = 1, 2, \ldots, n)$ *be same size positive definite matrices. Then we have the inequality*

$$\frac{\operatorname{tr}(\sum_{i=1}^n \mathbf{A}_i)^2}{\operatorname{tr}(\sum_{i=1}^n \mathbf{A}_i)} \le \sum_{i=1}^n \frac{\operatorname{tr}\mathbf{A}_i^2}{\operatorname{tr}\mathbf{A}_i}.$$

Proof When $n = 2$, according to (2) on the first page, we have

$$\begin{aligned}
&2\operatorname{tr}\mathbf{A}_1 \cdot \operatorname{tr}\mathbf{A}_2 \cdot \operatorname{tr}(\mathbf{A}_1\mathbf{A}_2) \\
&\quad\le 2\operatorname{tr}\mathbf{A}_1 \cdot \operatorname{tr}\mathbf{A}_2 \cdot \left(\operatorname{tr}\mathbf{A}_1^2\right)^{\frac{1}{2}}\left(\operatorname{tr}\mathbf{A}_2^2\right)^{\frac{1}{2}} \\
&\quad= 2\left[\operatorname{tr}\mathbf{A}_1\left(\operatorname{tr}\mathbf{A}_2^2\right)^{\frac{1}{2}}\right]\left[\operatorname{tr}\mathbf{A}_2\left(\operatorname{tr}\mathbf{A}_1^2\right)^{\frac{1}{2}}\right] \\
&\quad\le \left[\operatorname{tr}\mathbf{A}_1\left(\operatorname{tr}\mathbf{A}_2^2\right)^{\frac{1}{2}}\right]^2 + \left[\operatorname{tr}\mathbf{A}_2\left(\operatorname{tr}\mathbf{A}_1^2\right)^{\frac{1}{2}}\right]^2 \\
&\quad= (\operatorname{tr}\mathbf{A}_1)^2\operatorname{tr}\mathbf{A}_2^2 + (\operatorname{tr}\mathbf{A}_2)^2\operatorname{tr}\mathbf{A}_1^2.
\end{aligned} \tag{4.4}$$

That is,

$$2\operatorname{tr}(\mathbf{A}_1\mathbf{A}_2) \le \frac{\operatorname{tr}\mathbf{A}_1 \cdot \operatorname{tr}\mathbf{A}_2^2}{\operatorname{tr}\mathbf{A}_2} + \frac{\operatorname{tr}\mathbf{A}_2 \cdot \operatorname{tr}\mathbf{A}_1^2}{\operatorname{tr}\mathbf{A}_1}. \tag{4.5}$$

On the other hand,

$$\operatorname{tr}(\mathbf{A}_1 + \mathbf{A}_2)^2 = \operatorname{tr}\mathbf{A}_1^2 + 2\operatorname{tr}(\mathbf{A}_1\mathbf{A}_2) + \operatorname{tr}\mathbf{A}_2^2. \tag{4.6}$$

Note that

$$\begin{aligned}
&\left(\frac{\operatorname{tr}\mathbf{A}_1^2}{\operatorname{tr}\mathbf{A}_1} + \frac{\operatorname{tr}\mathbf{A}_2^2}{\operatorname{tr}\mathbf{A}_2}\right)\operatorname{tr}(\mathbf{A}_1 + \mathbf{A}_2) \\
&\quad= \left(\frac{\operatorname{tr}\mathbf{A}_1^2}{\operatorname{tr}\mathbf{A}_1} + \frac{\operatorname{tr}\mathbf{A}_2^2}{\operatorname{tr}\mathbf{A}_2}\right)(\operatorname{tr}\mathbf{A}_1 + \operatorname{tr}\mathbf{A}_2) \\
&\quad= \operatorname{tr}\mathbf{A}_1^2 + \operatorname{tr}\mathbf{A}_2^2 + \frac{\operatorname{tr}\mathbf{A}_1 \cdot \operatorname{tr}\mathbf{A}_2^2}{\operatorname{tr}\mathbf{A}_2} + \frac{\operatorname{tr}\mathbf{A}_2 \cdot \operatorname{tr}\mathbf{A}_1^2}{\operatorname{tr}\mathbf{A}_1}.
\end{aligned} \tag{4.7}$$

So, according to (4.5), (4.6), we deduce

$$\begin{aligned}
&\left(\frac{\operatorname{tr}\mathbf{A}_1^2}{\operatorname{tr}\mathbf{A}_1} + \frac{\operatorname{tr}\mathbf{A}_2^2}{\operatorname{tr}\mathbf{A}_2}\right)\operatorname{tr}(\mathbf{A}_1 + \mathbf{A}_2) \\
&\quad\ge \operatorname{tr}\mathbf{A}_1^2 + 2\operatorname{tr}(\mathbf{A}_1\mathbf{A}_2) + \operatorname{tr}\mathbf{A}_2^2 = \operatorname{tr}(\mathbf{A}_1 + \mathbf{A}_2)^2.
\end{aligned} \tag{4.8}$$

Hence,

$$\frac{\operatorname{tr}(\mathbf{A}_1 + \mathbf{A}_2)^2}{\operatorname{tr}(\mathbf{A}_1 + \mathbf{A}_2)} \le \frac{\operatorname{tr}\mathbf{A}_1^2}{\operatorname{tr}\mathbf{A}_1} + \frac{\operatorname{tr}\mathbf{A}_2^2}{\operatorname{tr}\mathbf{A}_2}. \tag{4.9}$$

Then the inequality holds.

Suppose that the inequality holds when $n = k$, *i.e.*,

$$\frac{\operatorname{tr}(\sum_{i=1}^{k}\mathbf{A}_i)^2}{\operatorname{tr}(\sum_{i=1}^{k}\mathbf{A}_i)} \le \sum_{i=1}^{k}\frac{\operatorname{tr}\mathbf{A}_i^2}{\operatorname{tr}\mathbf{A}_i}.$$

Then for $n = k + 1$, we have

$$\begin{aligned}\frac{\operatorname{tr}(\sum_{i=1}^{k+1}\mathbf{A}_i)^2}{\operatorname{tr}(\sum_{i=1}^{k+1}\mathbf{A}_i)} &= \frac{\operatorname{tr}(\sum_{i=1}^{k}\mathbf{A}_i + \mathbf{A}_{k+1})^2}{\operatorname{tr}(\sum_{i=1}^{k}\mathbf{A}_i + \mathbf{A}_{k+1})} \\ &\le \frac{\operatorname{tr}(\sum_{i=1}^{k}\mathbf{A}_i)^2}{\operatorname{tr}(\sum_{i=1}^{k}\mathbf{A}_i)} + \frac{\operatorname{tr}\mathbf{A}_{k+1}^2}{\operatorname{tr}\mathbf{A}_{k+1}} \\ &\le \sum_{i=1}^{k}\frac{\operatorname{tr}\mathbf{A}_i^2}{\operatorname{tr}\mathbf{A}_i} + \frac{\operatorname{tr}\mathbf{A}_{k+1}^2}{\operatorname{tr}\mathbf{A}_{k+1}}. \end{aligned} \tag{4.10}$$

That is, the inequality holds when $n = k + 1$. Thus we have finished the proof. □

Competing interests

The author declares that they have no competing interests.

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