

Research Article

Study on Triopoly Dynamic Game Model Based on Different Demand Forecast Methods in the Market

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The impact of inaccurate demand beliefs on dynamics of a Triopoly game is studied. We suppose that all the players make their own estimations on possible demand with errors. A dynamic Triopoly game with such demand belief is set up. Based on this model, existence and local stable region of the equilibriums are investigated by 3D stable regions of Nash equilibrium point. The complex dynamics, such as bifurcation scenarios and route to chaos, are displayed in 2D bifurcation diagrams, in which e_1 and α are negatively related to each other. Basins of attraction are investigated and we found that the attraction domain becomes smaller with the increase in price modification speed, which indicates that all the players' output must be kept within a certain range so as to keep the system stable. Feedback control method is used to keep the system at an equilibrium state.

1. Introduction

A Triopoly is a market structure dominated by three firms in the market. The market is known as Cournot game if firms choose quantities as their strategic variables to maximize their profits in an uncertain demand environment.

In conventional market games, players are supposed to have common accurate demand functions of market. The dynamics of such system with this assumption have been intensively investigated in literature [1–8].

Assuming cost function to be twice differentiable increasing, Elabbasy et al. [1] analyzed the dynamics of oligopoly games with three types of players: bounded rational, naive, and adaptive.

Ma and Liu [2] studied a generalized nonlinear Fokker-Planck diffusion equation with external force and absorption. They obtained the corresponding exact solution expressed by q -exponential function and the solutions can have a compact behavior or a long tailed behavior.

Yassen and Agiza [3] studied a repeated Cournot game model with delayed bounded rationality in the duopoly market and demonstrated that the lagged structure is helpful

to expand the stable region of the system via numerical simulations.

Ma and Ren [4] focused on the influence of parameters on the macroeconomics IS-LM model and improved the analysis capabilities of the traditional economic models to suit the actual macroeconomic environment. They found that the system order has an important influence on the running state of the system.

Tramontana and Elsadany [5] discussed a triopolistic market with heterogeneous firms when the demand function is isoelastic. He found that double routes lead to chaos, via period-doubling and Neimark–Sacker bifurcations.

Ma and Ji [6] built a Triopoly outputs game model in electric power market. They obtained that the Triopoly model is a chaotic system and it is better than the duopoly model in applications.

Ma and Wu [7] studied the complexity of a Triopoly price game model and influence of delayed decisions on the stability. All those approaches assume that there is one uniform and accurate market demand function available and shared by all player.

Ma and Pu [8] researched the Cournot-Bertrand duopoly model with the application of nonlinear dynamics theory. They analyzed the stability of the fixed points and gave the bifurcation diagram and Lyapunov exponent spectrum along with the corresponding chaotic attractor. The research results show that either the change of output modification speed or the change of price modification speed will lead the market to the chaotic state which is disadvantageous for both of the firms.

All those literatures assume that there is one accurate market demand function, which is shared by all players. In practice, demand functions may be influenced by lots of different factors and every player has to estimate his own market demand function on the basis of past experience.

Because it is impossible for the firms to get the whole information, they cannot know the accurate demand function, so all the players have to make an estimate for demand function. Compared to real market demand function, it is inevitable for all the players to make demand evaluation biases. Thus, it is important to study how dynamics of the Triopoly game will be influenced by those evaluation biases, in terms of equilibrium points, local stability, and system performance.

A few works have been done to investigate the system dynamics, equilibrium offset with inaccurate demand beliefs. Bischi et al. [9] studied a model of a quantity-setting duopoly market with misspecified demand, the global dynamics of this game was investigated, and the number of steady states and their welfare properties were characterized. The impact of misspecified demand on the steady state was also studied. However, the asymmetric system with heterogeneous players' behavior has not been considered.

Wang and Ma [10] considered a Cournot-Bertrand mixed duopoly game model with limited information about the market and opponent. They studied the local stability of the game model at the Nash equilibrium point and discussed the influences of the parameters on the system's performance.

Bischi et al. [11] considered a repeated oligopoly game in single product Cournot oligopolies and proposed a method to learn demand function in a repeated oligopoly game via a closed loop feedback of real market price, which adjusts the evaluated demand function.

Guo and Ma [12] built a collecting price game model for a close-loop supply chain system with a manufacturer and a retailer who have different rationalities. They analyzed the influences of parameters on complex dynamic phenomena, such as the bifurcation, chaos, and continuous power spectrum.

Qiu et al. [13] studied the impact of uncertain demand on dynamic output-setting market games. A dynamic game with uncertain demand for two heterogeneous players was built. Based on this model, the impact of uncertain demand on the game's complex dynamics was investigated. Sun and Ma [14] constructed the three-oligopoly game model and investigated the existence of the fixed points. The 3D stable regions were given. The complex dynamic behavior of the game model is studied and the chaos was successfully controlled.

In this paper, we analyze the complex dynamics of a Triopoly model with heterogeneous players and demand evaluation bias, focusing on the following perspectives:

- (1) Impact of demand estimate bias on equilibrium, stable region, and profits.
- (2) Impact of adjustment strategy on basins of attraction.

The paper is organized as follows. In Section 2, a Triopoly game model with inaccurate demand belief is established. In Section 3, the existence and local stability of equilibrium points are discussed. The effects of inaccurate demand on stable region, profit, and equilibriums are shown in Section 4. Dynamical behaviors of the game are investigated by numerical simulations using 2D bifurcation diagrams [15] in Section 5. Basins of attraction [16] of the model are given in Section 7. In Section 8, conclusions are drawn from our analysis.

2. The Cournot Triopoly Game Model

We consider a Cournot Triopoly game in which the price and the demand of firm i 's product are denoted by p_i and q_i , $i = 1, 2, 3$, and the demand functions for the three firms are as follows:

$$p = a - b(q_1 + q_2 + q_3) \quad (1)$$

in which a and b are both positive constants. Assume that all the three firms have nonlinear cost function considering that if q_i exceeds a certain level, the cost will increase quickly and the cost function of the i th firm has a quadratic form [3]:

$$C_i(q_i) = c_i q_i^2 \quad (i = 1, 2, 3). \quad (2)$$

While in practice, not all the firms can get the whole information, they may do not know demand function (1), so all the players have to make an estimate for demand function.

For each player, we assume that the actual demand function held by player i can be denoted by multiplying the demand function (1) with an error coefficient e_i . And it has the following form:

$$p = e_i(a - b(q_1 + q_2 + q_3)) \quad (3)$$

which is called its subjective demand function.

The error coefficient e_i which is between (0, 3) means the imperfection degree of player i about the market.

If $e_i = 1$, it indicates that the evaluated demand function is just the true demand function. If $e_i < 1$, it indicates that the demand is underestimated by player, while if $e_i > 1$, it indicates the case where the demand is overestimated.

So the firms can get their maximum profits according to the following profit functions:

$$\begin{aligned} \pi_1(t) &= e_1(a - b(q_1 + q_2 + q_3))q_1 - c_1 q_1^2 \\ \pi_2(t) &= e_2(a - b(q_1 + q_2 + q_3))q_2 - c_2 q_2^2 \\ \pi_3(t) &= e_3(a - b(q_1 + q_2 + q_3))q_3 - c_3 q_3^2 \end{aligned} \quad (4)$$

in which $\pi_i(t)$ is the profit of firm i . Hence, the marginal profit functions of firms in period t are given by

$$\begin{aligned}\frac{\partial \pi_1(t)}{\partial q_1(t)} &= e_1 (a - 2bq_1(t) - b(q_2(t) + q_3(t))) - 2c_1q_1 \\ \frac{\partial \pi_2(t)}{\partial q_2(t)} &= e_2 (a - 2bq_2(t) - b(q_1(t) + q_3(t))) - 2c_2q_2 \quad (5) \\ \frac{\partial \pi_3(t)}{\partial q_3(t)} &= e_3 (a - 2bq_3(t) - b(q_2(t) + q_1(t))) - 2c_3q_3,\end{aligned}$$

while in practice, firms usually cannot get the whole information. For example, they cannot know other firm's price in the next period in advance, for which they cannot compute the price by the marginal profit functions above. In this paper, we consider all the firms as bounded rational players and their next-period price decisions are made on the basis of the local estimate to their marginal profit in current period. So the players make their strategies as the following form:

$$q_i(t+1) = q_i(t) + k_i q_i(t) \frac{\partial \pi_i(t)}{\partial q_i(t)} \quad (k_i = \alpha, \beta, \gamma). \quad (6)$$

The equation means that if the marginal profit of the current period is positive, the firm will raise its price the next period; otherwise, it will reduce it. So the dynamical Triopoly system can be described as

$$\begin{aligned}q_1(t+1) &= q_1(t) + \alpha q_1(t) G_1(t) \\ q_2(t+1) &= q_2(t) + \beta q_2(t) G_2(t) \quad (7) \\ q_3(t+1) &= q_3(t) + \gamma q_3(t) G_3(t),\end{aligned}$$

where

$$\begin{aligned}G_1(t) &= e_1 (a - 2bq_1(t) - b(q_2(t) + q_3(t))) - 2c_1q_1 \\ G_2(t) &= e_2 (a - 2bq_2(t) - b(q_1(t) + q_3(t))) - 2c_2q_2 \quad (8) \\ G_3(t) &= e_3 (a - 2bq_3(t) - b(q_2(t) + q_1(t))) - 2c_3q_3.\end{aligned}$$

$0 < \alpha, \beta, \gamma < 1$ denote the players' adjustment speeds, respectively.

3. Equilibrium Points and Local Stability

3.1. Equilibrium Points. According to system (7), let $p_i(t+1) = p_i(t)$; then eight equilibrium points can be obtained:

$$\begin{aligned}E_1 &= (0, 0, 0) \\ E_2 &= \left(0, \frac{ae_2}{2(c_2 + be_2)}, 0\right) \\ E_3 &= \left(\frac{ae_1}{2(c_1 + be_1)}, 0, 0\right) \\ E_4 &= \left(0, 0, \frac{ae_3}{2(c_3 + be_3)}\right) \\ E_5 &= \left(0, \frac{ae_2(2c_3 + be_3)}{(4c_2c_3 + 4bc_2e_3 + 4bc_3e_2 + 3b^2e_2e_3)}, \frac{ae_3(2c_2 + be_2)}{(4c_2c_3 + 4bc_2e_3 + 4bc_3e_2 + 3b^2e_2e_3)}\right) \\ E_6 &= \left(\frac{ae_1(2c_2 + be_2)}{(4c_2c_1 + 4bc_1e_2 + 4bc_2e_1 + 3b^2e_2e_1)}, \frac{ae_2(2c_1 + be_1)}{(4c_2c_1 + 4bc_1e_2 + 4bc_2e_1 + 3b^2e_2e_1)}, 0\right) \\ E_7 &= \left(\frac{ae_1(2c_3 + be_3)}{(4c_3c_1 + 4bc_1e_3 + 4bc_3e_1 + 3b^2e_3e_1)}, 0, \frac{ae_3(2c_1 + be_1)}{(4c_3c_1 + 4bc_1e_3 + 4bc_3e_1 + 3b^2e_3e_1)}\right),\end{aligned} \quad (9)$$

and the Nash equilibrium point

$$E_8 = (q_1^*, q_2^*, q_3^*) \quad (10)$$

can be obtained, where

$$\begin{aligned}q_1^* &= \frac{(4ac_2c_3e_1 + 2abc_2e_1e_3 + 2abc_3e_1e_2 + ab^2e_1e_2e_3)}{(2(4c_1c_2c_3 + 3b^2c_1e_2e_3 + 3b^2c_2e_1e_3 + 3b^2c_3e_1e_2 + 2b^3e_1e_2e_3 + 4bc_1c_2e_3 + 4bc_1c_3e_2 + 4bc_2c_3e_1))} \\ q_2^* &= \frac{(ae_2(2c_1 + be_1)(2c_3 + be_3))}{(2(4c_1c_2c_3 + 3b^2c_1e_2e_3 + 3b^2c_2e_1e_3 + 3b^2c_3e_1e_2 + 2b^3e_1e_2e_3 + 4bc_1c_2e_3 + 4bc_1c_3e_2 + 4bc_2c_3e_1))} \\ q_3^* &= \frac{(ae_3(2c_1 + be_1)(2c_2 + be_2))}{(2(4c_1c_2c_3 + 3b^2c_1e_2e_3 + 3b^2c_2e_1e_3 + 3b^2c_3e_1e_2 + 2b^3e_1e_2e_3 + 4bc_1c_2e_3 + 4bc_1c_3e_2 + 4bc_2c_3e_1))}.\end{aligned} \quad (11)$$

It can be seen from above that E_8 is independent of the adjustment factors. From an economic point of view, that means

the value of the system local stability point in this dynamic game is independent of the players' adjustment speed, but just

determined by the characteristics of the system. We can find from (11) that if e_i of player i increases and e_j of other players remains the same, q_i^* will increase.

3.2. Nash Points in Error-Free System-Benchmark. If the players have perfect knowledge, their subjective demand functions totally coincide with the real ones. Setting $e_1 = e_2 = e_3 = 1$ in (11), we can get

$$\begin{aligned} q_1^* &= \frac{a(b+2c_2)(b+2c_3)}{(6b^2c_1+6b^2c_2+6b^2c_3+4b^3+8bc_1c_2+8bc_1c_3+8bc_2c_3+8c_1c_2c_3)} \\ q_2^* &= \frac{a(b+2c_1)(b+2c_3)}{(6b^2c_1+6b^2c_2+6b^2c_3+4b^3+8bc_1c_2+8bc_1c_3+8bc_2c_3+8c_1c_2c_3)} \\ q_3^* &= \frac{a(b+2c_1)(b+2c_2)}{(6b^2c_1+6b^2c_2+6b^2c_3+4b^3+8bc_1c_2+8bc_1c_3+8bc_2c_3+8c_1c_2c_3)}. \end{aligned} \quad (12)$$

Note that the denominator of q_1^* , q_2^* , and q_3^* is the same, the output depends on the cost of the players, and the greater the cost, the lower the yield. These results match the results in [17].

3.3. Local Stability of System Equilibriums. In order to analyze the stability of the preceding equilibrium points, the Jacobian matrix for discrete dynamic system (7) is found as follows:

$$J = \begin{pmatrix} J_1 & -\alpha be_1 q_1 & -\alpha be_1 q_1 \\ -\beta be_2 q_2 & J_2 & -\beta be_2 q_2 \\ -\gamma be_3 q_3 & -\gamma be_3 q_3 & J_3 \end{pmatrix} \quad (13)$$

in which

$$\begin{aligned} J_1 &= 1 - \alpha q_1 (2c_1 + 2be_1) \\ &\quad - \alpha (e_1 (b(q_2 + q_3) - a + 2bq_1) + 2c_1 q_1) \\ J_2 &= 1 - \beta q_2 (2c_2 + 2be_2) \\ &\quad - \beta (e_2 (b(q_1 + q_3) - a + 2bq_2) + 2c_2 q_2) \\ J_3 &= 1 - \gamma q_3 (2c_3 + 2be_3) \\ &\quad - \gamma (e_3 (b(q_1 + q_2) - a + 2bq_3) + 2c_3 q_3). \end{aligned} \quad (14)$$

According to Routh-Hurwitz condition, the necessary and sufficient conditions for equilibrium points to be asymptotically stable are that all roots of the characteristic equations have magnitudes of eigenvalues less than 1.

Remark 1. E_1, E_2, E_3, E_4 are unstable equilibrium points.

As for E_1 , $J_1 = 1 + \alpha a > 1$ is one eigenvalue which corresponds to E_1 , so E_1 is an unstable equilibrium point.

As for E_2 , $J_1 = 1 + \alpha e_1 a (1 - be_2 / (2c_2 + 2be_2)) > 1$ is one eigenvalue which corresponds to E_2 , so E_2 is an unstable equilibrium point. In the same way we can prove that E_3 and E_4 are unstable equilibrium points.

Remark 2. E_5, E_6 , and E_7 are unstable equilibrium points.

As for E_5 , $J_1 = 1 - \alpha (e_1 (b(q_2 + q_3) - a))$ is one eigenvalue which corresponds to E_1 ; set $q_2 = ae_2(2c_3 + be_3)/(4c_2c_3 +$

$4bc_2e_3 + 4bc_3e_2 + 3b^2e_2e_3)$, $q_3 = ae_3(2c_2 + be_2)/(4c_2c_3 + 4bc_2e_3 + 4bc_3e_2 + 3b^2e_2e_3)$, and then $b(q_2 + q_3) - a < 0$, so $J_1 = 1 - \alpha (e_1 (b(q_2 + q_3) - a)) > 1$.

So E_5 is an unstable equilibrium point. In the same way we can prove that E_6 and E_7 are unstable equilibrium points.

From an economic point of view, in the stable state of this dynamic game, no player is forced to withdraw from the market.

As for E_8 , the necessary and sufficient condition of asymptotic stability is that all the eigenvalues are inside the unit circle in complex plane. So a stable system must satisfy the following conditions:

$$\begin{aligned} f(1) &= 1 + A + B + C > 0 \\ -f(-1) &= 1 - A + B - C > 0 \\ C^2 - 1 &< 0 \end{aligned} \quad (15)$$

$$(1 - C^2)^2 - (B - AC)^2 > 0,$$

where $f(\lambda) = \lambda^3 + A\lambda^2 + B\lambda + C = 0$ is the characteristic polynomial at E_8 .

For convenience, we set the parameters as follows:

$$\begin{aligned} a &= 10, \\ b &= 0.2, \\ c_1 &= 0.1, \\ c_2 &= 0.1, \\ c_3 &= 0.1, \\ e_1 &= 0.9, \\ e_2 &= 1.0; \\ e_3 &= 1.1; \end{aligned} \quad (16)$$

and the initial values are chosen as $(1, 1, 1)$.

According to the parameters above,

$$E_8 = (9.4832, 10.0100, 10.4867). \quad (17)$$

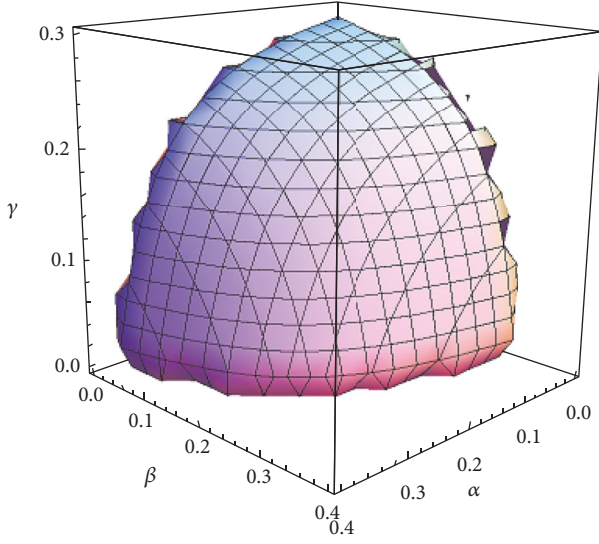


FIGURE 1: The stable region of Nash equilibrium point, $(e_1, e_2, e_3) = (0.9, 1.0, 1.1)$.

Its Jacobian matrix is

$$J(E_8) = \begin{pmatrix} 1.0 - 5.31\alpha & -1.71\alpha & -1.71\alpha \\ -2.0\beta & 1.0 - 6.01\beta & -2.0\beta \\ -2.31\gamma & -2.31\gamma & 1.0 - 6.71\gamma \end{pmatrix}. \quad (18)$$

The characteristic equation of Jacobian matrix (18) is

$$f(\lambda) = \lambda^3 + A_1\lambda^2 + B_1\lambda + C_1 = 0 \quad (19)$$

in which

$$\begin{aligned} A_1 &= (5.31\alpha + 6.01\beta + 6.71\gamma - 3.0) \\ B_1 &= ((6.71\gamma - 1.0)(5.31\alpha + 6.01\beta - 2.0) - 3.42\alpha\beta \\ &\quad - 3.94\alpha\gamma - 4.62\beta\gamma + (5.31\alpha - 1.0)(6.01\beta - 1.0)) \\ C_1 &= 2.31\gamma(3.42\alpha\beta + 1.71\alpha(5.31\alpha - 1.0)) - (3.94\alpha\gamma \\ &\quad + 4.62\beta\gamma)(5.31\alpha + 6.01\beta - 2.0) - (6.71\gamma - 1.0) \\ &\quad \cdot (3.42\alpha\beta - (5.31\alpha - 1.0)(6.01\beta - 1.0)) \\ &\quad + 2.31\gamma(2.0\beta(6.01\beta - 1.0) + 3.42\alpha\beta). \end{aligned} \quad (20)$$

As what can be shown in Figure 1, a stable region in the space of (α, β, γ) is determined by the above inequalities. In the stable region, the final prices of the three oligarchs will stay stable at E_8 after a number of games. From Figure 1, we can see that the market is stable when $\nu \in [0, 1]$, but the market may be unstable when α, β, γ increases. The economic meaning of the stable region is that if (α, β, γ) is in the stable region, prices of three firms will achieve the Nash equilibrium at last.

3.4. The Effects of Parameters e_i on Stable Region. In order to analyze the effects of parameter w on stable region, let

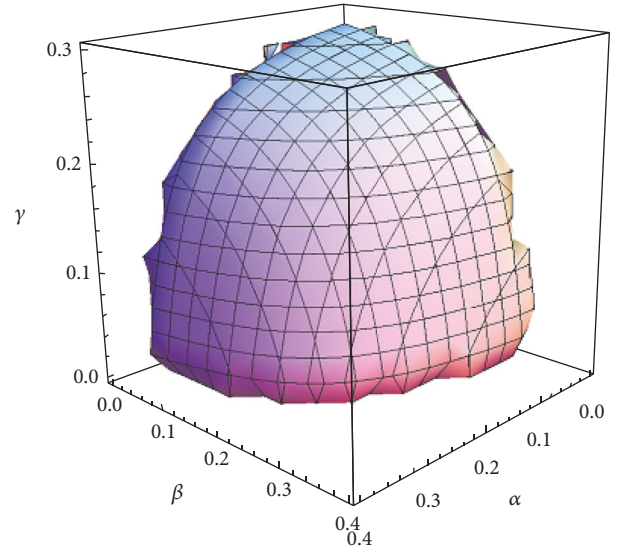


FIGURE 2: The stable region of Nash equilibrium point, $(e_1, e_2, e_3) = (1, 1.1, 1.2)$.

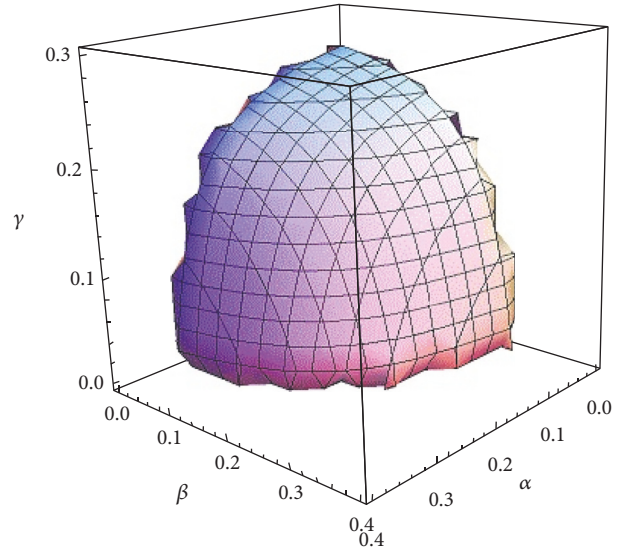


FIGURE 3: The stable region of Nash equilibrium point, $(e_1, e_2, e_3) = (0.8, 0.9, 1)$.

$(e_1, e_2, e_3) = (1, 1.1, 1.2)$ and $(0.8, 0.9, 1)$, respectively; then the corresponding stable region is shown in Figures 2 and 3. From the comparison we find that, with the increase of e_i , the stable region narrows.

From an economic point of view, if the players overestimate the demand, the range of price adjustment speed will be smaller; however if the players underestimate the demand, the range of price adjustment speed will be bigger.

4. Bifurcation Diagrams

4.1. 2D Bifurcation Diagrams and Interactive Relationships between e_1 and α, β , and γ . 2D bifurcation diagram is a more powerful tool in the numerical analysis of nonlinear

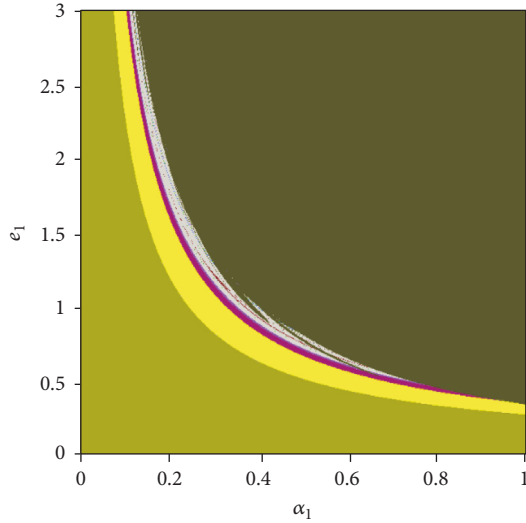


FIGURE 4: (e_1, α) -2D bifurcation diagram with $\beta = 0.2$ and $\gamma = 0.1$.

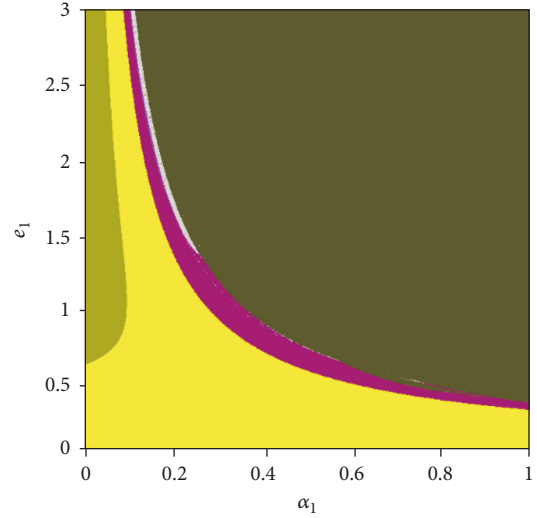


FIGURE 6: (e_1, α) -2D bifurcation diagram with $\beta = 0.3$ and $\gamma = 0.1$.

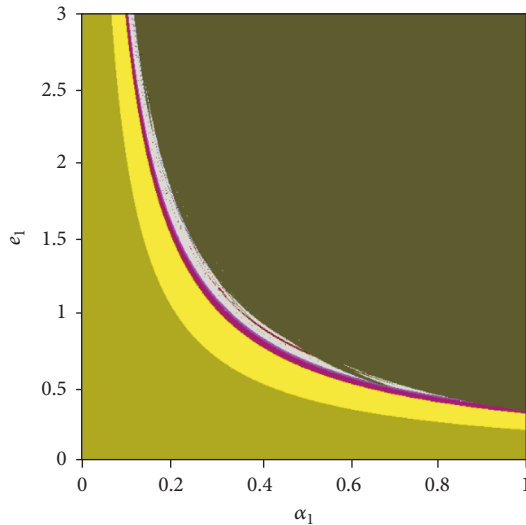


FIGURE 5: (e_1, α) -2D bifurcation diagram with $\beta = 0.25$ and $\gamma = 0.1$.

dynamics than a 1D bifurcation diagram. In the 2D bifurcation diagram, bifurcation scenarios and route to chaos can be displayed more clearly. In this section, the 2D bifurcation diagram will be used to analyze the effects of players' adjustment speeds and e_1 on system stability.

For convenience, we choose $e_2 = 1$ and $e_3 = 1.1$ and study the interactive relationships between e_1 and α by 2D bifurcation diagrams.

First let $\gamma = 0.1$ and $\beta = 0.2, 0.25, 0.3$, respectively; then three (α, e_1) 2D bifurcation diagrams are shown in Figures 4, 5, and 6.

In Figures 4–6, different colors are assigned to each region to show the particular behavior of system (7), that is, light green, stable states; yellow, period-2 stable cycles; purple, period-4; gray, chaotic state; dark green, escape.

In the 2D bifurcation diagrams, the system exhibits a sequence of flip bifurcations to chaos (which means that the

market will fall into chaos), then to divergence at last (which means the players will be out of the market).

As seen from Figure 4, if player 1's adjustment speed is relatively slow (in the brown area), the economic system will be in a stable state. Along with the increase in the adjustment speed parameters, the economic system will experience cyclical shocks, chaos, and even disappearance. Obviously, relatively large parameters are detrimental to the economic system.

We can find the following results from Figures 4, 5, and 6,

- (1) We find that, in the stable region in every figure, with the increase in α , the maximum of e_1 decreases; with the increase in e_1 , the maximum of α decreases.
- (2) We find that, with the increase in β , stable region reduces, while escape region does not expand obviously, but period-2 stable cycles expand obviously.

Secondly fix β at 0.1, let $\gamma = 0.2, 0.25, 0.3$, respectively, and then three (α, e_1) 2D bifurcation diagrams can be shown in Figures 7, 8, and 9. Comparing Figures 7, 8, and 9, we find that, with the increasing of γ , stable region reduces, while escape region does not expand obviously, but period-2 stable cycles and period-4 stable cycles expand obviously, from the comparison of Figures 7, 8, and 9, stable region of player 1 reduces when β and γ increase.

Let $\beta = 0.2$ and $\gamma = 0.2$, and then we can get Figure 10.

Comparing Figures 4 and 10, we can find that β and γ are nearly symmetrical and any parameter (β and γ) of changes will make stable region smaller.

4.2. 1D Bifurcation Diagrams and Interactive Attractors. We will display the consistency between 1D bifurcation diagrams and 2D bifurcation diagrams for Figure 10 and show the attractors.

In Figure 10, $\beta = 0.2$ and $\gamma = 0.2$; if e_1 is fixed at 0.9, then we can get the bifurcation diagrams with α in Figure 11, in which blue set of points denotes $p_1(t)$, red set of points

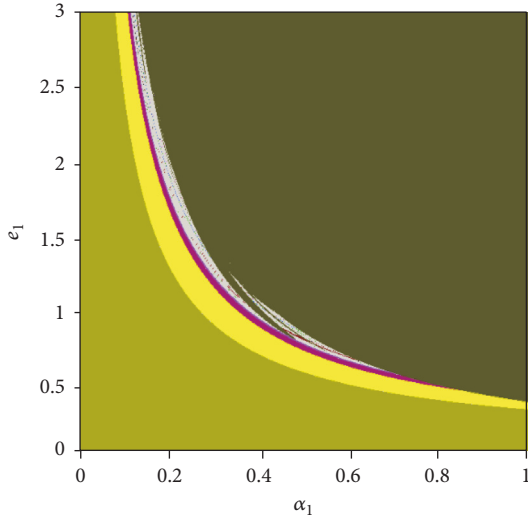


FIGURE 7: (e_1, α) -2D bifurcation diagram with $\beta = 0.1$ and $\gamma = 0.2$.

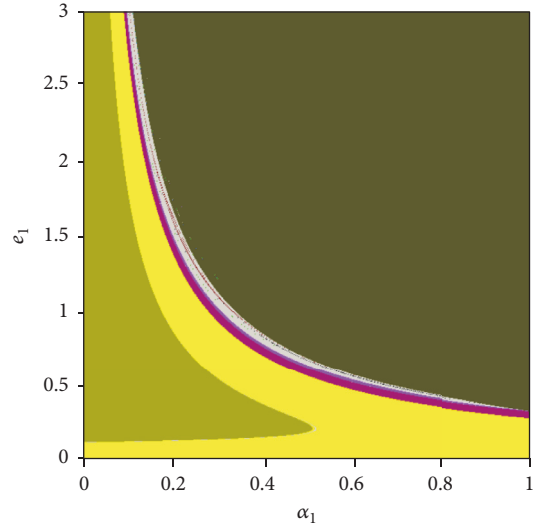


FIGURE 10: (e_1, α) -2D bifurcation diagram with $\beta = 0.2$ and $\gamma = 0.2$.

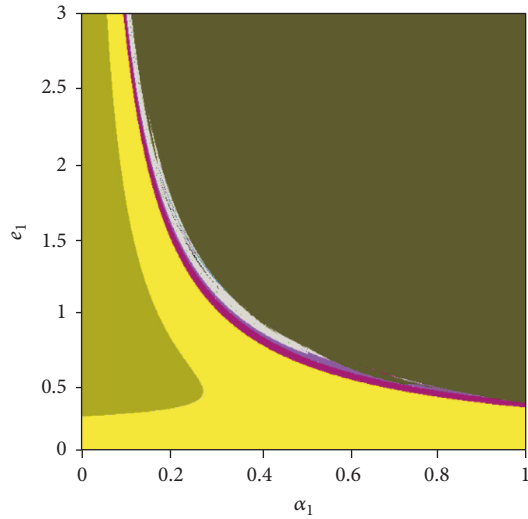


FIGURE 8: (e_1, α) -2D bifurcation diagram with $\beta = 0.1$ and $\gamma = 0.25$.

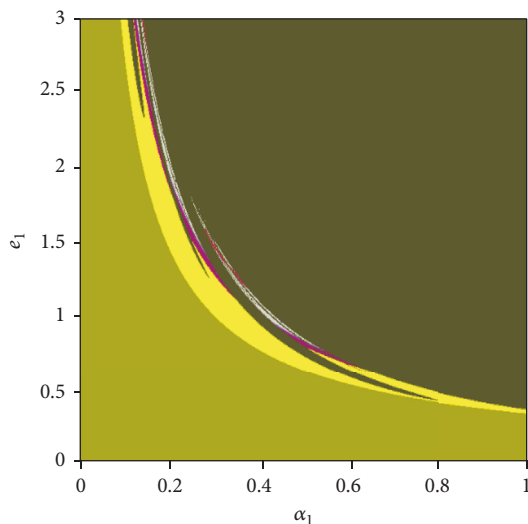


FIGURE 9: (e_1, α) -2D bifurcation diagram with $\beta = 0.1$ and $\gamma = 0.3$.

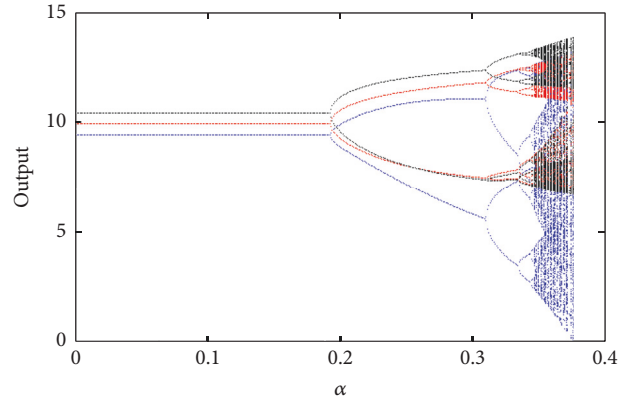


FIGURE 11: Bifurcation diagram with $\alpha, \beta = 0.2$ and $\gamma = 0.2$.

denotes $p_2(t)$, and black set of points denotes $p_3(t)$. As can be seen, system (7) loses its stability when $\alpha = 0.2$, and after a series of flip bifurcations, it falls into chaos when $\alpha = 0.36$, which is consistent with Figure 10. We can find that, with the same cost, the player i with bigger e_i has a higher equilibrium output, and player i with smaller e_i has a lower equilibrium output. We also give the corresponding largest Lyapunov exponent (LLE), which is consistent with Figure 12.

According to Figure 8, when $\alpha = 0.3$ and $\beta = 0.6$, LLE is positive, then system (7) is in chaos, and the chaotic attractor is shown in Figure 13.

From an economic point of view, the appearance of flip bifurcation means market gradually going into the chaotic state from the constant and violent fluctuations.

Under certain conditions, higher e_1 can improve the equilibrium result, as shown in Figure 11. However, it is not to say that the increase in e_1 will certainly increase the equilibrium output, if e_1 goes beyond the stability region. Increase in e_1 may lead to fluctuations in the system and not necessarily can play a role in improving production, which

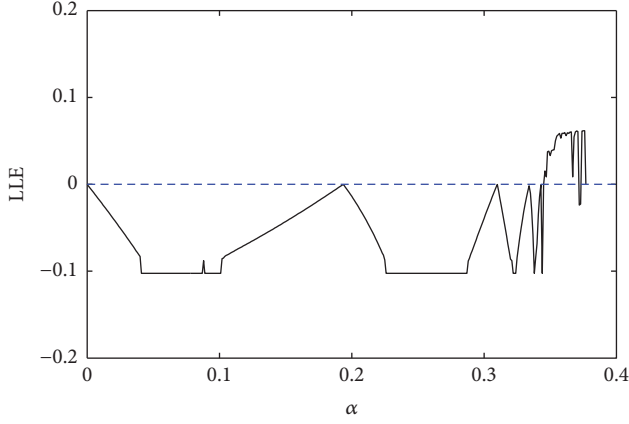


FIGURE 12: Largest Lyapunov exponent with $\alpha, \beta = 0.2$ and $\gamma = 0.2$.

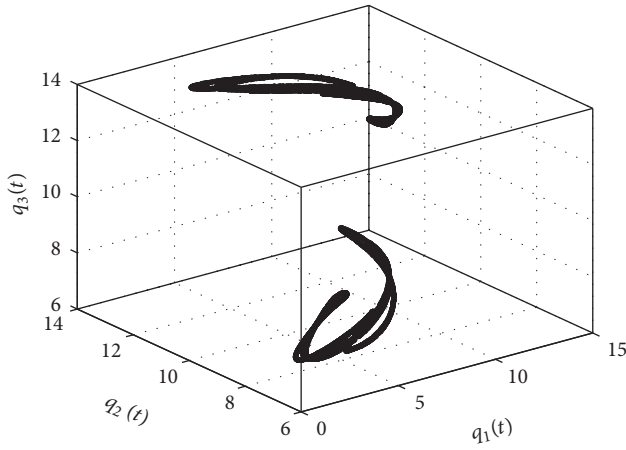


FIGURE 13: The chaotic attractor of system (7) with $\alpha = 0.36, \beta = 0.2$, and $\gamma = 0.2$.

can be seen in Figures 14, 15, and 16. We also find that the increase in e_1 will decrease output of other players.

5. The Effects of e_i on Profits

Next we will discuss the effects of e_i on profits; merge (4)–(7):

$$\begin{aligned}
 q_1(t+1) &= q_1(t) + \alpha q_1(t) \\
 [e_1(a - 2bq_1(t) - b(q_2(t) + q_3(t))) - 2c_1q_1] \\
 q_2(t+1) &= q_2(t) + \beta q_2(t) \\
 [e_2(a - 2bq_2(t) - b(q_1(t) + q_3(t))) - 2c_2q_2] \\
 q_3(t+1) &= q_3(t) + \gamma q_3(t) \\
 [e_3(a - 2bq_3(t) - b(q_2(t) + q_1(t))) - 2c_3q_3] \\
 \pi_1(t) &= e_1(a - b(q_1 + q_2 + q_3))q_1 - c_1q_1^2 \\
 \pi_2(t) &= e_2(a - b(q_1 + q_2 + q_3))q_2 - c_2q_2^2 \\
 \pi_3(t) &= e_3(a - b(q_1 + q_2 + q_3))q_3 - c_3q_3^2.
 \end{aligned} \tag{21}$$

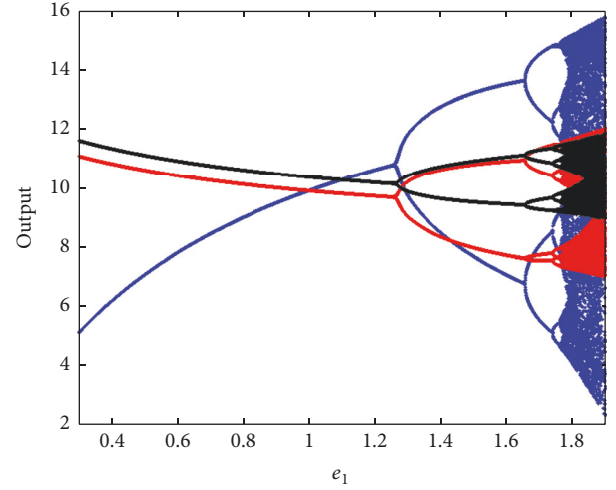


FIGURE 14: Bifurcation diagram with $e_1, \alpha = 0.2, \beta = 0.2$, and $\gamma = 0.1$.

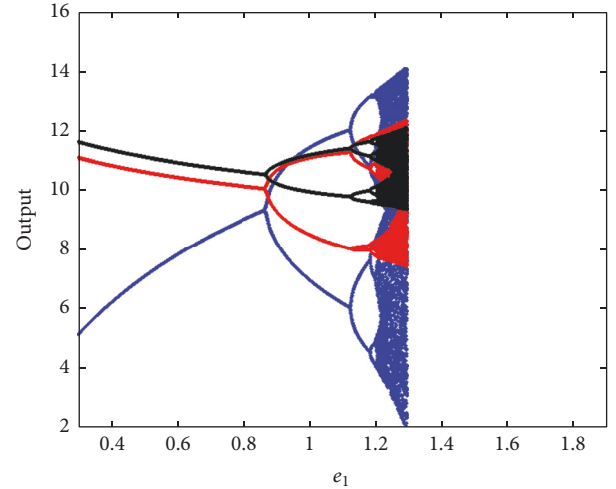


FIGURE 15: Bifurcation diagram with $e_1, \alpha = 0.3, \beta = 0.2$, and $\gamma = 0.1$.

The effects of e_1 on the three firms' profits can be shown in Figures 17–19.

The values of the parameters are the same as above, blue set of points denotes $\pi_1(t)$, red set of points denotes $\pi_2(t)$, and black set of points denotes $\pi_3(t)$.

We can conclude from Figure 17 that if $e_1 < 1.3$, with increase of e_1 , $\pi_1(t)$ increases, $\pi_2(t)$ and $\pi_3(t)$ decrease; when $e_1 > 1.3$, all the firms' profits will lose stability and even fall into chaos. Comparing Figures 15 and 16, we can get Figures 18 and 19.

When the speed of adjustment is accelerated, increase of e_1 may cause the system to lose stability, not necessarily to improve the profit.

An interesting phenomenon is that, compared with bifurcation diagram with e_1 , fluctuations in profits are smaller than fluctuations in output.

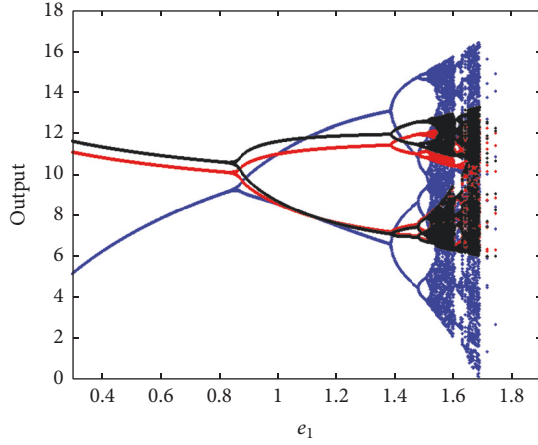


FIGURE 16: Bifurcation diagram with e_1 , $\alpha = 0.2$, $\beta = 0.2$, and $\gamma = 0.2$.

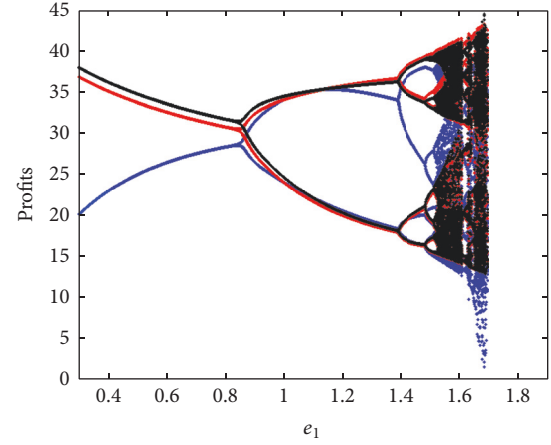


FIGURE 19: Effects of e_1 on profits, $\alpha = 0.2$, $\beta = 0.2$, and $\gamma = 0.2$.

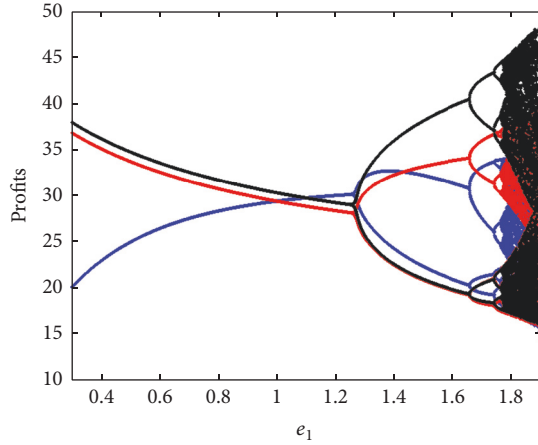


FIGURE 17: Effects of e_1 on profits, $\alpha = 0.2$, $\beta = 0.2$, and $\gamma = 0.1$.

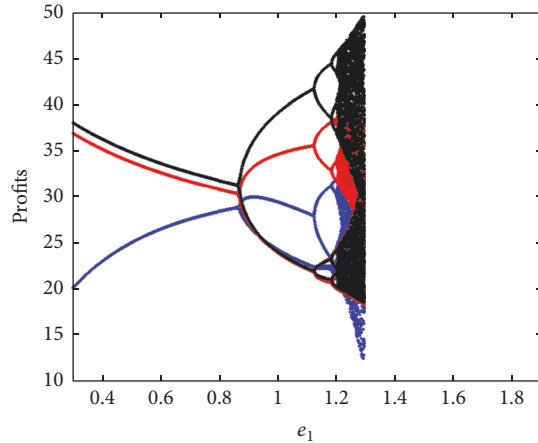


FIGURE 18: Effects of e_1 on profits, $\alpha = 0.3$, $\beta = 0.2$, and $\gamma = 0.1$.

6. Chaos Control

According to the above numerical simulation, we can see that if the firms' price adjustment speeds are beyond the stable region, the market will lose stability and even fall into chaos.

Chaos in the economic systems is harmful to all the firms. In order to avert the risk, it is expedient for prices chosen by Triopoly to maintain at Nash equilibrium.

Many methods for the chaos control have been proposed, such as time-delayed feedback method [18], modified straight-line stabilization method [19], OGY method [20], and pole placement method [21]. In this section, feedback control method proposed by Elabbasy et al. [1] is used, so the controlled system is given by

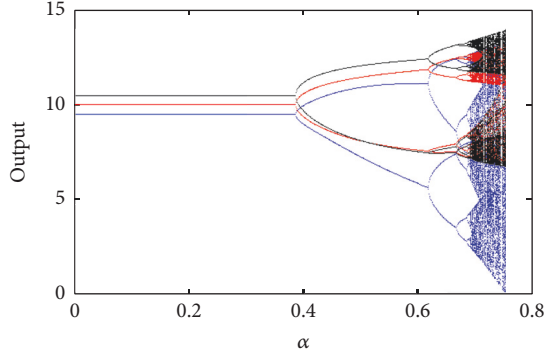
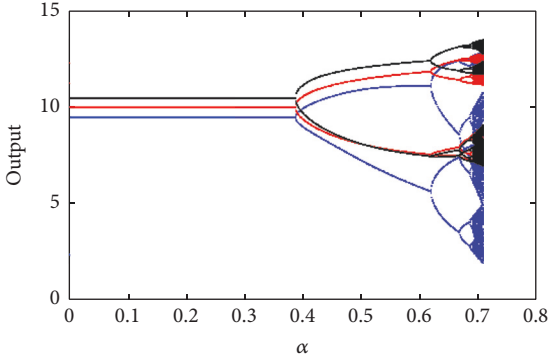
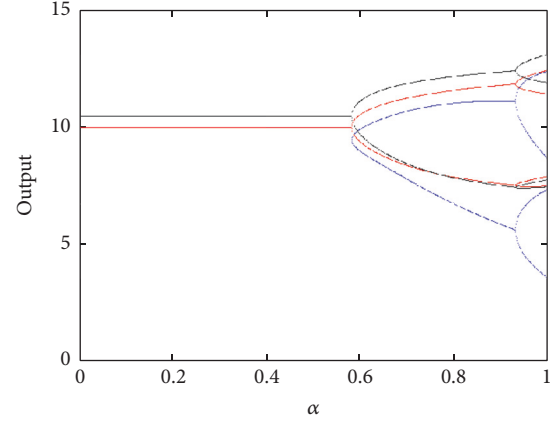
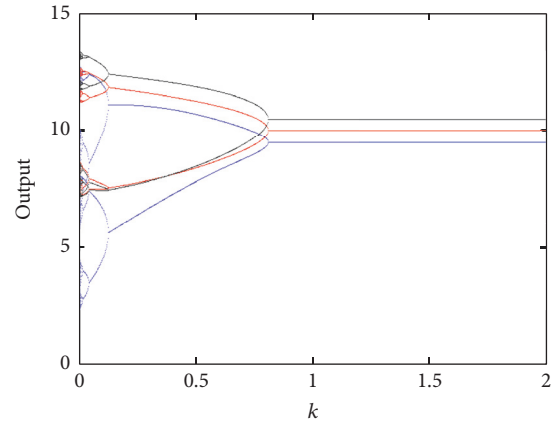
$$\begin{aligned}
 q_1(t+1) &= q_1(t) + \alpha q_1(t) \\
 [e_1(a - 2bq_1(t) - b(q_2(t) + q_3(t))) - 2c_1q_1] \\
 q_2(t+1) &= q_2(t) + \frac{\beta}{k+1}q_2(t) \\
 [e_2(a - 2bq_2(t) - b(q_1(t) + q_3(t))) - 2c_2q_2] \\
 q_3(t+1) &= q_3(t) + \gamma q_3(t) \\
 [e_3(a - 2bq_3(t) - b(q_2(t) + q_1(t))) - 2c_3q_3],
 \end{aligned} \tag{22}$$

where k is the controlling factor and the Jacobian matrix of (22) is given as

$$J = \begin{pmatrix} J_1 & -\alpha be_1 q_1 & -\alpha be_1 q_1 \\ -\frac{\beta}{k+1} be_2 q_2 & \frac{J_2}{k+1} & -\frac{\beta}{k+1} be_2 q_2 \\ -\gamma be_3 q_3 & -\gamma be_3 q_3 & J_3 \end{pmatrix}, \tag{23}$$

where

$$\begin{aligned}
 J_1 &= 1 - \alpha q_1 (2c_1 + 2be_1) \\
 &\quad - \alpha (e_1 (b(q_2 + q_3) - a + 2bq_1) + 2c_1 q_1) \\
 J_2 &= 1 - \beta q_2 (2c_2 + 2be_2) \\
 &\quad - \beta (e_2 (b(q_1 + q_3) - a + 2bq_2) + 2c_2 q_2) \\
 J_3 &= 1 - \gamma q_3 (2c_3 + 2be_3) \\
 &\quad - \gamma (e_3 (b(q_1 + q_2) - a + 2bq_3) + 2c_3 q_3).
 \end{aligned} \tag{24}$$

FIGURE 20: Bifurcation diagram with α , $k = 0.5$.FIGURE 21: Bifurcation diagram with α , $k = 1$.FIGURE 22: Bifurcation diagram with α , $k = 2$.FIGURE 23: Bifurcation diagram with the controlling factor k , $\alpha = 0.35$, $\beta = 0.2$, and $\gamma = 0.2$.

In the practice market, k can be considered as the learning ability or adaptability of firm 2. For example, firm 2 analyzed the information in the past and adjusted the speed of price. As what can be seen from Figures 20, 21, and 22, the chaos can be delayed and even eliminated with proper k . As for Figure 11, set $k = 0.5, 1, 2$, respectively; we can get Figures 20–22.

As can be seen, with the increase of the control factor k , the emergence of bifurcation is delayed. So if the second bounded rational player adopts this adjustment method, the price game can reach equilibrium state finally.

Let $\alpha = 0.35$, $\beta = 0.2$, and $\gamma = 0.2$, according to Figure 11; the system is in chaos, if α , β , and γ are fixed, and player 2 changes the controlling factor k ; we can get the following bifurcation diagram with the controlling factor k . As can see from Figure 23, when $k < 0.028$, the system is in a chaotic state, when $0.029 \leq k \leq 0.030$, the system is in a 16-period cycle, when $0.031 \leq k \leq 0.037$, the system is in 8-period cycle, when $0.038 \leq k \leq 0.16$, the system is in a 4-period cycle, when $0.16 \leq k \leq 0.76$, the system is in a 2-period cycle state, and the system reaches equilibrium state when $k \leq 0.76$.

7. Global Stability of the System

In order to investigate the impact of price adjustment speed on the global stability, we introduced basins of attraction, which include attraction domain, attractor, and escaping area. Let $(e_1, e_2) = (1, 1)$, and fix q_3 at 10, 7, 5. We make basins of attraction with initial output q_1 and q_2 .

The attraction domain is the set of initial output where the same attractor will emerge after a series of iteration if the initial price is taken from the attraction domain. As for the attractor, if it is one equilibrium point, from an economic point of view, the corresponding attraction domain will be a safe region, which means that if the initial output of two sides is in the safe region, the system will remain stable after iteration. If the initial price is in the escape area, the system will fall into divergence at last.

By fixing the system parameters as mentioned above and setting $(\alpha, \beta) = (0.3, 0.1), (0.3, 0.25)$, respectively, six basins of attraction about (p_1, p_2) of the system are shown in Figures 24–29 in which the green region denotes attraction domain, the red set of points denotes attractor, and the blue set of points denotes escape area.

In Figure 24, when $\alpha = 0.3$, $\beta = 0.1$, and $q_3 = 10$, the system is in 2-period cycle, and we can see that the attraction domain is an irregular hexagon. In Figure 25, when $\alpha = 0.3$, $\beta = 0.25$, and $q_3 = 10$, the system is in period-2 state, which means that if the initial prices of two sides are in this attraction domain, then outputs will oscillate between two points at last. The attraction domain is also an irregular pentagon area and chaotic attractor appears. In Figure 26, when $\alpha = 0.3$, $\beta = 0.1$, and $q_3 = 7$, the system is in 2-period

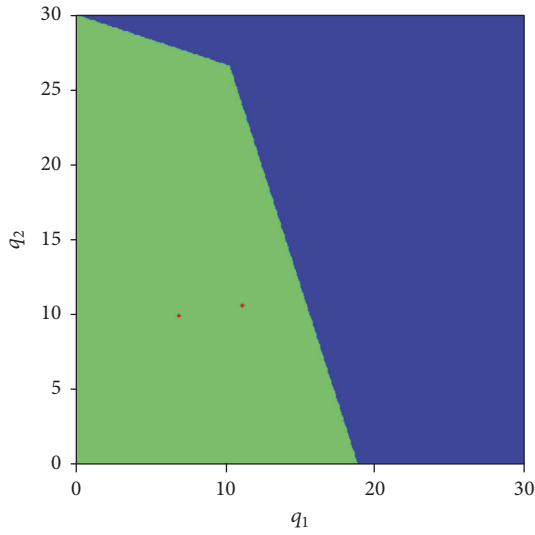


FIGURE 24: Basins of attraction, $\alpha = 0.3$, $\beta = 0.1$, and $q_3 = 10$.

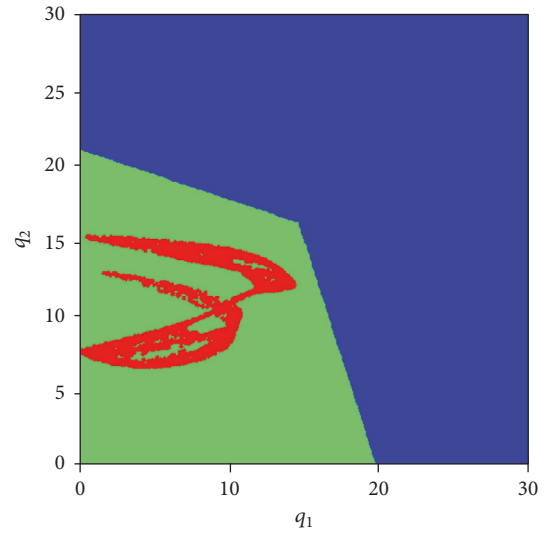


FIGURE 27: Basins of attraction, $\alpha = 0.3$, $\beta = 0.25$, and $q_3 = 7$.

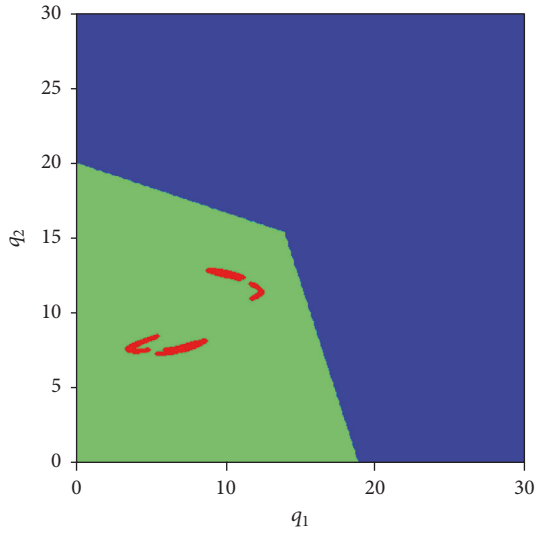


FIGURE 25: Basins of attraction, $\alpha = 0.3$, $\beta = 0.25$, and $q_3 = 10$.

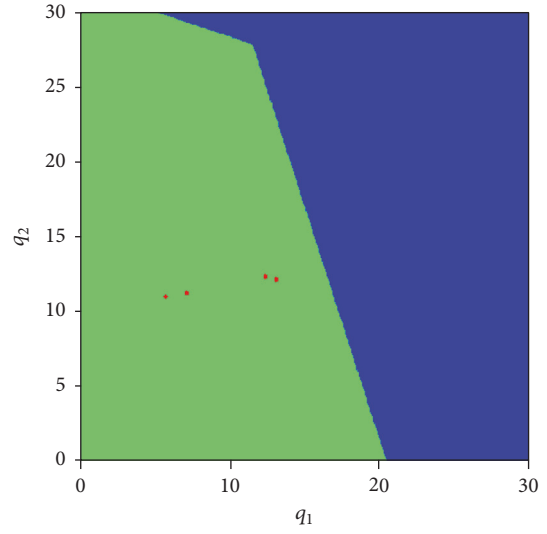


FIGURE 28: Basins of attraction, $\alpha = 0.3$, $\beta = 0.1$, and $q_3 = 5$.

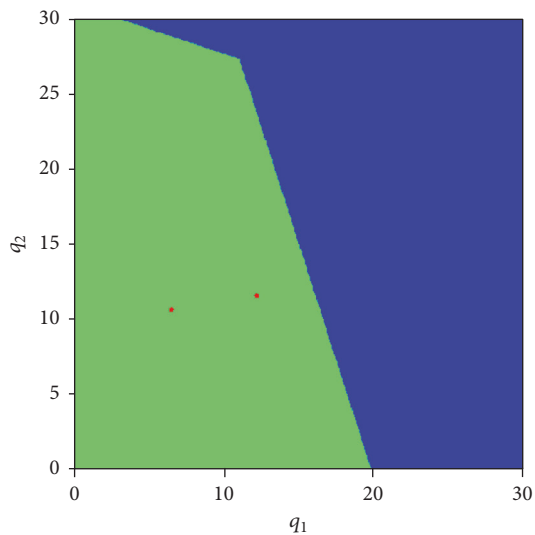


FIGURE 26: Basins of attraction, $\alpha = 0.3$, $\beta = 0.1$, and $q_3 = 7$.

cycle. In Figure 27, when $\alpha = 0.3$, $\beta = 0.25$, and $q_3 = 7$, the system is in chaos. In Figure 28, when $\alpha = 0.3$, $\beta = 0.1$, and $q_3 = 5$, the system is in 4-period cycle. In Figure 29, when $\alpha = 0.3$, $\beta = 0.25$, and $q_3 = 5$, the system is in chaos.

Comparing Figures 24–29, we find that the attraction domain reduces with the increase of price modification speed. Obviously, when β of player 2 increases, in the attraction domain, the range of q_2 reduces. We can also find that, with increase of q_3 , attraction domain reduces, although not so obvious.

From an economic perspective, the initial outputs of firm 1 and 2 should be lower in order to maintain market stability.

8. Conclusion

In this paper, Triopoly game with inaccurate demand beliefs is considered. Suppose all the firms as bounded rationally

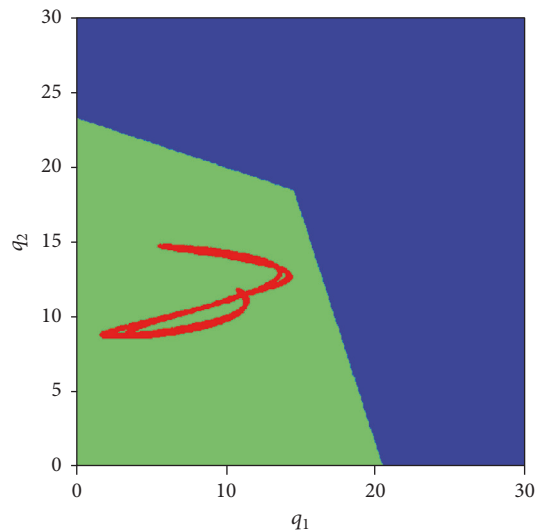


FIGURE 29: Basins of attraction, $\alpha = 0.3$, $\beta = 0.25$, and $q_3 = 5$.

players with demand evaluation bias. Equilibrium output is negatively related to players' costs; the value of e_i is negatively related to range of stability.

2D bifurcation diagram is introduced and we find that with the increase of output modification speed or e_i , the system will fall into chaos via period-doubling bifurcations. Increase in e_1 will increase equilibrium output q_1 while decreasing equilibrium output of other players.

Basins of attraction are investigated and results show that if a player speeds up his own output adjustment, the output of this player in the attraction domain will become smaller. In order to maintain market stability, a firm's output must be kept within a certain range. Feedback control method is used to help the system to keep at an equilibrium state.

Conflicts of Interest

The authors declare that they have no competing financial interests.

Acknowledgments

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