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INTUITIONISTIC FUZZY INTERIOR IDEALS OF SEMIGROUPS

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ABSTRACT. We consider the intuitionistic fuzzification of the concept of interior ideals in a semigroup S, and investigate some properties of such ideals. For any homomorphism f from a semigroup S to a semigroup T, if $B=(\mu_B,\gamma_B)$ is an intuitionistic fuzzy interior ideal of T, then the preimage $f^{-1}(B)=(f^{-1}(\mu_B),f^{-1}(\gamma_B))$ of B under f is an intuitionistic fuzzy interior ideal of S.

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1. Introduction. The idea of "intuitionistic fuzzy set" was first published by Atanassov [1, 2], as a generalization of the notion of fuzzy set. Jun et al. considered the fuzzification of interior ideals in semigroups [3]. In this paper, we introduce the notion of an intuitionistic fuzzy interior ideal of a semigroup S, and then some related properties are investigated. Characterizations of intuitionistic fuzzy interior ideals are given. Also for any homomorphism f from a semigroup S to a semigroup T, if $B=(\mu_B,\gamma_B)$ is an intuitionistic fuzzy interior ideal of T, then the preimage $f^{-1}(B)=(f^{-1}(\mu_B),f^{-1}(\gamma_B))$ of B under f is an intuitionistic fuzzy interior ideal of S.

2. Preliminaries. Let X be a nonempty fixed set. An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$$A=\{(x,\mu_A(x),\gamma_A(x)):x\in X\},\tag{2.1}$$

where the functions $\mu_A:X\to[0,1]$ and $\gamma_A:X\to[0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x\in X$ to the set A, respectively, and $0\le\mu_A(x)+\gamma_A(x)\le 1$ for all $x\in X$ (see Atanassov [1, 2]). For the sake of simplicity, we use the symbol $A=(\mu_A,\gamma_A)$ for the IFS $A=\{(x,\mu_A(x),\gamma_A(x)):x\in X\}$.

Let S be a semigroup. By a *subsemigroup* of S we mean a nonempty subset A of S such that $A^2\subseteq A$. A subsemigroup A of a semigroup S is called an *interior ideal* of S if $SAS\subseteq A$. A mapping f from a semigroup S to a semigroup T is called a *homomorphism* if $f(xy)=f(x)f(y)$ for all $x,y\in S$.

A fuzzy set μ in a semigroup S is called a *fuzzy subsemigroup* of S (see [3]) if $\mu(xy)\ge\mu(x)\wedge\mu(y)$ for all $x,y\in S$.

A fuzzy subsemigroup μ of a semigroup S is called a *fuzzy interior ideal* of S (see [3]) if $\mu(xay)\ge\mu(a)$ for all $a,x,y\in S$.

3. Intuitionistic fuzzy interior ideals. In what follows, S denotes a semigroup unless otherwise specified.

Definition 3.1. An IFS $A = (\mu_A, \gamma_A)$ in S is called an *intuitionistic fuzzy subsemigroup* of S if it satisfies
(IF1) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$,
(IF2) $\gamma_A(xy) \leq \gamma_A(x) \vee \gamma_A(y)$,
for all $x, y \in S$.

Example 3.2. Let $S = \{0, e, f, a, b\}$ be a set with the following Cayley table:

$\cdot$	0	e	f	a	b
0	0	0	0	0	0
e	0	e	0	a	0
f	0	0	f	0	b
a	0	a	0	0	e
b	0	0	b	f	0

Then S is a semigroup (see [4]). Define an IFS $A = (\mu_A, \gamma_A)$ in S by $\mu_A(0) = \mu_A(e) = \mu_A(f) = 1$, $\mu_A(a) = \mu_A(b) = 0$, $\gamma_A(0) = \gamma_A(e) = \gamma_A(f) = 0$, and $\gamma_A(a) = \gamma_A(b) = 1$. By routine calculations we know that $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy subsemigroup of S.

Definition 3.3. An intuitionistic fuzzy subsemigroup $A = (\mu_A, \gamma_A)$ of S is called an *intuitionistic fuzzy interior ideal* of S if
(IF3) $\mu_A(xay) \geq \mu_A(a)$,
(IF4) $\gamma_A(xay) \leq \gamma_A(a)$,
for all $x, y, a \in S$.

Example 3.4. The IFS $A = (\mu_A, \gamma_A)$ in Example 3.2 is an intuitionistic fuzzy interior ideal of S.

Theorem 3.5. *If $\{A_i\}_{i \in \Lambda}$ is a family of intuitionistic fuzzy interior ideals of S, then $\cap A_i$ is an intuitionistic fuzzy interior ideal of S, where $\cap A_i = (\wedge \mu_{A_i}, \vee \gamma_{A_i})$ and $\wedge \mu_{A_i}$ and $\vee \gamma_{A_i}$ are defined as follows:*

$$\begin{aligned} \wedge \mu_{A_i}(x) &= \inf\{\mu_{A_i}(x) \mid i \in \Lambda,\ x \in S\}, \\ \vee \gamma_{A_i}(x) &= \sup\{\gamma_{A_i}(x) \mid i \in \Lambda,\ x \in S\}. \end{aligned} \tag{3.1}$$

Proof. Let $x, y, a \in S$. Then

$$\begin{aligned} &\wedge \mu_{A_i}(xy) \geq \wedge(\mu_{A_i}(x) \wedge \mu_{A_i}(y)) = (\wedge \mu_{A_i}(x)) \wedge (\wedge \mu_{A_i}(y)), \\ &\vee \gamma_{A_i}(xy) \leq \vee(\gamma_{A_i}(x) \vee \gamma_{A_i}(y)) = (\vee \gamma_{A_i}(x)) \vee (\vee \gamma_{A_i}(y)), \\ &\wedge \mu_{A_i}(xay) \geq \wedge \mu_{A_i}(a), \qquad \vee \gamma_{A_i}(xay) \leq \vee \gamma_{A_i}(a). \end{aligned} \tag{3.2}$$

Hence $\cap A_i$ is an intuitionistic fuzzy interior ideal of S. □

Theorem 3.6. *If an IFS $A = (\mu_A, \gamma_A)$ in S is an intuitionistic fuzzy interior ideal of S, then so is $\square A := (\mu_A, \bar{\mu}_A)$, $\bar{\mu}_A = 1 - \mu_A$.*

Proof. It is sufficient to show that $\bar{\mu}_A$ satisfies conditions (IF2) and (IF4). For any $a, x, y \in S$, we have

$$\begin{aligned} \bar{\mu}_A(xy) &= 1 - \mu_A(xy) \le 1 - (\mu_A(x) \wedge \mu_A(y)) \\ &= (1 - \mu_A(x)) \vee (1 - \mu_A(y)) = \bar{\mu}_A(x) \vee \bar{\mu}_A(y) \end{aligned} \tag{3.3}$$

and $\bar{\mu}_A(xay) = 1 - \mu_A(xay) \le 1 - \mu_A(a) = \bar{\mu}_A(a)$. Therefore, A is an intuitionistic fuzzy interior ideal of S. □

Definition 3.7. Let $A = (\mu_A, \gamma_A)$ be an IFS in S and let $\alpha \in [0,1]$. Then the sets

$$\mu_{A,\alpha}^{\ge} := \{x \in S : \mu_A(x) \ge \alpha\}, \qquad \gamma_{A,\alpha}^{\le} := \{x \in S : \gamma_A(x) \le \alpha\} \tag{3.4}$$

are called a *μ-level α-cut* and a *γ-level α-cut* of A, respectively.

Theorem 3.8. *If an IFS $A = (\mu_A, \gamma_A)$ in S is an intuitionistic fuzzy interior ideal of S, then the μ-level α-cut $\mu_{A,\alpha}^{\ge}$ and γ-level α-cut $\gamma_{A,\alpha}^{\le}$ of A are interior ideals of S for every $\alpha \in \mathrm{Im}(\mu_A) \cap \mathrm{Im}(\gamma_A) \subseteq [0,1]$.*

Proof. Let $\alpha \in \mathrm{Im}(\mu_A) \cap \mathrm{Im}(\gamma_A) \subseteq [0,1]$ and let $x, y \in \mu_{A,\alpha}^{\ge}$. Then $\mu_A(x) \ge \alpha$ and $\mu_A(y) \ge \alpha$. It follows from (IF1) that

$$\mu_A(xy) \ge \mu_A(x) \wedge \mu_A(y) \ge \alpha \quad \text{so that } xy \in \mu_{A,\alpha}^{\ge}. \tag{3.5}$$

If $x, y \in \gamma_{A,\alpha}^{\le}$, then $\gamma_A(x) \le \alpha$ and $\gamma_A(y) \le \alpha$, and so

$$\gamma_A(xy) \le \gamma_A(x) \vee \gamma_A(y) \le \alpha, \quad \text{that is, } xy \in \gamma_{A,\alpha}^{\le}. \tag{3.6}$$

Hence $\mu_{A,\alpha}^{\ge}$ and $\gamma_{A,\alpha}^{\le}$ are subsemigroups of S. Now let $x, y \in S$ and $a \in \mu_{A,\alpha}^{\ge}$. Then $\mu_A(xay) \ge \mu_A(a) \ge \alpha$ and so $xay \in \mu_{A,\alpha}^{\ge}$. If $a \in \gamma_{A,\alpha}^{\le}$, then $\gamma_A(xay) \le \gamma_A(a) \le \alpha$ and thus $xay \in \gamma_{A,\alpha}^{\le}$. Therefore $\mu_{A,\alpha}^{\ge}$ and $\gamma_{A,\alpha}^{\le}$ are interior ideals of S. □

Theorem 3.9. *Let $A = (\mu_A, \gamma_A)$ be an IFS in S such that the nonempty sets $\mu_{A,\alpha}^{\ge}$ and $\gamma_{A,\alpha}^{\le}$ are interior ideals of S for all $\alpha \in [0,1]$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy interior ideal of S.*

Proof. Let $\alpha \in [0,1]$ and suppose that $\mu_{A,\alpha}^{\ge} (\ne \emptyset)$ and $\gamma_{A,\alpha}^{\le} (\ne \emptyset)$ are interior ideals of S. We must show that $A = (\mu_A, \gamma_A)$ satisfies conditions (IF1)–(IF4). If condition (IF1) is false, then there exist $x_0, y_0 \in S$ such that $\mu_A(x_0 y_0) < \mu_A(x_0) \wedge \mu_A(y_0)$. Taking

$$\alpha_0 := \frac{1}{2}(\mu_A(x_0 y_0) + \mu_A(x_0) \wedge \mu_A(y_0)), \tag{3.7}$$

we have $\mu_A(x_0 y_0) < \alpha_0 < \mu_A(x_0) \wedge \mu_A(y_0)$. It follows that $x_0, y_0 \in \mu_{A,\alpha_0}^{\ge}$ and $x_0 y_0 \notin \mu_{A,\alpha_0}^{\ge}$, which is a contradiction. Hence condition (IF1) is true. The proof of other conditions are similar to the case (IF1), we omit the proof. □

Theorem 3.10. *Let M be an interior ideal of S and let $A=(\mu_A,\gamma_A)$ be an IFS in S defined by*

$$\mu_A(x):=\begin{cases}\alpha_0 & \text{if } x\in M,\\ \alpha_1 & \text{otherwise,}\end{cases}\qquad \gamma_A(x):=\begin{cases}\beta_0 & \text{if } x\in M,\\ \beta_1 & \text{otherwise,}\end{cases}\tag{3.8}$$

for all $x\in S$ and $\alpha_i,\beta_i\in[0,1]$ such that $\alpha_0>\alpha_1$, $\beta_0<\beta_1$, and $\alpha_i+\beta_i\le 1$ for $i=0,1$. Then $A=(\mu_A,\gamma_A)$ is an intuitionistic fuzzy interior ideal of S and $\mu^{\gtrsim}_{A,\alpha_0}=M=\gamma^{\lesssim}_{A,\beta_0}$.

Proof. Let $x,y\in S$. If anyone of x and y does not belong to M, then

$$\begin{aligned}\mu_A(xy)&\ge\alpha_1=\mu_A(x)\wedge\mu_A(y),\\ \gamma_A(xy)&\le\beta_1=\gamma_A(x)\vee\gamma_A(y).\end{aligned}\tag{3.9}$$

Other cases are trivial, and we omit the proof. Hence $A=(\mu_A,\gamma_A)$ is an intuitionistic fuzzy subsemigroup of S. Now let $x,y,a\in S$. If $a\notin M$, then $\mu_A(xay)\ge\alpha_1=\mu_A(a)$ and $\gamma_A(xay)\le\beta_1=\gamma_A(a)$. Assume that $a\in M$. Since M is an interior ideal of S, it follows that $xay\in M$. Hence $\mu_A(xay)=\alpha_0=\mu_A(a)$ and $\gamma_A(xay)=\beta_0=\gamma_A(a)$. Therefore $A=(\mu_A,\gamma_A)$ is an intuitionistic fuzzy interior ideal of S. Obviously $\mu^{\gtrsim}_{A,\alpha_0}=M=\gamma^{\lesssim}_{A,\beta_0}$. □

Corollary 3.11. *Let χ_M be the characteristic function of an interior ideal M of S. Then the IFS $\tilde{M}=(\chi_M,\bar{\chi}_M)$ is an intuitionistic fuzzy interior ideal of S.*

Theorem 3.12. *If an IFS $A=(\mu_A,\gamma_A)$ is an intuitionistic fuzzy interior ideal of S, then*

$$\begin{aligned}\mu_A(x)&:=\sup\{\alpha\in[0,1]\mid x\in\mu^{\gtrsim}_{A,\alpha}\},\\ \gamma_A(x)&:=\inf\{\alpha\in[0,1]\mid x\in\gamma^{\lesssim}_{A,\alpha}\},\end{aligned}\tag{3.10}$$

for all $x\in S$.

Proof. Let $\delta:=\sup\{\alpha\in[0,1]\mid x\in\mu^{\gtrsim}_{A,\alpha}\}$ and let $\varepsilon>0$ be given. Then $\delta-\varepsilon<\alpha$ for some $\alpha\in[0,1]$ such that $x\in\mu^{\gtrsim}_{A,\alpha}$. It follows that $\delta-\varepsilon<\mu_A(x)$ so that $\delta\le\mu_A(x)$ since ε is arbitrary. We now show that $\mu_A(x)\le\delta$. Let $\mu_A(x)=\beta$. Then $x\in\mu^{\gtrsim}_{A,\beta}$ and so

$$\beta\in\{\alpha\in[0,1]\mid x\in\mu^{\gtrsim}_{A,\alpha}\}.\tag{3.11}$$

Hence $\mu_A(x)=\beta\le\sup\{\alpha\in[0,1]\mid x\in\mu^{\gtrsim}_{A,\alpha}\}=\delta$. Therefore

$$\mu_A(x)=\delta=\sup\{\alpha\in[0,1]\mid x\in\mu^{\gtrsim}_{A,\alpha}\}.\tag{3.12}$$

Now let $\eta=\inf\{\alpha\in[0,1]\mid x\in\gamma^{\lesssim}_{A,\alpha}\}$. Then

$$\inf\{\alpha\in[0,1]\mid x\in\gamma^{\lesssim}_{A,\alpha}\}<\eta+\varepsilon\quad\text{for any }\varepsilon<0,\tag{3.13}$$

and so $\alpha<\eta+\varepsilon$ for some $\alpha\in[0,1]$ with $x\in\gamma^{\lesssim}_{A,\alpha}$. Since $\gamma_A(x)\le\alpha$ and ε is arbitrary, it follows that $\gamma_A(x)\le\eta$. To prove $\gamma_A(x)\ge\eta$, let $\gamma_A(x)=\zeta$. Then $x\in\gamma^{\lesssim}_{A,\zeta}$ and thus $\zeta\in\{\alpha\in[0,1]\mid x\in\gamma^{\lesssim}_{A,\alpha}\}$. Hence

$$\inf\{\alpha\in[0,1]\mid x\in\gamma^{\lesssim}_{A,\alpha}\}\le\zeta,\quad\text{that is, }\eta\le\zeta=\gamma_A(x).\tag{3.14}$$

Consequently,

$$\gamma_A(x) = \eta = \inf\{\alpha \in [0,1] \mid x \in \gamma^{\leq}_{A,\alpha}\}. \tag{3.15}$$

This completes the proof. □

Theorem 3.13. *Let $\{C_\alpha \mid \alpha \in \Lambda\}$ be a collection of interior ideals of S such that*

(i) $S = \cup_{\alpha\in\Lambda} C_\alpha$,

(ii) $\beta > \alpha$ *if and only if* $C_\beta \subset C_\alpha$ *for all* $\beta, \alpha \in \Lambda$.

Then an IFS $A = (\mu_A, \gamma_A)$ in S defined by

$$\begin{aligned} \mu_A(x) &:= \sup\{\alpha \in \Lambda \mid x \in C_\alpha\}, \\ \gamma_A(x) &:= \inf\{\alpha \in \Lambda \mid x \in C_\alpha\}, \end{aligned} \tag{3.16}$$

for all $x \in S$, is an intuitionistic fuzzy interior ideal of S.

Proof. Following Theorem 3.9, it is sufficient to show that the nonempty level sets $\mu^{\geq}_{A,\alpha}$ and $\gamma^{\leq}_{A,\alpha}$ are interior ideals of S for every $\alpha \in [0,1]$. In order to prove that $\mu^{\geq}_{A,\alpha}(\neq \emptyset)$ is an interior ideal, we have the following two cases:

(i) $\alpha = \sup\{\delta \in \Lambda \mid \delta < \alpha\}$ and

(ii) $\alpha \neq \sup\{\delta \in \Lambda \mid \delta < \alpha\}$.

Case (i) implies that

$$x \in \mu^{\geq}_{A,\alpha} \Longleftrightarrow x \in C_\delta \quad \forall \delta < \alpha \Longleftrightarrow x \in \cap_{\delta<\alpha} C_\delta, \tag{3.17}$$

so that $\mu^{\geq}_{A,\alpha} = \cap_{\delta<\alpha} C_\delta$, which is an interior ideal of S. For the case (ii), we claim that $\mu^{\geq}_{A,\alpha} = \cup_{\delta\geq\alpha} C_\delta$. If $x \in \cup_{\delta\geq\alpha} C_\delta$, then $x \in C_\delta$ for some $\delta \geq \alpha$. It follows that $\mu_A(x) \geq \delta \geq \alpha$, so that $x \in \mu^{\geq}_{A,\alpha}$. This proves that $\cup_{\delta\geq\alpha} C_\delta \subseteq \mu^{\geq}_{A,\alpha}$. Now assume that $x \notin \cup_{\delta\geq\alpha} C_\delta$. Then $x \notin C_\delta$ for all $\delta \geq \alpha$. Since $\alpha \neq \sup\{\delta \in \Lambda \mid \delta < \alpha\}$, there exists $\varepsilon > 0$ such that $(\alpha-\varepsilon, \alpha) \cap \Lambda = \emptyset$. Hence $x \notin C_\delta$ for all $\delta > \alpha - \varepsilon$, which means that if $x \in C_\delta$ then $\delta \leq \alpha - \varepsilon$. Thus $\mu_A(x) \leq \alpha - \varepsilon < \alpha$, and so $x \notin \mu^{\geq}_{A,\alpha}$. Therefore $\mu^{\geq}_{A,\alpha} \subseteq \cup_{\delta\geq\alpha} C_\delta$, and thus $\mu^{\geq}_{A,\alpha} = \cup_{\delta\geq\alpha} C_\delta$ which is an interior ideal of S. Next we prove that $\gamma^{\leq}_{A,\alpha}(\neq \emptyset)$ is an interior ideal of S for all $\alpha \in [0,1]$. We consider the following two cases:

(iii) $\beta = \inf\{\delta \in \Lambda \mid \beta < \delta\}$ and

(iv) $\beta \neq \inf\{\delta \in \Lambda \mid \beta < \delta\}$.

For the case (iii) we have

$$x \in \gamma^{\leq}_{A,\beta} \Longleftrightarrow x \in C_\delta \quad \forall \beta < \delta \Longleftrightarrow x \in \cap_{\beta<\delta} C_\delta, \tag{3.18}$$

and hence $\gamma^{\leq}_{A,\beta} = \cap_{\beta<\delta} C_\delta$ which is an interior ideal of S. For the case (iv), there exists $\varepsilon > 0$ such that $(\beta, \beta+\varepsilon) \cap \Lambda = \emptyset$. We show that $\gamma^{\leq}_{A,\beta} = \cup_{\beta\geq\delta} C_\delta$. If $x \in \cup_{\beta\geq\delta} C_\delta$, then $x \in C_\delta$ for some $\beta \geq \delta$. It follows that $\gamma_A(x) \leq \delta \leq \beta$ so that $x \in \gamma^{\leq}_{A,\beta}$. Hence $\cup_{\beta\geq\delta} C_\delta \subseteq \gamma^{\leq}_{A,\beta}$. Conversely, if $x \notin \cup_{\beta\geq\delta} C_\delta$ then $x \notin C_\delta$ for all $\delta \leq \beta$, which implies that $x \notin C_\delta$ for all $\delta < \beta + \varepsilon$, that is, if $x \in C_\delta$ then $\delta \geq \beta + \varepsilon$. Thus $\gamma_A(x) \geq \beta + \varepsilon > \beta$, that is, $x \notin \gamma^{\leq}_{A,\beta}$. Therefore $\gamma^{\leq}_{A,\beta} \subseteq \cup_{\beta\geq\delta} C_\delta$ and consequently $\gamma^{\leq}_{A,\beta} = \cup_{\beta\geq\delta} C_\delta$ which is an interior ideal of S. This completes the proof. □

Theorem 3.14. *An IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy interior ideal of S if and only if the fuzzy sets μ_A and $\bar{\gamma}_A$ are fuzzy interior ideals of S.*

Proof. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy interior ideal of S. Then clearly μ_A is a fuzzy interior ideal of S. Let $x, a, y \in S$. Then

$$\begin{aligned} \bar{\gamma}_A(xy) &= 1 - \gamma_A(xy) \geq 1 - \gamma_A(x) \vee \gamma_A(y) \\ &= (1 - \gamma_A(x)) \wedge (1 - \gamma_A(y)) = \bar{\gamma}_A(x) \wedge \bar{\gamma}_A(y), \\ \bar{\gamma}_A(xay) &= 1 - \gamma_A(xay) \geq 1 - \gamma_A(a) = \bar{\gamma}_A(a). \end{aligned} \tag{3.19}$$

Hence $\bar{\gamma}_A$ is a fuzzy interior ideal of S.

Conversely, suppose that μ_A and $\bar{\gamma}_A$ are fuzzy interior ideals of S. Let $a, x, y \in S$. Then

$$\begin{aligned} 1 - \gamma_A(xy) &= \bar{\gamma}_A(xy) \geq \bar{\gamma}_A(x) \wedge \bar{\gamma}_A(y) \\ &= (1 - \gamma_A(x)) \wedge (1 - \gamma_A(y)) \\ &= 1 - \gamma_A(x) \vee \gamma_A(y), \\ 1 - \gamma_A(xay) &= \bar{\gamma}_A(xay) \geq \bar{\gamma}_A(a) = 1 - \gamma_A(a), \end{aligned} \tag{3.20}$$

which imply that $\gamma_A(xy) \leq \gamma_A(x) \vee \gamma_A(y)$ and $\gamma_A(xay) \leq \gamma_A(a)$. This completes the proof. □

Corollary 3.15. *An IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy interior ideal of S if and only if $\Box A = (\mu_A, \bar{\mu}_A)$ and $\Diamond A = (\bar{\gamma}_A, \gamma_A)$ are intuitionistic fuzzy interior ideals of S.*

Proof. The proof is straightforward by Theorem 3.14. □

Let f be a map from a set X to a set Y. If $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ are IFSs in X and Y, respectively, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is an IFS in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)), \quad \text{where } f^{-1}(\mu_B) = \mu_B(f). \tag{3.21}$$

Theorem 3.16. *Let $f : S \to T$ be a homomorphism of semigroups. If $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy interior ideal of T, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of B under f is an intuitionistic fuzzy interior ideal of S.*

Proof. Assume that $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy interior ideal of T and let $x, y \in S$. Then

$$\begin{aligned} f^{-1}(\mu_B)(xy) &= \mu_B(f(xy)) \\ &= \mu_B(f(x)f(y)) \\ &\geq \mu_B(f(x)) \wedge \mu_B(f(y)) \\ &= f^{-1}(\mu_B(x)) \wedge f^{-1}(\mu_B(y)), \\ f^{-1}(\gamma_B)(xy) &= \gamma_B(f(xy)) \\ &= \gamma_B(f(x)f(y)) \\ &\leq \gamma_B(f(x)) \vee \gamma_B(f(y)) \\ &= f^{-1}(\gamma_B(x)) \vee f^{-1}(\gamma_B(y)). \end{aligned} \tag{3.22}$$

Hence $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ is an intuitionistic fuzzy subsemigroup of S. For any $a, x, y \in S$, we have

$$\begin{aligned} f^{-1}(\mu_B)(xay) &= \mu_B(f(xay)) \\ &= \mu_B(f(x)f(a)f(y)) \\ &\geq \mu_B(f(a)) \\ &= f^{-1}(\mu_B(a)), \\ f^{-1}(\gamma_B)(xay) &= \gamma_B(f(xay)) \\ &= \gamma_B(f(x)f(a)f(y)) \\ &\leq \gamma_B(f(a)) \\ &= f^{-1}(\gamma_B(a)). \end{aligned} \tag{3.23}$$

Therefore $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ is an intuitionistic fuzzy interior ideal of S. □

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