

Research Article

Global Robust Exponential Stability and Periodic Solutions for Interval Cohen-Grossberg Neural Networks with Mixed Delays

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A class of interval Cohen-Grossberg neural networks with time-varying delays and infinite distributed delays is investigated. By employing H-matrix and M-matrix theory, homeomorphism techniques, Lyapunov functional method, and linear matrix inequality approach, sufficient conditions are established for the existence, uniqueness, and global robust exponential stability of the equilibrium point and the periodic solution to the neural networks. Our results improve some previously published ones. Finally, numerical examples are given to illustrate the feasibility of the theoretical results and further to exhibit that there is a characteristic sequence of bifurcations leading to a chaotic dynamics, which implies that the system admits rich and complex dynamics.

1. Introduction

In the past two decades, neural networks have received a great deal of attention due to the extensive applications in many areas such as signal processing, associative memory, pattern recognition, and parallel computation and optimization. It should be pointed out that the successful applications heavily rely on the dynamic behaviors of neural networks. Stability, as one of the most important properties of neural networks, is crucially required when designing neural networks.

In electronic implementation of neural networks, there exist inevitably some uncertainties caused by the existence of modeling errors, external disturbance, and parameter fluctuation, which would lead to complex dynamic behaviors. Thus, it is important to investigate the robustness of neural networks against such uncertainties and deviations (see [1–8] and references therein). In [4–6], employing homeomorphism techniques, Lyapunov method, H-matrix and M-matrix theory, and linear matrix inequality (LMI) approach, Shao et al. established some sufficient conditions for the existence, uniqueness, and global robust exponential stability

of the equilibrium point for the following interval Hopfield neural networks:

$$\begin{aligned} \dot{u}_i(t) = & -d_i u_i(t) + \sum_{j=1}^n a_{ij} f_j(u_j(t)) \\ & + \sum_{j=1}^n b_{ij} f_j(u_j(t - \tau_j(t))) - J_i, \end{aligned} \quad (1)$$
$$i = 1, 2, \dots, n,$$

where $\tau_j(t)$ is time-varying delay which is variable with time due to the finite switching speed of amplifiers. Recently, the stability of neural networks with time-varying delays has been extensively investigated, and various sufficient conditions have been established for the global asymptotic and exponential stability in [9–13]. Generally, neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths. It is desired to model them by introducing continuously distributed delays over a certain duration of time such that the distant past has less influence compared to the recent

behavior of the state (see [14–16]). However, the distributed delays were not taken into account in system (1).

As an important neural networks, Cohen-Grossberg neural networks (CGNNs) include Hopfield neural networks, cellular neural networks, and other neural networks. CGNNs have aroused a tremendous surge of investigation in these years. Whereas, for the interval CGNNs, fewer robust stability results have been reported in contrast to the results on Hopfield neural networks [17–19]. On the other hand, the research of neural networks involves not only the dynamic analysis of equilibrium point but also that of the periodic oscillatory solution, which is very important in learning theory due to the fact that learning usually requires repetition [20, 21]. Some important results for periodic solutions of neural networks have been obtained in [7, 22–27] and references therein. Motivated by the works of [4–6] and the discussions above, the objective of this paper is to investigate the global robust exponential stability and periodic solutions of the following CGNNs with time-varying and distributed delays:

$$\begin{aligned} \dot{u}_i(t) = & -\bar{\alpha}_i(u_i(t)) \\ & \times \left[\tilde{\beta}_i(u_i(t)) - \sum_{j=1}^n a_{ij} f_j(u_j(t)) \right. \\ & - \sum_{j=1}^n b_{ij} f_j(u_j(t - \tau_j(t))) \\ & \left. - \sum_{j=1}^n c_{ij} \int_{-\infty}^t k_j(t-s) f_j(u_j(s)) ds + J_i(t) \right], \\ & i = 1, 2, \dots, n, \end{aligned} \quad (2)$$

or equivalently

$$\begin{aligned} \dot{u}(t) = & -\bar{\alpha}(u(t)) \left[\tilde{\beta}(u(t)) - Af(u(t)) - Bf(u(t - \tau(t))) \right. \\ & \left. - C \int_{-\infty}^t K(t-s) f(u(s)) ds + J(t) \right], \end{aligned} \quad (3)$$

where

$$\begin{aligned} u(t) &= (u_1(t), \dots, u_n(t))^T, \\ \bar{\alpha}(u(t)) &= \text{diag}(\bar{\alpha}_1(u_1(t)), \dots, \bar{\alpha}_n(u_n(t))), \\ \tilde{\beta}(u(t)) &= (\tilde{\beta}_1(u_1(t)), \dots, \tilde{\beta}_n(u_n(t)))^T, \\ f(u(t)) &= (f_1(u_1(t)), \dots, f_n(u_n(t)))^T, \\ f(u(t - \tau(t))) &= (f_1(u_1(t - \tau_1(t))), \dots, f_n(u_n(t - \tau_n(t))))^T, \\ K(t) &= \text{diag}(k_1(t), \dots, k_n(t)), \\ A &= (a_{ij})_{n \times n}, \quad B = (b_{ij})_{n \times n}, \quad C = (c_{ij})_{n \times n}, \\ J(t) &= (J_1(t), \dots, J_n(t))^T, \end{aligned} \quad (4)$$

where $u_i(t)$ denotes the state of the i th neuron at time t , $\bar{\alpha}_i(u_i(t))$ denotes a positive, continuous, and bounded

amplification function; that is, $0 < \underline{\alpha}_i \leq \bar{\alpha}_i(u_i(t)) \leq \bar{\alpha}_i < +\infty$, $\tilde{\beta}_i(u_i(t))$ denotes an appropriate behaved function, $f_j(u_j(t))$ denotes the activation function, $\tau_j(t)$ denotes the time-varying delay associated with the j th neuron, satisfying $0 \leq \tau_j(t) \leq \tau$ and $0 \leq \dot{\tau}_j(t) \leq \delta < 1$, $k_j(t) > 0$ represents the delay kernel function, which is a real-valued continuous function, A is the connection weight matrix, B is the time-varying delayed connection weight matrix, C is the infinite distributed delayed connection weight matrix, $J_i(t)$ is the external input bias. The coefficients a_{ij} , b_{ij} , and c_{ij} can be intervalised as follows:

$$\begin{aligned} A_I &= [\underline{A}, \bar{A}] \\ &= \{A = (a_{ij})_{n \times n} : \underline{a}_{ij} \leq a_{ij} \leq \bar{a}_{ij}, i, j = 1, 2, \dots, n\}, \\ B_I &= [\underline{B}, \bar{B}] \\ &= \{B = (b_{ij})_{n \times n} : \underline{b}_{ij} \leq b_{ij} \leq \bar{b}_{ij}, i, j = 1, 2, \dots, n\}, \\ C_I &= [\underline{C}, \bar{C}] \\ &= \{C = (c_{ij})_{n \times n} : \underline{c}_{ij} \leq c_{ij} \leq \bar{c}_{ij}, i, j = 1, 2, \dots, n\}, \end{aligned} \quad (5)$$

where for $X = A, B, C$, $\underline{X} = (\underline{x}_{ij})_{n \times n}$, $\bar{X} = (\bar{x}_{ij})_{n \times n}$. Denote $B^* = (\bar{B} + \underline{B})/2$ and $B_* = (\bar{B} - \underline{B})/2$. Clearly, B_* is a nonnegative matrix and the interval matrix $[B, \bar{B}] = [B^* - B_*, B^* + B_*]$. Consequently, $B = B^* + \Delta B$, $\Delta B \in [-B_*, B_*]$. C^* and C_* are defined correspondingly.

Throughout this paper, we make the following assumptions.

(H1) For the behaved functions $\tilde{\beta}_i(\cdot)$ ($i = 1, 2, \dots, n$), there exist constants $\gamma_i > 0$ such that

$$\frac{\tilde{\beta}_i(x) - \tilde{\beta}_i(y)}{x - y} \geq \gamma_i > 0, \quad \forall x, y \in \mathbb{R}, x \neq y. \quad (6)$$

(H2) For the activation functions $f_i(\cdot)$ ($i = 1, 2, \dots, n$), there exist constants $l_i > 0$ such that

$$0 \leq \frac{f_i(x) - f_i(y)}{x - y} \leq l_i, \quad \forall x, y \in \mathbb{R}, x \neq y. \quad (7)$$

(H3) The delay kernels $k_j(\cdot)$ ($j = 1, 2, \dots, n$) satisfy

$$\int_0^\infty k_j(s) ds = 1, \quad \int_0^\infty k_j(s) e^{\mu s} ds < \infty, \quad (8)$$

for some positive constant μ .

A typical example of such delay kernels is given by $k_j(s) = s^r / r! \gamma_j^{r+1} e^{-\gamma_j s}$ for $s \in [0, \infty)$, where $\gamma_j \in [0, \infty)$, $r \in \{0, 1, \dots, n\}$, which are called the Gamma Memory Filter in [28].

The organization of this paper is as follows. In Section 2, some preliminaries are given. In Section 3, sufficient conditions are presented for the existence, uniqueness, and global robust exponential stability of the equilibrium point for system (2) with the external constant input bias (i.e., $J_i(t) \equiv J_i$, J_i is a constant). In Section 4, sufficient conditions are given which guarantee the uniqueness and global exponential

stability of periodic solutions for system (2) when the time-varying delay $\tau_i(t)$ and the external input bias $J_i(t)$ are continuously periodic functions. Numerical examples are provided to illustrate the effectiveness of the obtained results in Section 5. A concluding remark is given in Section 6 to end this work.

2. Preliminaries

We give some preliminaries in this section. Denote $\Gamma = \text{diag}(\gamma_1, \gamma_2, \dots, \gamma_n)$, $L = \text{diag}(l_1, l_2, \dots, l_n)$, $\underline{\alpha} = \text{diag}(\alpha_1, \alpha_2, \dots, \alpha_n)$, and $\hat{x}_{ij} = \max\{|\underline{x}_{ij}|, |\bar{x}_{ij}|\}$. For a vector $x = (x_1, x_2, \dots, x_n)$, $\|x\|_r = (\sum_{i=1}^n |x_i|^r)^{1/r}$ and for any $\varphi(s) = (\varphi_1(s), \dots, \varphi_n(s))$, $s \in (-\infty, 0]$, $\|\varphi(s)\|_r = \sup_{s \in (-\infty, 0]} (\sum_{i=1}^n |\varphi_i(s)|^r)^{1/r}$. For a matrix $A = (a_{ij})_{n \times n}$, A^T denotes the transpose; A^{-1} denotes the inverse; $A > (\geq) 0$ means that A is a symmetric positive definite (semidefinite) matrix; $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the largest and the smallest eigenvalues of A , respectively; and $\|A\|_2 = \sqrt{\lambda_{\max}(A^T A)}$ denotes the spectral norm of A . I denotes the identity matrix. $*$ denotes the symmetric block in a symmetric matrix.

Definition 1 (see [29]). The neural network (2) with the parameter ranges defined by (5) is globally robustly exponentially stable, if for each $A \in A_I, B \in B_I, C \in C_I$, and J , system (2) has a unique equilibrium point $u^* = (u_1^*, u_2^*, \dots, u_n^*)^T$, and there exist constants $a \geq 1$ and $\varepsilon > 0$ such that

$$\|u(t) - u^*\| \leq a \|\phi(\theta) - u^*\| e^{-\varepsilon t}, \quad \forall t > 0, \quad (9)$$

where $u(t) = (u_1(t), u_2(t), \dots, u_n(t))^T$ is a solution of system (2) with the initial value $u_i(\theta) = \phi_i(\theta)$, $i = 1, 2, \dots, n$, $\theta \in (-\infty, 0]$, and $\phi(\theta) = (\phi_1(\theta), \phi_2(\theta), \dots, \phi_n(\theta))$.

Definition 2 (see [30]). Let $Z_n = \{A = (a_{ij})_{n \times n} \in M_n(\mathbb{R}) : a_{ij} \leq 0 \text{ if } i \neq j, i, j = 1, 2, \dots, n\}$, where $M_n(\mathbb{R})$ denotes the set of all $n \times n$ matrices with entries from \mathbb{R} . Then a matrix A is called an M -matrix if $A \in Z_n$ and all successive principal minors of A are positive.

Definition 3 (see [30]). An $n \times n$ matrix $A = (a_{ij})_{n \times n}$ is said to be an H -matrix if its comparison matrix $M(A) = (m_{ij})_{n \times n}$ is an M -matrix, where $m_{ij} = \begin{cases} |a_{ii}|, & \text{if } i=j, \\ -|a_{ij}|, & \text{if } i \neq j. \end{cases}$

Lemma 4 (see [31]). For any vectors $x, y \in \mathbb{R}^n$ and positive definite matrix $G \in \mathbb{R}^{n \times n}$, the following inequality holds: $2x^T y \leq x^T G x + y^T G^{-1} y$.

Lemma 5 (see [30]). Let $A, B \in Z_n$. If A is an M -matrix and the elements of matrices A and B satisfy the inequalities $a_{ij} \leq b_{ij}$, $i, j = 1, 2, \dots, n$, then B is an M -matrix.

Lemma 6 (see [30]). The following LMI: $\begin{pmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{pmatrix} > 0$, where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$, is equivalent to $R(x) > 0$ and $Q(x) - S(x)R^{-1}(x)S^T(x) > 0$ or $Q(x) > 0$ and $R(x) - S^T(x)Q^{-1}(x)S(x) > 0$.

Lemma 7 (see [32]). Suppose that the neural network parameters are defined by (5), and

$$\Xi = \begin{pmatrix} \Phi - S & -PB_* & -PC_* \\ * & (1 - \delta)Q & 0 \\ * & * & R \end{pmatrix} > 0, \quad (10)$$

where $P = \text{diag}(p_1, p_2, \dots, p_n)$, $Q = \text{diag}(q_1, q_2, \dots, q_n)$, and $R = \text{diag}(r_1, r_2, \dots, r_n)$ are positive diagonal matrices, $\Phi = (\Phi_{ij})_{n \times n} = 2P\Gamma L^{-1} - ((2 - \delta)/(1 - \delta))\|PB_*\|_2 I - 2\|PC_*\|_2 I - Q - R$, $S = (s_{ij})_{n \times n}$ with

$$s_{ij} = \begin{cases} 2p_i \bar{a}_{ii}, & \text{if } i = j, \\ \max\{|p_i \underline{a}_{ij} + p_j \underline{a}_{ji}|, |p_i \bar{a}_{ij} + p_j \bar{a}_{ji}|\}, & \text{if } i \neq j. \end{cases} \quad (11)$$

Then, for all $A \in A_I, B \in B_I$, and $C \in C_I$, we have

$$\Theta = \begin{pmatrix} \Phi - S' & -P\Delta B & -P\Delta C \\ * & (1 - \delta)Q & 0 \\ * & * & R \end{pmatrix} > 0, \quad (12)$$

where $S' = (s'_{ij})_{n \times n} = PA + A^T P$.

3. Global Robust Exponential Stability of the Equilibrium Point

In this section, in system (2), we assume that the external input bias $J_i(t) \equiv J_i$, J_i is a constant ($i = 1, 2, \dots, n$), and we will give a new sufficient condition for the existence and uniqueness of the equilibrium point for system (2) and analyze the global robust exponential stability of the equilibrium point.

Theorem 8. Under assumptions (H1) and (H2), if there exist positive diagonal matrices $P = \text{diag}(p_1, p_2, \dots, p_n)$, $Q = \text{diag}(q_1, q_2, \dots, q_n)$, and $R = \text{diag}(r_1, r_2, \dots, r_n)$ such that $\Xi > 0$, where Ξ is defined by (10), then system (2) has a unique equilibrium point.

The proof of Theorem 8 is similar to that in [32], therefore we omit it here.

Let $u^* = (u_1^*, u_2^*, \dots, u_n^*)^T$ be the equilibrium point of system (2). By coordinate transformation $v(t) = u(t) - u^*$, one can transform system (2) into the following system

$$\begin{aligned} \dot{v}_i(t) &= -\alpha_i(v_i(t)) \\ &\times \left[\beta_i(v_i(t)) - \sum_{j=1}^n a_{ij} g_j(v_j(t)) \right. \\ &\quad \left. - \sum_{j=1}^n b_{ij} g_j(v_j(t - \tau_j(t))) \right. \\ &\quad \left. - \sum_{j=1}^n c_{ij} \int_{-\infty}^t k_j(t-s) g_j(v_j(s)) ds \right], \end{aligned} \quad (13)$$

$i = 1, 2, \dots, n,$

or equivalently

$$\begin{aligned} \dot{v}(t) = & -\alpha(v(t)) \\ & \times \left[\beta(v(t)) - Ag(v(t)) - Bg(v(t - \tau(t))) \right. \\ & \left. - C \int_{-\infty}^t K(t-s)g(v(s))ds \right], \end{aligned} \quad (14)$$

where $v(t) = (v_1(t), \dots, v_n(t))^T$, $\alpha(v(t)) = \text{diag}(\alpha_1(v_1(t)), \dots, \alpha_n(v_n(t)))$ with $\alpha_i(v_i(t)) = \tilde{\alpha}_i(v_i(t) + u_i^*)$, $\beta(v(t)) = (\beta_1(v_1(t)), \dots, \beta_n(v_n(t)))^T$ with $\beta_i(v_i(t)) = \tilde{\beta}_i(v_i(t) + u_i^*) - \tilde{\beta}_i(u_i^*)$, $g(v(t)) = (g_1(v_1(t)), \dots, g_n(v_n(t)))^T$, $g(v(t - \tau(t))) = (g_1(v_1(t - \tau_1(t))), \dots, g_n(v_n(t - \tau_n(t))))^T$ with $g_j(v_j(t)) = f_j(v_j(t) + u_j^*) - f_j(u_j^*)$.

Theorem 9. Under assumptions (H1)–(H3), if there exist positive diagonal matrices $P = \text{diag}(p_1, p_2, \dots, p_n)$, $Q = \text{diag}(q_1, q_2, \dots, q_n)$, and $R = \text{diag}(r_1, r_2, \dots, r_n)$ such that $\Xi > 0$, where Ξ is defined by (10), then the equilibrium point of system (2) is globally robustly exponentially stable.

Proof. Define a Lyapunov functional: $V(t) = \sum_{i=1}^4 V_i(t)$, where

$$\begin{aligned} V_1(t) &= 2he^{\epsilon t} \sum_{i=1}^n \int_0^{v_i(t)} \frac{s}{\alpha_i(s)} ds, \\ V_2(t) &= 2e^{\epsilon t} \sum_{i=1}^n p_i \int_0^{v_i(t)} \frac{g_i(s)}{\alpha_i(s)} ds, \\ V_3(t) &= \sum_{i=1}^n (q_i + \|PB^*\|_2) \int_{t-\tau_i(t)}^t e^{\epsilon(\tau+s)} g_i^2(v_i(s)) ds, \\ V_4(t) &= \sum_{i=1}^n (r_i + \|PC^*\|_2) \int_0^{+\infty} k_i(\xi) e^{\epsilon\xi} \int_{t-\xi}^t e^{\epsilon s} g_i^2(v_i(s)) ds d\xi. \end{aligned} \quad (15)$$

Calculating the derivative of $V(t)$ along the trajectories of system (13), we obtain that

$$\begin{aligned} \dot{V}_1(t) &= 2h\epsilon e^{\epsilon t} \sum_{i=1}^n \int_0^{v_i(t)} \frac{s}{\alpha_i(s)} ds - 2he^{\epsilon t} \\ & \times \sum_{i=1}^n v_i(t) \left[\beta_i(v_i(t)) - \sum_{j=1}^n a_{ij}g_j(v_j(t)) \right. \\ & \quad \left. - \sum_{j=1}^n b_{ij}g_j(v_j(t - \tau_j(t))) \right. \\ & \quad \left. - \sum_{j=1}^n c_{ij} \int_{-\infty}^t k_j(t-s)g_j(v_j(s))ds \right] \\ & \leq h\epsilon e^{\epsilon t} \sum_{i=1}^n \frac{1}{\alpha_i} v_i^2(t) - 2he^{\epsilon t} \sum_{i=1}^n \gamma_i v_i^2(t) + 2he^{\epsilon t} \end{aligned}$$

$$\begin{aligned} & \times \sum_{i=1}^n v_i(t) \left[\sum_{j=1}^n a_{ij}g_j(v_j(t)) \right. \\ & \quad \left. + \sum_{j=1}^n b_{ij}g_j(v_j(t - \tau_j(t))) \right. \\ & \quad \left. + \sum_{j=1}^n c_{ij} \int_{-\infty}^t k_j(t-s)g_j(v_j(s))ds \right] \\ & = h\epsilon e^{\epsilon t} v^T(t) \underline{\alpha}^{-1} v(t) + 2he^{\epsilon t} v^T(t) \\ & \quad \times \left[-\Gamma v(t) + Ag(v(t)) + Bg(v(t - \tau(t))) \right. \\ & \quad \left. + C \int_{-\infty}^t K(t-s)g(v(s))ds \right], \\ \dot{V}_2(t) &= 2\epsilon e^{\epsilon t} \sum_{i=1}^n p_i \int_0^{v_i(t)} \frac{g_i(s)}{\alpha_i(s)} ds - 2e^{\epsilon t} \\ & \quad \times \sum_{i=1}^n p_i g_i(v_i(t)) \\ & \quad \times \left[\beta_i(v_i(t)) - \sum_{j=1}^n a_{ij}g_j(v_j(t)) \right. \\ & \quad \left. - \sum_{j=1}^n b_{ij}g_j(v_j(t - \tau_j(t))) \right. \\ & \quad \left. - \sum_{j=1}^n c_{ij} \int_{-\infty}^t k_j(t-s)g_j(v_j(s))ds \right] \\ & \leq \epsilon e^{\epsilon t} \sum_{i=1}^n \frac{p_i l_i}{\alpha_i} v_i^2(t) - 2e^{\epsilon t} \sum_{i=1}^n p_i \gamma_i g_i(v_i(t)) v_i(t) + 2e^{\epsilon t} \\ & \quad \times \sum_{i=1}^n p_i g_i(v_i(t)) \\ & \quad \times \left[\sum_{j=1}^n a_{ij}g_j(v_j(t)) + \sum_{j=1}^n b_{ij}g_j(v_j(t - \tau_j(t))) \right. \\ & \quad \left. + \sum_{j=1}^n c_{ij} \int_{-\infty}^t k_j(t-s)g_j(v_j(s))ds \right] \\ & = \epsilon e^{\epsilon t} v^T(t) PL\underline{\alpha}^{-1} v(t) - 2e^{\epsilon t} g^T(v(t)) P\Gamma v(t) \\ & \quad + 2e^{\epsilon t} g^T(v(t)) \\ & \quad \times \left[PAg(v(t)) + PBg(v(t - \tau(t))) \right. \\ & \quad \left. + PC \int_{-\infty}^t K(t-s)g(v(s))ds \right] \\ & \leq \epsilon e^{\epsilon t} v^T(t) PL\underline{\alpha}^{-1} v(t) - 2e^{\epsilon t} g^T(v(t)) P\Gamma L^{-1} g(v(t)) \\ & \quad + 2e^{\epsilon t} g^T(v(t)) PAg(v(t)) + e^{\epsilon t} \|PB^*\|_2 \\ & \quad \times \left(\frac{1}{1-\delta} \|g(v(t))\|_2^2 + (1-\delta) \|g(v(t - \tau(t)))\|_2^2 \right) \\ & \quad + e^{\epsilon t} \|PC^*\|_2 \end{aligned}$$

$$\begin{aligned}
 & \times \left(\|g(v(t))\|_2^2 + \left\| \int_{-\infty}^t K(t-s)g(v(s))ds \right\|_2^2 \right) \\
 & + 2e^{\varepsilon t} g^T(v(t)) P\Delta B g(v(t-\tau(t))) \\
 & + 2e^{\varepsilon t} g^T(v(t)) P\Delta C \int_{-\infty}^t K(t-s)g(v(s))ds, \\
 \dot{V}_3(t) & = \sum_{i=1}^n (q_i + \|PB^*\|_2) \\
 & \times \left[e^{\varepsilon(t+\tau)} g_i^2(v_i(t)) - (1-\dot{\tau}_i(t)) e^{\varepsilon(t+\tau-\tau_i(t))} \right. \\
 & \quad \left. g_i^2(v_i(t-\tau_i(t))) \right] \\
 & \leq e^{\varepsilon(t+\tau)} g^T(v(t)) Qg(v(t)) + e^{\varepsilon(t+\tau)} \|PB^*\|_2 \|g(v(t))\|_2^2 \\
 & \quad - (1-\delta) e^{\varepsilon t} g^T(v(t-\tau(t))) Qg(v(t-\tau(t))) \\
 & \quad - (1-\delta) e^{\varepsilon t} \|PB^*\|_2 \|g(v(t-\tau(t)))\|_2^2, \\
 \dot{V}_4(t) & = \sum_{i=1}^n (r_i + \|PC^*\|_2) \\
 & \times \int_0^{+\infty} k_i(\xi) e^{\varepsilon(t+\xi)} \left[g_i^2(v_i(t)) - e^{-\varepsilon\xi} g_i^2(v_i(t-\xi)) \right] d\xi \\
 & = e^{\varepsilon t} \int_0^{+\infty} k_i(\xi) e^{\varepsilon\xi} d\xi g^T(v(t)) (R + \|PC^*\|_2 I) g(v(t)) \\
 & \quad - e^{\varepsilon t} \sum_{i=1}^n (r_i + \|PC^*\|_2) \int_{-\infty}^t k_i(t-s) g_i^2(v_i(s)) ds \\
 & \leq e^{\varepsilon t} \int_0^{+\infty} k_i(\xi) e^{\varepsilon\xi} d\xi g^T(v(t)) (R + \|PC^*\|_2 I) g(v(t)) \\
 & \quad - e^{\varepsilon t} \left(\int_{-\infty}^t K(t-s)g(v(s))ds \right)^T (R + \|PC^*\|_2 I) \\
 & \quad \times \left(\int_{-\infty}^t K(t-s)g(v(s))ds \right).
 \end{aligned} \tag{16}$$

Therefore, one can deduce that

$$\begin{aligned}
 \dot{V}(t) & \leq \varepsilon e^{\varepsilon t} v^T(t) (h\underline{\alpha}^{-1} + PL\underline{\alpha}^{-1}) v(t) \\
 & \quad + e^{\varepsilon(t+\tau)} g^T(v(t)) (Q + \|PB^*\|_2 I) g(v(t)) + e^{\varepsilon t} \\
 & \quad \times \int_0^{+\infty} k_i(s) e^{\varepsilon s} ds g^T(v(t)) (R + \|PC^*\|_2 I) g(v(t)) \\
 & \quad \quad - 2he^{\varepsilon t} v^T(t) \Gamma v(t) \\
 & \quad - e^{\varepsilon t} g^T(v(t)) \\
 & \quad \times \left(2P\Gamma L^{-1} - 2PA - \frac{1}{1-\delta} \|PB^*\|_2 - \|PC^*\|_2 \right) \\
 & \quad \times g(v(t)) - e^{\varepsilon t} (1-\delta) g^T(v(t-\tau(t))) Qg(v(t-\tau(t))) \\
 & \quad - e^{\varepsilon t} \left(\int_{-\infty}^t K(t-s)g(v(s))ds \right)^T \\
 & \quad \times R \left(\int_{-\infty}^t K(t-s)g(v(s))ds \right) + 2he^{\varepsilon t} v^T(t) \\
 & \quad \times \left(Ag(v(t)) + Bg(v(t-\tau(t))) \right) \\
 & \quad + C \int_{-\infty}^t K(t-s)g(v(s))ds
 \end{aligned}$$

$$\begin{aligned}
 & + 2e^{\varepsilon t} g^T(v(t)) \\
 & \times \left(P\Delta B g(v(t-\tau(t))) \right. \\
 & \quad \left. + P\Delta C \int_{-\infty}^t K(t-s)g(v(s))ds \right) \\
 & = \varepsilon e^{\varepsilon t} v^T(t) (h\underline{\alpha}^{-1} + PL\underline{\alpha}^{-1}) v(t) \\
 & \quad + e^{\varepsilon t} (e^{\varepsilon\tau} - 1) g^T(v(t)) (Q + \|PB^*\|_2 I) g(v(t)) \\
 & \quad + e^{\varepsilon t} \left(\int_0^{+\infty} k_i(s) e^{\varepsilon s} ds - 1 \right) g^T(v(t)) (R + \|PC^*\|_2 I) \\
 & \quad \times g(v(t)) - e^{\varepsilon t} w^T(t) \Psi w(t),
 \end{aligned} \tag{17}$$

where

$$\Psi = \begin{pmatrix} 2h\Gamma & -hA & -hB & -hC \\ * & \Phi - S' & -P\Delta B & -P\Delta C \\ * & * & (1-\delta)Q & 0 \\ * & * & * & R \end{pmatrix}, \tag{18}$$

$$\begin{aligned}
 w(t) & = \left(v^T(t) g^T(v(t)) g^T(v(t-\tau(t))) \right. \\
 & \quad \left. \times \left(\int_{-\infty}^t K(t-s)g(v(s))ds \right)^T \right)^T.
 \end{aligned} \tag{19}$$

Denote $\Upsilon = (A \ B \ C)$. From Lemma 6, $\Psi > 0$ is equivalent to $\Theta - (h/2)\Upsilon^T \Gamma^{-1} \Upsilon > 0$, where Θ is defined in (12). By Lemma 7, we have $\Theta > 0$. Letting $0 < h < 2\min_{1 \leq i \leq n} \{\gamma_i\} \lambda_{\min}(\Theta) / \lambda_{\max}(\Upsilon^T \Upsilon)$, we can derive that

$$\Theta - \frac{h}{2} \Upsilon^T \Gamma^{-1} \Upsilon \geq \Theta - \frac{h}{2\min_{1 \leq i \leq n} \{\gamma_i\}} \Upsilon^T \Upsilon > 0, \tag{20}$$

which yields $\Psi > 0$.

From assumption (H3), we can choose a constant ε_3 sufficiently small satisfying $0 < \varepsilon_3 < \mu$ and

$$\int_0^{+\infty} k_i(s) e^{\varepsilon_3 s} ds \leq \frac{\lambda_{\min}(\Psi)}{2\max_{1 \leq i \leq n} \{r_i + \|PC^*\|_2\}} + 1, \quad 1 \leq i \leq n. \tag{21}$$

Choosing $0 < \varepsilon < \min_{1 \leq i \leq 3} \{\varepsilon_i\}$ with

$$\begin{aligned}
 \varepsilon_1 & = \frac{\lambda_{\min}(\Psi)}{\max_{1 \leq i \leq n} \{\underline{\alpha}_i^{-1} (h + p_i l_i)\}}, \\
 \varepsilon_2 & = \frac{1}{\tau} \ln \left(\frac{\lambda_{\min}(\Psi)}{2\max_{1 \leq i \leq n} \{q_i + \|PB^*\|_2\}} + 1 \right),
 \end{aligned} \tag{22}$$

we get

$$\begin{aligned}
 \dot{V}(t) & \leq \varepsilon_1 e^{\varepsilon t} v^T(t) (h\underline{\alpha}^{-1} + PL\underline{\alpha}^{-1}) v(t) \\
 & \quad + e^{\varepsilon t} (e^{\varepsilon_2 \tau} - 1) g^T(v(t)) (Q + \|PB^*\|_2 I) g(v(t)) \\
 & \quad + e^{\varepsilon t} \left(\int_0^{+\infty} k_i(s) e^{\varepsilon_3 s} ds - 1 \right) g^T(v(t)) \\
 & \quad \times (R + \|PC^*\|_2 I) g(v(t)) - e^{\varepsilon t} w^T(t) \Psi w(t) \\
 & < e^{\varepsilon t} \lambda_{\min}(\Psi) \left[v^T(t) v(t) + g^T(v(t)) g(v(t)) \right] \\
 & \quad - e^{\varepsilon t} w^T(t) \Psi w(t) \leq 0.
 \end{aligned} \tag{23}$$

Consequently, $V(t) \leq V(0)$ for all $t \geq 0$.

On the other hand,

$$\begin{aligned}
V(0) &= 2he^{\varepsilon t} \sum_{i=1}^n \int_0^{v_i(0)} \frac{s}{\alpha_i(s)} ds + 2 \sum_{i=1}^n p_i \int_0^{v_i(0)} \frac{g_i(s)}{\alpha_i(s)} ds \\
&\quad + \sum_{i=1}^n (q_i + \|PB^*\|_2) \\
&\quad \times \int_{-\tau_i(0)}^0 e^{\varepsilon(\tau+s)} g_i^2(v_i(s)) ds + \sum_{i=1}^n (r_i + \|PC^*\|_2) \\
&\quad \times \int_0^{+\infty} k_i(\xi) e^{\varepsilon\xi} \int_{-\xi}^0 e^{\varepsilon s} g_i^2(v_i(s)) ds d\xi \\
&\leq \max_{1 \leq i \leq n} \left\{ \frac{h_i}{\alpha_i} \right\} \|v(0)\|_2^2 + \max_{1 \leq i \leq n} \left\{ \frac{p_i l_i}{\alpha_i} \right\} \|v(0)\|_2^2 \\
&\quad + \tau e^{\varepsilon\tau} \max_{1 \leq i \leq n} \{l_i^2 (q_i + \|PB^*\|_2)\} \times \sup_{-\tau \leq \theta \leq 0} \|v(\theta)\|_2^2 \\
&\quad + \frac{1}{2\varepsilon} \max_{1 \leq i \leq n} \{l_i^2\} \lambda_{\min}(\Psi) \sup_{-\infty < \theta \leq 0} \|v(\theta)\|_2^2 \\
&\leq a \sup_{-\infty < \theta \leq 0} \|v(\theta)\|_2^2,
\end{aligned} \tag{24}$$

where $a = \max_{1 \leq i \leq n} \{h_i/\alpha_i\} + \max_{1 \leq i \leq n} \{p_i l_i/\alpha_i\} + \tau e^{\varepsilon\tau} \max_{1 \leq i \leq n} \{l_i^2 (q_i + \|PB^*\|_2)\} + (1/2\varepsilon) \max_{1 \leq i \leq n} \{l_i^2\} \lambda_{\min}(\Psi)$. Hence, $(h/\max_{1 \leq i \leq n} \{\alpha_i\}) e^{\varepsilon t} \|v(t)\|_2^2 \leq V(t) \leq V(0) \leq a \sup_{-\infty < \theta \leq 0} \|v(\theta)\|_2^2$; that is,

$$\|u(t) - u^*\|_2 \leq \sqrt{\frac{a \max_{1 \leq i \leq n} \{\alpha_i\}}{h}} \|\phi(\theta) - u^*\|_2 e^{-\varepsilon t/2}, \quad t > 0. \tag{25}$$

Combining Theorem 8, we get that system (2) is globally robustly exponentially stable. The proof is complete. \square

Remark 10. Letting $P = pI$ be a positive scalar matrix in Theorem 9, we can get a robust exponential stability criterion based on LMI.

Remark 11. If $\tilde{\alpha}_i(u_i(t)) = 1$, $\tilde{\beta}_i(u_i(t)) = d_i u_i(t)$, $0 < \underline{d}_i \leq d_i \leq \bar{d}_i$, and $c_{ij} = 0$, system (2) turns into the interval Hopfield neural networks (1), which was studied in [4–6]. It can be seen that the main results in [4] is a special case of Theorem 9. Therefore, the obtained results in this paper improve the results in [4–6]. Also, our results generalize some previous ones in [33–35] as mentioned in [4]. In addition, in [19], the authors dealt with the robust exponential stability of CGNNs with time-varying delays. However, the distributed delays were not taken into account. Therefore, our results in this paper are more general than those reported in [19].

Remark 12. In previous works such as [6, 33–35], $\|B_*\|_2$ is often used as a part to estimate the bounds for $\|B\|_2$. Considering that B_* is a nonnegative matrix, we develop a new approach based on H-matrix theory. The obtained robust stability criterion is in terms of the matrices B_* and B_*^T , which can reduce the conservativeness of the robust results to some extent.

4. Periodic Solutions of Interval CGNNs

In this section, we consider the periodic solutions of system (2), in which $\tau_i(t)$ and $J_i(t)$ are continuously periodic functions with period ω ; that is, $\tau_i(t + \omega) = \tau_i(t)$, $J_i(t + \omega) = J_i(t)$ ($i = 1, 2, \dots, n$).

Theorem 13. Under assumptions (H1)–(H3), system (2) has an ω -periodic solution which is globally exponentially stable, if the following condition holds:

(H4) $\mathcal{M} = \Gamma - D$ is a nonsingular M -matrix, where

$$D = (d_{ij})_{n \times n}, \quad d_{ij} = l_j \left(\hat{a}_{ij} + \frac{\hat{b}_{ij}}{1 - \delta} + \hat{c}_{ij} \right). \tag{26}$$

Proof. Let $u_i(t, \phi)$ and $u_i(t, \psi)$ be two solutions of system (2) with initial values $\phi, \psi \in C((-\infty, 0], R^n)$, respectively. Since \mathcal{M} is a nonsingular M -matrix, \mathcal{M}^T is also a nonsingular M -matrix. It is well known that there exists a positive vector $p = (\mu_1, \dots, \mu_n)^T$ such that $\mathcal{M}^T p > 0$; that is,

$$\mu_i \gamma_i - \sum_{j=1}^n \mu_j l_j \left(\hat{a}_{ji} + \frac{\hat{b}_{ji}}{1 - \delta} + \hat{c}_{ji} \right) > 0, \quad i = 1, 2, \dots, n. \tag{27}$$

We can choose a constant $\varepsilon > 0$ sufficiently small such that

$$\begin{aligned}
F_i(\varepsilon) &= \mu_i \left(\gamma_i - \frac{\varepsilon}{\alpha_i} \right) \\
&\quad - \sum_{j=1}^n \mu_j l_j \left(\hat{a}_{ji} + \frac{\hat{b}_{ji}}{1 - \delta} e^{\varepsilon\tau} + \hat{c}_{ji} \int_0^{+\infty} k_j(\xi) e^{\varepsilon\xi} d\xi \right) \\
&> 0, \quad i = 1, 2, \dots, n.
\end{aligned} \tag{28}$$

Denote $X_i(t) = |u_i(t, \phi) - u_i(t, \psi)|$. Define a Lyapunov functional

$$W(t) = \sum_{i=1}^n \mu_i (W_{1i}(t) + W_{2i}(t) + W_{3i}(t)), \tag{29}$$

where

$$\begin{aligned}
W_{1i}(t) &= e^{\varepsilon t} \operatorname{sgn}(u_i(t, \phi) - u_i(t, \psi)) \int_{u_i(t, \psi)}^{u_i(t, \phi)} \frac{1}{\alpha_i(s)} ds, \\
W_{2i}(t) &= \sum_{j=1}^n \frac{l_j \hat{b}_{ij}}{1 - \delta} \int_{t-\tau_j(t)}^t e^{\varepsilon(\tau+s)} X_j(s) ds, \\
W_{3i}(t) &= \sum_{j=1}^n l_j \hat{c}_{ij} \int_0^{+\infty} k_j(\xi) e^{\varepsilon\xi} \int_{t-\xi}^t e^{\varepsilon s} X_j(s) ds d\xi.
\end{aligned} \tag{30}$$

Calculating the upper right derivative of $W(t)$ along the solution of (2), together with assumptions (H1)–(H3) and (28), we can derive that

$$\begin{aligned}
 D^+W(t) &\leq e^{\varepsilon t} \sum_{i=1}^n \mu_i \left[\left(\frac{\varepsilon}{\alpha_i} - \gamma_i \right) X_i(t) \right. \\
 &\quad \left. + \sum_{j=1}^n l_j \left(\widehat{a}_{ij} + \frac{\widehat{b}_{ij}}{1-\delta} e^{\varepsilon \tau} + \widehat{c}_{ij} \right. \right. \\
 &\quad \left. \left. \times \int_0^{+\infty} k_j(\xi) e^{\varepsilon \xi} d\xi \right) X_j(t) \right] \\
 &= -e^{\varepsilon t} \sum_{i=1}^n \left[\mu_i \left(\gamma_i - \frac{\varepsilon}{\alpha_i} \right) \right. \\
 &\quad \left. - \sum_{j=1}^n \mu_j l_j \left(\widehat{a}_{ji} + \frac{\widehat{b}_{ji}}{1-\delta} e^{\varepsilon \tau} \right. \right. \\
 &\quad \left. \left. + \widehat{c}_{ji} \int_0^{+\infty} k_i(\xi) e^{\varepsilon \xi} d\xi \right) \right] X_i(t) \\
 &= -e^{\varepsilon t} \sum_{i=1}^n F_i(\varepsilon) X_i(t) \leq 0.
 \end{aligned} \tag{31}$$

Then, we have $W(t) \leq W(0)$ for $t \geq 0$. On the other hand, it can be readily seen that

$$W(t) \geq m_0 e^{\varepsilon t} \sum_{i=1}^n |u_i(t, \phi) - u_i(t, \psi)|, \tag{32}$$

$$W(0) \leq M_0 \sup_{s \in (-\infty, 0]} \sum_{i=1}^n |\phi_i(s) - \psi_i(s)|,$$

in which

$$\begin{aligned}
 m_0 &= \min_{1 \leq i \leq n} \left\{ \frac{\mu_i}{\alpha_i} \right\}, \\
 M_0 &= \max_{1 \leq i \leq n} \left\{ \frac{\mu_i}{\alpha_i} + \sum_{j=1}^n \left[\frac{l_j \widehat{b}_{ij} (e^{\varepsilon \tau} - 1)}{\varepsilon (1 - \delta)} + \frac{l_j \widehat{c}_{ij}}{\varepsilon} \right. \right. \\
 &\quad \left. \left. \times \left(\int_0^{+\infty} k_j(\xi) e^{\varepsilon \xi} d\xi - 1 \right) \right] \right\}.
 \end{aligned} \tag{33}$$

Hence, $m_0 e^{\varepsilon t} \sum_{i=1}^n |u_i(t, \phi) - u_i(t, \psi)| \leq W(t) \leq W(0) \leq M_0 \sup_{s \in (-\infty, 0]} \sum_{i=1}^n |\phi_i(s) - \psi_i(s)|$. Let $M = M_0/m_0$, then

$$\|u(t, \phi) - u(t, \psi)\|_1 \leq M \|\phi - \psi\|_1 e^{-\varepsilon t}, \quad t > 0. \tag{34}$$

We can always choose a positive integer N such that $e^{-\varepsilon N \omega} M \leq 1/2$ and define a Poincaré mapping $P : C \rightarrow C$ by $P(\phi) = u_\omega(\phi)$. It follows from (34) that

$$\|P^N \phi - P^N \psi\|_1 \leq \frac{1}{2} \|\phi - \psi\|_1, \tag{35}$$

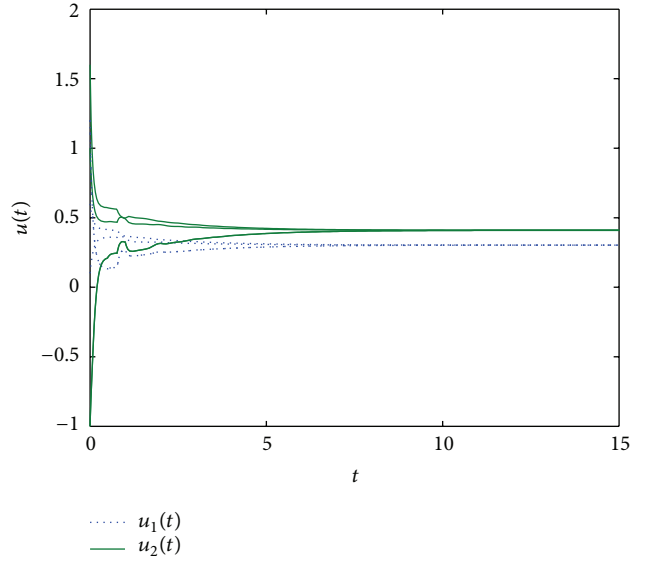


FIGURE 1: Time responses of the state variables $u(t)$ with different initial values in Example 1.

which implies that P^N is a contraction mapping. Thus, there exists a unique fixed point φ^* such that $P^N \varphi^* = \varphi^*$. Note that $P^N(P\varphi^*) = P(P^N \varphi^*) = P\varphi^*$. It means that $P\varphi^*$ is also a fixed point of P^N , then $P\varphi^* = \varphi^*$; that is, $u_\omega(\varphi^*) = \varphi^*$. Obviously, if $u(t, \varphi^*)$ is the solution of (2) through $(0, \varphi^*)$, $u(t + \omega, \varphi^*)$ is also a solution of (2) and $u_{t+\omega}(\varphi^*) = u_t(u_\omega(\varphi^*)) = u_t(\varphi^*)$ for $t > 0$. This shows that $u(t, \varphi^*)$ is exactly an ω -periodic solution of system (2) and all other solutions of (2) exponentially converge to it as $t \rightarrow +\infty$. This completes the proof. \square

Remark 14. The periodic oscillatory behavior of the neural networks is of great interest in many applications. For instance, this phenomena of periodic solutions for neural networks coincide with the fact that learning usually requires repetition and periodic sequences of neural impulse are also of fundamnent significance for the control of dynamic functions of the body such as heart beat and respiration which occur with great regularity.

Remark 15. In [23], the authors studied the existence and attractivity of periodic solutions for two class of CGNNs with discrete time delays or finite distributed time delays, respectively. In this paper, we incorporated time-varying delays and infinite distributed delays into CGNNs and derived the uniqueness and global exponential stability of periodic solutions. In [24, 27], two classes of CGNNs with distributed delays were investigated, and sufficient conditions were established to guarantee the uniqueness and global exponential stability of periodic solutions of such networks by using Lyapunov functional and the properties of M -matrix, whereas, the time-varying delays were ignored in the models. Thus, our results effectually improve or complement the results in [23, 24, 27].

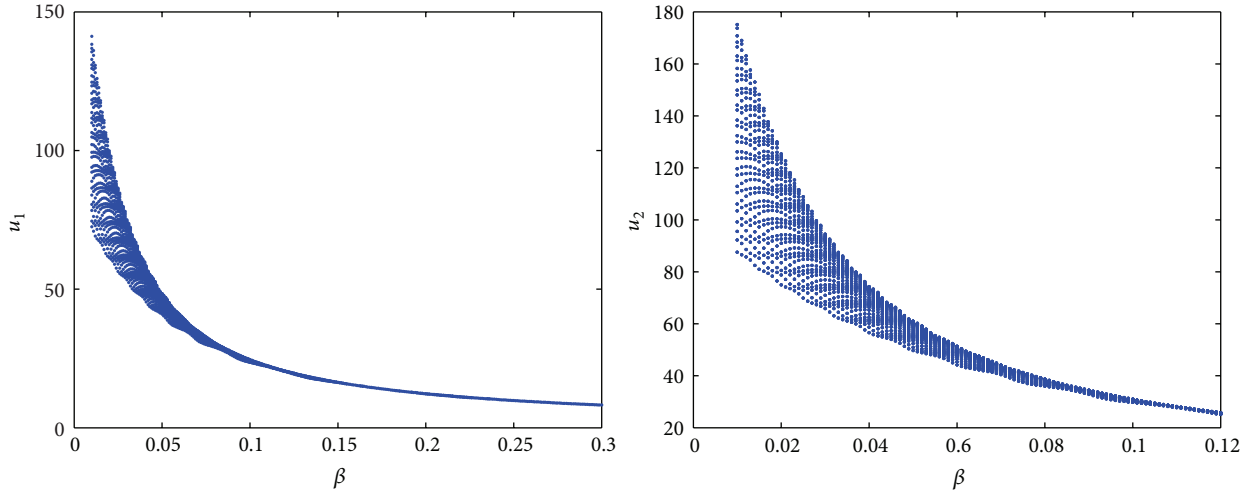


FIGURE 2: Bifurcation diagrams of system (2), and these show the effect of the parameter β on the dynamic behavior.

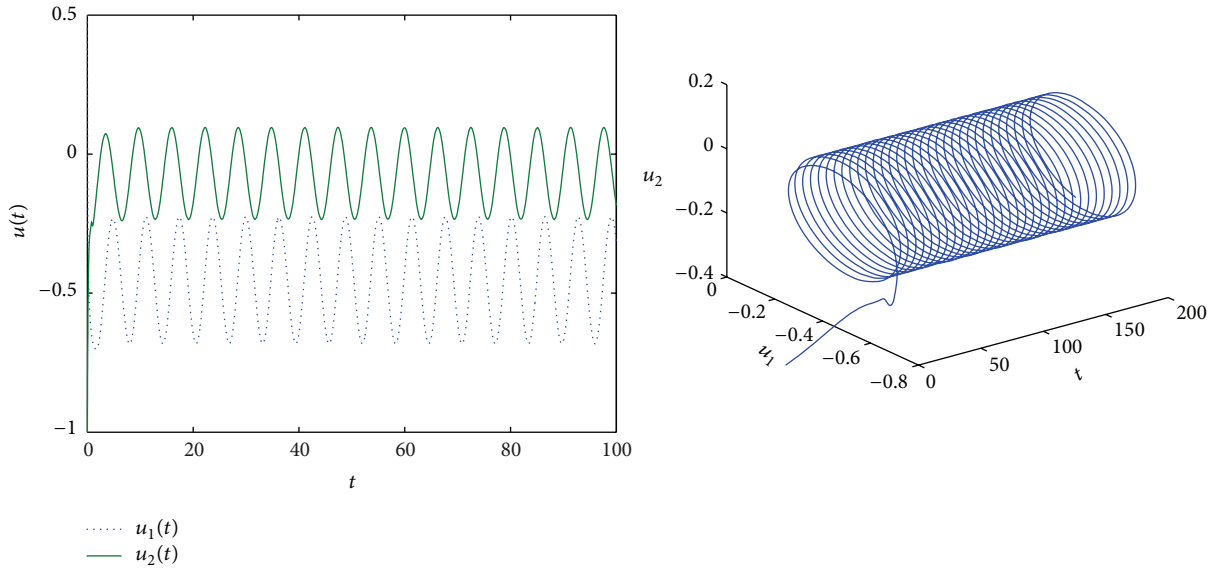


FIGURE 3: Time responses of the state variables $u(t)$ and phase plot in space (t, u_1, u_2) .

5. Numerical Simulation

In what follows, we give two examples to illustrate the results obtained in Sections 3 and 4.

Example 1. In system (2), we choose

$$\begin{aligned} \underline{A} &= \begin{pmatrix} -0.3 & -0.2 \\ -0.5 & -0.6 \end{pmatrix}, & \bar{A} &= \begin{pmatrix} 0.3 & 0.2 \\ 0.2 & 0.1 \end{pmatrix}, \\ \underline{B} &= \begin{pmatrix} -0.8 & -0.9 \\ -0.4 & -1 \end{pmatrix}, & \bar{B} &= \begin{pmatrix} 0.5 & 0.6 \\ 0.7 & 1 \end{pmatrix}, \\ \underline{C} &= \begin{pmatrix} -0.5 & -0.5 \\ -0.3 & -0.8 \end{pmatrix}, & \bar{C} &= \begin{pmatrix} 0.5 & 0.6 \\ 0.3 & 1 \end{pmatrix}, \end{aligned}$$

$$\tilde{\alpha}_i(u_i(t)) = 2 + \sin(u_i(t)), \quad \tilde{\beta}_i(u_i(t)) = 5u_i(t),$$

$$J_1 = -1, \quad J_2 = -1.5, \quad f_j(x) = \tanh(x),$$

$$\tau_j(t) = 1 - \frac{e^{-t}}{2}, \quad k_j(t) = te^{-t}, \quad i, j = 1, 2.$$

(36)

It is clear that $\gamma_1 = \gamma_2 = 5$, $l_1 = l_2 = 1$, $\tau = 1$, $\delta = 0.5$, $\mu = 1$. Using the optimization toolbox of Matlab and solving the optimization problem (10), we obtain

$$\begin{aligned} p_1 &= 1.6495, & p_2 &= 1.4828, & q_1 &= 2, \\ q_2 &= 2, & r_1 &= 1.2143, & r_2 &= 2. \end{aligned} \quad (37)$$

By Theorem 9, system (2) is globally robustly exponentially stable. To illustrate the theoretical result, we present a simulation with

$$\begin{aligned} A &= \begin{pmatrix} 0.2 & 0.1 \\ -0.1 & -0.4 \end{pmatrix}, & B &= \begin{pmatrix} 0.1 & 0.5 \\ 0.6 & 0.5 \end{pmatrix}, \\ C &= \begin{pmatrix} 0.2 & 0.36 \\ 0.2 & 0.8 \end{pmatrix}. \end{aligned} \quad (38)$$

We can find that the neuron vector $u(t) = (u_1(t), u_2(t))^T$ converges to the unique equilibrium point $x^* = (0.3031, 0.4095)^T$ (see Figure 1).

For system (2), we choose $\tilde{\beta}_i(u_i(t)) = \beta u_i(t)$. In Figure 2, we exhibit a typical bifurcation and chaos diagrams when we fix other parameters as (36) and (38) and choose β as a bifurcation parameter ($0.01 \leq \beta \leq 0.3$). It clearly shows that system (2) admits rich dynamics including period-doubling bifurcation and chaos.

Example 2. In system (2), we take $\tau_1(t) = \tau_2(t) = 1$, $J_1(t) = 2 + \sin t$, $J_2(t) = \cos t$, and the other parameters are the same as those in (36). One can obtain that $\mathcal{M} = \begin{pmatrix} 3.6 & -2.6 \\ -2.2 & 2.4 \end{pmatrix}$, which is a nonsingular M -matrix. According to Theorem 13, system (2) has a 2π -periodic solution which is globally exponentially stable. We present a simulation with the parameters in (38) (see Figure 3).

6. Conclusion

In this paper, we discussed a class of interval CGNNs with time-varying delays and infinite distributed delays. By employing H -matrix and M -matrix theory, Lyapunov functional method, and LMI approach, sufficient conditions were established for the existence, uniqueness, and global robust exponential stability of the equilibrium point and the periodic solution to the neural networks. It was shown that the obtained results improve or complement the previously published results. Numerical simulations demonstrated the main results and further showed that chaotic phenomena may occur for the system, which coincide with the fact of recognition character of human beings. On the other hand, it is well known that chaotic synchronization has been successfully applied to secure communication; chaotic behaviors of neural networks imply that they may be used to create secure communication systems.

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