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## Research Article

# Performance of Variable Negative Stiffness MRE Vibration Isolation System

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Magnetorheological elastomer (MRE) vibration isolation devices can improve a system's vibration response via adjustable stiffness and damping under different magnetic fields. Combined with negative stiffness design, these MRE devices can reduce a system's stiffness and improve the vibration control effect significantly. This paper develops a variable negative stiffness MRE isolation device by combining an improved separable iron core with laminated MREs. The relationship between the negative stiffness and the performance of the device is obtained by mathematical transformation. Its vibration response under simple harmonic excitation at small amplitude and the impact of the volume fraction of soft magnetic particles on the isolation system are also analyzed. The results show that the negative stiffness produced by the magnetic force is a major factor affecting the capacity of the isolation system. Compared to devices of the same size, the isolation system equipped with low-particle volume fraction MREs demonstrates better performance.

## 1. Introduction

The isolation system has been widely used in high-precision measurement, automotive suspension, shipping shaft systems, and disaster prevention and reduction systems to reduce environmental disturbances [1–4]. Environmental disturbances mainly come from the ground vibrations transferred by the support system, such as post and purlin, in which increased structural flexibility by a small-stiffness isolation system is a main method for protecting engineering structures or equipment. However, environmental disturbances are also caused by a variable weight or position of the structure or equipment, under which a large-stiffness isolation system is required to improve the structural stability and thus minimize the disturbance of the engineering structure or equipment [5]. Inevitably, this indicates that different engineering structures may place different stiffness requirements on the isolation system. The adjustable range of the isolation system thereby serves as a critical performance index, and an isolation system with a large isolation range can be applied

in a wide application area. Passive isolators are used most commonly in isolation because they are easily manufactured and cheaply equipped, but their limited frequency range and fixed stiffness have restricted their scope of application in the field. Fortunately, the stiffness of a system of two springs combined in series can be expressed as the ratio of their product to their sum. When a negative stiffness spring is connected in series with a positive one, the combined stiffness is infinite in theory. Therefore, the adjustment of the stiffness of each spring will directly affect the spring system's combined stiffness and the isolation system's bandwidth. Sarlis introduced a preloaded linear elastic spring into the passive isolation system and provided the damping system with negative stiffness by mounting an anchor frame and a movable frame [6]. Because the preloaded linear spring combination can provide negative stiffness by a certain connection method, Mochida et al. hinged three linear springs to form a combination that can be applied to foundation isolation, which provides negative stiffness [7]. Mizuno et al. used a voice coil motor as a linear actuator to generate a suspension system with negative

stiffness and a pair of plate springs with normal stiffness and damping; the control strategy based on the Laplace transform was also introduced [8]. In the above literature, the system's adjustable stiffness range is restricted by the fixed stiffness values of springs. This paper, using adjustable stiffness MREs as viscoelastic springs, devises a variable negative stiffness isolation unit and presents research conducted on the isolation performance.

Magnetorheological elastomers (MREs) are an intelligent composite material with actively controllable mechanical properties. The main ingredients of MREs are micron-sized soft magnetic particles and viscoelasticity polymer matrix. Micron-sized soft magnetic particles are dispersed directionally or distributed randomly in the polymer matrix curing without magnetic field or with that for higher material properties. The mechanical properties of MREs, such as damping feature, Young's modulus, and shear modulus, may be transiently reversed under an external magnetic field. Being free of sedimentation and easily encapsulated, MREs are widely used in dampers, brakes, and deflection sensors [9–11]. Because conventional passive rubber isolators cannot change their mechanical properties, which results in problems pertaining to their lack of adaptability, many new options have been tested. A novel MRE seismic isolator with a laminated structure of steel and MRE layers was developed for real-time controllability of stiffness and damping [12], but the flexibility of this device is limited because the MR effect of MREs was increased only by the magnetic field. Yang et al. [13] assembled two permanent magnets into the MRE seismic isolator up and down. The main contribution mentioned above is the magnetic control. However, magnetic force is another method for the large range of excitation frequencies because wideband natural frequencies make the seismic isolator adaptable.

Based on the magnetic force between an electromagnet and a permanent magnet, Mizuno et al. probed into the negative stiffness effects on the active isolation system [5]. Kim et al. [14] investigated the isolation of a support system with a single degree of freedom by using an electromagnet connected in series with a linear spring. The result showed that an adjustable passive isolation system with negative stiffness has a large range for adjusting stiffness on the whole; thus, the limitation of conventional passive vibration isolation systems—that is, being inapplicable to broadband excitation—can be resolved. Because the electromagnetic force supplying an active force can provide the vibration isolation system with negative stiffness, Li et al. [15] developed a vibration control system with adjustable negative stiffness using the stiffness and damping of a common rubber. A combination of passive isolation and semiactive isolation is also a novel method wherein the MREs with controllable stiffness and damping serve as a semiactive control component [16]. MRE isolation units, by use of an electromagnet or permanent magnets, provided variable stiffness to give the isolation system a wideband variation range of stiffness, optimizing the isolation performance of the system. Incontestably, the focus of the isolation system is on the variation of stiffness and damping. Adding constant-stiffness rubber or springs is a conventional isolator strategy capable of mitigating the effects

of one type of earthquake [12]. Further, the feasibility and practicality of laminated MRE isolators, similar to laminated rubber bearing structures, have been developed by specialists and scholars. The electromagnet was utilized for generating a magnetic field and controlling MRE stiffness in the literature [12, 13]. The concept of “negative changing stiffness” was first mentioned in the context of MRE isolator systems. In the literature, the stiffness of the MRE structure can be increased and decreased by adjusting a hybrid magnetic system such that the natural frequency of the system dodges the natural frequency [13]. In this study, however, an equivalent negative spring whose stiffness equals the ratio of an added magnetic force and displacement is joined to an equivalent normal spring whose stiffness is controlled by a hybrid magnetic system (two electromagnets) in series. The hybrid magnetic system not only changes the stiffness of MREs decreasingly and increasingly but also accommodates the added magnetic force for negative stiffness. Thus, the performance and the influence factor of this variable negative stiffness MRE vibration isolation system should be researched and confirmed in theory before the manufacturing process.

The mechanical properties of MREs are sensibly influenced by the particle volume fraction, such as storage Young's modulus, storage shear modulus, loss factors [17], linear strain and the tensions induced in the MRE [18], and strain energy [19]. Because the added force is supplied by magnetic force and MREs, stiffness is directly related to the magnetic flux density inside MRE material. Thus, the effect of magnetic flux density and the volume fraction of MREs is researched and analyzed as the major influence factor. It is worth mentioning that setting a compact structure consisting of steel layers and laminated MREs inside the electromagnetic coil may increase the magnetic flux density; however, the air gap cannot be avoided because of the vertical displacement loaded by the mass or disturbance acting on the isolation that results in a decreased magnetic field inside the MRE material [12, 13]. This is also the reason why the iron core is separable.

In this study, the performance of variable negative stiffness MRE vibration isolation systems is analyzed in theory; further work will be given in next part. After the introduction, the structure and workflow are detailed and expatiated. Second, the Fourier transformation is used to deduce the impact of the magnetic force that produced the negative stiffness on the MRE isolation system's performance. In Section 3, the impacts of particle volume fractions of MREs on the isolation system's vibration characteristics are analyzed. Finally, the vibration characteristics of the isolation system under simple harmonic motion at a small amplitude are investigated.

## 2. System Sketch

The schematic diagram of the tunable negative stiffness MRE isolation device is depicted in Figure 1. The MRE isolation device mainly consists of isolated plates made of steel, laminated MREs, separable iron cores with telescopic strut/sleeve, and two electromagnets. The intermediate plate is connected with two cylindrical hollow laminated MRE blocks (upper and lower). Each laminated MRE block is composed of a circular iron plate and laminated MREs. The electromagnet, via

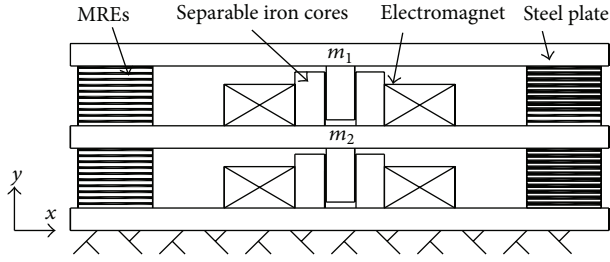


FIGURE 1: Schematic diagram of MRE isolation system.

a separable iron core, attracts the neighboring plate to produce a magnetic force that provides the system with negative stiffness as an actuator.

The alterable magnetic flux density can drive the two MRE layer blocks with variable stiffness  $k_{\text{MREs}}$ ; also, there is a linear relation between the magnetic force generated by the magnetic fields and the displacement of the steel plate. When the displacement of the steel plate is small enough, the stiffness  $k_{\text{add}}$  can be defined by the added magnetic force. The special frameworks of the MRE isolation system can be simplified into two springs in series, and the total stiffness  $k_t$  of this MRE isolation system becomes a reciprocal value [8]:

$$k_t = \frac{k_{\text{MREs}}k_{\text{add}}}{k_{\text{MREs}} + k_{\text{add}}}. \quad (1)$$

The theoretical stiffness  $k_{\text{add}}$  would be negative when the direction of relative displacement with respect to that of the added magnetic force is below zero. The initial equilibrium state is shown as in Figure 2(a); the distance between the isolated plate  $m$  and the base plate is  $l$ , equal to that of a shaking plate. A generalized force  $Q$ , which may be an increment of a force or a mass, is hypothetically applied to the isolated plate as shown in Figure 2(b), and the distance increases or decreases by  $\Delta l$ . At the same time, the electromagnets generate an added magnetic force in a direction opposite to the hypothetically generalized force, and the absolute value is  $Q$ ; the stiffness is then  $-(Q/\Delta l)$  for negative stiffness. The total stiffness of the system becomes extremely large if the absolute value of the stiffness  $k_{\text{MREs}}$  is close to that of the negative stiffness  $k_{\text{add}} = -(Q/\Delta l)$ ; thus, the isolation plate is safe from vibration. When the system is subjected to a generalized force, a displacement  $(Q/k_{\text{MREs}})$  in the positive direction appears as shown in Figure 2(b); in the meantime, the added magnetic force makes the isolated plate move backward  $(Q/k_{\text{add}})$  in the reverse or negative direction. The same state is then turned up on the isolation system as shown in Figure 2(c) if the absolute value of the stiffness  $k_{\text{MREs}}$  is close to that of the negative stiffness  $k_{\text{add}}$  because the above negative displacement and the forward displacement cancel each other out.

### 3. Magnetic Force

An equivalent physical model of variable negative stiffness MRE isolation systems is illustrated in Figure 3. The equivalent model consists of two masses modeled by the isolated

plate, two parallel springs and parallel dashpots modeled by laminated MREs, and two parallel magnetic forces generated by the electromagnets applied to the plates. Considering kinetic equilibrium, the equation of the equivalent model can be written as follows:

$$m_1 u_1'' + c_1 (u_1' - u_2') + k_1 (u_1 - u_2) = f_{\text{mag}}, \quad (2)$$

$$m_2 u_2'' + c_2 u_2' + k_2 u_2 + c_1 (u_2' - u_1') + k_1 (u_2 - u_1) = f_{\text{ext}} - 2f_{\text{mag}}, \quad (3)$$

where  $m_1$  and  $m_2$  represent the masses of the upper and lower isolated plates, respectively,  $u_1$  and  $u_2$  are the mass displacements relative to the equilibrium position,  $c_1$  and  $c_2$  are the damping coefficients of the laminated MRE block,  $k_1$  and  $k_2$  are the stiffness values of the laminated MRE block,  $f_{\text{mag}}$  is the magnetic force applied to the plate, and  $f_{\text{ext}}$  is the inputted excitation load (Figure 3).

From (2) and (3), we obtain a Fourier transform representation:

$$U_1 (-m_1 \omega^2 + i\omega c_1 + k_1) + U_2 (-i\omega c_1 - k_1) = F_{\text{mag}},$$

$$U_1 (-i\omega c_1 - k_1) + U_2 (-m_2 \omega^2 + i\omega c_2 + k_2 + i\omega c_1 + k_1) = F_{\text{ext}} - 2F_{\text{mag}}. \quad (4)$$

It can be obtained as follows:

$$U_2 = \frac{(-m_1 \omega^2 + i\omega c_1 + k_1) F_{\text{ext}} + (2m_1 \omega^2 - i\omega c_1 - k_1) F_{\text{mag}}}{(-m_1 \omega^2 + i\omega c_1 + k_1) (-m_2 \omega^2 + i\omega c_2 + k_2) - m_1 \omega^2 (i\omega c_1 + k_1)}. \quad (5)$$

If the displacement of the intermediate plate  $m_2$  of the damping target object decreases to null, then the control strategy is as follows:

$$\frac{F_{\text{ext}}}{F_{\text{mag}}} = -\frac{2m_1 \omega^2 - i\omega c_1 - k_1}{-m_1 \omega^2 + i\omega c_1 + k_1}, \quad (6)$$

$$\frac{f_{\text{ext}}}{f_{\text{mag}}} = \frac{u(2m_1 \omega^2 - k_1) - u'c_1}{u(m_1 \omega^2 - k_1) - u'c_1}.$$

From the equation mentioned above, the magnetic field force mainly falls into two parts: the first part is the displacement feedback part  $(m_1 \omega^2 - k_1)u$  related to the equivalent resonant frequency and elastic force; the other is the velocity feedback part  $c_1 u'$  related to the damping force. The stiffness of the MREs is adjusted by changing the external magnetic field strength and setting  $m_1 \omega^2 = k_1$ . This is meant to make the isolator's nondamping natural frequency equal to the external exciting frequency [20]. At this time, the system's isolation efficiency can be expressed as a ratio of external

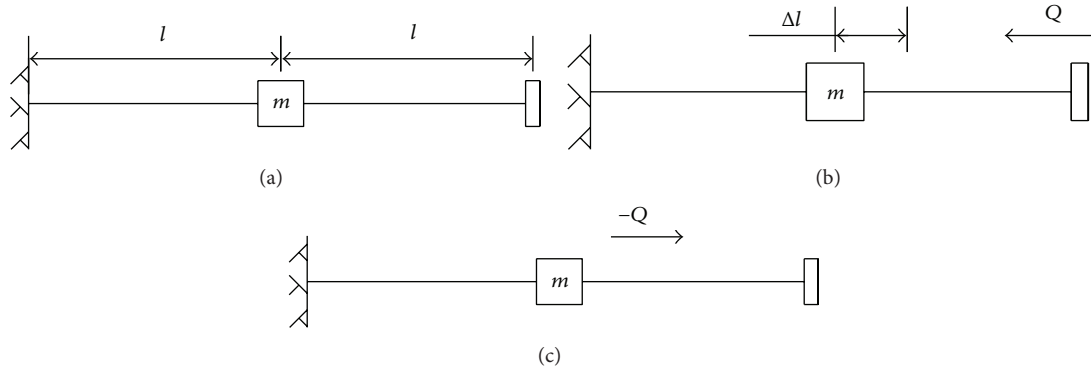


FIGURE 2: Operation chart of negative stiffness process.

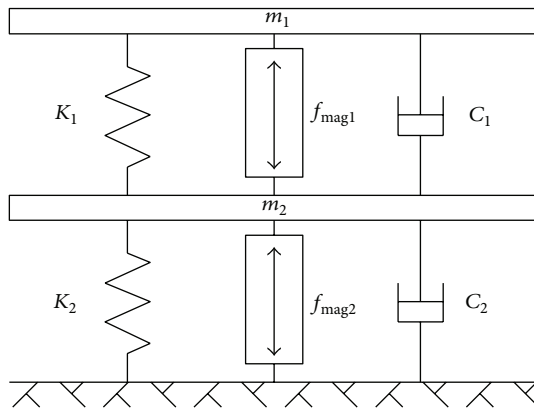


FIGURE 3: The phenomenological model.

exciting power to the power of the magnetic force applied to the plates. Consider

$$\beta = \frac{f_{\text{ext}} u}{f_{\text{mag}} u} = \frac{F_{\text{ext}} U}{F_{\text{mag}} U} = \frac{m_1 \omega^2 - i \omega c_1}{i \omega c_1} \quad (7)$$

$$= \frac{(\omega/\omega_n)^4 + 4\xi^2 (\omega/\omega_n)^2}{4\xi^2 (\omega/\omega_n)^2},$$

where  $\xi = \sqrt{c^2/(4km)}$  is the damping ratio and  $\omega_n = \sqrt{k_{\text{equ}}/m_2}$  is the natural frequency.

The system's isolation efficiency can be used to illustrate the capacity of the system's energy dissipation. The system's energy dissipation varies with damping ratio as shown in Figure 4. It can be seen from the diagram that the negative stiffness produced by the magnetic force contributes to the system's energy dissipation as the frequency ratio increases, which satisfies the control strategy for minimal displacement of the intermediate plate in the damping target object mentioned above. The isolation system shows a lower overall stiffness under the negative stiffness effect resulting from the magnetic force; also, the negative stiffness generated by the magnetic force and positive stiffness resulting from the MRE deformation should be in harmony with each other in the spring system. Thus, the influences of magnetic force on the

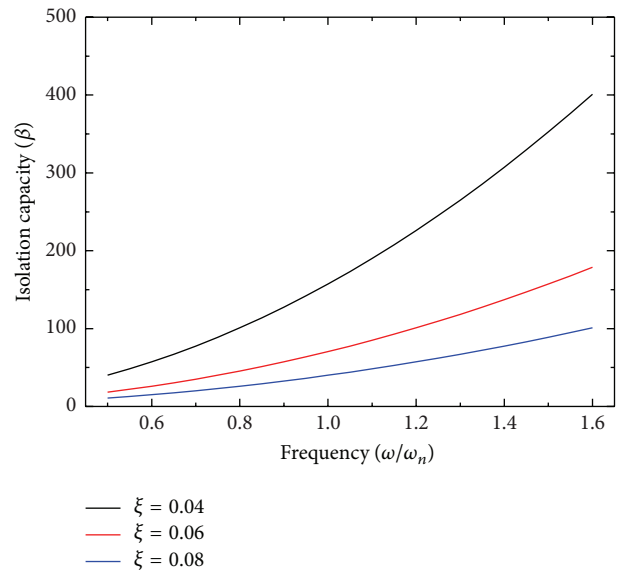


FIGURE 4: The vibration isolation capacity of the system.

vibration isolation capacity should be analyzed further. Many factors can affect the magnitude of the magnetic force in this system, such as current intensity, number of windings, gap size, and material permeability. In this part, the magnetic field strength is selected as the independent variable on vibration isolation capacity because the magnetic field strength can be changed by the above elements.

Now suppose that an external excitation acts upon the above plate  $m_1$ , and the magnetic force applied to the intermediate plate of the damping target object is  $f_{\text{mag}}$ . The following power balance equation for the intermediate plate is then obtained from (3):

$$m_2 u_2'' + c_2 u_2' + k_2 u_2 + c_1 (u_2' - u_1') + k_1 (u_2 - u_1) = f_{\text{mag}} \quad (8)$$

Under an external magnetic field  $H$  with average magnetic flux density  $B$ , the magnetic energy between the positive and negative displacements of the intermediate plate during vibration is  $E$ , and the magnetic force can be expressed as

the magnetic energy's differential in the direction of the plate displacement  $\delta$ . Consider

$$f_{\text{mag}0} = \frac{dE}{d\delta} = \frac{d(BH(2S\delta))}{d\delta} = \frac{B^2S}{2\mu_0K_f^2} = \frac{\mu_0S(NI)^2}{4\Delta^2K_f^2}, \quad (9)$$

where  $\mu_0$ ,  $S$ ,  $N$ ,  $I$ ,  $K_f$ , and  $\Delta$  are vacuum magnetic conductivity, MRE cross section area, number of turns in the coil, current in the coil, leakage coefficient of the coil, and original distance between the surrounding plates, respectively. The magnetic forces can then be expressed as follows:

$$\begin{aligned} f_{\text{mag}1} &= \frac{\mu_0S(NI)^2}{4(\delta_{\text{max}} - \delta)^2 K_f^2} \\ &\approx \frac{\mu_0S(NI)^2}{4\Delta^2K_f^2} \left( 1 + \frac{2\delta}{\delta_{\text{max}}} + \frac{3\delta^2}{\delta_{\text{max}}^2} + \dots \right), \\ f_{\text{mag}2} &= \frac{\mu_0S(NI)^2}{4(\delta_{\text{max}} + \delta)^2 K_f^2} \\ &\approx \frac{\mu_0S(NI)^2}{4\Delta^2K_f^2} \left( 1 - \frac{2\delta}{\delta_{\text{max}}} + \frac{3\delta^2}{\delta_{\text{max}}^2} + \dots \right). \end{aligned} \quad (10)$$

If the displacements are small, the magnetic forces can be seen as linear functions of the relative displacement, and the resulting negative stiffness can be expressed as

$$k_n = \frac{f_{\text{mag}}}{\delta} = -\frac{f_{\text{mag}2} - f_{\text{mag}1}}{\delta} = -\frac{\mu_0S(NI)^2}{4\Delta^2K_f^2} \cdot \frac{4}{\delta_{\text{max}}}. \quad (11)$$

Assuming that the stiffness values of the intermediate plate's upper and lower support systems are equal,  $k_1 = k_2 = k$ . The isolation system's equivalent magnetic force can then be expressed as

$$\begin{aligned} f_{\text{equ}} &= k \cdot \delta + f_{\text{mag}} = k_{\text{equ}} \cdot \delta \\ &= \left( k - \frac{\mu_0S(NI)^2}{4\Delta^2K_f^2} \cdot \frac{4}{\delta_{\text{max}}} \right) \cdot \delta, \end{aligned} \quad (12)$$

where  $k_1$  and  $k_2$  are stiffness values of the upper and lower support systems, respectively—namely, the stiffness values of MREs, each of which is a function of the external magnetic fields  $k_1(NI)$  and  $k_2(NI)$ .  $k_{\text{equ}}$  is the equivalent stiffness.

Equation (8) can be simplified as

$$\begin{aligned} m_2 u_2'' + c_{\text{equ}} u_2' + k_{\text{equ}} u_2 + c_{\text{equ}} (u_2' - u_1') \\ + k_{\text{equ}} (u_2 - u_1) = 0, \end{aligned} \quad (13)$$

where  $c_{\text{equ}} = c_1 = c_2$  points to the equivalent viscous damping coefficient. If the displacement of the upper plate is the sine exciting displacement ( $u_1 = A \sin(\bar{\omega}t - v)$ ), then

$$\begin{aligned} m_2 u_2'' + 2c_{\text{equ}} u_2' + 2k_{\text{equ}} u_2 \\ = \sqrt{k_{\text{equ}}^2 A^2 + c_{\text{equ}}^2 \bar{\omega}^2 A^2} \sin(\bar{\omega}t - v + \varepsilon), \end{aligned} \quad (14)$$

where the phase angle is  $\tan \varepsilon = k_{\text{equ}}/c_{\text{equ}}\bar{\omega}$ ; thus, the transmissibility of the isolation system is

$$\begin{aligned} \eta &= \sqrt{\frac{k_{\text{equ}}^2 + c_{\text{equ}}^2 \bar{\omega}^2}{(2k_{\text{equ}} - m_2 \bar{\omega}^2)^2 + (2c_{\text{equ}} \bar{\omega})^2}} \\ &= \sqrt{\frac{1 + 4\xi^2 (\bar{\omega}/\omega_n)^2}{(2 - (\bar{\omega}/\omega_n)^2)^2 + (4\xi \bar{\omega}/\omega_n)^2}}. \end{aligned} \quad (15)$$

Chen et al. [21] proposed a kind of MRE-based natural rubber. The test results indicated that the mechanical properties of prestructured natural rubber-based MREs, such as absolute modulus and relative modulus, are relatively good. In this paper, the impact of the external magnetic field with respect to the system isolation transmissibility is studied assuming that the MREs are equipped within. The relation between the system's transmissibility and dimensionless frequency ratio is shown in Figure 5. It is shown that the damping characteristics correlated to peak transfer rate are affected by magnetic flux density, and the natural frequency is negatively correlated with the external magnetic field. As the external magnetic field increases, the value of the transmissibility decreases, which can be explained by the fact that the system stiffness is reduced by the negative stiffness produced by the magnetic force. The transmissibility of the isolation system with negative stiffness unconsidered is shown in Figure 6 for comparison. Contrary to the case given in Figure 5, the smaller relative change in peak frequency is observed with the increased magnetic flux density in this nonnegative stiffness case, and the damping related to the peak transmissibility is dependent on the magnetic flux density as seen in the literature [16]. The most likely reason for the similar tendency of transmissibility is that the system considered here can be reduced as MRE isolators when the negative effects mentioned above are ignored. When the prestructured magnetic field strength is 600 mT and the external magnetic field strength is 0.4 mT, the soft magnetic particles of the MREs would reach magnetic saturation [21]. This is also one of the reasons why the variation of the phase angle is minimized by further increasing the magnetic flux density in Figure 7. As seen from (12), the negative stiffness of the system is a function of magnetic field, which dominates the mechanical properties of MREs introduced into the isolation system for a viscoelasticity component. Thus, it is necessary to discuss the impact of soft magnetic particles in MREs on the isolation performance.

#### 4. The Particle Volume Fraction of MREs

MREs prepared with a magnetic field character have a higher equivalent damping coefficient and energy dissipation than those prepared without a magnetic field [22], so the MREs prepared with a magnetic field in the isolation system should be chosen as viscoelastic components in the isolation system. When the magnetic particle chains in MREs are perpendicular to the plane of plates, Young's modulus of MREs is

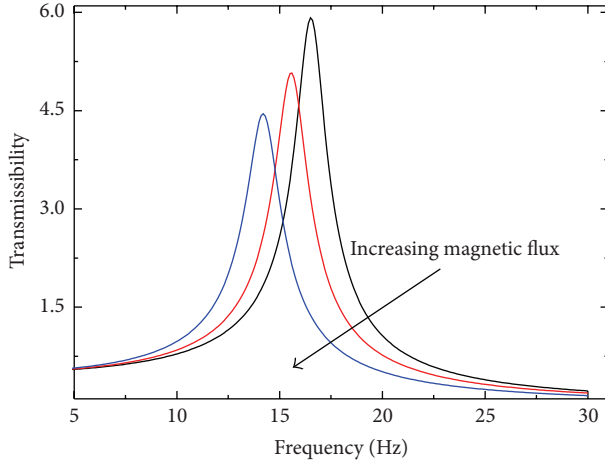


FIGURE 5: Transmissibility of the isolation system.

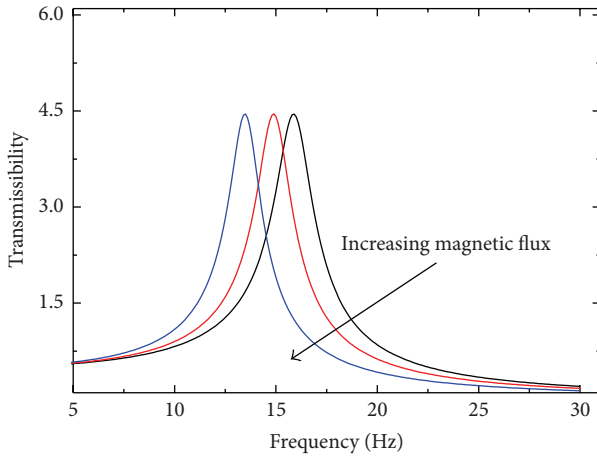


FIGURE 6: Transmissibility of the isolation system without negative stiffness.

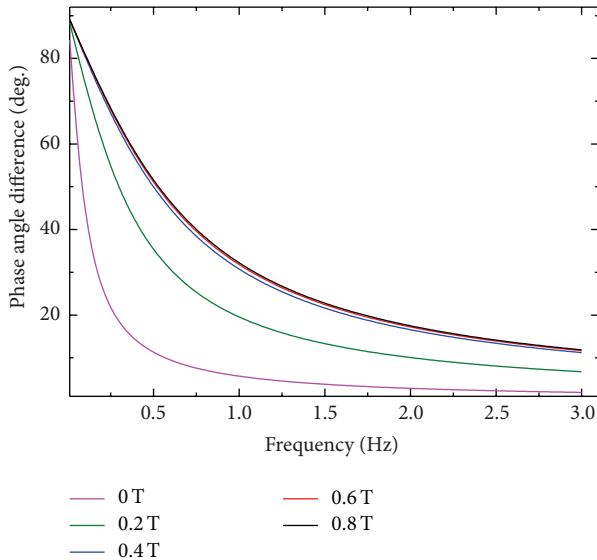


FIGURE 7: Phase angle difference of the isolation system.

correlated with the soft magnetic particles in MREs [23]. Consider

$$E_{\text{abs}} = \frac{3N_p\mu_0m^2}{\pi r_0^3}, \quad (16)$$

where  $N_p$  is the number of magnetic particles per unit of MREs,  $r_0$  denotes the center distance of adjacent particles in a particle chain, and  $m$  is the magnetic dipole moment [24]:

$$m = \mu_0\mu_1\chi VH \left( \frac{1}{1 - (4/3)\chi C (r/r_0)^3} \right), \quad (17)$$

where  $\mu_1$  is the magnetic conductivity of MREs,  $\chi$  is the susceptibility of soft magnetic particles,  $r$  and  $V$  are the average radius and volume of the soft magnetic particles, respectively,  $H$  is the external magnetic intensity, and  $C$  is the mathematical constant.

The magnetic dipole moment can be expressed as  $\mu_0\mu_1VH$  in the first-order expression. Because the volume fraction of MREs can directly affect the susceptibility of soft magnetic particles, the mechanical properties of MREs under the external magnetic field, such as damping controllability and shear modulus, vary greatly with different volume fractions [11, 25, 26]. Assuming that the MRE particle chain is a straight chain [27], the volume fraction can be expressed as the ratio of the particle volume to the sum of the particle volume and the surrounding matrix as well as the ratio of the total volume of the particles to that of MREs:

$$\phi = \frac{4\pi r^3/3}{4\pi r_0^3/3}, \quad (18)$$

$$\phi = \frac{N_p V}{(\Delta S)}. \quad (19)$$

MRE stiffness can be expressed as a ratio of the product of its modulus and cross section area to unit thickness. According to the equations mentioned above, the stiffness of MREs can be obtained as follows:

$$k = \frac{8\mu_0^3\mu_1^2(NI)^2 S^2 \phi^2}{\Delta^2}. \quad (20)$$

Here, the magnetic flux leakage is neglected; from (12) and (20), the natural frequency of the isolation system is a function of the MRE volume fraction. Consider

$$\begin{aligned} \omega_n(\phi) &= \frac{1}{\sqrt{m_2}} \sqrt{\frac{8\mu_0^3\mu_1^2(NI)^2 S^2 \phi^2}{\Delta^2} - \frac{\mu_0(NI)^2 S}{\Delta^2 \delta_m}} \\ &= \frac{NI}{\Delta} \sqrt{\frac{\mu_0 S}{m_2}} \sqrt{8\mu_0^2\mu_1^2 \phi^2 S - \frac{1}{\delta_m}}. \end{aligned} \quad (21)$$

The impact of MRE particle volume fraction on the transmissibility is obtained from (12) and (18). Figure 8 indicates that transmissibility varies with frequency. The dimensions of the system are as follows: the damping ratio is 0.05, the MRE cross section area is 31,400 mm<sup>2</sup>, and the maximum

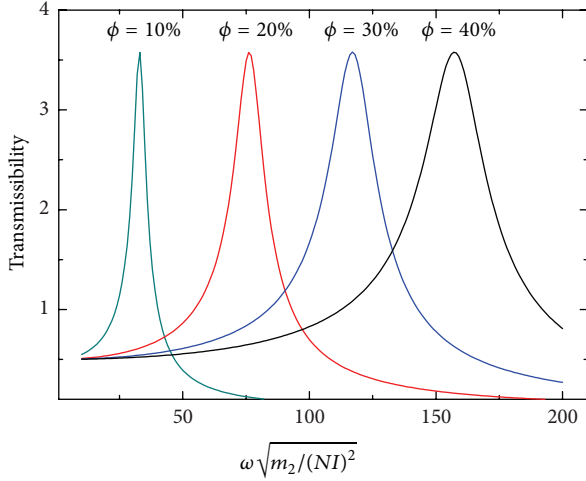


FIGURE 8: The impact of different volume fractions on the transmissibility.

displacement is 4 mm. It can be seen from the figure that an increased particle volume fraction results in an increased peak frequency; for isolation systems with identical size, the volume fractions had a remarkable influence on their natural frequencies. This can be explained because the particle volume fraction can affect the stiffness and modulus of MREs, and the system's negative stiffness can lead to softened overall stiffness.

According to (21), the system's natural frequency was negatively correlated to the MRE volume fraction. This is because the MREs show a more significant magnetorheological effect and magnetic anisotropy with a larger volume fraction. Such a phenomenon is more obvious if the external magnetic field is parallel to MRE particle chains [28, 29]. When the external magnetic field is comparatively larger and the induced field of soft magnetic particles reaches saturation, the maximum magnetic dipole moment of the soft magnetic particles could be approximately taken as 2.1 V [30]. At this time, the equivalent stiffness of the isolation system can be expressed as  $26.46\mu_0 S^2 \phi^2 - \mu_0 (NI)^2 S / \delta_m^3$ . If the magnetic flux leakage is neglected, the isolation system's natural frequency is a function of MRE volume fraction. Consider

$$\begin{aligned} \omega_n(\phi) &= \frac{1}{\sqrt{m_2}} \sqrt{26.46\mu_0 S^2 \phi^2 - \frac{\mu_0 (NI)^2 S}{\delta_m^3}} \\ &= \sqrt{\frac{\mu_0 S}{m_2}} \sqrt{26.46S\phi^2 - \frac{(NI)^2}{\delta_m^3}}. \end{aligned} \quad (22)$$

## 5. Conclusion

This paper proposed an isolation system for MREs with negative stiffness whose separable iron core is replaced by a conventional cylindrical core, which provides negative stiffness for the conventional laminated MRE isolation core. To verify the effectiveness of the negative stiffness and the laminated MRE, the transmissibility with and without negative stiffness

and that with different particle volume fractions were discussed; the following conclusions can be obtained:

- (i) The frequency and the system transmissibility decrease as the magnetic strength increases and there is lower transmissibility without negative stiffness.
- (ii) The vibration isolation performance of the one equipped with MREs with a low volume fraction is better than the one with high particle fraction.
- (iii) The negative stiffness effect produced by the magnetic force is conducive to the energy dissipation for the isolation system.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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