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Research Article

A Quasiphysical and Dynamic Adjustment Approach for Packing the Orthogonal Unequal Rectangles in a Circle with a Mass Balance: Satellite Payload Packing

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Packing orthogonal unequal rectangles in a circle with a mass balance (BCOURP) is a typical combinational optimization problem with the NP-hard nature. This paper proposes an effective quasiphysical and dynamic adjustment approach (QPDAA). Two embedded degree functions between two orthogonal rectangles and between an orthogonal rectangle and the container are defined, respectively, and the extruded potential energy function and extruded resultant force formula are constructed based on them. By an elimination of the extruded resultant force, the dynamic rectangle adjustment, and an iteration of the translation, the potential energy and static imbalance of the system can be quickly decreased to minima. The continuity and monotony of two embedded degree functions are proved to ensure the compactness of the optimal solution. Numerical experiments show that the proposed QPDAA is superior to existing approaches in performance.

1. Introduction

2D rectangle packing problems are derived from the industry and antiaircraft field [1–3]. They occur in logistics packing, plate cutting, the layout design of the very large scale integration (VLSI), and satellite modules. They can be divided into unconstrained rectangle packing problems [1] and constrained ones [3]. Both are NP-hard problems and are difficult to be solved. However they have attracted much attention and some packing approaches for different containers have been reported in literatures.

For the 2D rectangle container, the packing approaches mainly include graph theories [4–7], branch-and-bound methods [8–10], dynamic planning [11], heuristics [12–15], artificial intelligent [16], evolutionary approaches [17], and hybrid approaches [18–20]. Regarding the strip container, main packing approaches are branch-and-bound methods [21], heuristics [22, 23], and evolutionary approaches [24, 25]. On the 2D polygon or 3D polyhedron container, the existing packing approaches have heuristics [26], evolutionary approaches [27–29], and integer programming [30]. Some scholars are interested in the packing problem of the convex region and have proposed heuristics [26] and branch-and-bound approaches [31].

The layout design problem of the satellite module described in [32] is an important packing problem, which can be transformed into the problem of packing 2D orthogonal unequal rectangles within a circular container with the mass balance (BCOURP). In 1999, Feng et al. [33] built a mathematical model of this problem and analyzed the isomorphism and equivalent intrinsic properties among its layout schemes by using the graph theory and group theory and proposed a theoretical global optimization algorithm. In 2007, Xu et al. [34] defined embedded degree functions between two rectangles and between the rectangle and circular container and presented a compaction algorithm with the particle swarm local search (CA-PSLS). Their idea is that a feasible solution with a smaller envelope radius obtained through the gradient method is taken as an elite individual and the optimal solution is obtained by the PSO iteration. In 2010, Xu

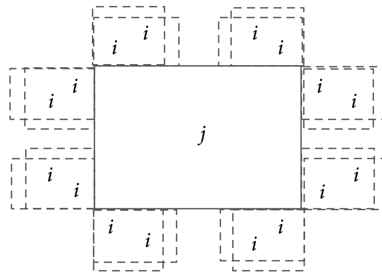


Figure 1: The sketch map for available positions of the rectangle.

et al. [35] suggested a heuristic algorithm ordered by GA (GA-HA, see its algorithm steps in Appendix A and Figure 1). Its key technology is the positioning strategy of constructing the feasible solution. By combining it with GA, the computational efficiency and solution quality are improved.

Generally, there exist strong points and deficiencies for each type of approaches.

(i) For approaches based on the graph theory, there exists combinatorial explosion when the adjacent topological relation is transformed into the layout diagraph without a size limit for the large-scale layout problem. This is because only two limited relations called the vicinity and distance can be used in pruning branch.

(ii) The heuristic method can be used to quickly construct a feasible solution. But it is generally not easy to devise a good heuristic strategy, unless the designer makes a long time painstaking effort and has good luck.

(iii) Stochastic algorithms have the global search ability, but there exists the bottleneck of time-costing overlapping area calculation for them [33]. By combining the heuristic method with the stochastic algorithm, their respective advantages can be exerted to the utmost. Based on this mechanism, CA-PSLS and GA-HA are consecutively proposed to solve this problem. According to No Free Lunch Theory [36], how to obtain the knowledge from the problem itself and its area and fuse it into the heuristic and stochastic search mechanism is a way of designing a high performance approach for this problem.

Huang et al. [37–40] presented a quasiphysical and quasi-human heuristic algorithm and its variants for the circle packing problem. They obtained excellent results. For BCOURP, Xu et al. [34] proposed CA-PSLS based on embedded degrees between circumcircles of two rectangles and between the container and rectangle's circumcircle. But due to the discontinuity of the two embedded degree functions, it is difficult to obtain a high quality solution by using CA-PSLS. That is, constructing the continuous rectangular embedded function and exploring a better optimized mechanism are necessary for this problem. Therefore, in this paper, we consider two definitions of monotonous and continuous embedded degrees between two orthogonal rectangles and between an orthogonal rectangle and the container and suggest a dynamic adjustment strategy. We merge them into the proposed QPDAA to improve the solution quality of BCOURP. Numerical experiments will test effectiveness of the considered QPDAA.

The remainder of this paper is organized as follows. The problem statement and mathematical model are in Section 2. The compact and feasible solution strategy and dynamic adjustment strategy are given in Sections 3 and 4, respectively. This algorithm is presented in Section 5. Section 6 is experiments and analysis. The conclusion is shown in Section 7. The final part is acknowledgment.

2. Problem Statement and Mathematical Model

Consider the following two related definitions where $\mathbf{I}_n = \{1, 2, \ldots, n\}$ and n is the number of rectangles.

Definition 1. As shown in Figure 2, let the origin of the Cartesian coordinate system be the center of the container. Let R_i denote the ith ($i = 1, 2, \ldots, n$) rectangle and let (x_i, y_i), l_i, w_i, m_i, and θ_i be its center, length, width, mass, and direction angle between its long side and the positive direction of the x-axis, respectively. Then a layout scheme of n rectangles $R_i\ (x_i, y_i, l_i, w_i, m_i, \theta_i)$ $(i = 1, 2, \ldots, n)$ can be denoted by $\mathbf{X}'\ (x_i, y_i, \theta_i \mid i = 1, 2, \ldots, n)$.

Definition 2. For a layout scheme $\mathbf{X}'$, if $\theta_i = 0^\circ$ or 90°, then $\mathbf{X}'$ is an orthogonal rectangle packing scheme, R_i $(i \in \mathbf{I}_n)$ is an orthogonal rectangle, and the packing is the orthogonal rectangle packing (see Figure 3).

Herein this paper considers only orthogonal rectangle packing schemes.

Suppose that the center of each rectangle coincides with its mass center. Let T be a rectangle set $\{R_1, R_2, \ldots, R_n\}$; then the mathematical model of this problem can be described as follows. Find a solution $\mathbf{X} = (x_i, y_i, \theta_i \mid i = 1, 2, \ldots, n)^T$ and $\mathbf{X}$ satisfies Formulas (1)–(4). Consider

$$\min\ f(\mathbf{X}) \tag{1}$$

$$\text{s.t.}\quad \operatorname{int}(R_i) \cap \operatorname{int}(R_j) = \emptyset, \quad i, j = 1, 2, \ldots, n,\ i \neq j \tag{2}$$

$$\operatorname{int}(R_i) \cap \operatorname{int}(R_c) = \operatorname{int}(R_i) \quad i = 1, 2, \ldots, n \tag{3}$$

$$J(X) = \sqrt{\left(\sum_{i=1}^{n} m_i x_i\right)^2 + \left(\sum_{i=1}^{n} m_i y_i\right)^2} \leq \delta. \tag{4}$$

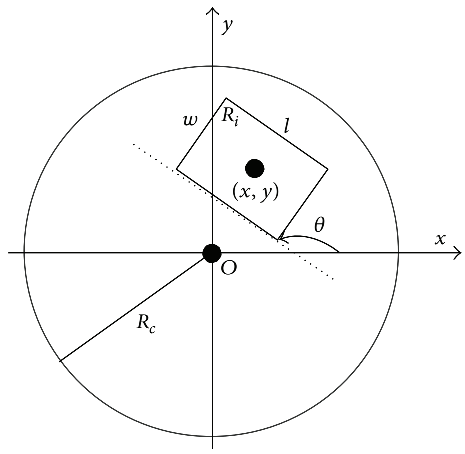


FIGURE 2: The definition of a rectangle.

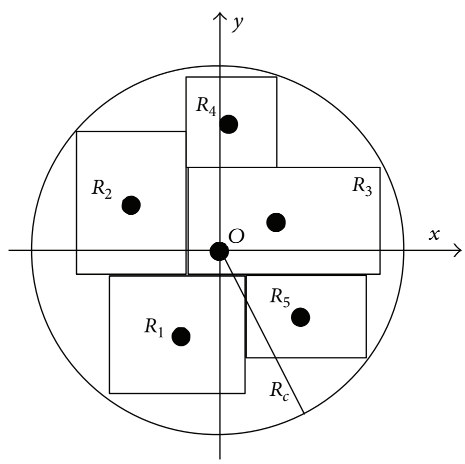


FIGURE 3: The orthogonal rectangle packing scheme.

In Formula (1), $f(\mathbf{X})$ denotes the radius of the enveloping circle of the scheme $\mathbf{X}$ whose circular center is at $(0, 0)$. Formula (2) indicates that there is no overlap region between two rectangles R_i and R_j. Formula (3) indicates that all rectangles are contained in the container. In Formulas (2) and (3), $\text{int}(R_i)$ denotes the interior region of the rectangle R_i. Formula (4) means that the static imbalance $J(\mathbf{X})$ of the solution $\mathbf{X}$ is less than its threshold δ, where $\delta > 0$.

3. Compact and Feasible Solution Strategy

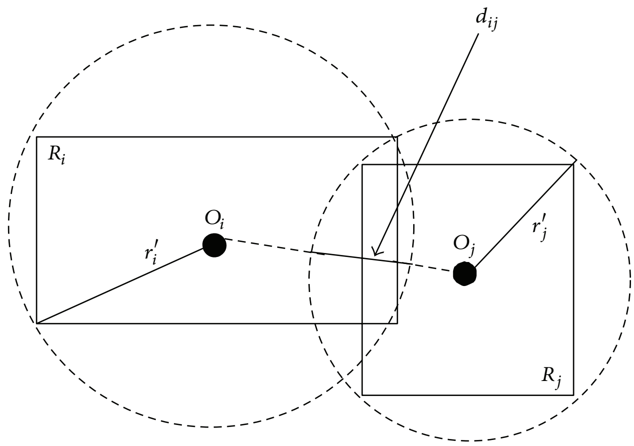
Based on the potential energy function of the embedded degree between two circles, Huang et al. [38–40] proposed the quasiphysical strategy and its variants for the circle packing problem. Inspired by the quasiphysical idea, we suggest a compact and feasible strategy for BCOURP.

3.1. Embedded Degree Function and Related Properties. Xu et al. [34] defined the embedded degrees between two rectangles and between the rectangle and container by Definitions 3 and 4, respectively.

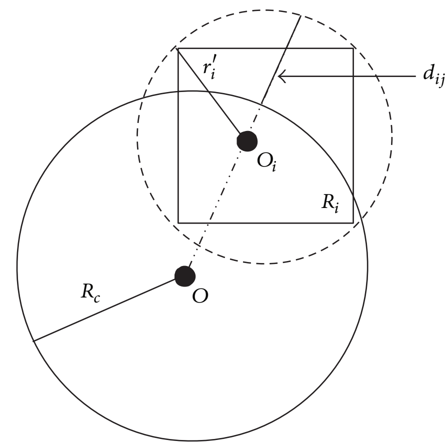
Definition 3. Let r_i' and r_j' ($i, j = 1, 2, \ldots, n$ and $i \neq j$) denote the radii of the circumscribed circles of two rectangles R_i and R_j, respectively, and d_{ij} (see Figure 4(a)) the embedded degree between them (see Figure 4(a)); then d_{ij} can be calculated by

$$d_{ij} = \begin{cases} r_i' + r_j' - \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2} & \text{if } R_i \text{ and } R_j \text{ overlap } (i, j = 1, 2, \ldots, n,\ i \neq j) \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Definition 4. Let R_c and r_i' ($i = 1, 2, \ldots, n$) denote the radii of the container and circumscribed circle of the rectangle R_i, respectively, and d_{0i} (see Figure 4(b)) the embedded



(a) The embedded degree between two rectangles



(b) The embedded degree between the rectangle and container

FIGURE 4: Two embedded degree definitions in [34].

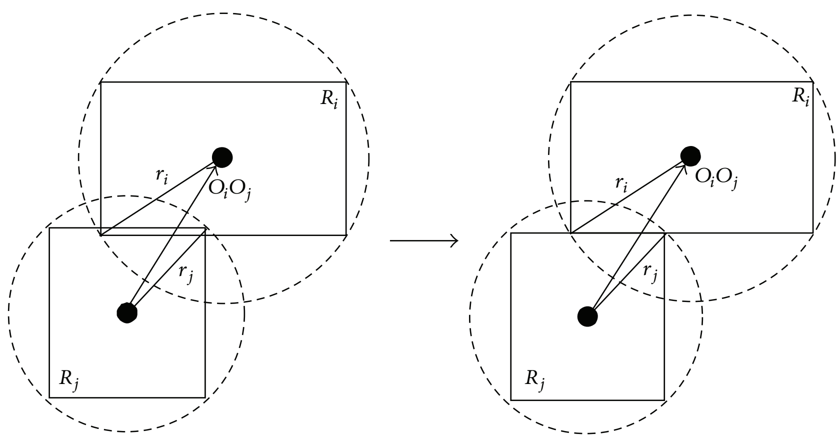


FIGURE 5: The embedded degree between two rectangles in the critical state.

degree between the rectangle and container; then d_{0i} can be calculated by

$$d_{0i} = \begin{cases} \sqrt{x_i^2 + y_i^2} + r_i' - R_c & \text{if } R_i \text{ overlap with the container} \\ & (i = 1, 2, \ldots, n) \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

Both of the above two embedded degree functions are discontinuous in the critical state from overlapping to separating, as has been discussed by Stoyan and Yaskov [41]. For example, as shown in Figure 5, by moving one rectangle along a direction, two rectangles with the overlap area of 10^{-5} (state 1) are changed into $R_i(2.5, 5, 6, 5, 30, 0^\circ)$ and $R_j(0, 0, 5, 4, 20, 90^\circ)$ (state 2). By Formula (5), we know that $d_{ij} = r_i + r_j - O_iO_j \approx 1.52$ for state 1, but $d_{ij} = 0$ for state 2. So, d_{ij} in Definition 3 is discontinuous where $d_{ij} = 0$ ($i, j = 1, 2, \ldots, n$ and $i \neq j$). Similarly, d_{0i} ($i = 1, 2, \ldots, n$) in Definition 4 is also discontinuous (see Figure 6). Owing to their discontinuity, it is difficult to select an appropriate step length to obtain the feasible and compact layout scheme for the gradient iteration of CA-PSLS. Inspired by [41], Definitions 5 and 6 are given for the considered QPDAA.

Definition 5. For two rectangles R_i and R_j ($\theta_i, \theta_j = 0^\circ$ or 90°, $i, j = 1, 2, \ldots, n$ and $i \neq j$) (shown in Figure 7), let r_i and r_j be the radii of their circumscribed circles, respectively, and let d_{ij} denote their embedded degree; then d_{ij} can be calculated by

$$d_{ij} = \begin{cases} \dfrac{(r_i + r_j)}{\left(\left(\log^{u}_{(a_i+a_j)}\right)^2 + \left(\log^{v}_{(b_i+b_j)}\right)^2 + 1\right)}, & \text{If } 0 < u \leq 1, \\ & 0 < v \leq 1 \\ 0 & \text{Otherwise.} \end{cases} \quad (7)$$

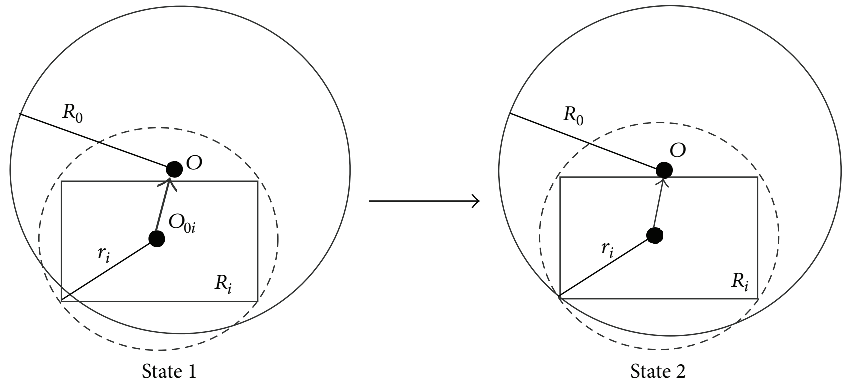


FIGURE 6: The embedded degree between the rectangle and container in the critical state.

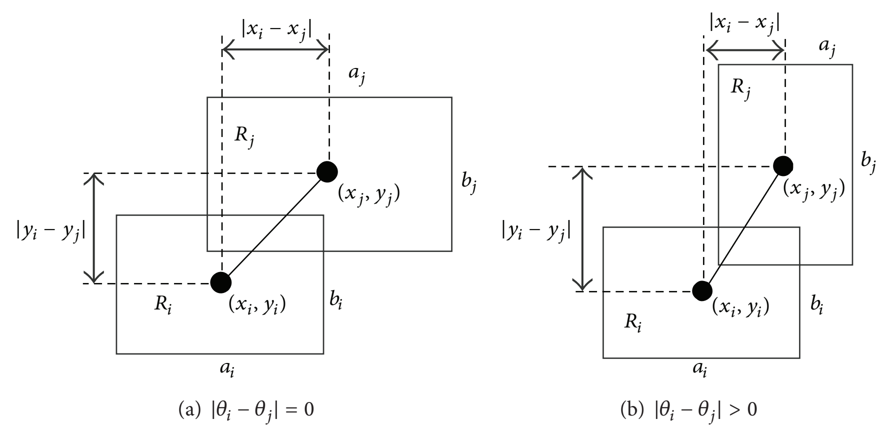


FIGURE 7: Two cases of the embedded degree definition between two rectangles.

In Formula (7), $u = (a_i + a_j - 2|x_i - x_j|)/(a_i + a_j)$, $v = (b_i + b_j - 2|y_i - y_j|)/(b_i + b_j)$. Here, if $\theta_i = 0^\circ$, $a_i = l_i$, $a_j = l_j$, $b_i = w_i$, and $b_j = w_j$ (see Figure 7(a)); otherwise, $a_i = w_i$, $a_j = w_j$, $b_i = l_i$, and $b_j = l_j$ (see Figure 7(b)).

In Formula (7), the embedded degree between two rectangles is the moving distance of the rectangle R_j from an overlap state with the stationary R_i to the separation state along the direction from the center (x_i, y_i) to the center (x_j, y_j). If R_i and R_j are two squares, and the center of R_j is on the diagonal line of R_i and enough close to its center (i.e., $x_i \to x_j$ and $y_i \to y_j$), the moving distance of R_j from the initial state (shown in Figure 8(a)) to the separate state (shown in Figure 8(b)) along the direction of their diagonal lines is about $r_i + r_j$. Thus in the initial state, their embedded degree d_{ij} is close to the maximal value $r_i + r_j$. In addition, when $2|x_i - x_j| \to a_i + a_j$ and/or $2|y_i - y_j| \to b_i + b_j$, $u \to 0$ and/or $v \to 0$. That is, $d_{ij} \to 0$.

Definition 6. For the rectangle R_i ($\theta_i = 0^\circ$ or 90°, $i = 1, 2, \ldots, n$) and the circle container $(0, 0, R_c)$ as shown in Figure 7, let d_{0i} denote the embedded degree between the rectangle R_i and container; then d_{0i} can be calculated by

$$d_{0i} = \begin{cases} \sqrt{\left(|x_i| + \frac{l_i}{2}\right)^2 + \left(|y_i| + \frac{w_i}{2}\right)^2} - R_c & \sqrt{\left(|x_i| + \frac{l_i}{2}\right)^2 + \left(|y_i| + \frac{w_i}{2}\right)^2} > R_c \\ 0 & \sqrt{\left(|x_i| + \frac{l_i}{2}\right)^2 + \left(|y_i| + \frac{w_i}{2}\right)^2} \le R_c \end{cases} \quad (i = 1, 2, \ldots, n). \tag{8}$$

The geometric interpretation of Definition 6 is that when the farthest vertex of the rectangle R_i ($i = 1, 2, \ldots, n$) from the coordinate origin is within the container, their embedded degree $d_{0i} = 0$; otherwise it is the length of the straight line segment pointed by d_{0i} in Figure 9.

For embedded degree functions in Definitions 3 and 5, their geometric figures are two curved semi-cone surfaces

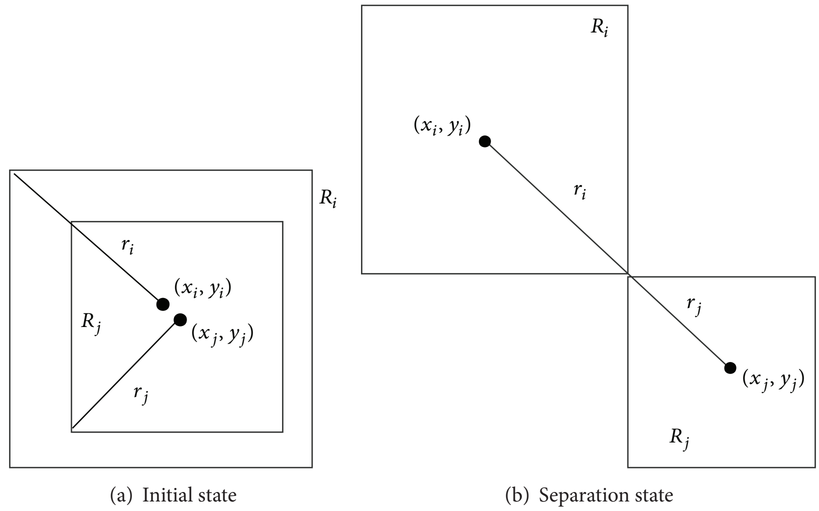


FIGURE 8: The geometric interpretation of definition of the embedded degree between two rectangles.

shown in Figures 10(a) and 10(b), respectively, where it is obvious that there is a gap between the semi-cone surface and *xoy* plane in Figure 10(a) but there is no gap between them in Figure 10(b). It is not difficult to assert that the difference of geometric figures of two embedded degree functions in Definitions 4 and 6 is the same as the above one. After describing Lemma 7, we propose properties of two embedded degree functions in Definitions 5 and 6, respectively.

Lemma 7. *If a binary function $f(x, y)$ is continuous for each variable in a domain, respectively, and is monotonous for the variable x or y, then the function $f(x, y)$ is continuous in the domain.*

Property 1. $\forall i, j \in \mathbf{I}_n$, let R_i and R_j ($i \neq j$ and $\theta_i, \theta_j = 0^\circ$ or 90°) be two rectangles, and the domain $\mathbf{D} = \{u \leq 1 \text{ and } v \leq 1\}$. Then $d_{ij}(u, v)$ in Definition 5 is a continuous binary function in the domain $\mathbf{D}$.

Proof. $\forall i, j \in \mathbf{I}_n$ and $i \neq j$, set $\mathbf{D}_1 = \{(u, v) \mid 0 < u \leq 1 \text{ and } 0 < v \leq 1\}$. From Definition 5, we know that the function $d_{ij}(u, v)$ is continuous on both $\mathbf{D}_1$ and $\mathbf{D} - \mathbf{D}_1$. Here, we prove that it is continuous on the domain $\mathbf{D}_2\{(u, v) \mid (u, v) \in \mathbf{D} \text{ and } u = 0 \text{ or } v = 0\}$.

For $(u, v) \in \mathbf{D}_1$,

$$\lim_{u \to 0} d_{ij}(u, v) = \lim_{u \to 0} \frac{r_i + r_j}{\left(\log_{a_i + a_j}^{u}\right)^2 + \left(\log_{b_i + b_j}^{v}\right)^2 + 1} = 0. \quad (9)$$

Simultaneously,

$$\lim_{v \to 0} d_{ij}(u, v) = \lim_{u \to 0, v \to 0} d_{ij}(u, v) = 0 \quad \text{for } (u, v) \in \mathbf{D}_1. \quad (10)$$

This is because

$$\frac{\partial d_{ij}(u, v)}{\partial u} = \frac{-\left(r_i + r_j\right)\left(a_i + a_j\right)\log_{a_i + a_j}^{u}}{u \ln\left(a_i + a_j\right)\left(\left(\log_{a_i + a_j}^{u}\right)^2 + \left(\log_{b_i + b_j}^{v}\right)^2 + 1\right)^2} > 0 \quad \text{for } (u, v) \in \mathbf{D}_1. \quad (11)$$

According to Lemma 7, the binary function $d_{ij}(u, v)$ is continuous on the domain $\mathbf{D}_2$. Therefore, $d_{ij}(u, v)$ is continuous on the domain $\mathbf{D}$. □

Property 2. $\forall i, j \in \mathbf{I_n}$ and $i \neq j$, for the container with the radius R_c and rectangle R_i ($\theta_i = 0^\circ$ or 90°, $i = 1, 2, \ldots, n$), set $u = x_i$ and $v = y_i$; then the binary function d_{0i} in Definition 6 is continuous on the domain $\mathbf{D}_3\{(u, v) \mid -\infty < u < +\infty \text{ and } -\infty < v < +\infty\}$.

3.2. Extruded Force and Energy Function. In order to quickly decrease the overlapping area of the rectangle packing system, we define the extruded forces between two rectangles and between the rectangle and container.

Definition 8. Let R_i and R_j be two rectangles with the embedded degree d_{ij} ($i, j = 1, 2, \ldots, n$ and $i \neq j$). Then the extruded force $\overrightarrow{F_{ji}}$ between R_i and R_j is calculated by

$$\overrightarrow{F_{ji}} = \frac{\alpha d_{ji} \overrightarrow{d_{ji}}}{\left|\overrightarrow{d_{ji}}\right|} \quad (i, j = 1, 2, \ldots, n, \ i \neq j). \quad (12)$$

Definition 9. Let R_i ($i = 1, 2, \ldots, n$) and R_c denote the rectangle i and container. Then the extruded force $\overrightarrow{F_{0i}}$ between R_i and R_c can be calculated by Formula (13), whose direction

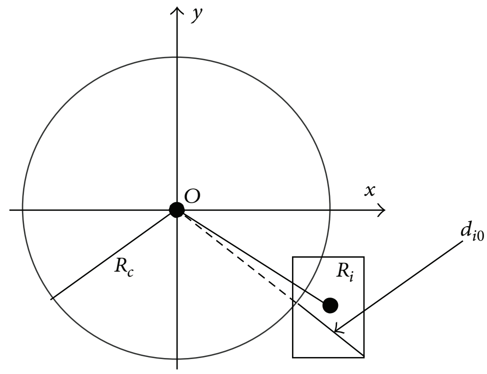


Figure 9: The schematic diagram overlapped between the orthogonal rectangle and container.

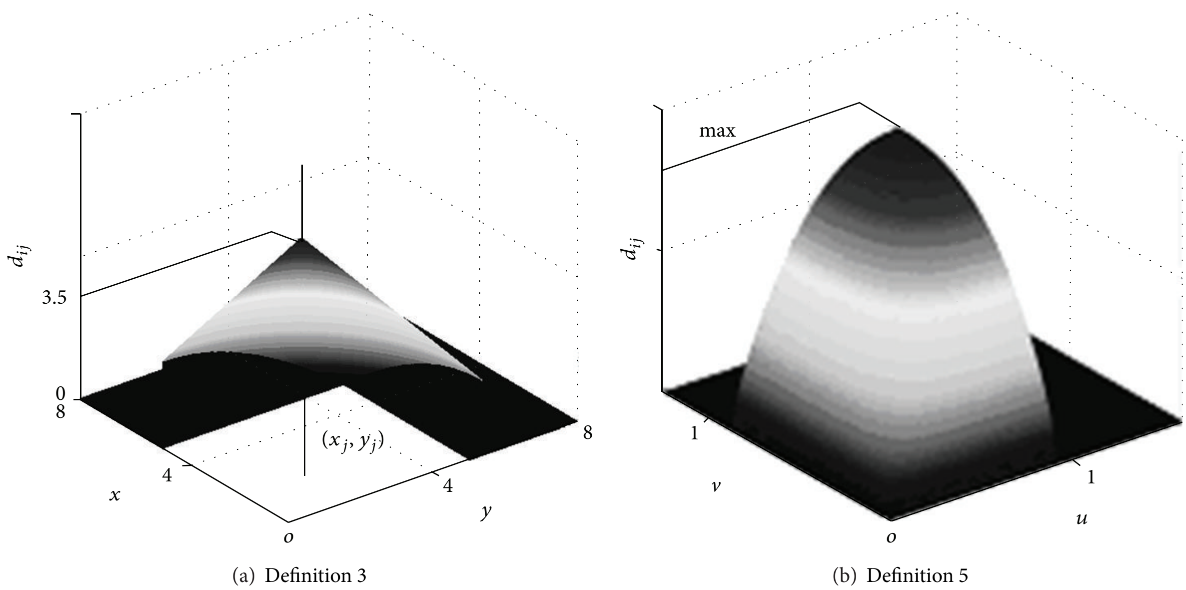


Figure 10: The curved semi-cone surface of the embedded degree of two orthogonal rectangles.

is the direction from the center of the container to the furthermost rectangular vertex (see Figure 6):

$$\overrightarrow{F_{0i}} = \frac{\beta d_{0i}\overrightarrow{d_{0i}}}{\left|\overrightarrow{d_{0i}}\right|} \quad (i = 1, 2, \ldots, n). \tag{13}$$

By experiments, we can know that $\alpha > \beta$.

So, the extruded resultant force $\overrightarrow{F_i}$ of R_i $(i = 1, 2, \ldots, n)$ in the rectangle packing scheme can be calculated by

$$\overrightarrow{F_i} = \sum_{j=0, i \neq j}^{n} \overrightarrow{F_{ji}}. \tag{14}$$

Definition 10. Let E_{ji} and E_{0i} $(i, j = 1, 2, \ldots, n$ and $i \neq j)$ denote extruded potential energies of R_i with respect to R_j and the container, respectively. Then E_{ji} can be calculated by Formula (15), where d_{ij} and d_{i0} denote two embedded degrees between two rectangles R_i and R_j and between the rectangles R_i and container:

$$E_{ji} = d_{ji}^2 \quad (i = 1, 2, \ldots, n,\ j = 0, 1, \ldots, n,\ i \neq j). \tag{15}$$

Definition 11. The total extruded potential energy E_i of R_i can be calculated by

$$E_i = \sum_{j=0, j \neq i}^{n} E_{ji} \quad (i, j = 1, 2, \ldots, n,\ i \neq j). \tag{16}$$

Let S_i be the area of the rectangle R_i; then E_i and E_i/S_i $(i = 1, 2, \ldots, n)$ are its absolute and relative extruded potential energies, respectively.

3.3. Compact and Feasible Solution Strategy. By predetermining envelope radius R_0 of this problem, the extruded force and direction can be calculated by Formula (13). The extruded force of the envelope circle makes each rectangle close to the center of the container in the direction. For each rectangle,

Table 1: Parameters of rectangles for five layout examples.

Example 1	(length, width, mass) (8, 6, 12), (8, 8, 16), (10, 6, 15), (12, 4, 12), (6, 6, 9)
Example 2	(length, width, mass) (8, 6, 12), (8, 8, 16), (10, 6, 15), (10, 8, 20), (10, 10, 25), (12, 6, 18)
Example 3	(length, width, mass) (8, 6, 12), (8, 8, 16), (10, 6, 15), (10, 8, 20), (10, 10, 25), (12, 6, 18), (12, 4, 12), (12, 8, 24) (12, 10, 30)
Example 4	(length, width, mass) (8, 5, 10), (4, 8, 8), (10, 6, 15), (7, 8, 14), (10, 3, 7.5), (12, 6, 18), (12, 4, 12), (12, 6, 18), (8, 10, 20), (7, 3, 6), (8, 6, 6), (8, 3, 15), (10, 6, 20), (10, 8, 17.5), (10, 7, 15), (12, 5, 15), (12, 4, 12), (10, 8, 20), (12, 10, 30), (6, 6, 9)
Example 5	(length, width, mass) (26, 26, 8), (22, 28, 16), (20, 30, 16), (20, 34, 15), (34, 22, 17), (20, 32, 15), (30, 20, 14), (28, 26, 15), (20, 28, 14), (26, 38, 13), (34, 30, 12), (26, 36, 17), (30, 34, 12), (28, 24, 17) (32, 24, 10), (30, 38, 11), (30, 20, 18), (22, 22, 17), (30, 32, 11), (28, 28, 10), (20, 38, 18), (22, 30, 13) (32, 32, 14), (26, 26, 17), (34, 20, 14), (26, 32, 18), (38, 22, 19), (36, 34, 18), (36, 24, 13), (26, 30, 15) (26, 38, 12), (30, 38, 19), (28, 24, 15), (38, 34, 19), (26, 20, 14), (32, 36, 16), (28, 22, 13), (24, 28, 19), (38, 22, 17), (20, 38, 16)

the extruded forces of all others with respect to the rectangle drive away themselves to relieve the pressure in respective direction and their extruded resultant force is calculated by Formula (12). In this paper, the above two steps are used to decrease the overlapping area of the packing scheme and make it compact.

4. Dynamic Adjustment Strategy

Considering the problem of the low efficiency and possible local optimum (e.g., large static imbalance) of the iteration of two steps in Section 3.3, we propose Property 3 and dynamic adjustment strategy for QPDAA.

4.1. Related Property. For optimizing the static imbalance of the layout scheme, we introduce Property 3.

Property 3. Assume $A_1(x_m, y_m)$ is the mass center of an orthogonal packing scheme $\mathbf{X}_1$ of this problem and $A_1(x_m, y_m) \neq (0,0)$ and $\mathbf{X}_2$ is another scheme obtained by interchanging centers of two rectangles R_i with a mass m_i and R_j $(i, j \in \mathbf{I}_n$ and $i \neq j)$ with a mass m_j in $\mathbf{X}_1$. If $m_j > m_i$, $d_j > d_i$, centers (x_i, y_i) and (x_i, y_i) satisfy Formula (17), then $J(\mathbf{X}_1) > J(\mathbf{X}_2)$, where $d_k = (x_k^2 + y_k^2)^{1/2}$ $(k = i, j)$:

$$2\left(x_j - x_i\right) x_m \sum_{k=1}^{n} m_k + 2\left(y_j - y_i\right) y_m \sum_{k=1}^{n} m_k > \left(m_j - m_i\right) d_{ij}^2. \quad (17)$$

In Formula (17), $d_{ij} = ((x_j - x_i)^2 + (y_j - y_i)^2)^{1/2}$.

Proof. Consider

$$\because J^2\left(X_1\right) = \left(\sum_{k=1}^{n} m_k x_k\right)^2 + \left(\sum_{k=1}^{n} m_k y_k\right)^2$$

$$J^2\left(X_2\right) = \left(\sum_{k=1}^{n} m_k x_k - m_i x_i - m_j x_j + m_i x_j + m_j x_i\right)^2 + \left(\sum_{k=1}^{n} m_k y_k - m_i y_i - m_j y_j + m_i y_j + m_j y_i\right)^2$$

$$\begin{aligned}
\therefore J^2\left(X_1\right) - J^2\left(X_2\right) &= \left(m_i x_i + m_j x_j - m_i x_j - m_j x_i\right) \\
&\quad \times \left(2\sum_{k=1}^{n} m_k x_k - m_i x_i - m_j x_j + m_i x_j + m_j x_i\right) \\
&\quad + \left(m_i y_i + m_j y_j - m_i y_j - m_j y_i\right) \\
&\quad \times \left(2\sum_{k=1}^{n} m_k y_k - m_i y_i - m_j y_j + m_i y_j + m_j y_i\right)^2 \\
&= \left(m_i - m_j\right)\left(x_i - x_j\right) \\
&\quad \times \left[2x_m \sum_{k=1}^{n} m_k x_k - \left(m_i - m_j\right)\left(x_i - x_j\right)\right] \\
&\quad + \left(m_i - m_j\right)\left(y_i - y_j\right) \\
&\quad \times \left[2y_m \sum_{k=1}^{n} m_k y_k - \left(m_i - m_j\right)\left(y_i - y_j\right)\right] \\
&= 2\left(x_j - x_i\right) x_m \sum_{k=1}^{n} m_k + 2\left(y_j - y_i\right) y_m \sum_{k=1}^{n} m_k \\
&\quad - \left(m_j - m_i\right) d_{ij}^2 > 0.
\end{aligned} \quad (18)$$

Property 3 indicates that, for selecting rectangle $R_i \in \mathbf{X}_1$, we can find a rectangle R_j in the sector area $A_1 O A_2$ with an angle φ (shown as in Figure 11) and interchange them to obtain $\mathbf{X}_2$ whose static imbalance is less than $\mathbf{X}_1$. Here, the angle φ satisfies

$$2\sum_{k=1}^{n} m_k \left(x_m^2 + y_m^2\right)^{1/2} \left[\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2\right]^{1/2} \cos\left(\varphi\right) = \left(m_j - m_i\right) d_{ij}^2. \quad (19)$$

□

4.2. Dynamic Adjustment Strategy. Let $\mathbf{X} = (x_k, y_k, \theta_k \mid k = 1, 2, \ldots, n)$; we consider the following dynamic adjustment strategy.

Table 2: Performance comparisons of four algorithms.

Example number	Size	Algorithm	Average r	Standard deviation	Minimal r	Maximal r	Average t/s
1	5	GA + HA [35]	11.7703	0.0335	11.7597	11.8655	0.242
		CA-PSLS [34]	12.2181	0.5214	11.5443	13.0982	11.657
		QPDAA	11.6756	0.0235	11.4737	11.8533	0.953
2	6	IGA [44]	—	—	14.62000	—	18.000
		CA-PSLS [34]	15.19402	—	14.39625	—	19.904
		QPDAA	14.68374	—	14.344996	14.909841	4.963
3	9	GA + HA [35]	18.1709	0.2361	17.8988	18.6041	1.262
		CA-PSLS [34]	20.0899	1.1957	18.3758	22.0803	58.126
		QPDAA	18.2461	0.0694	17.6880	18.5983	1.110
4	20	GA + HA [35]	23.2713	0.4728	22.6396	24.0605	9.989
		CA-PSLS [34]	32.7216	4.0065	27.2863	38.5048	321.071
		QPDAA	22.8368	0.0222	22.3510	23.7884	2.625
5	40	GA [35]	119.2396	1.5087	115.836	120.7099	62.795
		CA-PSLS [34]	253.4165	43.8096	195.5914	306.2740	1367.573
		QPDAA	119.1383	0.6887	115.6252	120.5102	3.375

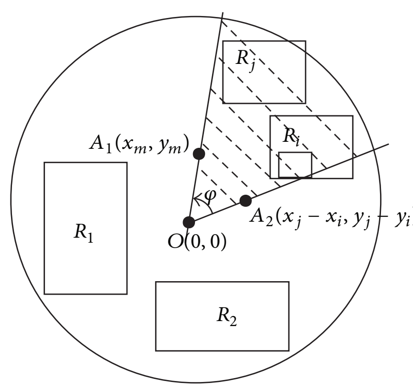


Figure 11: The geometric area of Formula (17).

Table 3: The layout schemes of the proposed QPDAA for Examples 1 and 2.

Example 1			Example 2		
i	x_i	y_i	i	x_i	y_i
1	5.017966	−4.093750	1	−5.810001	−7.386930
2	4.932835	2.906250	2	5.686532	6.537550
3	−3.982034	−4.093750	3	8.827469	−0.519116
4	−5.067165	0.906250	4	3.371405	−7.538003
5	−2.067165	5.906250	5	−3.346021	6.667169
			6	−7.636339	−1.338337

Table 4: The layout schemes of the proposed QPDAA for Example 3.

i	x_i	y_i
1	0.074698	−3.832386
2	2.329571	3.285391
3	4.944505	−10.373944
4	3.514680	11.503878
5	11.343280	1.762618
6	−6.074076	−9.903838
7	10.077353	−5.359918
8	−10.295647	−2.839607
9	−7.690402	6.187573

4.2.1. Rectangle-Interchanging. According to Property 3, the static imbalance of the packing scheme can be decreased by interchanging positions of two rectangles.

Two rectangles, R_i with a smaller mass and R_j with a larger mass ($i, j \in \mathbf{I}_n, d_i < 0.5R_0, 0.5R_0 < d_j < R_0$), are selected from $\mathbf{X}$. If R_i and R_j satisfy Formula (17), then we update $\mathbf{X}$ by $x_i \leftrightarrow x_j$, $y_i \leftrightarrow y_j$, $\theta_i \leftrightarrow \theta_j$.

4.2.2. Rotation and Off-Trap. (i) A rectangle with a larger pain degree is found out from $\mathbf{X}$ and is rotated 90° round its center counterclockwise direction to relieve its pain. (ii) A rectangle with a larger pain degree is found out from $\mathbf{X}$ and is moved to such a place in the container where its pain degree is smaller.

Table 5: The layout scheme of the proposed QPDAA for Example 4.

i	x_i	y_i
1	2.275791	−8.061370
2	0.693375	15.930070
3	8.143939	8.438630
4	−4.806625	15.930070
5	−5.021757	−17.061370
6	5.978243	−15.573636
7	0.275791	−3.561370
8	−12.856061	7.930070
9	−5.724209	−10.561370
10	6.193375	17.938630
11	−16.608718	1.930070
12	7.143939	3.938630
13	−7.608718	1.930070
14	16.143939	1.426364
15	−1.856061	8.430070
16	9.143939	13.938630
17	3.391282	0.438630
18	−14.724209	−5.069930
19	12.275791	−7.573636
20	−12.724209	−12.069930

We know that the role of (ii) is similar to a construction of the nonisomorphic layout pattern [42, 43].

4.2.3. Center Translation. If its mass center $P_m(x_m, y_m) \neq O(0,0)$, then $\mathbf{X} = (x_k - x_m, y_k - y_m, \theta_k \mid k = 1, 2, \ldots, n)$.

5. The Proposed Algorithm

Through an organic combination of the compact and feasible solution strategy and dynamic adjustment strategy, we present QPDAA for BCOURP.

Let R_0 and r^* be the predetermined value and an allowable maximum of the envelope radius of the solution, respectively; n is the number of rectangles. $P_m(x_m, y_m)$, $f(\mathbf{X})$, and $E(\mathbf{X})$ denote the mass center, envelope radius, and extruded potential energy of the packing scheme $\mathbf{X}$, respectively. $E_k(\mathbf{X})$ $(k = 1, 2, \ldots, n)$ is the extruded potential energy of the rectangle R_k; h is the step length; N is the maximum translation times. Then steps of the proposed QPDAA are shown in Algorithm 1.

6. Experiments and Analysis

6.1. Experiments. The proposed QPDAA is coded in VC++ 6.0 and carried on a Pentium 3 GHZ PC with 512 MB memory. CA-PSLS [34], GA-HA [35] are coded in VC++ 6.0 and two algorithms are carried on a Pentium 1.83 GHZ with 512 MB memory; IGA [44] is carried on an IBM 586 166 MHz.

Experiment 1. Five examples are taken from [33, 35] and are used in testing the performance of the proposed QPDAA.

Table 6: The layout scheme of the proposed QPDAA for Example 5.

i	x_i	y_i
1	−20.484058	−97.670716
2	96.802177	−23.135827
3	−32.957579	−3.670716
4	−17.484058	−67.670716
5	−34.765848	92.329284
6	−16.197823	−34.670716
7	49.515942	−85.946702
8	−5.847798	30.463901
9	24.515942	−86.536099
10	−55.957579	−24.941525
11	−93.957579	1.531453
12	22.042421	4.053298
13	23.234152	85.053298
14	71.802177	−25.946702
15	−71.484058	−59.468547
16	92.042421	9.864173
17	23.152202	57.053298
18	3.515942	−83.536099
19	−32.847798	65.329284
20	−41.484058	−57.941525
21	−32.957579	30.329284
22	66.042421	1.053298
23	9.802177	−56.536099
24	−4.765848	88.463901
25	−59.957579	4.058475
26	−4.847798	59.463901
27	12.802177	−29.536099
28	−86.957579	−30.468547
29	52.515942	−63.946702
30	−64.765848	69.531453
31	44.802177	−32.946702
32	−85.957579	35.531453
33	86.152202	40.864173
34	57.234152	69.864173
35	21.152202	32.053298
36	−6.957579	−0.536099
37	−56.957579	25.058475
38	−45.484058	−85.941525
39	53.152202	35.053298
40	45.042421	5.053298

Data of all examples are shown in Table 1. For the proposed algorithm, we take $\alpha = 18$, $\beta = 56$, $\varepsilon = 10^{-20}$, $h = 0.8$, $\delta = 6$, and $N = 120$, respectively. For Examples 1–5, we take $R_0 = 11.4, 14.3, 17.5, 22.3$, and 115.5 and take $r^* = 1.05R_0$, $1.05R_0$, $1.07R_0$, $1.07R_0$, and $1.05R_0$, respectively. Running the proposed QPDAA 30 times for each example (the success rate is 100%), we show its average running time, average envelope radius, standard variance of the radius, and the maximum and minimum envelope radii in Table 2; for each example, its layout scheme diagraph is shown in Figure 12; the other data

Table 7: The effect of parameters R_0 and r^* on the optimal radius and running time for the proposed QPDAA.

Experiment number	1	2	3	4	5
Size	5	6	9	20	40
R_0	11.4	14.3	17.5	22.3	115.5
r^*	$1.05R_0$	$1.05R_0$	$1.07R_0$	$1.07R_0$	$1.05R_0$
The optimal radius	11.4737	14.344996	17.6880	22.3510	115.6252
Running time	0.953	3.5872	1.110	2.625	2.625
R_0	11.4	14.3	17.5	22.06	114.8
r^*	$1.02R_0$	$1.02R_0$	$1.03R_0$	$1.01R_0$	$1.01R_0$
The optimal radius	11.450458	14.309924	17.597856	22.141009	115.042385
Running time	1.297	8.963	7.906	12.920	17.125

```
Definite X, X_1, X_2, R_0, r^*, h, ε, k, j, N, α, β, δ and initialize δ = 6, h = 0.8, ε = 10^{-20}, α = 18, β = 56, k = 0, j = 0, N = 120;
Do
{
  Randomly generate X_1 with the envelope radius R_0 and calculate E(X_1); j = 0;
  If (E(X_1) < ε)
    Continue;
  Else break;
  While (1)
  {   Do
      {  j = j + 1; for i = 1, 2, ..., n, calculate \vec{F_i} of R_i and X_{2i} = X_{1i} + h · \vec{F_i};
         Calculate E(X_2) and f(X_2);
         If (E(X_2) < ε and f(X_2) ≤ r^*) {X = X_2; j = N + 1}
         Else if (E(X_2) ≥ ε){Calculate |∇E| = ((1/n) \sum_{i=1}^{n} | \vec{F_{ji}} |^2)^{1/2}; X_1 = X_2;}
              Else if (f(X_2) > r^*) {j = −1; break;}
      } While (|∇E| < 1 and j < N);
      If (j = −1 or j > N) break;
      If (J(X_2) > δ) find R_i and R_j which satisfy formula (17) in X_2 and x_i ↔ x_j, y_i ↔ y_j, θ_i ↔ θ_j;
      Find p so that E_p(X_2)/S_p = max{E_i(X_2)/S_i, i = 1, 2, ..., n};
      If (k ≠ p) {by rotating R_p, X_1 is obtained;
                 If (E_p(X_1) > 0.75E_p(X_2)) update X_1 by moving (x_p, y_p), k = p;}
      Else find q so that E_q(X_2) = min{E_i(X_2), i = 1, 2, ..., n}, and update X_1 by rotating R_p;
      If (E_q(X_1) > 0.75E_q(X_2)), update X_1 by moving (x_q, y_q) out, k = q;
  }
} while (f(X_1) > r^*);
Calculate the mass center P_m(x_m, y_m), and X = (x_k − x_m, y_k − y_m, θ_k | k = 1, 2, ..., n), and calculate f(X);
Output the layout scheme X and envelope radius f(X);
End the algorithm.
```

Algorithm 1

Table 8: The optimal layout schemes of Examples 1 and 2 for the proposed QPDAA.

i	x_i	y_i	i	x_i	y_i
1	−2.954693	−5.470039	1	4.948526	8.054106
2	5.045307	−2.429991	2	4.903437	−6.945894
3	−2.408844	4.570009	3	−4.140424	−8.010294
4	−4.954693	−0.429991	4	5.911271	1.054106
5	5.591156	4.570009	5	−4.088729	5.989706
			6	−5.096563	−2.010294

in Table 2 is taken from [33–35]. The optimal layout schemes of the proposed QPDAA are shown in Tables 3, 4, 5, and 6 for 5 examples.

Experiment 2. For testing the effects of R_0 and r^* on the minimal radius and running time with the proposed QPDAA, we take another set of R_0 and r^* (five examples and other parameters are the same as those of Experiment 1) and run the proposed QPDAA procedure 30 times for each example. The minimal radii and running times are given in Table 7. Their layout schemes and layout diagraphs are shown in Tables 8, 9, 10, and 11 and Figure 13. It can be found from

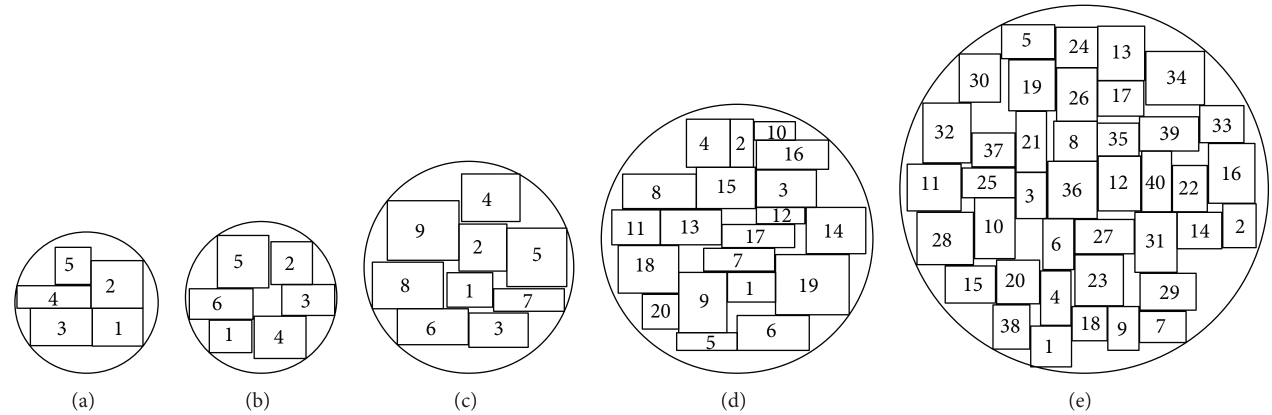


Figure 12: The layout diagraphs of five examples for the proposed QPDAA.

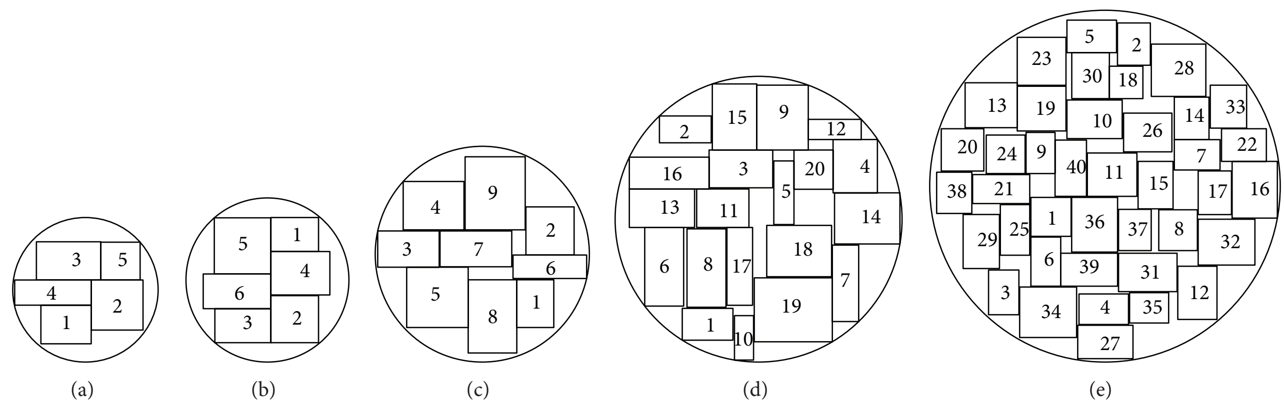


Figure 13: The optimal layout scheme diagraphs of five examples for the proposed QPDAA.

Table 9: The optimal layout schemes of Example 3 for the proposed QPDAA.

i	x_i	y_i
1	8.952825	−8.127957
2	11.192355	3.872043
3	−11.893145	0.866506
4	−7.807645	7.896834
5	−7.161996	−7.133494
6	11.084588	−2.235259
7	−0.893145	0.866506
8	1.838004	−10.127957
9	2.125081	10.168689

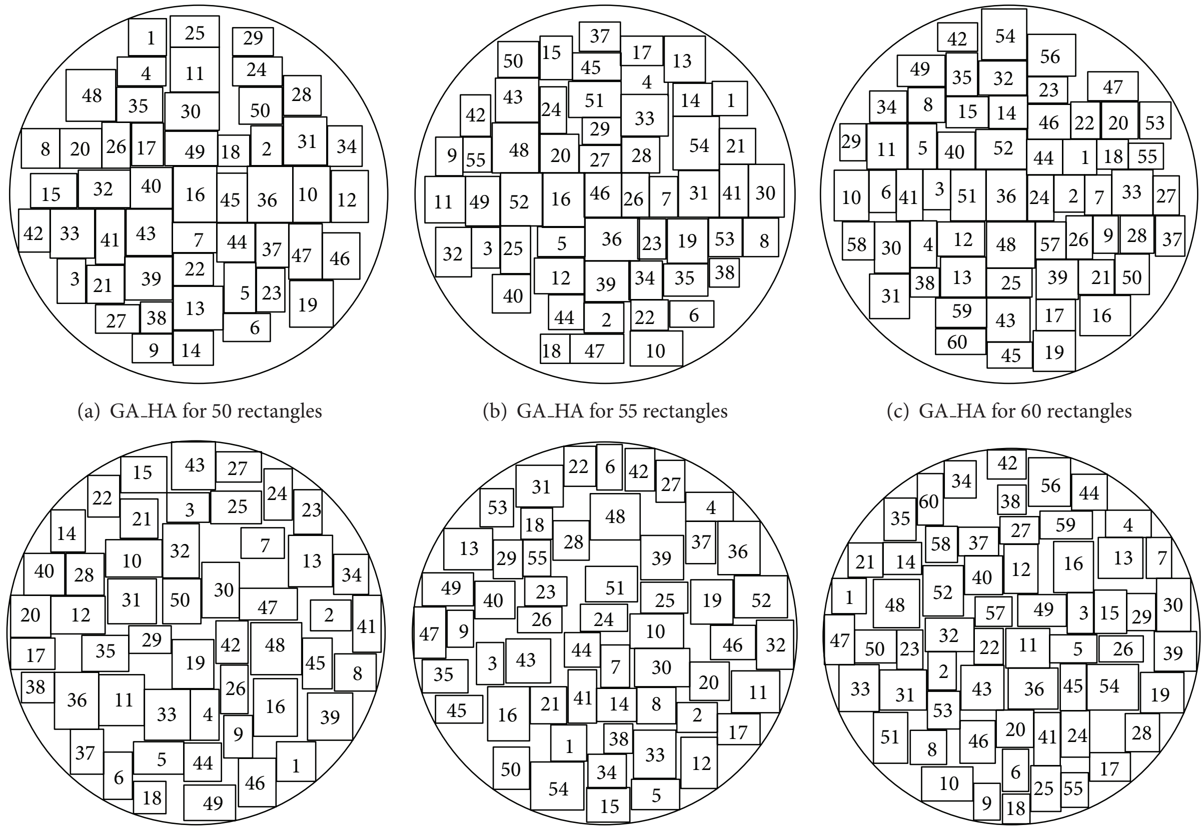
Table 7 that changing values of R_0 and r^* we can obtain the packing scheme with a smaller envelop radius, but it costs more time for each example. So, values R_0 and r^* in Experiment 1 can be applied to make a tradeoff between the computational effectiveness and solution quality.

In order to further test the effectiveness of the proposed QPDAA, we consider Experiment 3.

Experiment 3. Numbers of rectangles (generated randomly) of three examples are 50, 55, and 60 and their lengths and widths are between 20 and 40. For the proposed QPDAA, we take $\alpha = 18$, $\beta = 56$, $\varepsilon = 10^{-20}$, $h = 0.8$, $\delta = 10$, $N = 120$, $R_0 = 125.9, 133.8, 137.2$, and $r^* = 126.8, 134.6, 138.8$ for Examples 1–3, respectively. For GA + HA, the population size, mutation probability, and max number of the iteration are 30, 0.125, and 50, respectively. By running HA + GA and the proposed QPDAA 30 times for three examples, respectively, their optimal envelop radii and average times are shown in Table 12, respectively. The optimal packing scheme diagrams of GA + HA and the proposed QPDAA are shown in Figures 14(a)–14(c) and Figures 14(d)–14(f). We can know from Table 12 that both the solution quality and computational quality of the proposed QPDAA are obviously higher than those of GA + HA.

Note that, in Experiment 3, the procedure of GA + HA is coded by author and is carried on a Pentium 3 GHZ with 512 MB memory.

6.2. Analysis. From data of Tables 2, 7, and 12, we know that the solution quality of the proposed QPDAA algorithm is higher than those of CA-PSLS and GA-HA. Compared



(a) GA_HA for 50 rectangles (b) GA_HA for 55 rectangles (c) GA_HA for 60 rectangles

(d) QPDAA for 50 rectangles (e) QPDAA for 55 rectangles (f) QPDAA for 60 rectangles

FIGURE 14: Packing scheme diagrams of GA + HA and the proposed QPDAA for Experiment 3.

with those of CA-PSLS, embedded degree functions of the proposed QPDAA can make the layout scheme more compact. Due to the fixed candidate positions of GA + HA, it is difficult to find the best position for some rectangles close to the marginal region of the container. The deficiency of a mechanism of decreasing the static balance in the process of packing rectangles also limits the solution quality of GA + HA. The experimental results illustrate the effectiveness of the proposed QPDAA.

The computational efficiency of the proposed QPDAA is one magnitude higher than those of CA-PSLS. There are two reasons. (i) Owing to orthogonal packing, searching the optimal solution in the 2D solution of the proposed QPDAA is easier than that in the 3D solution space of CA-PSLS. (ii) In order to improve the solution quality of CA-PSLS, PSO is used to optimize the feasible solution obtained through the gradient method based on two discontinuous embedded degree functions, but the proposed QPDAA need not do the PSO optimization without reducing its solution quality. Except for Example 1, the computational efficiency of the proposed QPDAA is higher than that of GA + HA. And with the increase of the size of the packing problem, the advantage of the proposed QPDAA is more obvious. This is because computational complexities of the extruded resultant force and potential energy are $O(n)$ in this paper, but the computational complexity of noninterference judgment is $O(n^2)$. In addition, for GA + HA, with the increasing of the number of rectangles, the number of candidate positions of each rectangle increases dramatically. These reasons lead to the computational efficiency of the proposed QPDAA higher than that of GA + HA for the BCOURP with a large size.

7. Conclusions

Taking the layout design of a satellite module as the application background, we have proposed the QPDAA for the BCOURP problem in this paper. Two continuous embedded functions between orthogonal rectangles and between the rectangle and container are constructed to overcome the weakness of embedded functions in [34]. And the suggestion of the extruded resultant force formula and the potential energy function of the rectangle packing system based on the proposed embedded functions make solving the BCOURP problem simple and effective as solving the circle packing problem [37–40]. The proposed dynamic adjustment strategy can quickly decrease the static imbalance of the packing scheme and make the iteration skip the local optimum. The experiment results show that the proposed QPDAA is superior to existing algorithms in performance for the BCOURP problem, especially for the BCOURP problem with the large size. The next work is to extend the above algorithm into solving the 3D satellite module payload packing problem.

Table 10: The optimal layout scheme of Example 4 for the proposed QPDAA.

i	x_i	y_i
1	−7.913018	−16.1629110
2	−11.4301555	13.766710
3	−2.797531	7.599405
4	14.854397	8.176985
5	3.751568	4.136671
6	−14.476432	−7.582581
7	13.305438	−9.941647
8	−8.088653	−7.608034
9	3.747621	15.611128
10	−2.290583	−18.311460
11	−5.543514	1.554382
12	11.758080	13.883226
13	−14.800586	1.537640
14	16.662225	0.074950
15	−3.889690	15.762332
16	−13.867726	7.136671
17	−2.96.2478	−7.489430
18	6.210196	−4.937357
19	5.257463	−14.032250
20	8.273430	7.563706

Appendices

A. HA + GA [35]

Input the length, width, and mass of the rectangle F_i ($i = 1, 2, \ldots, n$) in turn, initialize the number N of the maximal iteration times, and generate their placing sequence set $\{t(i),\ i = 1, 2, \ldots, n\}$.

Step 1. Set $k = 1$.

Step 2. Set the center of $F_{t(1)}$ at $(0, 0)$ and its long side is parallel to x_1-axis, $i = 2$, $j = 1$.

Step 3. For $j = 1, 2, \ldots, i - 1$, calculate centers and direction angles of 16 candidate positions (see Figure 1) of the rectangle $F_{t(i)}$ with respect to the rectangle $F_{t(j)}$. From its candidate positions eliminate unfeasible ones and calculate the optimal one (i.e., compared with other feasible candidate positions, it makes the packing scheme of the first i rectangles have less envelop radius).

Step 4. If $i < n$ then update the current packing scheme, $i{+}{+}$, and go to Step 3; otherwise, go to Step 5.

Step 5. If $k < N$ then update the optimal packing scheme and use GA to generate a new placing sequence set $\{t(i),\ i = 1, 2, \ldots, n\}$ and go to Step 2; otherwise, go to Step 6.

Step 6. Output the optimal packing scheme and envelop radius; algorithm ends.

Table 11: The optimal layout scheme of Example 5 for the proposed QPDAA.

i	x_i	y_i
1	−35.716569	−20.219336
2	19.295868	92.929139
3	−66.464390	−69.791544
4	−0.799905	−80.612101
5	−8.704132	97.929139
6	−38.754325	−49.219336
7	60.543358	20.832873
8	47.933196	−29.082316
9	−42.456642	21.911290
10	−6.576451	43.929139
11	4.543358	7.350623
12	60.568942	−69.995791
13	−74.792262	52.886007
14	57.004507	44.832873
15	33.5433588	0.917684
16	97.933196	−2.928148
17	71.933196	−4.167127
18	14.886008	67.929139
19	−41.5764518	50.911290
20	−93.27781098	23.736930
21	−67.7165699	−2.088710
22	90.543358	27.071852
23	−41.704132	81.911290
24	−65.456642	20.911290
25	−58.7543259	−29.088710
26	28.423549	35.350623
27	0.535610	−101.612101
28	48.2958689	75.832873
29	−80.754325	−36.263070
30	−9.113992	71.929139
31	28.245675	−56.649377
32	79.933196	−36.928148
33	81.004507	52.071852
34	−37.464390	−82.649377
35	29.200095	−79.649377
36	−6.716569	−25.649377
37	20.283431	−29.082316
38	−98.716569	−4.263070
39	−9.754325	−54.649377
40	−22.456642	11.780664

B. Results of Experiment 2

See Tables 8, 9, 10, and 11 and Figure 13.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Table 12: The effect of R_0 and r^* on the optimal radius and running time for the proposed QPDAA.

Example number	Size	HA + GA		QPDAA	
		The optimal radius	Running time (s)	The optimal radius	Running time (s)
1	50	127.667244	30.170088	126.567311	13.345431
2	55	134.844782	42.654031	134.105091	19.056438
3	60	139.072606	66.908203	137.833657	37.01790

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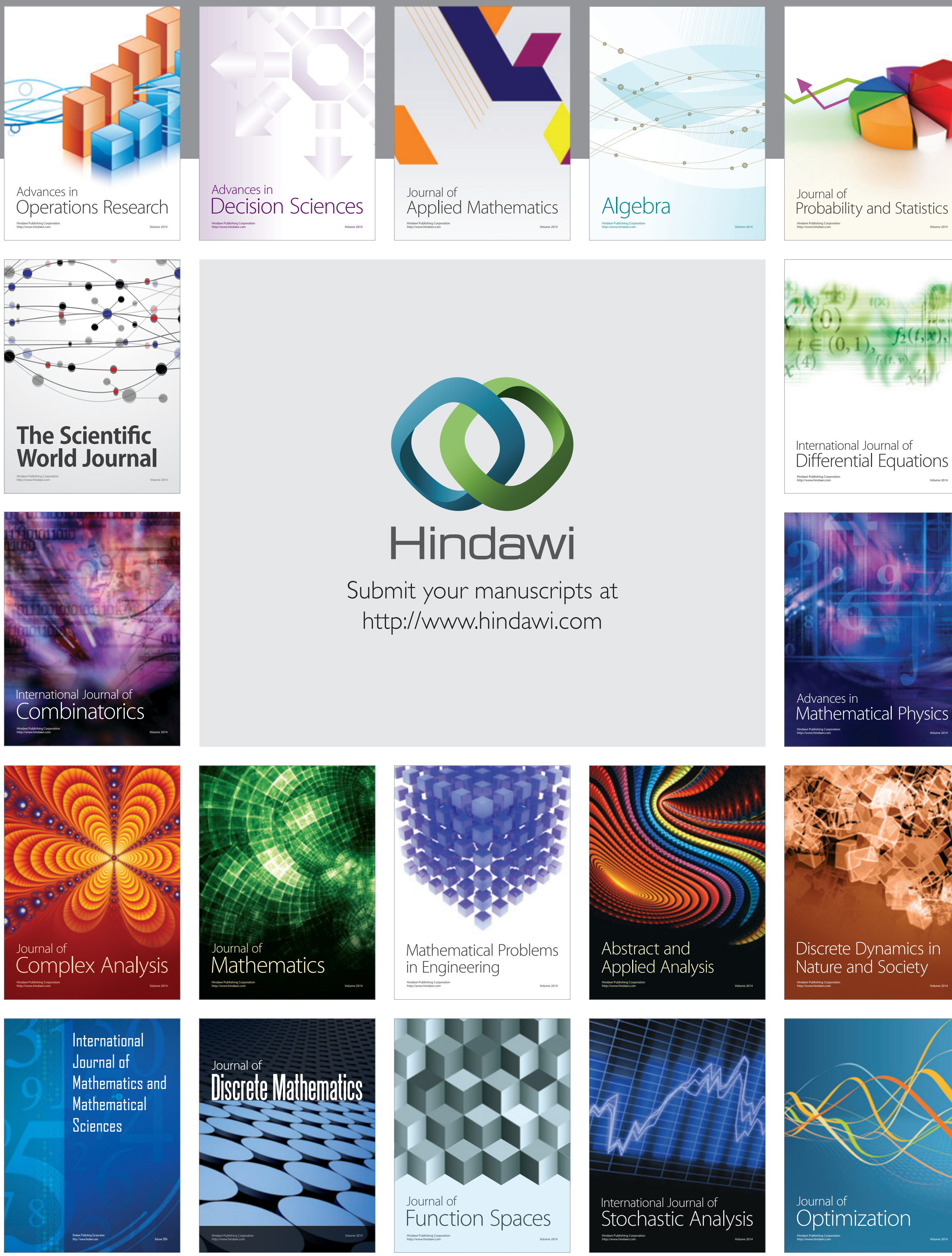
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