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Research Article

Adaptive Robust Backstepping Control of Permanent Magnet Synchronous Motor Chaotic System with Fully Unknown Parameters and External Disturbances

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The chaotic behavior of permanent magnet synchronous motor is directly related to the parameters of chaotic system. The parameters of permanent magnet synchronous motor chaotic system are frequently unknown. Hence, chaotic control of permanent magnet synchronous motor with unknown parameters is of great significance. In order to make the subject more general and feasible, an adaptive robust backstepping control algorithm is proposed to address the issues of fully unknown parameters estimation and external disturbances inhibition on the basis of associating backstepping control with adaptive control. Firstly, the mathematical model of permanent magnet synchronous motor chaotic system with fully unknown parameters is constructed, and the external disturbances are introduced into the model. Secondly, an adaptive robust backstepping control technology is employed to design controller. In contrast with traditional backstepping control, the proposed controller is more concise in structure and avoids many restricted problems. The stability of the control approach is proved by Lyapunov stability theory. Finally, the effectiveness and correctness of the presented algorithm are verified through multiple simulation experiments, and the results show that the proposed scheme enables making permanent magnet synchronous motor operate away from chaotic state rapidly and ensures the tracking errors to converge to a small neighborhood within the origin rapidly under the full parameters uncertainties and external disturbances.

1. Introduction

In recent years, the permanent magnet synchronous motor (PMSM) is utilized widely in various industrial fields due to its constantly dropping production cost, simple structure, high torque, and high efficiency. However, Hemati found that PMSM would generate chaotic behavior with system parameters entering into a certain region [1]. Previous studies have shown that the chaotic movement of PMSM will produce irregular oscillations of torque and speed, exacerbate current noise, and worsen operation performance and may even damage the entire drive system. Therefore, research on PMSM chaos phenomenon has attracted extensive attention worldwide [2–5], and further studying on the control method of PMSM chaos is of extreme significance [6–8].

The nonlinear characteristics of PMSM, such as multivariability, strong coupling, and high dimension, make it difficult to control for traditional linear control theory. Hence, a variety of modern and nonlinear control algorithms are introduced to suppress PMSM chaotic behavior. In terms of these control algorithms whether or not relying on the model parameters, the previous control methods can primarily be divided into two categories. The first type is on the basis of accurate model parameters, such as entrainment and migration control [9], exact feedback linearization control [10], and decoupling control [11]. However, the accuracy of these control methods directly depends on PMSM model parameters; if the system parameters deviate from the rated values, the control performance will go bad. The second type is based on unknown parameters, which have become the research focus of PMSM chaos suppression recently, mainly including sliding mode variable structure control [12, 13], fuzzy control [14], and H_∞ control [15]. However, sliding mode variable structure control requires uncertain parameters to satisfy certain matching conditions, fuzzy control is dependent on the fuzzification of Takagi-Sugeno, and H_∞ control is inclined to ignore the operating states under special conditions [16]. In essence, PMSM chaotic system is highly sensitive to initial states and parameters, and PMSM model parameters are susceptible to the temperature and humidity of the surrounding environment. Therefore, PMSM chaotic repression with unknown model parameters has applicability to a broader field and is more in line with reality [17]. Actually, the adaptive control (AC) provides a natural routine for PMSM chaotic control with unknown parameters, which has been presented in literatures [12, 13, 18].

Backstepping control (BC) is one of the most popular nonlinear control methods newly proposed to address parameter uncertainty, specifically the uncertainty not satisfying matching condition, which has been successfully applied to many engineering fields such as motor drive, temperature control of boiler main steam, and rocket location tracking. The core idea of BC is that complex high-dimensional nonlinear systems are decomposed into many simple low-dimensional subsystems and virtual control variables are introduced to backstepping process to design concrete controllers. In addition, BC has been successfully applied to suppress Liu chaotic system [19] and Chen chaotic system [20]. Therefore, the idea of combining BC with AC provides a useful and feasible train of thought to control PMSM chaotic system with unknown parameters. Literatures [21, 22] have exactly practiced this idea.

However, the conventional backstepping approach is confronted with two major problems of solving complicated "regression matrix" [23] and encountering "explosion of terms" [24]. In [25], the complexity of regression matrix is sufficiently manifested, which almost occupies one full page. Nevertheless, explosion of terms is an inherent shortcoming and is induced by repeated differentiations of virtual variables, particularly in design of adaptive backstepping controller [26]. Additionally, integration of BC with AC is frequently faced with the singularity arising from any estimation term emerging as a denominator of any control input. The overparameterization caused by the number of estimations larger than actual system parameters hinders the conventional adaptive backstepping control.

In addition to the above problems, to the extent of our knowledge, mostly existing literatures on PMSM chaotic control only concentrate on the cases of single unknown parameter and partial unknown parameters [21, 22], and there is no way to address the issue of fully unknown parameters. Furthermore, the existing researches mainly aim at the situation of sudden power failure during PMSM operation [16]; the existing conclusions lack the generality. Hence, through combination of BC and AC, not only does this paper study the control issue of PMSM chaos suppression with fully unknown parameters, but also the external disturbances are taken into account in PMSM chaos model. Newly adaptive updating laws of unknown parameters are designed to totally estimate unknown parameters of PMSM chaotic model, and adaptive robust backstepping controllers on the basis of adaptive estimations and external disturbances are developed to drive PMSM to escape out of chaotic state quickly, inhibit the external disturbances, and accomplish the given signals tracking rapidly. The method proposed in this paper expands the applied range of backstepping control theory in PMSM chaotic system. Moreover, the study of chaos control problem with totally unknown parameters and external disturbances is more general and practical, and the results and conclusions obtained are more applicable.

2. PMSM Chaotic Model with Fully Unknown Parameters

For a PMSM, its mathematical model in dq axis coordinate system can be described as follows [16]:

$$\begin{aligned}\dot{\widetilde{\omega}} &= \frac{3p\phi_m}{2J}\widetilde{i}_q - \frac{B}{J}\widetilde{\omega} - \frac{\widetilde{\tau}_l}{J},\\ \dot{\widetilde{i}}_q &= -\frac{R}{L_q}\widetilde{i}_q - p\widetilde{\omega}\cdot\widetilde{i}_d - \frac{p\phi_m}{L_q}\cdot\widetilde{\omega} + \frac{1}{L_q}\widetilde{u}_q,\\ \dot{\widetilde{i}}_d &= -\frac{R}{L_d}\widetilde{i}_d + p\widetilde{\omega}\cdot\widetilde{i}_q + \frac{1}{L_d}\widetilde{u}_d,\end{aligned} \tag{1}$$

where $\widetilde{\omega}$ is the mechanical angular velocity of the rotating rotor, $\widetilde{i}_d$ and $\widetilde{i}_q$ are d axis and q axis currents of stator winding, respectively, $\widetilde{u}_d$ and $\widetilde{u}_q$ are d axis and q axis voltages of stator winding, p is the number of rotor pole pairs, ϕ_m is the flux generated by permanent magnets, J is the moment of inertia, B is the viscous damping coefficient, $\widetilde{\tau}_l$ is the load torque, R is the phase resistance of the stator windings, and L_d and L_q are d axis and q axis inductances of stator winding, respectively. For a PMSM with uniform air gap, $L_d = L_q$. Hence, we use L to substitute L_d and L_q in the following paper.

Selecting the affine transformation $\begin{bmatrix}\widetilde{\omega}\\ \widetilde{i}_q\\ \widetilde{i}_d\end{bmatrix} = \begin{bmatrix}1/p\tau & 0 & 0\\ 0 & k & 0\\ 0 & 0 & k\end{bmatrix}\begin{bmatrix}\omega\\ i_q\\ i_d\end{bmatrix}$ and time scale transformation $\widetilde{t} = \tau t$, the PMSM mathematical model described in (1) can be converted into dimensionless form as follows:

$$\begin{aligned}\dot{\omega} &= \frac{1}{\delta}\left(i_q - \omega\right) - \tau_l,\\ \dot{i}_q &= -i_q - \omega\cdot i_d + \gamma\cdot\omega + u_q,\\ \dot{i}_d &= -i_d + \omega\cdot i_q + u_d,\end{aligned} \tag{2}$$

where $\tau = L/R$, $k = 2B/3p^2\tau\phi_m$, $u_d = (1/kR)\widetilde{u}_d$, $\gamma = -\phi_m/kL$, $u_q = (1/kR)\widetilde{u}_q$, $\delta = J/B\tau$, and $\tau_l = p\tau^2\widetilde{\tau}_l/J$.

As presented in (2), the dynamic performance of PMSM depends on three parameters δ, γ, and τ_l. Considering the most general case, let $\delta = 0.2$, $\gamma = 50$, $\tau_l = 3.2$, $u_d = -0.6$, and $u_q = 0.8$. If the initial state is selected as $(\omega, i_q, i_d) = (0, 0, 0)$, PMSM system will run on a chaotic state and display the chaotic behavior. A typical chaotic attractor of PMSM is manifested in Figure 1.

In reality, the three parameters δ, γ, and τ_l in (2) tend to be unknown or to have uncertainties resulting from operating conditions. In other words, when all the parameters δ, γ, and τ_l cannot be determined, (2) actually represents PMSM chaotic system model with fully unknown parameters.

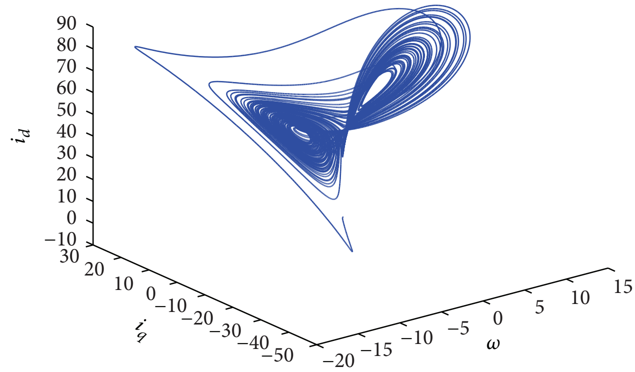

Figure 1: Chaotic attractor of PMSM system.

3. Design of Adaptive Robust Controller with Backstepping Approach

Taking a more general situation into account, the PMSM chaotic model described in (2) is immersed by external disturbances. The model can be rewritten as follows:

$$\begin{aligned} \dot{\omega} &= \frac{1}{\delta}\left(i_q - \omega\right) - \tau_l, \\ \dot{i}_q &= -i_q - \omega \cdot i_d + \gamma \cdot \omega + u_q + \Delta_1(\mathbf{x}, t), \\ \dot{i}_d &= -i_d + \omega \cdot i_q + u_d + \Delta_2(\mathbf{x}, t), \end{aligned} \tag{3}$$

where $\Delta_1(\mathbf{x}, t)$ and $\Delta_2(\mathbf{x}, t)$ represent the external disturbances, $\mathbf{x}$ indicates the system states, and $\mathbf{x} = (x_1, x_2, x_3) = (\omega, i_q, i_d)$.

3.1. Control Objective and Assumptions. Control problem in the paper can be described as follows: for PMSM chaotic system (3) with fully unknown parameters δ, γ, and τ_l and external disturbances Δ_1 and Δ_2, adaptive laws of unknown parameters δ, γ, and τ_l are designed and adaptive robust controllers u_d and u_q are constructed to ensure PMSM breaks away from chaos rapidly and runs into an expected orbit. Simultaneously, the fully unknown parameters δ, γ, and τ_l can be estimated accurately and the external disturbances can be inhibited effectively.

For convenience of controller design, the control system is supposed to hold some reasonable assumptions as follows.

Assumption 1. The state variables for PMSM chaotic system (ω, i_q, i_d) are observable.

Assumption 2. The external disturbances $\Delta_i(\mathbf{x}, t)$ satisfy the condition $|\Delta_i(\mathbf{x}, t)| \leq d_i(\mathbf{x}) f_i(t)$, $i = 1, 2$, where $d_i(\mathbf{x})$ is a known function, $f_i(t)$ is an unknown but bounded time-varying function, and $|f_i(t)| \leq f_{i\max}$, where $f_{i\max}$ is a constant.

Assumption 3. The desired speed and d axis current reference signals ω^* and i_d^* and their derivatives are known and bounded.

The estimated values of unknown system parameters are described as $\hat{\delta}$, $\hat{\gamma}$, and $\hat{\tau}_l$; then, the estimation errors $\tilde{\delta}$, $\tilde{\gamma}$, and $\tilde{\tau}_l$ can be expressed as follows:

$$\begin{aligned} \tilde{\delta} &= \hat{\delta} - \delta, \\ \tilde{\gamma} &= \hat{\gamma} - \gamma, \\ \tilde{\tau}_l &= \hat{\tau}_l - \tau_l. \end{aligned} \tag{4}$$

3.2. Controller Design. The essence of adaptive robust backstepping controller is to design controller through combination of backstepping method and adaptive approach; then, a reasonably stable function is built in accordance with Lyapunov stability theory to guarantee error variables to be effectively stabilized and meanwhile ensure the output of closed loop system tracks reference signals quickly. On the basis of this, the adaptive robust backstepping controller is designed as follows.

Step 1. For the speed reference signal ω^*, define the tracking error e_ω as follows:

$$e_\omega = \omega - \omega^*. \tag{5}$$

Taking PMSM chaotic system model (3) into account, the derivative of (5) can be written as

$$\dot{e}_\omega = \dot{\omega} - \dot{\omega}^* = \frac{1}{\delta}\left(i_q - \omega\right) - \tau_l - \dot{\omega}^*. \tag{6}$$

Define the tracking error e_q of q axis stator current i_q as follows:

$$e_q = i_q - i_q^*, \tag{7}$$

where i_q^* is the expected output value of i_q.

For the d axis current reference signal i_d^*, its tracking error e_d is defined as follows:

$$e_d = i_d - i_d^*. \tag{8}$$

By substitution of (7) into (6), we can obtain

$$\begin{aligned} \dot{e}_\omega &= \frac{1}{\delta}\left(e_q + i_q^* - \omega\right) - \tau_l - \dot{\omega}^* \\ &= \frac{1}{\delta}\left(i_q^* - \omega\right) - \hat{\tau}_l + \tilde{\tau}_l - \dot{\omega}^*. \end{aligned} \tag{9}$$

Let

$$i_q^* = \omega + \hat{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right), \tag{10}$$

$$i_d^* = 0, \tag{11}$$

where k_1 represents the positive control gain.

Through substitution of (10) into (9), (12) can be obtained:

$$\begin{aligned} \dot{e}_\omega &= \frac{1}{\delta} e_q + \frac{\hat{\delta}}{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) - \hat{\tau}_l + \tilde{\tau}_l - \dot{\omega}^* \\ &= \frac{1}{\delta} e_q + \frac{\delta + \tilde{\delta}}{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) - \hat{\tau}_l + \tilde{\tau}_l - \dot{\omega}^* \\ &= \frac{1}{\delta} e_q + \frac{\tilde{\delta}}{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) + \tilde{\tau}_l - k_1 e_\omega. \end{aligned} \tag{12}$$

Lyapunov function V_1 is selected as follows:

$$V_1 = \frac{1}{2}e_\omega^2. \tag{13}$$

Then, the derivative of V_1 can be described as

$$\begin{aligned}\dot{V}_1 &= e_\omega \dot{e}_\omega \\ &= e_\omega \left(\frac{1}{\delta} e_q + \frac{\hat{\delta}}{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) + \tilde{\tau}_l - k_1 e_\omega \right).\end{aligned} \tag{14}$$

Step 2. To stabilize the output q axis current of PMSM, the derivative of e_q is conducted as follows:

$$\begin{aligned}\dot{e}_q &= \dot{i}_q - \dot{i}_q^* = -i_q - \omega \cdot i_d + \gamma \cdot \omega + u_q + \Delta_1 - \Big[\dot{\omega} \\ &\quad + \dot{\hat{\delta}}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) + \hat{\delta}\left(\dot{\hat{\tau}}_l + \ddot{\omega}^* - k_1 \cdot \dot{e}_\omega\right)\Big] \\ &= -i_q - \omega \cdot i_d + \gamma \cdot \omega + u_q + \Delta_1 - \Big[\dot{\omega} \\ &\quad + \dot{\hat{\delta}}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) + \hat{\delta}\left(\dot{\hat{\tau}}_l + \ddot{\omega}^* - k_1 \cdot \dot{e}_\omega\right)\Big] \\ &= -i_q - \omega \cdot i_d + \hat{\gamma} \cdot \omega - \tilde{\gamma} \cdot \omega + u_q + \Delta_1 - \dot{\hat{\delta}}\left(\hat{\tau}_l \right. \\ &\quad \left. + \dot{\omega}^* - k_1 \cdot e_\omega\right) - \dot{\omega} - \hat{\delta}\left(\dot{\hat{\tau}}_l + \ddot{\omega}^* - k_1 \cdot \dot{e}_\omega\right) = -i_q \\ &\quad - \omega \cdot i_d + \hat{\gamma} \cdot \omega - \tilde{\gamma} \cdot \omega + u_q + \Delta_1 - \dot{\hat{\delta}}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \right. \\ &\quad \left. \cdot e_\omega\right) - \dot{e}_\omega - \dot{\omega}^* - \hat{\delta}\left(\dot{\hat{\tau}}_l + \ddot{\omega}^*\right) + \hat{\delta} \cdot k_1 \left(\omega - \dot{\omega}^*\right).\end{aligned} \tag{15}$$

By substitution of (12) into (15), we can get

$$\begin{aligned}\dot{e}_q &= -i_q - \omega \cdot i_d + \hat{\gamma} \cdot \omega - \tilde{\gamma} \cdot \omega + u_q + \Delta_1 \\ &\quad - \dot{\hat{\delta}}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) - \frac{1}{\delta} e_q \\ &\quad - \frac{\tilde{\delta}}{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) - \hat{\tau}_l + k_1 \cdot e_\omega - \dot{\omega}^* \\ &\quad - \hat{\delta}\left(\dot{\hat{\tau}}_l + \ddot{\omega}^*\right) + \hat{\delta} \cdot k_1 \left(\omega - \dot{\omega}^*\right).\end{aligned} \tag{16}$$

Combined with the mathematical model of PMSM chaotic system, (16) can be further calculated as follows:

$$\begin{aligned}\dot{e}_q &= -i_q - \omega \cdot i_d + \hat{\gamma} \cdot \omega - \tilde{\gamma} \cdot \omega + u_q + \Delta_1 \\ &\quad - \dot{\hat{\delta}}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) - \frac{1}{\delta} e_q \\ &\quad - \frac{\tilde{\delta}}{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) - \hat{\tau}_l + k_r \cdot e_\omega - \dot{\omega}^* \\ &\quad - \hat{\delta}\left(\dot{\hat{\tau}}_l + \ddot{\omega}^*\right) + \hat{\delta} \cdot k_1 \cdot \frac{i_q - \omega - \tau_l}{\delta} - \hat{\delta} \cdot k_1 \cdot \dot{\omega}^* \\ &= -i_q - \omega \cdot i_d + \hat{\gamma} \cdot \omega - \tilde{\gamma} \cdot \omega + u_q + \Delta_1 \\ &\quad - \dot{\hat{\delta}}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) - \frac{1}{\delta} e_q \\ &\quad - \frac{\tilde{\delta}}{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) - \hat{\tau}_l + k_1 \cdot e_\omega - \dot{\omega}^* \\ &\quad - \hat{\delta}\left(\dot{\hat{\tau}}_l + \ddot{\omega}^*\right) + \frac{\hat{\delta}}{\delta} \cdot k_1 \cdot \left(i_q - \omega\right) - \hat{\delta} \cdot k_1 \cdot \tau_l - \hat{\delta} \\ &\quad \cdot k_1 \cdot \dot{\omega}^* \\ &= -i_q - \omega \cdot i_d + \hat{\gamma} \cdot \omega - \tilde{\gamma} \cdot \omega + u_q + \Delta_1 \\ &\quad - \dot{\hat{\delta}}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) - \frac{1}{\delta} e_q \\ &\quad - \frac{\tilde{\delta}}{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) - \hat{\tau}_l + k_1 \cdot e_\omega - \dot{\omega}^* \\ &\quad - \hat{\delta}\left(\dot{\hat{\tau}}_l + \ddot{\omega}^*\right) + \frac{\hat{\delta}}{\delta} \cdot k_1 \cdot \left(i_q - \omega\right) - \hat{\delta} \cdot k_1 \\ &\quad \cdot \left(\hat{\tau}_l - \tilde{\tau}_l\right) - \hat{\delta} \cdot k_1 \cdot \dot{\omega}^* \\ &= -i_q - \omega \cdot i_d + \hat{\gamma} \cdot \omega - \tilde{\gamma} \cdot \omega + u_q + \Delta_1 \\ &\quad - \dot{\hat{\delta}}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) - \frac{1}{\delta} e_q \\ &\quad - \frac{\tilde{\delta}}{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 \cdot e_\omega\right) - \hat{\tau}_l + k_1 \cdot e_\omega - \dot{\omega}^* \\ &\quad - \hat{\delta}\left(\dot{\hat{\tau}}_l + \ddot{\omega}^*\right) + \frac{\hat{\delta}}{\delta} \cdot k_1 \cdot \left(i_q - \omega\right) - \hat{\delta} \cdot k_1 \cdot \hat{\tau}_l + \hat{\delta} \\ &\quad \cdot k_1 \cdot \tilde{\tau}_l - \hat{\delta} \cdot k_1 \cdot \dot{\omega}^*.\end{aligned} \tag{17}$$

By substitution of $\hat{\delta} = \tilde{\delta} + \delta$ into (17), the following equation can be obtained:

$$\begin{aligned}\dot{e}_q &= -i_q - \omega \cdot i_d + \hat{\gamma} \cdot \omega - \tilde{\gamma} \cdot \omega + u_q + \Delta_1 \\ &\quad - \dot{\hat{\delta}}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) - \frac{1}{\delta} e_q \\ &\quad - \frac{\tilde{\delta}}{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) - \hat{\tau}_l + k_1 e_\omega - \dot{\omega}^* \\ &\quad - \hat{\delta}\left(\dot{\hat{\tau}}_l + \ddot{\omega}^*\right) + \frac{\delta + \tilde{\delta}}{\delta} \cdot k_1 \cdot \left(i_q - \omega\right) - \hat{\delta} \cdot k_1 \cdot \hat{\tau}_l \\ &\quad + \hat{\delta} \cdot k_1 \cdot \tilde{\tau}_l - \hat{\delta} \cdot k_1 \cdot \dot{\omega}^* \\ &= -i_q - \omega \cdot i_d + \hat{\gamma} \cdot \omega - \tilde{\gamma} \cdot \omega + u_q + \Delta_1 \\ &\quad - \dot{\hat{\delta}}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) - \frac{1}{\delta} e_q \\ &\quad - \frac{\tilde{\delta}}{\delta}\left(\hat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) - \hat{\tau}_l + k_1 e_\omega - \dot{\omega}^* \\ &\quad - \hat{\delta}\left(\dot{\hat{\tau}}_l + \ddot{\omega}^*\right) + k_1 \cdot \left(i_q - \omega\right) + \frac{\tilde{\delta}}{\delta} \cdot k_1 \cdot \left(i_q - \omega\right) \\ &\quad - \hat{\delta} \cdot k_1 \cdot \hat{\tau}_l + \hat{\delta} \cdot k_1 \cdot \tilde{\tau}_l - \hat{\delta} \cdot k_1 \cdot \dot{\omega}^*.\end{aligned} \tag{18}$$

Lyapunov function V_2 is chosen as follows:

$$V_2 = V_1 + \frac{1}{2} e_q^2. \tag{19}$$

Then, the derivative of V_2 can be described as

$$\dot{V}_2 = \dot{V}_1 + e_q\dot{e}_q = e_\omega\left(\frac{1}{\delta}e_q + \frac{\widehat{\delta}}{\delta}\left(\widehat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) + \widetilde{\tau}_l - k_1 e_\omega\right) + e_q\left(-i_q - \omega\cdot i_d + \widehat{\gamma}\cdot\omega - \widetilde{\gamma}\cdot\omega + u_q + \Delta_1 - \dot{\widehat{\delta}}\left(\widehat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) - \frac{1}{\delta}e_q - \frac{\widetilde{\delta}}{\delta}\left(\widehat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) - \widetilde{\tau}_l + k_1 e_\omega - \dot{\omega}^* - \widehat{\delta}\left(\dot{\widehat{\tau}}_l + \ddot{\omega}^*\right) + k_1\cdot\left(i_q - \omega\right) + \frac{\widetilde{\delta}}{\delta}\cdot k_1\cdot\left(i_q - \omega\right) - \widehat{\delta}\cdot k_1\cdot\widehat{\tau}_l + \widehat{\delta}\cdot k_1\cdot\widetilde{\tau}_l - \widehat{\delta}\cdot k_1\cdot\dot{\omega}^*\right). \tag{20}$$

The first control variable is selected as

$$u_q = u_{qs} + u_{qr}, \tag{21}$$

where u_{qs} and u_{qr} are the model compensation and robust control inputs, respectively.

Then, u_{qs} and u_{qr} can be, respectively, chosen as

$$u_{qs} = -k_2\cdot e_q + i_q + \omega\cdot i_d - \widehat{\gamma}\cdot\omega + \dot{\widehat{\delta}}\left(\widehat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) + \dot{\omega}^* + \widehat{\delta}\left(\dot{\widehat{\tau}}_l + \ddot{\omega}^*\right) + \widehat{\delta}\cdot k_1\cdot\widehat{\tau}_l + \widehat{\delta}\cdot k_r\cdot\dot{\omega}^* - k_1\left(i_q - \omega\right), \tag{22}$$

$$u_{qr} = -\frac{d_1^2(\mathbf{x})}{4\varepsilon_1}e_q, \tag{23}$$

where k_2 is another positive control gain and ε_1 is a positive number chosen arbitrarily.

By substitution of (21) and (22) into (20), we can acquire

$$\dot{V}_2 = \dot{V}_1 + e_q\dot{e}_q = e_\omega\left(\frac{1}{\delta}e_q + \frac{\widehat{\delta}}{\delta}\left(\widehat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) + \widetilde{\tau}_l - k_1 e_\omega\right) + e_q\left(-k_2 e_q - \frac{1}{\delta}\cdot e_q + k_1 e_\omega - \widetilde{\gamma}\cdot\omega + u_{qr} + \Delta_1 - \frac{\widetilde{\delta}}{\delta}\left(\left(\widehat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) - k_1\cdot\left(i_q - \omega\right)\right) + \widetilde{\tau}_l\left(\widehat{\delta}\cdot k_1 - 1\right)\right) = e_\omega\left(\frac{1}{\delta}e_q + \frac{\widehat{\delta}}{\delta}\left(\widehat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) + \widetilde{\tau}_l - k_1 e_\omega\right) + e_q\left(-k_2 e_q - \frac{1}{\delta}\cdot e_q + k_1 e_\omega - \widetilde{\gamma}\cdot\omega - \frac{d_1^2(\mathbf{x})}{4\varepsilon_1}e_q + \Delta_1 - \frac{\widetilde{\delta}}{\delta}\left(\left(\widehat{\tau}_l + \dot{\omega}^* - k_1 e_\omega\right) - k_1\cdot\left(i_q - \omega\right)\right) + \widetilde{\tau}_l\left(\widehat{\delta}\cdot k_1 - 1\right)\right). \tag{24}$$

Additionally,

$$e_q\left(-\frac{d_1^2(\mathbf{x})}{4\varepsilon_1}e_q + \Delta_1\right) = -\frac{d_1^2(\mathbf{x})}{4\varepsilon_1}e_q^2 + \Delta_1 e_q \le -\frac{d_1^2(\mathbf{x})}{4\varepsilon_1}e_q^2 + d_1(\mathbf{x})f_{1\max}\left|e_q\right| = -\left(\frac{d_1(\mathbf{x})\left|e_q\right|}{2\sqrt{\varepsilon_1}} - \sqrt{\varepsilon_1}f_{1\max}\right)^2 + \varepsilon_1 f_{1\max}^2. \tag{25}$$

Step 3. Differentiating the tracking error e_d of d axis current i_d, we can get

$$\dot{e}_d = \dot{i}_d - \dot{i}_d^* = -i_d + \omega\cdot i_q + u_d + \Delta_2 - \dot{i}_d^*. \tag{26}$$

Lyapunov function V_3 is chosen as

$$V_3 = V_2 + \frac{1}{2}e_d^2. \tag{27}$$

Then, the derivative of V_3 can be represented as

$$\dot{V}_3 = \dot{V}_2 + e_d\dot{e}_d = \dot{V}_2 + e_d\left(-i_d + \omega\cdot i_q + u_d + \Delta_2 - \dot{i}_d^*\right). \tag{28}$$

In terms of (28), d axis output stator voltage u_d can be calculated:

$$u_d = u_{ds} + u_{dr}, \tag{29}$$

where

$$u_{ds} = -k_3\cdot e_d + i_d - \omega\cdot i_q + \dot{i}_d^*, \tag{30}$$

$$u_{dr} = -\frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d, \tag{31}$$

where k_3 is the positive control gain and ε_2 is a positive number chosen arbitrarily.

By substitution of (30) and (31) into (26) and (28), respectively, the following equations can be acquired:

$$\dot{e}_d = -k_3\cdot e_d + u_{dr} + \Delta_2 = -k_3\cdot e_d - \frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d + \Delta_2, \tag{32}$$

$$\dot{V}_3 = \dot{V}_2 + e_d\dot{e}_d = \dot{V}_2 + e_d\left(-k_3\cdot e_d - \frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d + \Delta_2\right), \tag{33}$$

$$e_d\left(-\frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d+\Delta_2\right)=-\frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d^2+\Delta_2 e_d \le -\frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d^2+d_2(\mathbf{x})f_{2\max}\left|e_d\right| = -\left(\frac{d_2(\mathbf{x})\left|e_d\right|}{2\sqrt{\varepsilon_2}}-\sqrt{\varepsilon_2}f_{2\max}\right)^2+\varepsilon_2 f_{2\max}^2. \tag{34}$$

Step 4. Lyapunov function V of PMSM chaotic system with fully unknown parameters and external disturbances is selected as follows:

$$V=V_3+\frac{1}{2\theta_1}\tilde{\tau}_l^2+\frac{1}{2\theta_2}\tilde{\gamma}^2+\frac{1}{2\delta\cdot\theta_3}\tilde{\delta}^2, \tag{35}$$

where θ_1, θ_2, and θ_3 represent positive adaptive gains.

Combined with equations $\dot{\tilde{\delta}}=\dot{\hat{\delta}}$, $\dot{\tilde{\gamma}}=\dot{\hat{\gamma}}$, and $\dot{\tilde{\tau}}_l=\dot{\hat{\tau}}_l$, derivative of selected Lyapunov function V can be calculated as follows:

$$\begin{aligned}\dot{V}&=\dot{V}_3+\frac{\tilde{\tau}_l}{\theta_1}\dot{\hat{\tau}}_l+\frac{\tilde{\gamma}}{\theta_2}\dot{\hat{\gamma}}+\frac{\tilde{\delta}}{\delta\cdot\theta_3}\dot{\hat{\delta}}=\dot{V}_2+e_d\left(-k_3\cdot e_d\right.\\&\left.+u_{dr}+\Delta_2\right)=\dot{V}_1+e_q\left(-k_2e_q-\frac{1}{\delta}\cdot e_q+k_1e_\omega-\tilde{\gamma}\right.\\&\cdot\omega+u_{qr}+\Delta_1\\&-\frac{\tilde{\delta}}{\delta}\left(\left(\hat{\tau}_l+\dot{\omega}^*-k_1e_\omega\right)-k_1\cdot\left(i_q-\omega\right)\right)\\&\left.+\tilde{\tau}_l\left(\hat{\delta}\cdot k_1-1\right)\right)+e_d\left(-k_3\cdot e_d+u_{dr}+\Delta_2\right)\\&=e_\omega\left(\frac{1}{\delta}e_q+\frac{\hat{\delta}}{\delta}\left(\hat{\tau}_l+\dot{\omega}^*-k_1e_\omega\right)+\tilde{\tau}_l-k_1e_\omega\right)\\&+e_q\left(-k_2e_q-\frac{1}{\delta}\cdot e_q+k_1e_\omega-\tilde{\gamma}\cdot\omega+u_{qr}+\Delta_1\right.\\&-\frac{\tilde{\delta}}{\delta}\left(\left(\hat{\tau}_l+\dot{\omega}^*-k_1e_\omega\right)-k_1\cdot\left(i_q-\omega\right)\right)\\&\left.+\tilde{\tau}_l\left(\hat{\delta}\cdot k_1-1\right)\right)+e_d\left(-k_3\cdot e_d+u_{dr}+\Delta_2\right)=\frac{1}{\delta}\\&\cdot e_qe_\omega+\frac{\tilde{\delta}}{\delta}\left(\hat{\tau}_l+\dot{\omega}^*-k_1e_\omega\right)\cdot e_\omega+\tilde{\tau}_le_\omega-k_1e_\omega^2\\&-k_3e_d^2-k_2e_q^2-\tilde{\gamma}\omega e_q-\frac{1}{\delta}e_q^2+e_q\left(u_{qr}+\Delta_1\right)\\&+e_d\left(u_{dr}+\Delta_2\right)-\frac{\tilde{\delta}}{\delta}\left(\left(\hat{\tau}_l+\dot{\omega}^*-k_1e_\omega\right)\right.\\&\left.-k_1\left(i_q-\omega\right)\right)\cdot e_q+\tilde{\tau}_l\left(\hat{\delta}\cdot k_1-1\right)e_q+k_1e_\omega e_q\\&+\frac{\tilde{\tau}_l}{\theta_1}\dot{\hat{\tau}}_l+\frac{\tilde{\gamma}}{\theta_2}\dot{\hat{\gamma}}+\frac{\tilde{\delta}}{\delta\theta_3}\dot{\hat{\delta}}=\frac{1}{\delta}e_qe_\omega+k_1e_\omega e_q\\&+\frac{\tilde{\delta}}{\delta}\left[\left(\hat{\tau}_l+\dot{\omega}^*-k_1e_\omega\right)e_\omega-\left(\hat{\tau}_l+\dot{\omega}^*-k_1e_\omega\right)e_q\right.\\&\left.+k_1\left(i_q-\omega\right)e_q+\frac{\dot{\hat{\delta}}}{\theta_3}\right]-k_1e_\omega^2-k_2e_d^2-k_3e_q^2\\&+\tilde{\tau}_l\left[e_\omega-e_q+\hat{\delta}k_1e_q+\frac{\dot{\hat{\tau}}_l}{\theta_1}\right]+\tilde{\gamma}\left[-\omega e_q+\frac{\dot{\hat{\gamma}}}{\theta_2}\right]\\&-\frac{1}{\delta}e_q^2+e_q\left(-\frac{d_1^2(\mathbf{x})}{4\varepsilon_1}e_q+\Delta_1\right)+e_d\left(-\frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d\right.\\&\left.+\Delta_2\right).\end{aligned} \tag{36}$$

In terms of (36), the adaptive laws of unknown parameters δ, γ, and τ_l can be selected, respectively, as follows:

$$\dot{\hat{\delta}}=-\theta_3\left[\left(\hat{\tau}_l+\dot{\omega}^*-k_1e_\omega\right)\cdot\left(e_\omega-e_q\right)+k_1\left(i_q-\omega\right)e_q\right], \tag{37}$$

$$\dot{\hat{\gamma}}=\theta_2\omega e_q, \tag{38}$$

$$\dot{\hat{\tau}}_l=-\theta_1\left[e_\omega-e_q+\hat{\delta}k_1e_q\right]. \tag{39}$$

By substitution of (37), (38), and (39) into (36), (36) can be simplified as follows:

$$\begin{aligned}\dot{V}&=\frac{1}{\delta}e_qe_\omega+k_1e_qe_\omega-k_1e_\omega^2-k_3e_d^2-k_2e_q^2-\frac{1}{\delta}e_q^2\\&+e_q\left(-\frac{d_1^2(\mathbf{x})}{4\varepsilon_1}e_q+\Delta_1\right)\\&+e_d\left(-\frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d+\Delta_2\right).\end{aligned} \tag{40}$$

3.3. Stability Analysis

Theorem 4. *For PMSM chaotic system model (3) with fully unknown parameters and external disturbances, design of adaptive control laws (37), (38), and (40) and selection of suitable controller gains k_1, k_2, k_3, ε_1, and ε_2 and adaptive gains θ_1, θ_2, and θ_3, the proposed adaptive robust backstepping controllers (21) and (29) can ensure the tracking error signals (5), (7), and (8) of PMSM chaotic systems are asymptotically stable. That is to say, PMSM chaotic system can run out of chaos quickly through the proposed controllers (21) and (29) and track the given reference signals.*

Through stability analysis, we want to verify the correctness of the theorem.

According to (40), new expression can be obtained as follows through some mathematical computations:

$$\begin{aligned}\dot{V} &= \frac{1}{\delta}e_q e_\omega + k_1 e_q e_\omega - k_1 e_\omega^2 - k_3 e_d^2 - k_2 e_q^2 - \frac{1}{\delta}e_q^2 \\ &\quad + e_q\left(-\frac{d_1^2(\mathbf{x})}{4\varepsilon_1}e_q + \Delta_1\right) + e_d\left(-\frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d + \Delta_2\right) \\ &= -\frac{1}{2\delta}\left(e_\omega - e_q\right)^2 + \frac{1}{2\delta}e_\omega^2 + \frac{1}{2\delta}e_q^2 - \frac{k_1}{2}\left(e_\omega - e_q\right)^2 \\ &\quad + \frac{k_1}{2}e_\omega^2 + \frac{k_1}{2}e_q^2 - k_1 e_\omega^2 - k_3 e_d^2 - k_2 e_q^2 - \frac{1}{\delta}e_q^2 \\ &\quad + e_q\left(-\frac{d_1^2(\mathbf{x})}{4\varepsilon_1}e_q + \Delta_1\right) + e_d\left(-\frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d + \Delta_2\right) \\ &= -\frac{1}{2\delta}\left(e_\omega - e_q\right)^2 - \frac{k_1}{2}\left(e_\omega - e_q\right)^2 - k_3 e_d^2 \\ &\quad + \left(\frac{1}{2\delta} + \frac{k_1}{2} - k_1\right)e_\omega^2 + \left(\frac{1}{2\delta} + \frac{k_1}{2} - k_2 - \frac{1}{\delta}\right)e_q^2 \\ &\quad + e_q\left(-\frac{d_1^2(\mathbf{x})}{4\varepsilon_1}e_q + \Delta_1\right) \\ &\quad + e_d\left(-\frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d + \Delta_2\right).\end{aligned} \tag{41}$$

Appropriate controller gains k_1 and k_2 are selected as follows:

$$\begin{aligned}\left(\frac{1}{2\delta} + \frac{k_1}{2} - k_1\right) &< 0, \\ \left(\frac{1}{2\delta} + \frac{k_1}{2} - k_2 - \frac{1}{\delta}\right) &< 0.\end{aligned} \tag{42}$$

Equation (42) can be replaced by the following:

$$\begin{aligned}k_1 &> \frac{1}{\delta}, \\ k_1 - 2k_2 &< \frac{1}{\delta}.\end{aligned} \tag{43}$$

Then, by substitution of (43) into (41), we can obtain

$$\begin{aligned}\dot{V} &= -\frac{1}{2\delta}\left(e_\omega - e_q\right)^2 - \frac{k_1}{2}\left(e_\omega - e_q\right)^2 - k_3 e_d^2 \\ &\quad + \left(\frac{1}{2\delta} + \frac{k_1}{2} - k_1\right)e_\omega^2 + \left(\frac{1}{2\delta} + \frac{k_1}{2} - k_2 - \frac{1}{\delta}\right)e_q^2 \\ &\quad + e_q\left(-\frac{d_1^2(\mathbf{x})}{4\varepsilon_1}e_q + \Delta_1\right) \\ &\quad + e_d\left(-\frac{d_2^2(\mathbf{x})}{4\varepsilon_2}e_d + \Delta_2\right) \\ &\le -\frac{1}{2\delta}\left(e_\omega - e_q\right)^2 - \frac{k_1}{2}\left(e_\omega - e_q\right)^2 - k_3 e_d^2 - k_4 e_\omega^2 \\ &\quad - k_5 e_q^2 - \left(\frac{d_1(\mathbf{x})\left|e_q\right|}{2\sqrt{\varepsilon_1}} - \sqrt{\varepsilon_1} f_{1\max}\right)^2 + \varepsilon_1 f_{1\max}^2 \\ &\quad - \left(\frac{d_2(\mathbf{x})\left|e_d\right|}{2\sqrt{\varepsilon_2}} - \sqrt{\varepsilon_2} f_{2\max}\right)^2 + \varepsilon_2 f_{2\max}^2 \\ &\le -\frac{1}{2\delta}\left(e_\omega - e_q\right)^2 - \frac{k_1}{2}\left(e_\omega - e_q\right)^2 - k_3 e_d^2 - k_4 e_\omega^2 \\ &\quad - k_5 e_q^2 - \left(\frac{d_1(\mathbf{x})\left|e_q\right|}{2\sqrt{\varepsilon_1}} - \sqrt{\varepsilon_1} f_{1\max}\right)^2 \\ &\quad - \left(\frac{d_2(\mathbf{x})\left|e_d\right|}{2\sqrt{\varepsilon_2}} - \sqrt{\varepsilon_2} f_{2\max}\right)^2 + \varepsilon_0,\end{aligned} \tag{44}$$

where $k_4 = -(1/2\delta + k_1/2 - k_1) \geq 0$, $k_5 = -(1/2\delta + k_1/2 - k_2 - 1/\delta) \geq 0$, and $\varepsilon_0 = \varepsilon_1 f_{1\max}^2 + \varepsilon_2 f_{2\max}^2$.

Let

$$\begin{aligned}W(i(t)) &= -\frac{1}{2\delta}\left(e_\omega - e_q\right)^2 - \frac{k_1}{2}\left(e_\omega - e_q\right)^2 - k_3 e_d^2 \\ &\quad - k_4 e_\omega^2 - k_5 e_q^2 \\ &\quad - \left(\frac{d_1(\mathbf{x})\left|e_q\right|}{2\sqrt{\varepsilon_1}} - \sqrt{\varepsilon_1} f_{1\max}\right)^2 \\ &\quad - \left(\frac{d_2(\mathbf{x})\left|e_d\right|}{2\sqrt{\varepsilon_2}} - \sqrt{\varepsilon_2} f_{2\max}\right)^2 + \varepsilon_0,\end{aligned} \tag{45}$$

where $i(t) = (e_\omega, e_q, e_d)$.

By integration of (45), we can get

$$\begin{aligned}\int_{t_0}^{t} W(i(t))\,dt &= -\int_{t_0}^{t} \dot{V}(i(t))\,dt \Longrightarrow \\ \int_{t_0}^{t} W(i(t))\,dt &= V(t_0) - V(t).\end{aligned} \tag{46}$$

Since $V(t_0)$ is bounded and $V(t)$ is bounded and nonincreasing, hence

$$\lim_{t\to\infty} \int_{t_0}^{t} W(i(t))\,dt < \infty. \tag{47}$$

Moreover, $W(i(t))$ is uniform continuous and $\dot{W}(i(t))$ is bounded. In accordance with Barbalat's Lemma, the following equation can be obtained:

$$\lim_{t\to\infty} W(i(t)) = 0. \tag{48}$$

Apparently, through selection of suitable controller gains k_1, k_2, k_3, ε_1, and ε_2, $\dot{V}$ can be ensured to be negative definite.

The above derivation has proved that the selected suitable controller gains k_1, k_2, k_3, ε_1, and ε_2 and adaptive gains θ_1, θ_2,

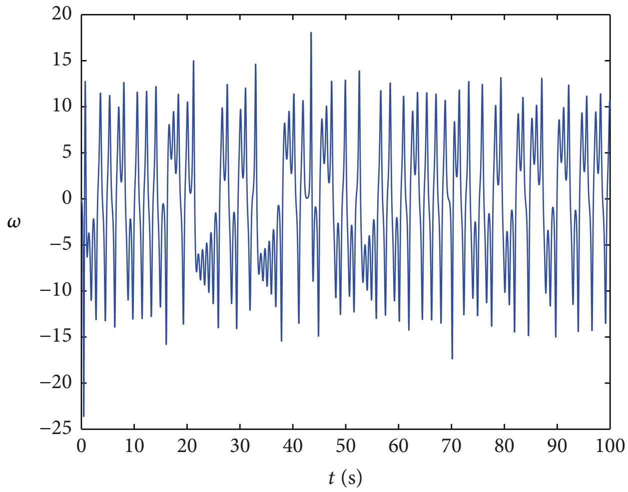

Figure 2: The ω curve of PMSM chaotic system with no control inputs u_d and u_q.

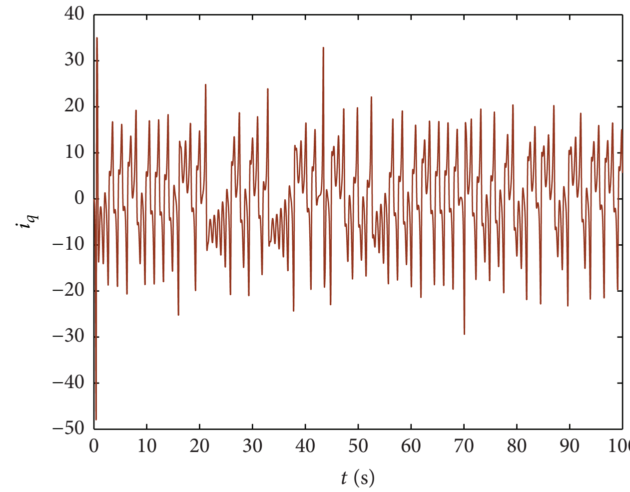

Figure 3: The i_q curve of PMSM chaotic system with no control inputs u_d and u_q.

and θ_3 can make the inequalities of $V \geq 0$ and $\dot{V} < 0$ hold. In addition, equation of $V = 0$ is not satisfied until $e_\omega = e_q = e_d = \tilde{\tau}_l = \tilde{\gamma} = \tilde{\delta} = 0$. In summary, PMSM chaotic system is globally asymptotically stable at the equilibrium point of $(e_\omega, e_q, e_d) = (0, 0, 0)$.

4. Numerical Simulation and Discussions

In order to illustrate the superiority of the proposed approach adequately, the simulation is carried out in MATLAB environment for three cases under the initial condition of $(\omega, i_q, i_d) = (0, 0, 0)$. Let $(\omega, i_q, i_d) = (x_1, x_2, x_3)$ and the control parameters are selected as $k_1 = 10$, $k_2 = 30000$, $k_3 = 5$, and $\varepsilon_1 = \varepsilon_2 = 0.01$; the adaptive gains are chosen as $\theta_1 = 6.2$, $\theta_2 = 100$, and $\theta_3 = 0.06$. The simulation time is chosen as 100 s and the designed controller is put into effect at the time of 20 s.

4.1. Test-I. The PMSM chaotic system is tested with the parameters $\delta = 0.2$, $\gamma = 50$, and $\tau_l = 3.2$. In order to be consistent with the reality better, we assume that the three parameters of PMSM chaotic system are all unknown with the initial condition of $(\delta, \gamma, \tau_l) = (0, 0, 0)$, and the expected reference signals are set as $\omega^* = 10$ and $i_d^* = 1$. Furthermore, the external disturbances $\Delta_1(\mathbf{x}, t) = 20x_3 \sin(5t)$ and $\Delta_2(\mathbf{x}, t) = 10 \sin(5t)$ are injected into the PMSM chaotic system. The simulation results given in Figures 2–4 apparently show PMSM runs in a chaotic state with no control inputs. Therefore, introduction of the presented control approach to suppress chaos in PMSM system will be of great importance and necessity. Figures 5–12 show that the proposed controller is utilized to control the PMSM chaotic system, where Figures 5–7 display the curves of state variables changing over time for PMSM chaotic system, which demonstrate the PMSM system stays away from the previous chaotic state when the designed controller is added to PMSM chaotic system, and track the desired signals accurately and rapidly. Furthermore,

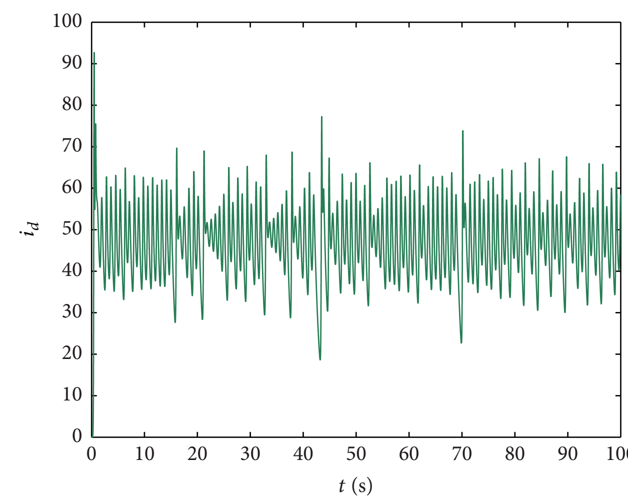

Figure 4: The i_d curve of PMSM chaotic system with no control inputs u_d and u_q.

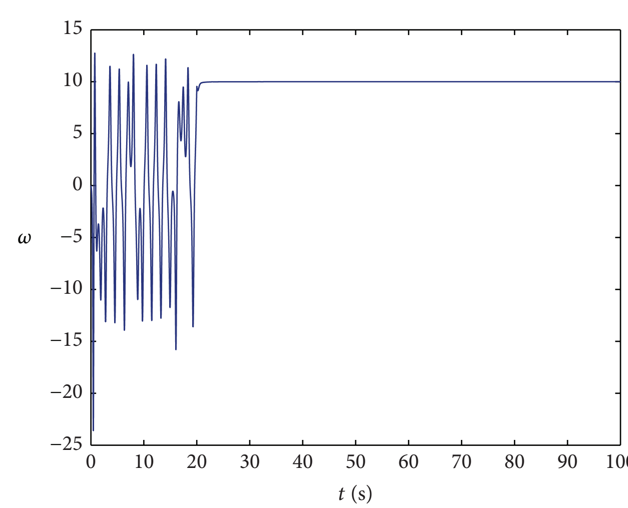

Figure 5: The ω curve of PMSM chaotic system added the controller inputs u_d and u_q.

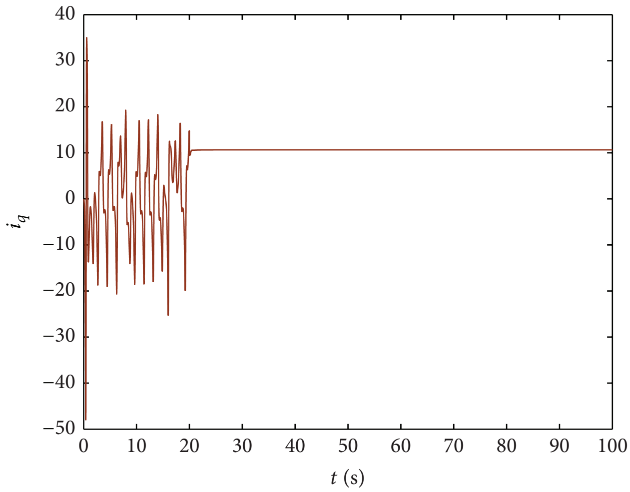

FIGURE 6: The i_q curve of PMSM chaotic system added the controller inputs u_d and u_q.

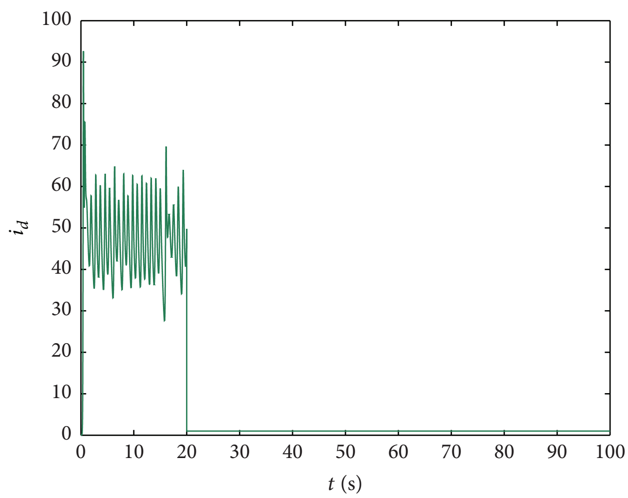

FIGURE 7: The i_d curve of PMSM chaotic system added the controller inputs u_d and u_q.

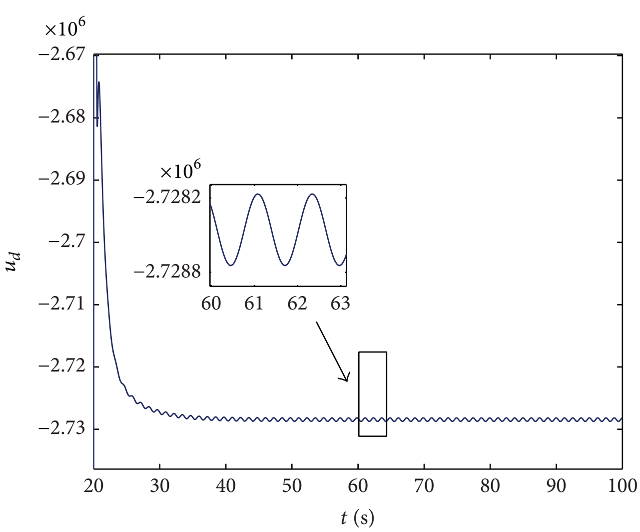

FIGURE 8: The controller input u_d.

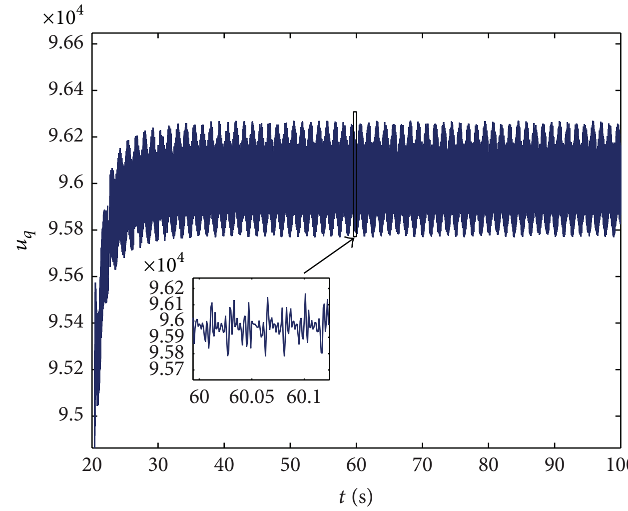

FIGURE 9: The controller input u_q.

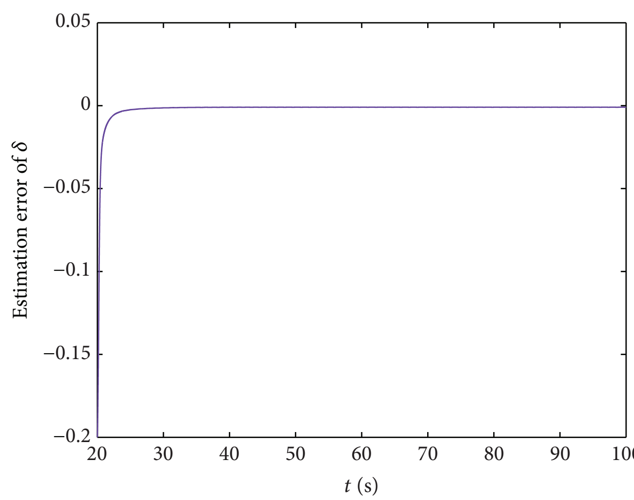

FIGURE 10: The $\tilde{\delta}$ curve of estimation error of δ.

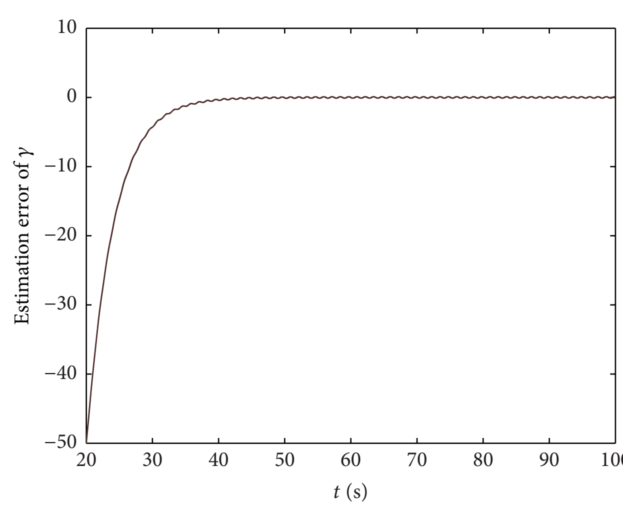

FIGURE 11: The $\tilde{\gamma}$ curve of estimation error of γ.

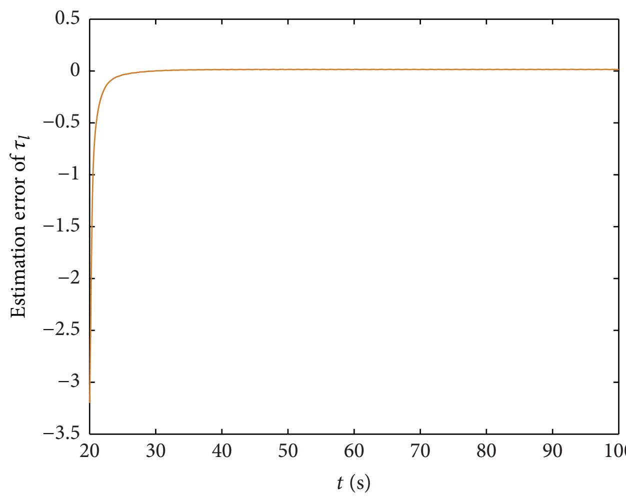

Figure 12: The $\tilde{\tau}_l$ curve of estimation error of τ_l.

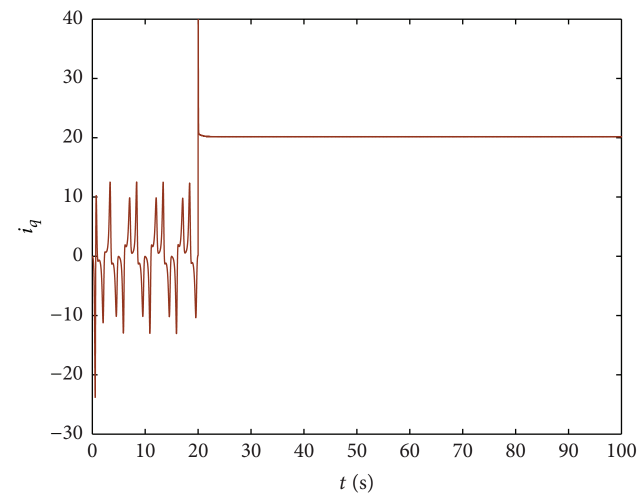

Figure 14: The i_q curve of PMSM chaotic system added the controller inputs u_d and u_q.

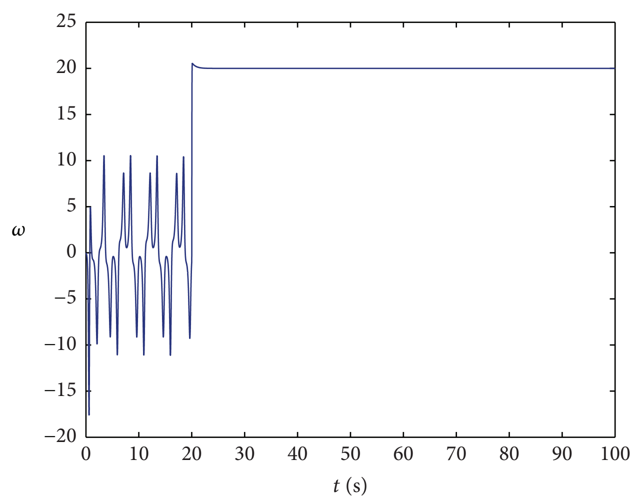

Figure 13: The ω curve of PMSM chaotic system added the controller inputs u_d and u_q.

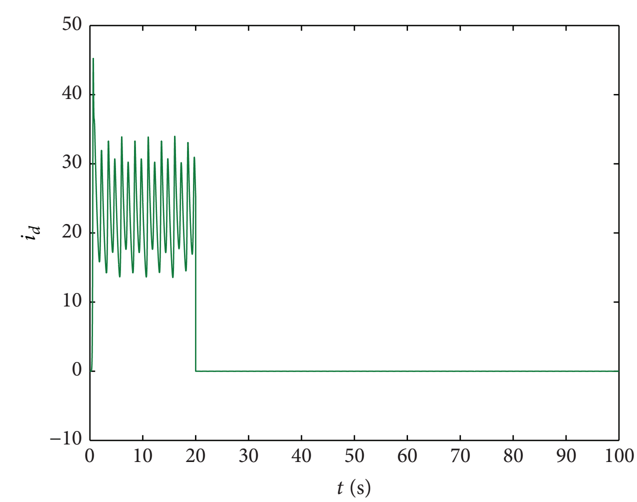

Figure 15: The i_d curve of PMSM chaotic system added the controller inputs u_d and u_q.

Figures 5–7 indicate the proposed controller shown in Figures 8-9 can inhibit the external disturbances.

Figures 10–12 show the estimated errors $\tilde{\delta}$, $\tilde{\gamma}$, and $\tilde{\tau}_l$ of unknown parameters δ, γ, and τ_l for PMSM chaotic system, which testify the effectiveness of constructed adaptive laws and demonstrate the proposed approach has a good robustness against the uncertainties in system parameters.

4.2. Test-II. In reality, the motor parameters are frequently varying with the design values. As a result, the parameters δ, γ, and τ_l in Test-I are changed into $\delta = 0.1$, $\gamma = 25$, and $\tau_l = 1.6$ in Test-II, respectively. Simultaneously, the expected reference signals are also changed and set as $\omega^* = 20$ and $i_d^* = 0$. In a word, the unknown motor parameters and expected reference signals all differ from Test-I, which is able to validate the proposed control algorithm better. The simulation results are shown in Figures 13–20. Figures 13–15 indicate the motor's output states ω, i_q, and i_d added the

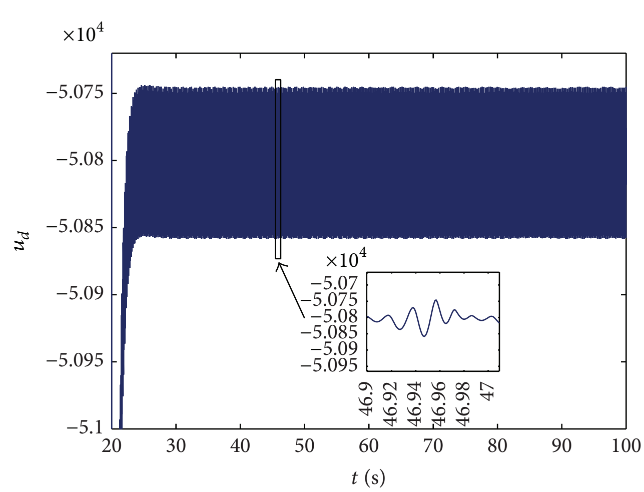

Figure 16: The controller input u_d.

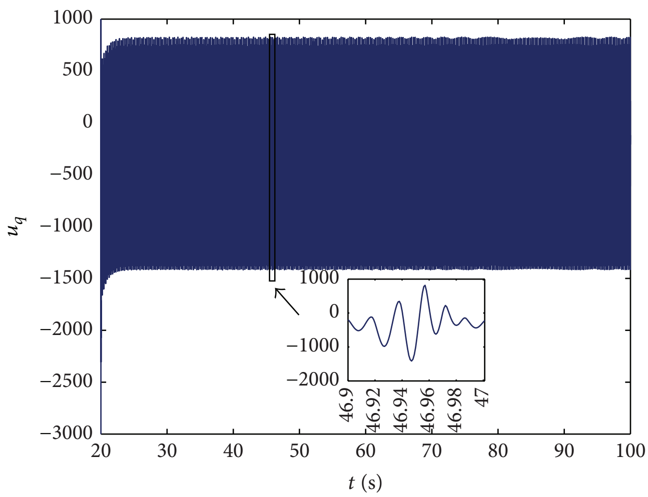

Figure 17: The controller input u_q.

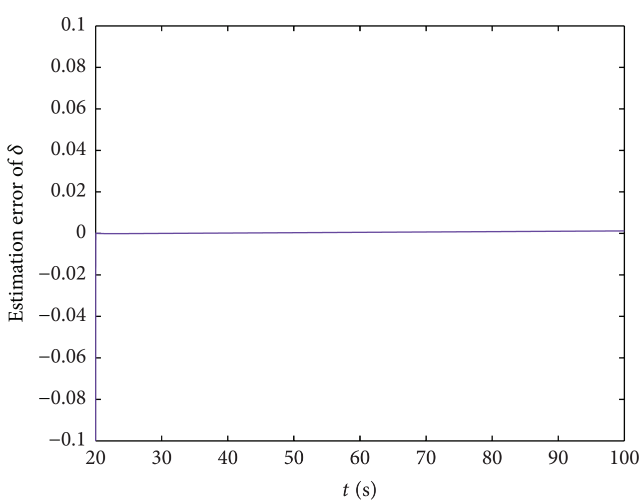

Figure 18: The $\tilde{\delta}$ curve of estimation error of δ.

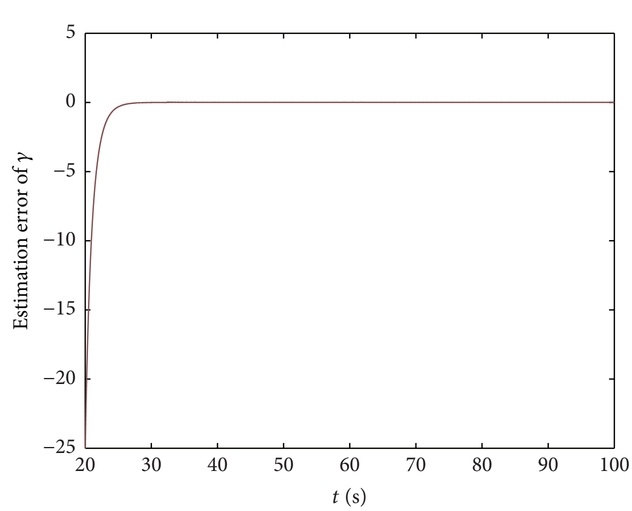

Figure 19: The $\tilde{\gamma}$ curve of estimation error of γ.

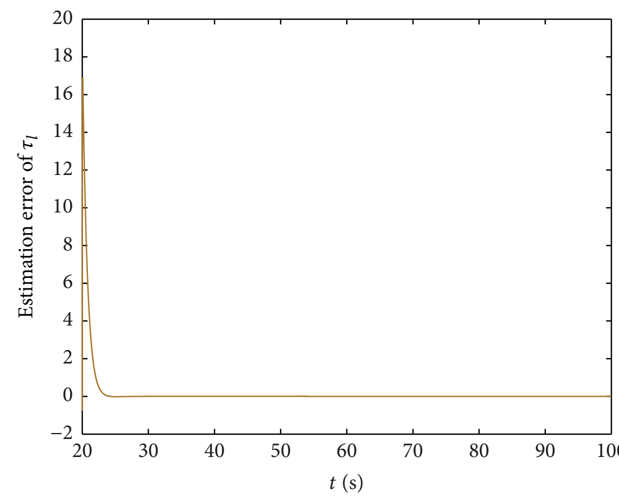

Figure 20: The $\tilde{\tau}_l$ curve of estimation error of τ_l.

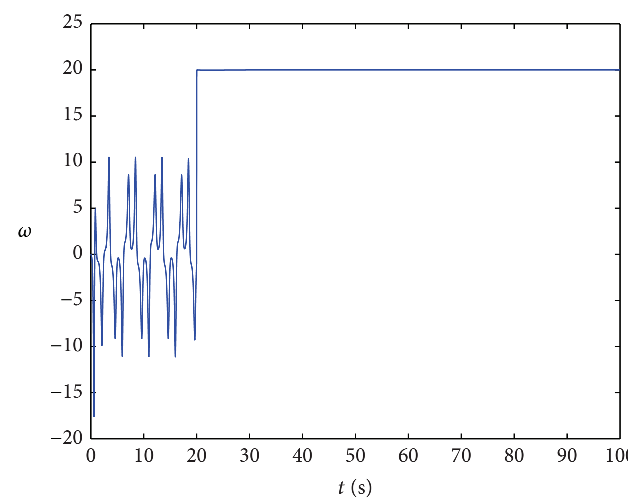

Figure 21: The ω curve of PMSM chaotic system added the controller inputs u_d and u_q.

controller inputs u_d and u_q shown in Figures 16-17, which demonstrate that the designed controller can guarantee the outputs track references well and suppress the external disturbances effectively. Figures 18–20 indicate that the designed adaptive law can estimate the fully unknown parameters precisely even if the fully unknown parameters are changed.

4.3. Test-III. For Test-III, the external disturbances are enlarged in addition to changing the unknown motor parameters and expected reference signals on the basis of Test-II, which are described as $\Delta_1(\mathbf{x}, t) = 40x_3 \sin(5t)$ and $\Delta_2(\mathbf{x}, t) = 20\sin(5t)$. The control difficulty in Test-III is larger than the previous two experiments and Test-III is a more general instance to verify the controller's performance. The simulation results are shown in Figures 21–28. Figures 21–23 give the curves of the state variables ω, i_q, and i_d, which manifest these variables are controlled to their references and chaos is eliminated when adding the proposed controllers

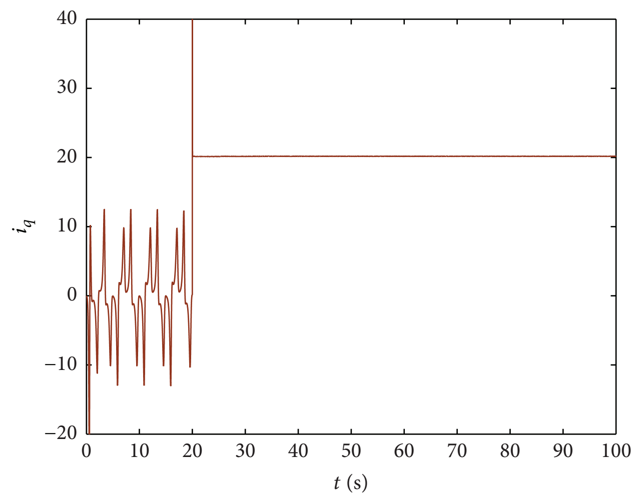

FIGURE 22: The i_q curve of PMSM chaotic system added the controller inputs u_d and u_q.

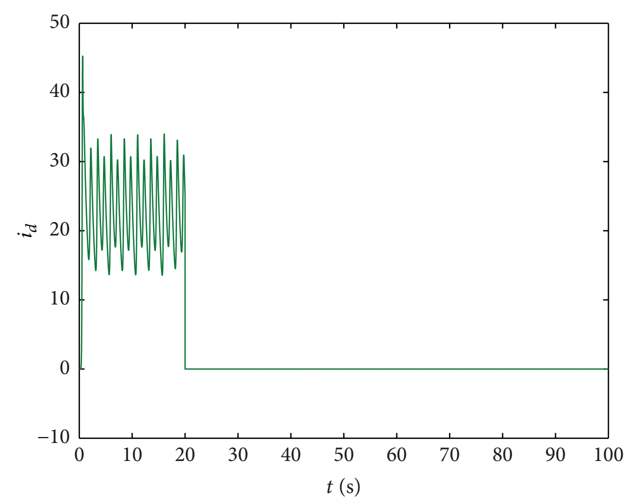

FIGURE 23: The i_d curve of PMSM chaotic system added the controller inputs u_d and u_q.

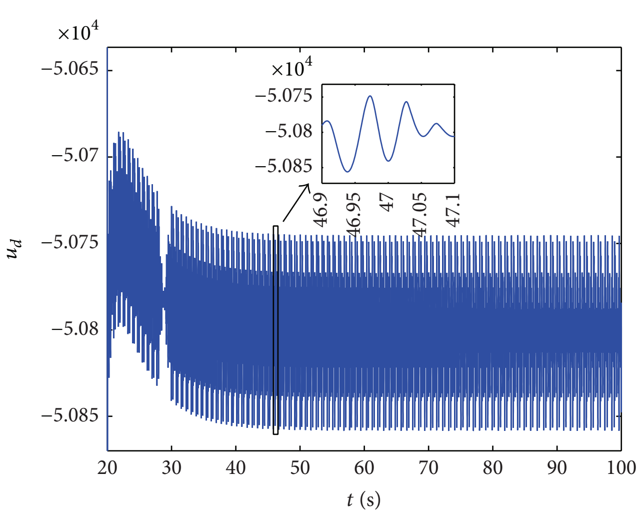

FIGURE 24: The controller input u_d.

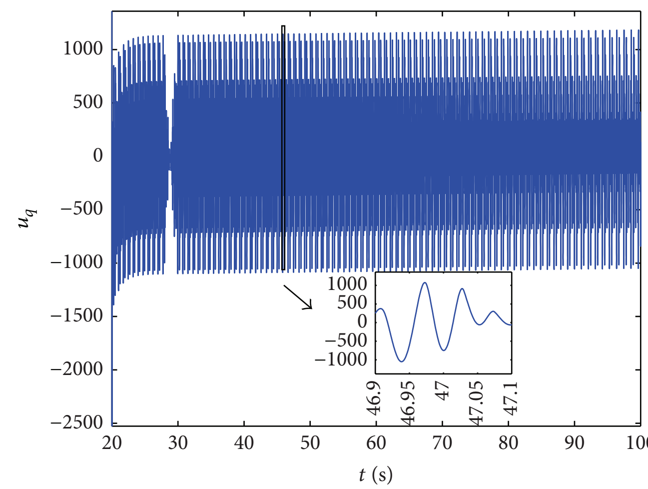

FIGURE 25: The controller input u_q.

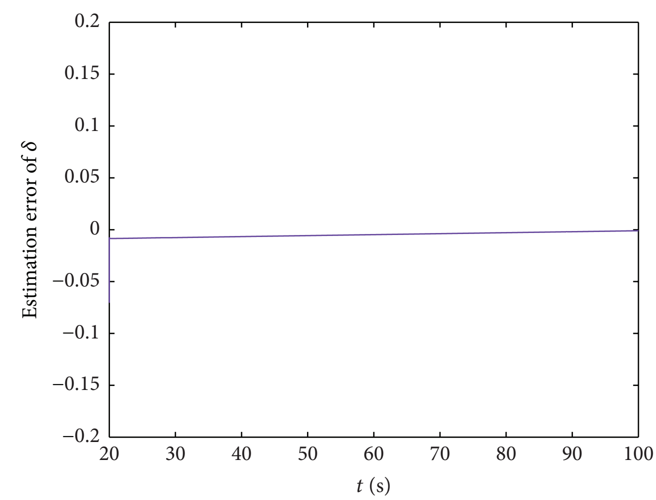

FIGURE 26: The $\tilde{\delta}$ curve of estimation error of δ.

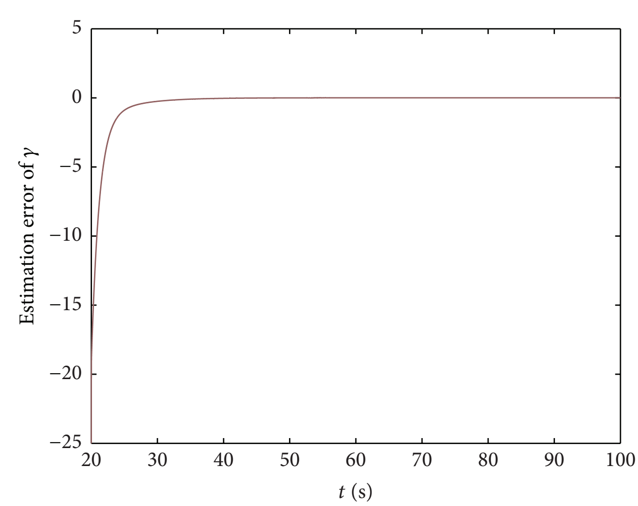

FIGURE 27: The $\tilde{\gamma}$ curve of estimation error of γ.

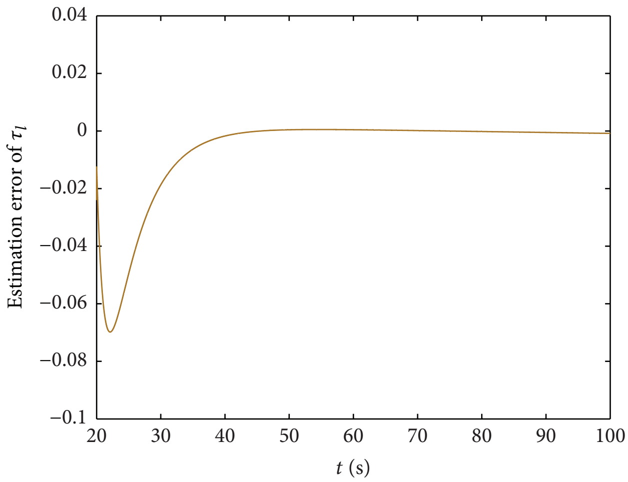

Figure 28: The $\tilde{\tau}_l$ curve of estimation error of τ_l.

u_d and u_q shown in Figures 24-25. Figures 21–23 also illustrate the enlarged external disturbances are restrained by the controllers. The estimation errors of the fully unknown parameters are provided in Figures 26–28, which proves the effectiveness of the adaptive laws again.

Remark 5. Previous researches on parameter estimation of PMSM chaotic system mostly assumed that only partial parameters of the system are unknown. The paper takes fully nondeterministic parameters δ, γ, and τ_l into account; it undoubtedly extends the theory of parameter estimation for PMSM chaotic system.

Remark 6. The action time of control inputs is 20 s in the simulation. The aim of doing this is to observe the effect of the control approach better. In reality, as long as the chaos occurs, the controller will be put into effect.

Remark 7. On the basis of considering fully unknown parameters, the external disturbances are introduced into the PMSM chaotic model. Hence, the designed control consists of two parts. One is to guarantee the state variables to track the reference signals; another is to suppress the external disturbances. In general, the simultaneous consideration of fully unknown parameters and external disturbances makes the research results more general and practical.

5. Conclusions

In this paper, a control approach is proposed to address the control issue of chaos in PMSM system with fully unknown parameters and external disturbances. Main conclusions are acquired as the following:

(1) Through combination of adaptive control with backstepping control, the presented adaptive robust backstepping control scheme resolves the main problems of the conventional backstepping algorithm encountered. And the stability of the designed controller is proved by Lyapunov theory.

(2) The simulation results show that the designed controller is able to make the PMSM operate out of chaotic state quickly, and the adaptive laws are established to estimate the unknown parameters accurately. Furthermore, the proposed algorithm can ensure the unknown parameters converge to the actual values fast and restrain the external disturbance effectively.

(3) The design method in this paper is simple and effective. For PMSM chaotic system with fully unknown parameters, the control variables in proposed approach can be self-adjusted with the changing of system parameters. Therefore, our findings are more practical and more convenient for engineering applications. Future research will discuss the application of the proposed control approach into practical implementation.

Competing Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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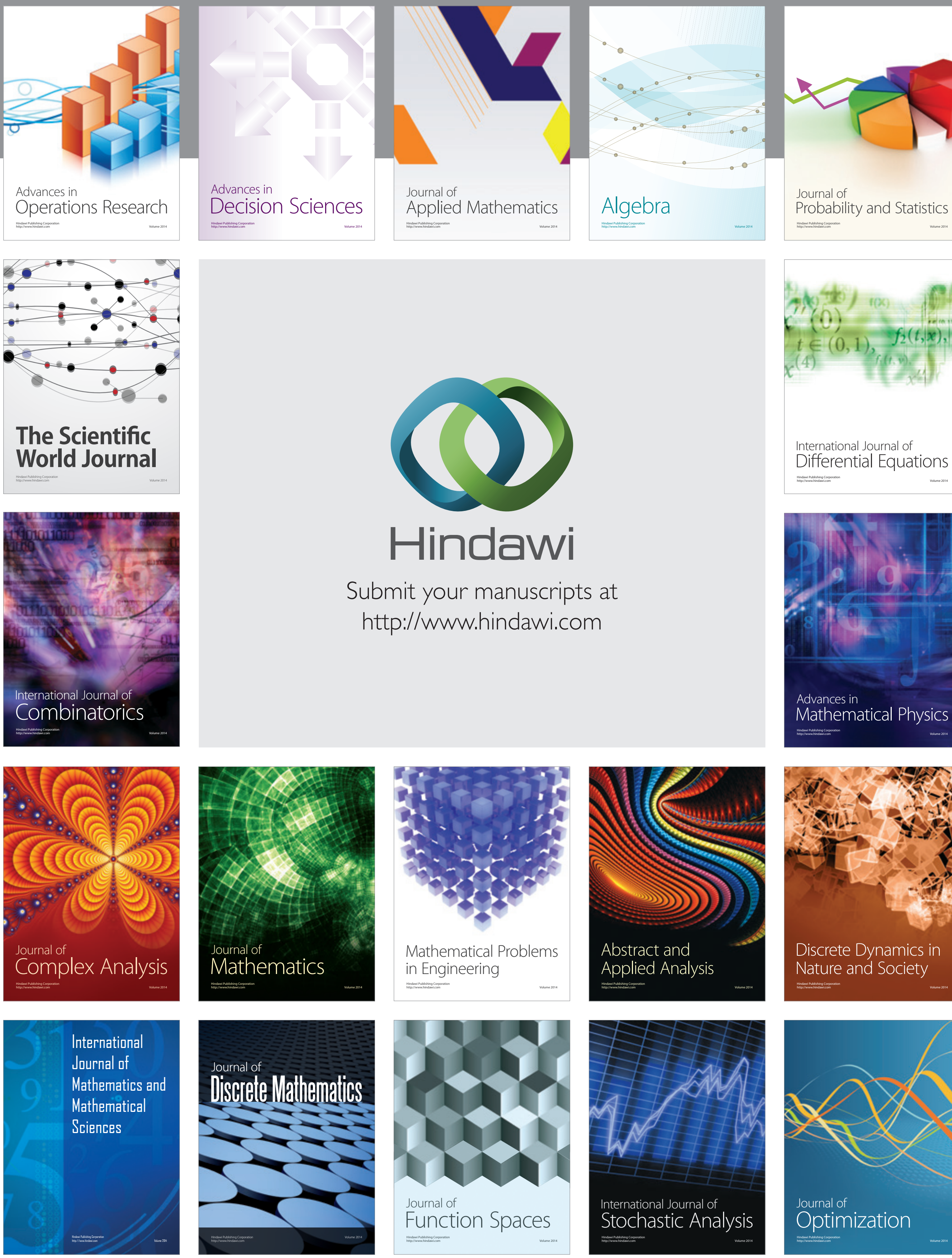
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