Hindawi
Mathematical Problems in Engineering
Volume 2017, Article ID 6987436, 22 pages
https://doi.org/10.1155/2017/6987436

Research Article

Adaptive Exponential Synchronization for Stochastic Competitive Neural Networks with Time-Varying Leakage Delays and Reaction-Diffusion Terms

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Received 20 February 2017; Accepted 2 May 2017; Published 20 August 2017

Academic Editor: Marco Mussetta

We study the exponential synchronization problem for a class of stochastic competitive neural networks with different timescales, as well as spatial diffusion, time-varying leakage delays, and discrete and distributed time-varying delays. By introducing several important inequalities and using Lyapunov functional technique, an adaptive feedback controller is designed to realize the exponential synchronization for the proposed competitive neural networks in terms of p-norm. According to the theoretical results obtained in this paper, the influences of the timescale, external stimulus constants, disposable scaling constants, and controller parameters on synchronization are analyzed. Numerical simulations are presented to show the feasibility of the theoretical results.

1. Introduction

Neural networks are mathematical models that are inspired by the structure and functional aspects of biological neural networks. Meyer-Baese et al. [1] proposed competitive neural networks with different timescales, which describe the dynamics of cortical cognitive maps with unsupervised synaptic modifications. In the competitive neural networks model, there are two types of state variables: the short-term-memory (STM) variables describing the fast neural activity and the long-term-memory (LTM) variables describing the slow unsupervised synaptic modifications. Hence, there are two timescales in the competitive neural networks, one of which corresponds to the fast change of the state and the other to the slow change of the synapse by external stimuli. The above competitive neural networks are described by the following differential equations:

$$\begin{aligned}\text{STM: } \varepsilon\frac{\mathrm{d}y_i(t)}{\mathrm{d}t} &= -c_i y_i(t) + \sum_{j=1}^{n} a_{ij} f\left(y_j(t)\right) \\ &\quad + H_i \sum_{l=1}^{r} m_{il}(t)\, w_l, \\ \text{LTM: } \frac{\mathrm{d}m_{il}(t)}{\mathrm{d}t} &= -m_{il}(t) + w_l f\left(y_i(t)\right),\end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, n$, $y_i(t)$ is the neuron current activity level, $m_{il}(t)$ is the synaptic efficiency, $f(y_j(t))$ is the output of neurons, $c_i > 0$ is the time constant of the neuron, a_{ij} denotes the connection strength of the jth neuron on the ith neuron, H_i is the strength of the external stimulus, w_l is the constant external stimulus, r is the number of the constant external stimuli, and $\varepsilon > 0$ is the timescale of the STM state.

Synchronization problems of neural networks have been widely researched because of their extensive applications in secure communication, information processing, and chaos generators design. Synchronization of competitive neural networks with different timescales has attracted a great interest [2–7]. In [7], Gan et al. studied the adaptive synchronization for a class of competitive neural networks with different timescales and stochastic perturbation by constructing a Lyapunov-Krasovskii functional:

$$\begin{aligned} \text{STM: } \varepsilon \frac{\mathrm{d} y_i(t)}{\mathrm{d} t} &= -c_i y_i(t) + \sum_{j=1}^{n} a_{ij} f_j\left(y_j(t)\right) \\ &+ \sum_{j=1}^{n} b_{ij} f_j\left(y_j(t-\tau(t), x)\right) \\ &+ \sum_{j=1}^{n} d_{ij} \int_{t-\tau^*(t)}^{t} f_j\left(y_j(s)\right) \mathrm{d}s \\ &+ H_i \sum_{l=1}^{r} m_{il}(t) w_l, \\ \text{LTM: } \frac{\mathrm{d} m_{il}(t)}{\mathrm{d} t} &= -K_i m_{il}(t) + w_l f_i\left(y_i(t)\right), \end{aligned} \tag{2}$$

where $\tau(t)$ and $\tau^*(t)$ are the discrete time-varying delay and the distributed time-varying delay, respectively; b_{ij} and d_{ij} are, respectively, the discrete time-varying delay connection strength and the distributed time-varying delay connection strength of the jth neuron on the ith neuron; K_i is the disposable scaling constant.

The first term $y_i(t)$ in each of the right sides of (2) is called leakage term corresponding to a stabilizing negative feedback of the system [8, 9]. In real world, the transmission delays often appear in leakage terms, which are called leakage delays [10]. It is well known that leakage delays have been incorporated into neural networks by many researchers [11–14]. However, leakage delays of neural networks in most bibliographies listed above are constants. As pointed out in [15–18], the delays in neural networks are usually time-varying. Hence, the results about the neural networks with constant delays in the leakage term are imperfect.

In addition, dynamic behaviors of neural networks derive from the interactions of neurons, which is dependent on not only the time of each neuron but also its space position [19, 20]. From this point, diffusion phenomena should not be ignored in neural networks. Many good results about reaction-diffusion neural networks have been obtained [21–25]. The boundary conditions in most literatures listed are assumed to be Dirichlet boundary conditions. In engineering applications, such as thermodynamics, Neumann boundary conditions need to be considered. As far as we know, there are few results concerning the synchronization of competitive neural networks with reaction-diffusion term under Neumann boundary conditions.

Based on the above discussion, we are concerned with the combined effects of time-varying leakage delays, stochastic perturbation, and spatial diffusion on the synchronization of competitive neural networks with Neumann boundary conditions in terms of p-norm via an adaptive feedback controller to improve the previous results. To this end, we discuss the following neural networks:

$$\begin{aligned} \text{STM: } \varepsilon \frac{\partial y_i(t,x)}{\partial t} &= D_i \Delta y_i(t,x) - c_i y_i\left(t-\delta_i(t), x\right) \\ &+ \sum_{j=1}^{n} a_{ij} f_j\left(y_j(t,x)\right) \\ &+ \sum_{j=1}^{n} b_{ij} f_j\left(y_j\left(t-\tau_{ij}(t), x\right)\right) \\ &+ \sum_{j=1}^{n} d_{ij} \int_{t-\tau_{ij}^*(t)}^{t} f_j\left(y_j(s,x)\right) \mathrm{d}s \\ &+ H_i \sum_{l=1}^{r} m_{il}(t,x) w_l, \\ \text{LTM: } \frac{\partial m_{il}(t,x)}{\partial t} &= -K_i m_{il}(t,x) + w_l f_i\left(y_i(t,x)\right), \end{aligned} \tag{3}$$

where $x = (x_1, x_2, \ldots, x_m)^T \in \Omega \subset \mathbb{R}^m$ and $\Omega = \{x = (x_1, x_2, \ldots, x_m)^T \mid |x_k| < m_k\}$ is a bound compact set with smooth boundary $\partial\Omega$ and mes $\Omega > 0$ in space $\mathbb{R}^m$; $y(t,x) = (y_1(t,x), y_2(t,x), \ldots, y_n(t,x))$ with $y_i(t,x)$ denotes the state of the ith neuron at time t and in space x; $\Delta = \sum_{k=1}^{m}(\partial^2/\partial x_k^2)$ is the Laplace operator; $0 < \tau_{ij}(t) \le \tau$ and $0 < \tau_{ij}^*(t) \le \tau^*$ are the discrete time-varying delay and the distributed time-varying delay, respectively; $0 < \delta_i(t) \le \delta$ is the time-varying leakage delay; $D_i > 0$ corresponds to the transmission diffusion coefficient along the ith neuron.

Let $s_i(t,x) = \sum_{l=1}^{r} m_{il}(t,x) w_l = m_i^T(t,x) w$, where $m_i(t,x) = (m_{i1}(t,x), m_{i2}(t,x), \ldots, m_{ir}(t,x))^T$ and $w = (w_1, w_2, \ldots, w_r)^T$, and then then system (3) can be rewritten as

$$\begin{aligned} \text{STM: } \varepsilon \frac{\partial y_i(t,x)}{\partial t} &= D_i \Delta y_i(t,x) - c_i y_i\left(t-\delta_i(t), x\right) \\ &+ \sum_{j=1}^{n} a_{ij} f_j\left(y_j(t,x)\right) \\ &+ \sum_{j=1}^{n} b_{ij} f_j\left(y_j\left(t-\tau_{ij}(t), x\right)\right) \\ &+ \sum_{j=1}^{n} d_{ij} \int_{t-\tau_{ij}^*(t)}^{t} f_j\left(y_j(s,x)\right) \mathrm{d}s \\ &+ H_i s_i(t,x), \\ \text{LTM: } \frac{\partial s_i(t,x)}{\partial t} &= -K_i s_i(t,x) + |w|^2 f_i\left(y_i(t,x)\right), \end{aligned} \tag{4}$$

where $|w|^2 = w_1^2 + w_2^2 + \cdots + w_r^2$. Without loss of generality, the input stimulus vector w is assumed to be normalized with magnitude $|w|^2 = 1$. System (4) is simplified to

$$\begin{aligned}
\text{STM: } \varepsilon \frac{\partial y_i(t,x)}{\partial t} &= D_i \Delta y_i(t,x) - c_i y_i(t - \delta_i(t), x) \\
&\quad + \sum_{j=1}^{n} a_{ij} f_j(y_j(t,x)) \\
&\quad + \sum_{j=1}^{n} b_{ij} f_j(y_j(t - \tau_{ij}(t), x)) \\
&\quad + \sum_{j=1}^{n} d_{ij} \int_{t-\tau_{ij}^{*}(t)}^{t} f_j(y_j(s,x))\, \mathrm{d}s \\
&\quad + H_i s_i(t,x), \\
\text{LTM: } \frac{\partial s_i(t,x)}{\partial t} &= -K_i s_i(t,x) + f_i(y_i(t,x)).
\end{aligned} \tag{5}$$

The boundary condition of system (5) takes the form

$$\begin{aligned}
\frac{\partial y_i(t,x)}{\partial \mathbf{n}} &:= \left(\frac{\partial y_i(t,x)}{\partial x_1}, \frac{\partial y_i(t,x)}{\partial x_2}, \ldots, \frac{\partial y_i(t,x)}{\partial x_m} \right)^T \\
&= \mathbf{0}, \quad (t,x) \in [-\overline{\tau}, +\infty) \times \partial\Omega, \\
\frac{\partial s_i(t,x)}{\partial \mathbf{n}} &:= \left(\frac{\partial s_i(t,x)}{\partial x_1}, \frac{\partial s_i(t,x)}{\partial x_2}, \ldots, \frac{\partial s_i(t,x)}{\partial x_m} \right)^T = \mathbf{0}, \\
&\qquad (t,x) \in [-\overline{\tau}, +\infty) \times \partial\Omega.
\end{aligned} \tag{6}$$

The initial value of system (5) takes the form

$$\begin{aligned}
y(\theta,x) &= \phi^{y}(\theta,x), \\
s(\theta,x) &= \phi^{s}(\theta,x), \\
&\qquad (\theta,x) \in [-\overline{\tau}, 0] \times \Omega,
\end{aligned} \tag{7}$$

where $\overline{\tau} = \max\{\delta, \tau, \tau^{*}\}$, $s(\theta,x) = (s_1(\theta,x), s_2(\theta,x), \ldots, s_n(\theta,x))^T$, $\phi^{y}(\theta,x) = (\phi_1^{y}(\theta,x), \phi_2^{y}(\theta,x), \ldots, \phi_n^{y}(\theta,x))^T$, $\phi^{s}(\theta,x) = (\phi_1^{s}(\theta,x), \phi_2^{s}(\theta,x), \ldots, \phi_n^{s}(\theta,x))^T \in \mathscr{C}([-\overline{\tau},0) \times \Omega, \mathbb{R}^n)$, and $\mathscr{C}([-\overline{\tau},0) \times \Omega, \mathbb{R}^n)$ is the Banach space of continuous functions which maps $[-\overline{\tau},0) \times \Omega$ into $\mathbb{R}^n$ with the topology of uniform converge and p-norm (p is a positive integer) defined by

$$\begin{aligned}
\|\phi^{y}\|_p &= \left(\int_{\Omega} \sum_{i=1}^{n} \sup_{-\overline{\tau} \le \theta \le 0} |\phi_i^{y}(\theta,x)|^p \, \mathrm{d}x \right)^{1/p}, \\
\|\phi^{s}\|_p &= \left(\int_{\Omega} \sum_{i=1}^{n} \sup_{-\overline{\tau} \le \theta \le 0} |\phi_i^{s}(\theta,x)|^p \, \mathrm{d}x \right)^{1/p}.
\end{aligned} \tag{8}$$

In order to observe the exponential synchronization behavior of system (5), the response system with stochastic perturbation is designed as

$$\begin{aligned}
\text{STM: } \mathrm{d}z_i(t,x) &= \frac{1}{\varepsilon} \Bigg[D_i \Delta z_i(t,x) - c_i z_i(t - \delta_i(t), x) \\
&\quad + \sum_{j=1}^{n} a_{ij} f_j(z_j(t,x)) + \sum_{j=1}^{n} b_{ij} f_j(z_j(t - \tau_{ij}(t), x)) \\
&\quad + \sum_{j=1}^{n} d_{ij} \int_{t-\tau_{ij}^{*}(t)}^{t} f_j(z_j(s,x))\, \mathrm{d}s + H_i h_i(t,x) + u_i(t, \\
&\quad x) \Bigg] \mathrm{d}t + \sum_{j=1}^{n} \sigma_{ij} \big(e_j(t,x), e_j(t - \delta_i(t), x), \\
&\quad e_j(t - \tau_{ij}(t), x), e_j(t - \tau_{ij}^{*}(t), x) \big)\, \mathrm{d}\omega_j(t), \\
\text{LTM: } \frac{\partial h_i(t,x)}{\partial t} &= -K_i h_i(t,x) + f_i(z_i(t,x)),
\end{aligned} \tag{9}$$

where $z(t,x) = (z_1(t,x), z_2(t,x), \ldots, z_n(t,x))^T$ and $h(t,x) = (h_1(t,x), h_2(t,x), \ldots, h_n(t,x))^T$ denote the state of the response system; $e(t,x) = z(t,x) - y(t,x)$ and $R(t,x) = h(t,x) - s(t,x)$ are the synchronization error system; $\sigma = (\sigma_{ij})_{n\times n}$ is the noise intensity matrix and the stochastic disturbance $\omega(t) = [\omega_1(t), \omega_2(t), \ldots, \omega_n(t)]^T \in \mathbb{R}^n$ is a Brownian motion defined on $(\Omega, \mathscr{F}, \mathscr{P})$ (where Ω is the sample, $\mathscr{F}$ is the σ-algebra of subsets of the sample space, and $\mathscr{P}$ is the probability measure on $\mathscr{F}$), and

$$\begin{aligned}
\mathbf{E}\{\mathrm{d}\omega(t)\} &= 0, \\
\mathbf{E}\{\mathrm{d}\omega^2(t)\} &= 0,
\end{aligned} \tag{10}$$

where $\mathbf{E}\{\cdot\}$ is the mathematical expectation operator with respect to the given probability measure $\mathscr{P}$; $u(t,x) = (u_1(t,x), u_2(t,x), \ldots, u_n(t,x))^T$ is a feedback controller of the following form:

$$u(t,x) = \epsilon e(t,x). \tag{11}$$

The feedback strength $\epsilon = \operatorname{diag}(\epsilon_1, \epsilon_2, \ldots, \epsilon_n)$ is updated by the following law:

$$\frac{\partial \epsilon_i}{\partial t} = -\nu_i |e_i(t,x)|^p e^{\mu t}, \tag{12}$$

where $\mu > 0$ and $\nu_i > 0$ $(i = 1, 2, \ldots, n)$ are arbitrary positive constants.

The boundary condition and initial condition for response system (9) are given in the following forms:

$$\begin{aligned}
\frac{\partial z_i(t,x)}{\partial \mathbf{n}} &:= \left(\frac{\partial z_i(t,x)}{\partial x_1}, \frac{\partial z_i(t,x)}{\partial x_2}, \ldots, \frac{\partial z_i(t,x)}{\partial x_m} \right)^T \\
&= \mathbf{0}, \quad (t,x) \in [-\overline{\tau}, +\infty) \times \partial\Omega,
\end{aligned}$$

$$\frac{\partial h_i(t,x)}{\partial \mathbf{n}} := \left(\frac{\partial h_i(t,x)}{\partial x_1}, \frac{\partial h_i(t,x)}{\partial x_2}, \dots, \frac{\partial h_i(t,x)}{\partial x_m}\right)^T = \mathbf{0}, \quad (t,x) \in [-\overline{\tau}, +\infty) \times \partial\Omega, \tag{13}$$

$$z(\theta,x) = \psi^z(\theta,x), \quad h(\theta,x) = \psi^h(\theta,x), \quad (\theta,x) \in [-\overline{\tau}, 0] \times \Omega, \tag{14}$$

where $\psi^z(\theta,x) = (\psi_1^z(\theta,x), \psi_2^z(\theta,x), \dots, \psi_n^z(\theta,x))^T$ and $\psi^h(\theta,x) = (\psi_1^h(\theta,x), \psi_2^h(\theta,x), \dots, \psi_n^h(\theta,x))^T \in \mathscr{C}([-\overline{\tau}, 0) \times \Omega, \mathbb{R}^n)$.

Subtracting (5) from (9) yields the error system as follows:

$$\begin{aligned}
\text{STM: } \mathrm{d}e_i(t,x) &= \frac{1}{\varepsilon}\Bigg[D_i \Delta e_i(t,x) - c_i e_i(t-\delta_i(t),x) \\
&\quad + \sum_{j=1}^n a_{ij} f_j^*(e_j(t,x)) + \sum_{j=1}^n b_{ij} f_j^*(e_j(t-\tau_{ij}(t),x)) \\
&\quad + \sum_{j=1}^n d_{ij} \int_{t-\tau_{ij}^*(t)}^t f_j^*(e_j(s,x))\,\mathrm{d}s + H_i R_i(t,x) \\
&\quad + u_i(t,x)\Bigg]\mathrm{d}t + \sum_{j=1}^n \sigma_{ij}\big(e_j(t,x), e_j(t-\delta_i(t),x), \\
&\quad e_j(t-\tau_{ij}(t),x), e_j(t-\tau_{ij}^*(t),x)\big)\,\mathrm{d}\omega_j(t), \\
\text{LTM: } \frac{\partial R_i(t,x)}{\partial t} &= -K_i R_i(t,x) + f_i^*(e_i(t,x)),
\end{aligned} \tag{15}$$

where $f_j^*(e_j(\cdot,x)) = f_j(z_j(\cdot,x)) - f_j(y_j(\cdot,x))$.

In this paper, we give the following hypotheses.

(H_1) There exists a positive constant L_i such that the neuron activation function f_i satisfies the following conditions:

$$|f_i(\widehat{v}_i) - f_i(\check{v}_i)| \le L_i |\widehat{v}_i - \check{v}_i|, \tag{16}$$

where $\widehat{v}_i, \check{v}_i \in \mathbb{R}$, $i = 1, 2, \dots, n$.

(H_2) There exists a positive constant η_{ij} such that

$$\begin{aligned}
&\left|\sigma_{ij}(\widetilde{v}_1, \widehat{v}_1, \check{v}_1, \breve{v}_1) - \sigma_{ij}(\widetilde{v}_2, \widehat{v}_2, \check{v}_2, \breve{v}_2)\right|^2 \\
&\quad \le \eta_{ij}\left(|\widetilde{v}_1 - \widetilde{v}_2|^2 + |\widehat{v}_1 - \widehat{v}_2|^2 + |\check{v}_1 - \check{v}_2|^2 + |\breve{v}_1 - \breve{v}_2|^2\right),
\end{aligned} \tag{17}$$

for all $\widetilde{v}_1, \widetilde{v}_2, \widehat{v}_1, \widehat{v}_2, \check{v}_1, \check{v}_2, \breve{v}_1, \breve{v}_2 \in \mathbb{R}$, and $\sigma_{ij}(0,0,0,0) = 0$, $i, j = 1, 2, \dots, n$.

(H_3) There exist positive constants ρ' and ρ'' such that $\dot{\delta}_i(t) \le \rho' < 1$ or $\dot{\delta}_i(t) \ge \rho'' > 1$ for all $t, i = 1, 2, \dots, n$.

(H_4) There exist positive constants ϱ' and ϱ'' such that $\dot{\tau}_{ij}(t) \le \varrho' < 1$ or $\dot{\tau}_{ij}(t) \ge \varrho'' > 1$ for all $t, i, j = 1, 2, \dots, n$.

(H_5) There exist positive constants $\varrho^{*\prime}$ and $\varrho^{*\prime\prime}$ such that $\dot{\tau}_{ij}^*(t) \le \varrho^{*\prime} < 1$ or $\dot{\tau}_{ij}(t) \ge \varrho^{*\prime\prime} > 1$ for all $t, i, j = 1, 2, \dots, n$.

The paper is organized as follows. In the next section, we introduce some definitions and state several lemmas which will be essential to our proofs. In Section 3, by constructing a suitable Lyapunov functional, some new criteria are obtained to ensure the exponential synchronization of systems (5) and (9) under the adaptive feedback controller (11) and (12). Numerical simulations are carried out in Section 4 to illustrate the feasibility of the main theoretical results. A brief conclusion is given in Section 5.

2. Preliminary

In this section, we introduce some notations and lemmas which will be useful in the next section.

Definition 1. The noise-perturbed response system (9) and the drive system (5) can be exponentially synchronized under the adaptive controller (11) and (12) based on p-norm, if there exist constants $\gamma, \gamma^* > 0$ and $M, M^* \ge 1$ such that

$$\begin{aligned}
&\mathbf{E}\left\{\|z(t,x) - y(t,x)\|_p\right\} + \mathbf{E}\left\{\|h(t,x) - s(t,x)\|_p\right\} \\
&\quad \le M\mathbf{E}\left\{\|\psi^z - \phi^y\|_p\right\} e^{-\gamma t} + M^*\mathbf{E}\left\{\|\psi^h - \phi^s\|_p\right\} e^{-\gamma^* t}, \\
&\qquad (t,x) \in [0, +\infty) \times \Omega,
\end{aligned} \tag{18}$$

where $z(t,x), h(t,x)$ and $y(t,x), s(t,x)$ are solutions of systems (9) and (5) with differential initial functions (14) and (7), respectively, and

$$\begin{aligned}
\|z(t,x) - y(t,x)\|_p &= \left(\int_\Omega \sum_{i=1}^n |z_i(t,x) - y_i(t,x)|^p \mathrm{d}x\right)^{1/p}, \\
\|h(t,x) - s(t,x)\|_p &= \left(\int_\Omega \sum_{i=1}^n |h_i(t,x) - s_i(t,x)|^p \mathrm{d}x\right)^{1/p}.
\end{aligned} \tag{19}$$

Lemma 2 (Wang [26], Itô's formula). *Let $x(t)$ $(t \ge 0)$ be Itô processes, and*

$$\mathrm{d}x(t) = f(t)\,\mathrm{d}t + g(t)\,\mathrm{d}B_t, \tag{20}$$

where $f \in \mathscr{L}^1(R^+, R^n)$ ($\mathscr{L}^1$ is the space of absolutely integrable function) and $g \in \mathscr{L}^2(R^+, R^{n\times m})$ ($\mathscr{L}^2$ is the space of square integrable function). If $V(x,t) \in C^{2,1}(R^n \times R^+; R)$, ($C^{2,1}(R^n \times R^+; R)$ is the family of all nonnegative functions on $R^+ \times R^n$ which are continuously twice differentiable in x and once differentiable in t), then $V(x(t),t)$ are still Itô processes, and

$$\begin{aligned}
\mathrm{d}V(x(t),t) &= \Big[V_t(x(t),t) + V_x(x(t),t) f(t) \\
&\quad + \frac{1}{2}\mathrm{tr}\left(g^T(t) V_{xx}(x(t),t) g(t)\right)\Big]\mathrm{d}t \\
&\quad + V_x(x(t),t) g(t)\,\mathrm{d}B_t,
\end{aligned} \tag{21}$$

where

$$V_t(x(t),t)=\frac{\partial V(x(t),t)}{\partial t},$$
$$V_x(x(t),t)=\left(\frac{\partial V(x(t),t)}{\partial x_1},\dots,\frac{\partial V(x(t),t)}{\partial x_n}\right), \quad (22)$$
$$V_{xx}(x(t),t)=\left(\frac{\partial^2 V(x(t),t)}{\partial e_i \partial e_j}\right)_{n\times n}.$$

Lemma 3 (Mei et al. [27]). *Let $p\geq 2$ and let $a,b,h>0$. Then*

$$a^{p-1}b\leq\frac{(p-1)ha^p}{p}+\frac{b^p}{ph^{p-1}}, \quad (23)$$

$$a^{p-2}b^2\leq\frac{(p-2)ha^p}{p}+\frac{2b^p}{ph^{(p-2)/2}}. \quad (24)$$

Lemma 4 (Mao [28]). *Let $F(x),G(x):[a,b]\rightarrow\mathbb{R}$ be continuous functions. Suppose that positive constants p and q satisfy*

$$\frac{1}{p}+\frac{1}{q}=1. \quad (25)$$

Then

$$\int_a^b|F(x)G(x)|\,\mathrm{d}x\leq\left[\int_a^b|F(x)|^p\,\mathrm{d}x\right]^{1/p}\left[\int_a^b|G(x)|^q\,\mathrm{d}x\right]^{1/q}. \quad (26)$$

Lemma 5 (Gu et al. [29]). *Suppose that Ω is a bound domain of R^m with a smooth boundary $\partial\Omega$. $u(x),v(x)$ are real-valued functions belonging to $\mathscr{C}^2(\Omega\cup\partial\Omega)$. Then*

$$\int_\Omega u(x)\Delta v(x)\,\mathrm{d}x=\int_{\partial\Omega}u(x)\frac{\partial v(x)}{\partial\mathbf{n}}\mathrm{d}x-\int_\Omega(\nabla u(x))^T\nabla v(x)\,\mathrm{d}x, \quad (27)$$

where $\nabla=(\partial/\partial x_1,\partial/\partial x_2,\dots,\partial/\partial x_m)^T$ is the gradient operator.

Lemma 6. *Let $p\geq 2$ be a positive integer and let Ω be a bound domain of R^{l^*} with a smooth boundary $\partial\Omega$. $\varphi(x)\in\mathscr{C}^1(\Omega)$ is a real-valued function and $(\partial\varphi(x)/\partial\mathbf{n})|_{\partial\Omega}=\mathbf{0}$. Then*

$$\int_\Omega|\varphi(x)|^p\,\mathrm{d}x\leq\frac{p-1}{\lambda_1}\int_\Omega|\varphi(x)|^{p-2}|\nabla\varphi(x)|^2\,\mathrm{d}x, \quad (28)$$

where λ_1 is the smallest positive eigenvalue of the Neumann boundary problem:

$$-\Delta\vartheta(x)=\lambda\vartheta(x),\quad x\in\Omega,$$
$$\frac{\partial\vartheta(x)}{\partial\mathbf{n}}=0,\quad x\in\partial\Omega. \quad (29)$$

The proof of Lemma 6 is attached in Appendix.

Remark 7. If $p=2$, the integral inequality (28) is the Poincaré integral inequality in [30]. The smallest eigenvalue λ_1 of the Neumann boundary problem (29) is determined by the boundary of Ω [30]. If $\Omega=\{x=(x_1,x_2,\dots,x_{l^*})^T\mid m_k^-\leq x_k\leq m_k^+,\ k=1,2,\dots,l^*\}\subset R^{l^*}$, then

$$\lambda_1=\min\left\{\left(\frac{\pi}{m_1^+-m_1^-}\right)^2,\left(\frac{\pi}{m_2^+-m_2^-}\right)^2,\dots,\left(\frac{\pi}{m_{l^*}^+-m_{l^*}^-}\right)^2\right\}. \quad (30)$$

3. Exponential Synchronization Criterion

In this section, the exponential synchronization criterion of the drive system (5) and the response system (9) is obtained under the adaptive feedback controller (11) and (12). For convenience, the following denotations are introduced.

Denote

$$\alpha_i=\frac{1}{\kappa|1-\rho|e^{\mu\delta}}\left[\frac{1}{\varepsilon}\frac{1}{\xi_i^{**p-1}}+(p-1)\sum_{j=1}^n\frac{\eta_{ji}}{\varsigma_i^{*((p-2)/2)}}\right],$$

$$\beta_{ij}=\frac{1}{e^{\mu\tau}\kappa|1-\varrho|}\left[\frac{(p-1)\eta_{ji}}{\varsigma_i^{**((p-2)/2)}}+\frac{1}{\varepsilon}\frac{|b_{ji}|L_i}{\varpi_i^{*p-1}}\right],$$

$$\gamma_{ij}=\frac{(p-1)\eta_{ji}}{e^{\mu\tau^*}\kappa|1-\varrho^*|\varsigma_i^{***((p-2)/2)}},$$

$$\alpha_i^*=\alpha_i-\kappa\alpha_i\operatorname{sgn}(1-\rho),$$

$$\beta_{ij}^*=\beta_{ij}-\kappa\beta_{ij}\operatorname{sgn}(1-\varrho),$$

$$\gamma_{ij}^*=\gamma_{ij}-\kappa\gamma_{ij}\operatorname{sgn}(1-\varrho^*),$$

$$\gamma_{ij}^{**}=\frac{1}{\varepsilon}\frac{(\tau^*)^{p-1}L_i|d_{ji}|}{\varpi_i^{**p-1}},$$

$$l_i=\frac{\varepsilon}{p}\left[-\frac{1}{\varepsilon}p\lambda_1D_i+\mu+\alpha_ie^{\mu\delta}+\frac{1}{\varepsilon}c_i(p-1)\xi_i^{**}+\frac{L_i}{\xi_i^{p-1}}+\frac{1}{\varepsilon}|H_i|(p-1)\xi_i^*+e^{\mu\tau}\sum_{j=1}^n\beta_{ij}+e^{\mu\tau^*}\sum_{j=1}^n\gamma_{ij}+\tau^*e^{\mu\tau^*}\sum_{j=1}^n\gamma_{ij}^{**}+\frac{1}{\varepsilon}\sum_{j=1}^n\frac{1}{\varpi_i^{p-1}}|a_{ji}|L_i\right.$$

$$+\frac{1}{\varepsilon}(p-1)\sum_{j=1}^{n}\left|b_{ij}\right|L_j\varpi_j^{*}$$

$$+\frac{1}{\varepsilon}(p-1)\sum_{j=1}^{n}\left|d_{ij}\right|L_j\varpi_j^{**}$$

$$+\frac{1}{\varepsilon}(p-1)\sum_{j=1}^{n}\left|a_{ij}\right|L_j\varpi_j+\frac{(p-1)(p-2)}{2}\sum_{j=1}^{n}\eta_{ij}\varsigma_j$$

$$+\frac{(p-1)(p-2)}{2}\sum_{j=1}^{n}\eta_{ij}\varsigma_j^{*}$$

$$+\frac{(p-1)(p-2)}{2}\sum_{j=1}^{n}\eta_{ij}\varsigma_j^{**}$$

$$+\frac{(p-1)(p-2)}{2}\sum_{j=1}^{n}\eta_{ij}\varsigma_j^{***}+(p-1)\sum_{j=1}^{n}\frac{\eta_{ji}}{\varsigma_i^{(p-2)/2}}\Bigg], \tag{31}$$

where $0<\kappa<1$, $|1-\rho|=\max\{|1-\rho'|,|1-\rho''|\}$, $|1-\varrho|=\max\{|1-\varrho'|,|1-\varrho''|\}$, $|1-\varrho^*|=\max\{|1-\varrho^{*\prime}|,|1-\varrho^{*\prime\prime}|\}$, and ξ_i, ξ_i^*, ξ_i^{**}, ς_i, ς_i^*, ς_i^{**}, ς_i^{***}, ϖ_i, ϖ_i^*, and ϖ_i^{**} are nonnegative real numbers, respectively.

Theorem 8. *Under assumptions* (H_1)–(H_5), *the nonlinear couple neural networks (9) and (5) can be exponentially synchronized under the adaptive feedback controller (11) and (12) based on p-norm, if the following condition is also satisfied.*

(H_6) $\mu-pK_i+L_i(p-1)\xi_i+(1/\varepsilon)(|H_i|/\xi_i^{*p-1})\le 0$.

Proof. Define

$$\begin{aligned}V(t)=\int_\Omega\sum_{i=1}^{n}\Bigg[&V_i(t,x)+\alpha_i e^{\mu\delta}\int_{t-\delta_i(t)}^{t}V_i(s,x)\,ds\\&+\alpha_i^{*}e^{\mu\delta}\int_{t-\delta}^{t-\delta_i(t)}V_i(s,x)\,ds\\&+e^{\mu\tau}\sum_{j=1}^{n}\beta_{ij}\int_{t-\tau_{ij}(t)}^{t}V_i(s,x)\,ds\\&+e^{\mu\tau}\sum_{j=1}^{n}\beta_{ij}^{*}\int_{t-\tau}^{t-\tau_{ij}(t)}V_i(s,x)\,ds\\&+e^{\mu\tau^*}\sum_{j=1}^{n}\gamma_{ij}\int_{t-\tau_{ij}^*(t)}^{t}V_i(s,x)\,ds\\&+e^{\mu\tau^*}\sum_{j=1}^{n}\gamma_{ij}^{*}\int_{t-\tau^*}^{t-\tau_{ij}^*(t)}V_i(s,x)\,ds\\&+e^{\mu\tau^*}\sum_{j=1}^{n}\gamma_{ij}^{**}\int_{-\tau^*}^{0}\int_{t+s}^{t}V_i(\eta,x)\,d\eta ds\\&+e^{\mu t}\left|R_i(t,x)\right|^{p}+\frac{p}{2\varepsilon\nu_i}(\epsilon_i+l_i)^2\Bigg]dx,\end{aligned} \tag{32}$$

where $V_i(t,x)=e^{\mu t}|e_i(t,x)|^p$.

By (10), Itô's differential formula, and Dini derivation, it can be deduced that

$$\begin{aligned}D^{+}\mathbf{E}\{V(t)\}=\mathbf{E}\Bigg\{\int_\Omega\sum_{i=1}^{n}\Big\{&\mu V_i(t,x)+\alpha_i e^{\mu\delta}\left[V_i(t,x)-V_i(t-\delta_i(t),x)\left(1-\dot{\delta}_i(t)\right)\right]\\&+\alpha_i^{*}e^{\mu\delta}\left[V_i(t-\delta_i(t),x)\left(1-\dot{\delta}_i(t)\right)-V_i(t-\delta,x)\right]\\&+e^{\mu\tau}\sum_{j=1}^{n}\beta_{ij}\left[V_i(t,x)-V_i\left(t-\tau_{ij}(t),x\right)\left(1-\dot{\tau}_{ij}(t)\right)\right]\\&+e^{\mu\tau}\sum_{j=1}^{n}\beta_{ij}^{*}\left[V_i\left(t-\tau_{ij}(t),x\right)\left(1-\dot{\tau}_{ij}(t)\right)-V_i(t-\tau,x)\right]\\&+e^{\mu\tau^*}\sum_{j=1}^{n}\gamma_{ij}\left[V_i(t,x)-V_i\left(t-\tau_{ij}^*(t),x\right)\left(1-\dot{\tau}_{ij}^*(t)\right)\right]\\&+e^{\mu\tau^*}\sum_{j=1}^{n}\gamma_{ij}^{*}\left[V_i\left(t-\tau_{ij}^*(t),x\right)\left(1-\dot{\tau}_{ij}^*(t)\right)-V_i(t-\tau^*,x)\right]\\&+e^{\mu\tau^*}\sum_{j=1}^{n}\gamma_{ij}^{**}\int_{-\tau^*}^{0}\left[V_i(t,x)-V_i(t+s,x)\right]ds+\mu e^{\mu t}\left|R_i(t,x)\right|^{p}\\&+pe^{\mu t}\left|R_i(t,x)\right|^{p-1}\left[-K_iR_i(t,x)+f_i^{*}(e_i(t,x))\right]\end{aligned}$$

$$
\begin{aligned}
& - \frac{p}{\varepsilon}\left(\epsilon_i + l_i\right)\left|e_i(t,x)\right|^p e^{\mu t} \\
& + \frac{1}{\varepsilon} p e^{\mu t}\left|e_i(t,x)\right|^{p-1}\left[D_i \Delta e_i(t,x) - c_i e_i\left(t-\delta_i(t),x\right)\right. \\
& + \sum_{j=1}^{n} a_{ij} f_j^*\left(e_j(t,x)\right) \\
& + \sum_{j=1}^{n} b_{ij} f_j^*\left(e_j\left(t-\tau_{ij}(t),x\right)\right) \\
& \left. + H_i R_i(t,x) + \epsilon_i e_i(t,x)\right] \\
& + \frac{p(p-1) e^{\mu t}\left|e_i(t,x)\right|^{p-2}}{2} \\
& \cdot \sum_{j=1}^{n} \sigma_{ij}^2\left(e_j(t,x), e_j\left(t-\delta_i(t),x\right),\right. \\
& \left.\left. e_j\left(t-\tau_{ij}(t),x\right), e_j\left(t-\tau_{ij}^*(t),x\right)\right)\right\} \mathrm{d}x \Bigg\} \\
& \leq \mathbf{E}\left\{\int_\Omega \sum_{i=1}^{n}\left\{\mu V_i(t,x) + \alpha_i e^{\mu\delta} V_i(t,x)\right.\right. \\
& - \kappa\alpha_i\left|1-\rho\right| e^{\mu\delta} V_i\left(t-\delta_i(t),x\right) \\
& + e^{\mu\tau}\sum_{j=1}^{n}\beta_{ij} V_i(t,x) - e^{\mu\tau}\sum_{j=1}^{n}\kappa\beta_{ij}\left|1-\varrho\right| V_i\left(t-\tau_{ij}(t),x\right) \\
& + e^{\mu\tau^*}\sum_{j=1}^{n}\gamma_{ij} V_i(t,x) - e^{\mu\tau^*}\sum_{j=1}^{n}\kappa\gamma_{ij}\left|1-\varrho^*\right| V_i\left(t-\tau_{ij}^*(t),x\right) \\
& + \tau^* e^{\mu\tau^*}\sum_{j=1}^{n}\gamma_{ij}^{**} V_i(t,x) - e^{\mu\tau^*}\sum_{j=1}^{n}\gamma_{ij}^{**}\int_{t-\tau^*}^{t} V_i(s,x)\,\mathrm{d}s \\
& + \left(\mu - pK_i\right) e^{\mu t}\left|R_i(t,x)\right|^p + p e^{\mu t}\left|R_i(t,x)\right|^{p-1} L_i\left|e_i(t,x)\right| \\
& - \frac{p}{\varepsilon}\left(\epsilon_i + l_i\right)\left|e_i(t,x)\right|^p e^{\mu t} + \frac{1}{\varepsilon} p e^{\mu t}\left|e_i(t,x)\right|^{p-1}\left[D_i \Delta\left|e_i(t,x)\right| + c_i\left|e_i\left(t-\delta_i(t),x\right)\right|\right. \\
& + \sum_{j=1}^{n}\left|a_{ij}\right| L_j\left|e_j(t,x)\right| + \sum_{j=1}^{n}\left|b_{ij}\right| L_j\left|e_j\left(t-\tau_{ij}(t),x\right)\right| \\
& + \sum_{j=1}^{n}\left|d_{ij}\right|\int_{t-\tau_{ij}^*(t)}^{t} L_j\left|e_j(s,x)\right|\mathrm{d}s \\
& \left. + \left|H_i\right|\left|R_i(t,x)\right| + \epsilon_i\left|e_i(t,x)\right|\right] + \frac{p(p-1) e^{\mu t}\left|e_i(t,x)\right|^{p-2}}{2} \\
& \sum_{j=1}^{n}\eta_{ij}\left(\left|e_j(t,x)\right|^2 + \left|e_j\left(t-\delta_i(t),x\right)\right|^2\right. \\
& \left.\left.\left. + \left|e_j\left(t-\tau_{ij}(t),x\right)\right|^2 + \left|e_j\left(t-\tau_{ij}^*(t),x\right)\right|^2\right)\right\}\mathrm{d}x\right\}.
\end{aligned}
\tag{33}
$$

From the boundary conditions (6) and (13) and Lemma 6, we get

$$\begin{aligned} & p\int_{\Omega}\left|e_i(t,x)\right|^{p-1}D_i\Delta\left|e_i(t,x)\right|\mathrm{d}x \\ & = p\left(\int_{\partial\Omega}\left|e_i(t,x)\right|^{p-1}\right. \\ & \cdot D_i\sum_{k=1}^{m}\frac{\partial\left|e_i(t,x)\right|}{\partial x_k}\cos\left(x_k,n\right)\mathrm{d}s \\ & \left.-\int_{\Omega}\sum_{k=1}^{m}D_i\frac{\partial\left|e_i(t,x)\right|}{\partial x_k}\cdot\frac{\partial\left|e_i(t,x)\right|^{p-1}}{\partial x_k}\mathrm{d}x\right) \\ & = -p(p-1)D_i\int_{\Omega}\left|e_i(t,x)\right|^{p-2}\sum_{k=1}^{m}\frac{\partial\left|e_i(t,x)\right|}{\partial x_k} \\ & \cdot\frac{\partial\left|e_i(t,x)\right|}{\partial x_k}\mathrm{d}x = -p(p-1) \\ & \cdot D_i\int_{\Omega}\left|e_i(t,x)\right|^{p-2}\mid\nabla\left|e_i(t,x)\right|^2\mathrm{d}x \\ & \leq -p\lambda_1 D_i\int_{\Omega}\left|e_i(t,x)\right|^p\mathrm{d}x. \end{aligned} \tag{34}$$

By Lemma 4, we obtain

$$\begin{aligned} & \int_{t-\tau^*}^{t}V_i(s,x)\,\mathrm{d}s \geq e^{\mu t}e^{-\mu\tau^*}\int_{t-\tau_{ij}^*(t)}^{t}\left|e_i(s,x)\right|^p\mathrm{d}s \\ & \geq e^{\mu t}e^{-\mu\tau^*}\frac{\left(\int_{t-\tau_{ij}^*(t)}^{t}\left|e_i(s,x)\right|\mathrm{d}s\right)^p}{\left(\tau_{ij}^*(t)\right)^{p/q}} \\ & \geq e^{\mu t}e^{-\mu\tau^*}\left(\tau^*\right)^{1-p}\left(\int_{t-\tau_{ij}^*(t)}^{t}\left|e_i(s,x)\right|\mathrm{d}s\right)^p. \end{aligned} \tag{35}$$

It follows from (23) that

$$\begin{aligned} & p\left|R_i(t,x)\right|^{p-1}\left|e_i(t,x)\right| \\ & \quad\leq (p-1)\,\xi_i\left|R_i(t,x)\right|^p + \frac{\left|e_i(t,x)\right|^p}{\xi_i^{p-1}}, \\ & p\left|e_i(t,x)\right|^{p-1}\left|R_i(t,x)\right| \\ & \quad\leq (p-1)\,\xi_i^*\left|e_i(t,x)\right|^p + \frac{\left|R_i(t,x)\right|^p}{\xi_i^{*\,p-1}}, \\ & p\left|e_i(t,x)\right|^{p-1}\left|e_i(t-\delta_i(t),x)\right| \\ & \quad\leq (p-1)\,\xi_i^{**}\left|e_i(t,x)\right|^p + \frac{\left|e_i(t-\delta_i(t),x)\right|^p}{\xi_i^{**\,p-1}}, \\ & p\left|e_i(t,x)\right|^{p-1}\sum_{j=1}^{n}\left|a_{ij}\right|L_j\left|e_j(t,x)\right| \\ & \quad\leq (p-1)\left|e_i(t,x)\right|^p\sum_{j=1}^{n}\left|a_{ij}\right|L_j\varpi_j \\ & \qquad+\sum_{j=1}^{n}\frac{L_j\left|a_{ij}\right|\left|e_j(t,x)\right|^p}{\varpi_j^{p-1}}, \\ & p\left|e_i(t,x)\right|^{p-1}\sum_{j=1}^{n}\left|b_{ij}\right|L_j\left|e_j\left(t-\tau_{ij}(t),x\right)\right| \\ & \quad\leq (p-1)\left|e_i(t,x)\right|^p\sum_{j=1}^{n}\left|b_{ij}\right|L_j\varpi_j^* \\ & \qquad+\sum_{j=1}^{n}\frac{L_j\left|b_{ij}\right|\left|e_j\left(t-\tau_{ij}(t),x\right)\right|^p}{\varpi_j^{*\,p-1}}, \\ & p\left|e_i(t,x)\right|^{p-1}\sum_{j=1}^{n}\left|d_{ij}\right|L_j\int_{t-\tau_{ij}^*(t)}^{t}\left|e_j(s,x)\right|\mathrm{d}s \\ & \quad\leq (p-1)\left|e_i(t,x)\right|^p\sum_{j=1}^{n}\left|d_{ij}\right|L_j\varpi_j^{**} \\ & \qquad+\sum_{j=1}^{n}\frac{\left|d_{ij}\right|L_j}{\varpi_j^{**\,p-1}}\left(\int_{t-\tau_{ij}^*(t)}^{t}\left|e_j(s,x)\right|\mathrm{d}s\right)^p. \end{aligned} \tag{36}$$

It follows from (24) that

$$\begin{aligned} & p\left|e_i(t,x)\right|^{p-2}\sum_{j=1}^{n}\eta_{ij}\left|e_j(t,x)\right|^2 \\ & \quad\leq (p-2)\left|e_i(t,x)\right|^p\sum_{j=1}^{n}\eta_{ij}\varsigma_j + \sum_{j=1}^{n}\frac{2\eta_{ij}\left|e_j(t,x)\right|^p}{\varsigma_j^{(p-2)/2}}, \\ & p\left|e_i(t,x)\right|^{p-2}\sum_{j=1}^{n}\eta_{ij}\left|e_j\left(t-\delta_i(t)\right)\right|^2 \\ & \quad\leq (p-2)\left|e_i(t,x)\right|^p\sum_{j=1}^{n}\eta_{ij}\varsigma_j^* \end{aligned}$$

$$+\sum_{j=1}^{n}\frac{2\eta_{ij}\left|e_j\left(t-\delta_i\left(t\right)\right)\right|^p}{\varsigma_j^{*((p-2)/2)}},$$

$$p\left|e_i\left(t,x\right)\right|^{p-2}\sum_{j=1}^{n}\eta_{ij}\left|e_j\left(t-\tau_{ij}\left(t\right)\right)\right|^2$$

$$\leq\left(p-2\right)\left|e_i\left(t,x\right)\right|^p\sum_{j=1}^{n}\eta_{ij}\varsigma_j^{**}$$

$$+\sum_{j=1}^{n}\frac{2\eta_{ij}\left|e_j\left(t-\tau_{ij}\left(t\right)\right)\right|^p}{\varsigma_j^{**((p-2)/2)}},$$

$$p\left|e_i\left(t,x\right)\right|^{p-2}\sum_{j=1}^{n}\eta_{ij}\left|e_j\left(t-\tau_{ij}^{*}\left(t\right)\right)\right|^2$$

$$\leq\left(p-2\right)\left|e_i\left(t,x\right)\right|^p\sum_{j=1}^{n}\eta_{ij}\varsigma_j^{***}$$

$$+\sum_{j=1}^{n}\frac{2\eta_{ij}\left|e_j\left(t-\tau_{ij}^{*}\left(t\right)\right)\right|^p}{\varsigma_j^{***((p-2)/2)}}. \tag{37}$$

Substituting (34)–(37) into (33), it follows from (31) and (H_5) that

$$D^{+}\mathbf{E}\left\{V\left(t\right)\right\}\leq\mathbf{E}\left\{\int_{\Omega}\sum_{i=1}^{n}\left\{\left[\mu+\alpha_i e^{\mu\delta}+e^{\mu\tau}\sum_{j=1}^{n}\beta_{ij}\right.\right.\right.$$

$$+e^{\mu\tau^*}\sum_{j=1}^{n}\gamma_{ij}+\tau^{*}e^{\mu\tau^*}\sum_{j=1}^{n}\gamma_{ij}^{**}$$

$$+\frac{L_i}{\xi_i^{p-1}}-\frac{p}{\varepsilon}l_i-\frac{1}{\varepsilon}p\lambda_1 D_i$$

$$+\frac{1}{\varepsilon}c_i\left(p-1\right)\xi_i^{**}+\frac{1}{\varepsilon}\left(p-1\right)\sum_{j=1}^{n}\left|a_{ij}\right|L_j\varpi_j$$

$$+\frac{1}{\varepsilon}\left(p-1\right)\sum_{j=1}^{n}\left|b_{ij}\right|L_j\varpi_j^{*}$$

$$+\frac{1}{\varepsilon}\left(p-1\right)\sum_{j=1}^{n}\left|d_{ij}\right|L_j\varpi_j^{**}$$

$$+\frac{1}{\varepsilon}\left|H_i\right|\left(p-1\right)\xi_i^{*}+\frac{1}{\varepsilon}\sum_{j=1}^{n}\frac{1}{\varpi_i^{p-1}}\left|a_{ji}\right|L_i$$

$$+\frac{1}{2}\left(p-1\right)\left(p-2\right)\sum_{j=1}^{n}\eta_{ij}\varsigma_j$$

$$+\frac{1}{2}\left(p-1\right)\left(p-2\right)\sum_{j=1}^{n}\eta_{ij}\varsigma_j^{*}$$

$$+\frac{1}{2}\left(p-1\right)\left(p-2\right)\sum_{j=1}^{n}\eta_{ij}\varsigma_j^{**}$$

$$+\frac{1}{2}\left(p-1\right)\left(p-2\right)\sum_{j=1}^{n}\eta_{ij}\varsigma_j^{***}$$

$$\left.+\left(p-1\right)\sum_{j=1}^{n}\frac{\eta_{ji}}{\varsigma_i^{(p-2)/2}}\right]V_i\left(t,x\right)$$

$$
\begin{aligned}
&+\Bigg[\frac{1}{\varepsilon}\frac{1}{\xi_i^{**p-1}}+(p-1)\sum_{j=1}^{n}\frac{\eta_{ji}}{\varsigma_i^{*((p-2)/2)}}\\
&-\kappa\alpha_i\left|1-\rho\right|e^{\mu\delta}\Bigg]V_i\left(t-\delta_i\left(t\right),x\right)\\
&+\Bigg[\left(p-1\right)\sum_{j=1}^{n}\frac{\eta_{ji}}{\varsigma_i^{**((p-2)/2)}}+\frac{1}{\varepsilon}\sum_{j=1}^{n}\frac{\left|b_{ji}\right|L_i}{\varpi_i^{*p-1}}\\
&-e^{\mu\tau}\sum_{j=1}^{n}\kappa\beta_{ij}\left|1-\varrho\right|\Bigg]V_i\left(t-\tau_{ij}\left(t\right),x\right)\\
&+\Bigg[\left(p-1\right)\sum_{j=1}^{n}\frac{\eta_{ji}}{\varsigma_i^{***((p-2)/2)}}\\
&-e^{\mu\tau^*}\sum_{j=1}^{n}\kappa\gamma_{ij}\left|1-\varrho^*\right|\Bigg]V_i\left(t-\tau_{ij}^*\left(t\right),x\right)\\
&+\left[\frac{1}{\varepsilon}\sum_{j=1}^{n}\frac{\left|d_{ji}\right|L_i}{\varpi_i^{**p-1}}-\sum_{j=1}^{n}\gamma_{ij}^{**}\left(\tau^*\right)^{1-p}\right]\\
&\cdot e^{\mu t}\left(\int_{t-\tau_{ij}^*(t)}^{t}\left|e_i\left(s,x\right)\right|\mathrm{d}s\right)^p\\
&+\left(\frac{1}{\varepsilon}\frac{\left|H_i\right|}{\xi_i^{*p-1}}+\mu-pK_i+L_i\left(p-1\right)\xi_i\right)\\
&e^{\mu t}\left|R_i\left(t,x\right)\right|^p\Bigg\}\,\mathrm{d}x\Bigg\}\\
&=\mathbf{E}\left\{\int_\Omega\sum_{i=1}^{n}\left\{\left(\frac{1}{\varepsilon}\frac{\left|H_i\right|}{\xi_i^{*p-1}}+\mu-pK_i+L_i\left(p-1\right)\xi_i\right)e^{\mu t}\left|R_i\left(t,x\right)\right|^p\right\}\mathrm{d}x\right\}\leq 0
\end{aligned}
\tag{38}
$$

which implies that

$$\mathbf{E}\left\{V\left(t\right)\right\}\leq\mathbf{E}\left\{V\left(0\right)\right\}. \tag{39}$$

Note that

$$
\begin{aligned}
\mathbf{E}\left\{V\left(0\right)\right\}\leq\mathbf{E}\Bigg\{\int_\Omega\sum_{i=1}^{n}\Bigg[&\left|e_i\left(0,x\right)\right|^p\\
&+\alpha_i e^{\mu\delta}\int_{-\delta_i(0)}^{0}e^{\mu s}\left|e_i\left(s,x\right)\right|^p\mathrm{d}s
\end{aligned}
$$

$$
\begin{aligned}
&+ \alpha_i^* e^{\mu\delta} \int_{-\delta}^{-\delta_i(0)} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ e^{\mu\tau} \sum_{j=1}^{n} \beta_{ij} \int_{-\tau_{ij}(0)}^{0} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ e^{\mu\tau} \sum_{j=1}^{n} \beta_{ij}^* \int_{-\tau}^{-\tau_{ij}(0)} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ e^{\mu\tau^*} \sum_{j=1}^{n} \gamma_{ij} \int_{-\tau_{ij}^*(0)}^{0} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ e^{\mu\tau^*} \sum_{j=1}^{n} \gamma_{ij}^* \int_{-\tau^*}^{-\tau_{ij}^*(0)} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ e^{\mu\tau^*} \sum_{j=1}^{n} \tau^* \gamma_{ij}^{**} \int_{-\tau^*}^{0} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ \left|R_i(0,x)\right|^p + \frac{p}{2\varepsilon\nu_i} \left(\epsilon_i(0,x) + l_i\right)^2 \Big] \mathrm{d}x \Big\} \\
&\leq \mathbf{E} \Big\{ \int_{\Omega} \sum_{i=1}^{n} \Big[\left|e_i(0,x)\right|^p \\
&+ \alpha_i e^{\mu\delta} \int_{-\delta_i(0)}^{0} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ \alpha_i^* e^{\mu\delta} \int_{-\delta}^{-\delta_i(0)} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ e^{\mu\tau} \max_{i=1,2,\ldots,n} \left\{ \sum_{j=1}^{n} \beta_{ij} \right\} \sum_{j=1}^{n} \int_{-\tau_{ij}(0)}^{0} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ e^{\mu\tau} \max_{i=1,2,\ldots,n} \left\{ \sum_{j=1}^{n} \beta_{ij}^* \right\} \sum_{j=1}^{n} \int_{-\tau}^{-\tau_{ij}(0)} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ e^{\mu\tau^*} \max_{i=1,2,\ldots,n} \left\{ \sum_{j=1}^{n} \gamma_{ij} \right\} \sum_{j=1}^{n} \int_{-\tau_{ij}^*(0)}^{0} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ e^{\mu\tau^*} \max_{i=1,2,\ldots,n} \left\{ \sum_{j=1}^{n} \gamma_{ij}^* \right\} \sum_{j=1}^{n} \int_{-\tau^*}^{-\tau_{ij}^*(0)} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ \tau^* e^{\mu\tau^*} \max_{i=1,2,\ldots,n} \left\{ \sum_{j=1}^{n} \gamma_{ij}^{**} \right\} \sum_{j=1}^{n} \int_{-\tau^*}^{0} e^{\mu s} \left|e_i(s,x)\right|^p \mathrm{d}s \\
&+ \left|R_i(0,x)\right|^p + \frac{p}{2\varepsilon\nu_i} \left(\epsilon_i(0,x) + l_i\right)^2 \Big] dx \Big\}.
\end{aligned}
\tag{40}
$$

Since

$$\left\|\psi^{z}-\phi^{y}\right\|_{p} = \left(\int_{\Omega}\sum_{i=1}^{n}\sup_{-\overline{\tau}\leq\theta\leq 0}\left|\psi^{z}(\theta,x)-\phi_{i}^{y}(\theta,x)\right|^{p}\mathrm{d}x\right)^{1/p}, \tag{41}$$

then

$$\begin{aligned}\int_{\Omega}\sum_{i=1}^{n}\int_{-\tau_{ij}(0)}^{0}&e^{\mu s}\left|e_{i}(s,x)\right|^{p}\mathrm{d}s\,\mathrm{d}x\\ &\leq\int_{\Omega}\sum_{i=1}^{n}\int_{-\tau}^{0}\left|e_{i}(s,x)\right|^{p}\mathrm{d}s\,\mathrm{d}x\\ &\leq\int_{\Omega}\sum_{i=1}^{n}\tau\sup_{-\overline{\tau}\leq s\leq 0}\left|e_{i}(s,x)\right|^{p}\mathrm{d}x=\tau\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p}.\end{aligned} \tag{42}$$

Similarly,

$$\begin{aligned}&\int_{\Omega}\sum_{i=1}^{n}\left|e_{i}(0,x)\right|^{p}\mathrm{d}x\leq\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p},\\ &\int_{\Omega}\sum_{i=1}^{n}\left|R_{i}(0,x)\right|^{p}\mathrm{d}x\leq\left\|\psi^{h}-\phi^{s}\right\|_{p}^{p},\\ &\int_{\Omega}\sum_{i=1}^{n}\int_{-\tau^{*}}^{0}e^{\mu s}\left|e_{i}(s,x)\right|^{p}\mathrm{d}s\,\mathrm{d}x\leq\tau^{*}\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p},\\ &\int_{\Omega}\sum_{i=1}^{n}\int_{-\delta}^{-\delta_{i}(0)}e^{\mu s}\left|e_{i}(s,x)\right|^{p}\mathrm{d}s\,\mathrm{d}x\leq\delta\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p},\\ &\int_{\Omega}\sum_{i=1}^{n}\int_{-\delta_{i}(0)}^{0}e^{\mu s}\left|e_{i}(s,x)\right|^{p}\mathrm{d}s\,\mathrm{d}x\leq\delta\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p},\\ &\int_{\Omega}\sum_{i=1}^{n}\int_{-\delta}^{-\delta_{i}(0)}e^{\mu s}\left|e_{i}(s,x)\right|^{p}\mathrm{d}s\,\mathrm{d}x\leq\delta\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p},\\ &\int_{\Omega}\sum_{i=1}^{n}\int_{-\tau}^{-\tau_{ij}(0)}e^{\mu s}\left|e_{i}(s,x)\right|^{p}\mathrm{d}s\,\mathrm{d}x\leq\tau\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p},\\ &\int_{\Omega}\sum_{i=1}^{n}\int_{-\tau_{ij}^{*}(0)}^{0}e^{\mu s}\left|e_{i}(s,x)\right|^{p}\mathrm{d}s\,\mathrm{d}x\leq\tau^{*}\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p},\\ &\int_{\Omega}\sum_{i=1}^{n}\int_{-\tau^{*}}^{-\tau_{ij}^{*}(0)}e^{\mu s}\left|e_{i}(s,x)\right|^{p}\mathrm{d}s\,\mathrm{d}x\leq\tau^{*}\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p}.\end{aligned} \tag{43}$$

Applying (42)-(43) into (40), we have

$$\begin{aligned}\mathbf{E}\{V(0)\}\leq\Bigg[&1+\alpha_{i}\delta e^{\mu\delta}+\alpha_{i}^{*}\delta e^{\mu\delta}\\ &+n\tau e^{\mu\tau}\max_{i=1,2,\dots,n}\left\{\sum_{j=1}^{n}\beta_{ij}\right\}+n\tau e^{\mu\tau}\max_{i=1,2,\dots,n}\left\{\sum_{j=1}^{n}\beta_{ij}^{*}\right\}\\ &+n\tau^{*}e^{\mu\tau^{*}}\max_{i=1,2,\dots,n}\left\{\sum_{j=1}^{n}\gamma_{ij}\right\}\\ &+n\tau^{*}e^{\mu\tau^{*}}\max_{i=1,2,\dots,n}\left\{\sum_{j=1}^{n}\gamma_{ij}^{*}\right\}\\ &+n{\tau^{*}}^{2}e^{\mu\tau^{*}}\max_{i=1,2,\dots,n}\left\{\sum_{j=1}^{n}\gamma_{ij}^{**}\right\}+\frac{N}{\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p}}\Bigg]\\ &\cdot\mathbf{E}\left\{\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p}\right\}+\mathbf{E}\left\{\left\|\psi^{h}-\phi^{s}\right\|_{p}^{p}\right\}\\ &=M\mathbf{E}\left\{\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p}\right\}+M^{*}\mathbf{E}\left\{\left\|\psi^{h}-\phi^{s}\right\|_{p}^{p}\right\},\end{aligned} \tag{44}$$

where

$$\begin{aligned}M^{*}&=1,\\ N&=\int_{\Omega}\sum_{i=1}^{n}\frac{p}{2\varepsilon\nu_{i}}\left(\epsilon_{i}(0,x)+l_{i}\right)^{2}\mathrm{d}x,\\ M&=1+\alpha_{i}\delta e^{\mu\delta}+\alpha_{i}^{*}\delta e^{\mu\delta}+n\tau e^{\mu\tau}\max_{i=1,2,\dots,n}\left\{\sum_{j=1}^{n}\beta_{ij}\right\}\\ &\quad+n\tau e^{\mu\tau}\max_{i=1,2,\dots,n}\left\{\sum_{j=1}^{n}\beta_{ij}^{*}\right\}\\ &\quad+n\tau^{*}e^{\mu\tau^{*}}\max_{i=1,2,\dots,n}\left\{\sum_{j=1}^{n}\gamma_{ij}\right\}\\ &\quad+n\tau^{*}e^{\mu\tau^{*}}\max_{i=1,2,\dots,n}\left\{\sum_{j=1}^{n}\gamma_{ij}^{*}\right\}\\ &\quad+n{\tau^{*}}^{2}e^{\mu\tau^{*}}\max_{i=1,2,\dots,n}\left\{\sum_{j=1}^{n}\gamma_{ij}^{**}\right\}+\frac{N}{\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p}}\\ &\geq 1.\end{aligned} \tag{45}$$

Therefore,

$$\begin{aligned}\mathbf{E}\{V(t)\}&\leq\mathbf{E}\{V(0)\}\\ &\leq M\mathbf{E}\left\{\left\|\psi^{z}-\phi^{y}\right\|_{p}^{p}\right\}+M^{*}\mathbf{E}\left\{\left\|\psi^{h}-\phi^{s}\right\|_{p}^{p}\right\}.\end{aligned} \tag{46}$$

Further, we obtain

$$\begin{aligned}\mathbf{E}\{V(t)\} &\geq \mathbf{E}\left\{\int_{\Omega}\sum_{i=1}^{n}\left[V_i(t,x)+e^{\mu t}\left|R_i(t,x)\right|^p\right]\mathrm{d}x\right\}\\ &= e^{\mu t}\mathbf{E}\left\{\int_{\Omega}\sum_{i=1}^{n}\left[\left|z_i(t,x)-y_i(t,x)\right|^p+\left|h_i(t,x)-s_i(t,x)\right|^p\right]\mathrm{d}x\right\}\\ &= e^{\mu t}\mathbf{E}\left\{\|z(t,x)-y(t,x)\|^p\right\}+e^{\mu t}\mathbf{E}\left\{\|h(t,x)-s(t,x)\|^p\right\}.\end{aligned} \tag{47}$$

From (46) and (47), we have

$$\begin{aligned}&\mathbf{E}\left\{\|z(t,x)-y(t,x)\|^p\right\}+\mathbf{E}\left\{\|h(t,x)-s(t,x)\|^p\right\}\\ &\quad\leq M\mathbf{E}\left\{\left\|\psi^z-\phi^y\right\|_p^p\right\}e^{-\mu t}\\ &\qquad+M^*\mathbf{E}\left\{\left\|\psi^h-\phi^s\right\|_p^p\right\}e^{-\mu t}.\end{aligned} \tag{48}$$

Hence, the nonlinear couple neural networks (9) and (5) can be exponentially synchronized under the adaptive feedback controller (11) and (12) based on p-norm. The proof of Theorem 8 is complete. □

Remark 9. It is the first time to consider the combined effects of time-varying leakage delays, the discrete time-varying delay, the distributed time-varying delay, stochastic perturbation, and spatial diffusion on the exponential synchronization of competitive neural networks under an adaptive feedback controller. The neural networks discussed in [6, 7, 31] are the special cases of the model in this paper. From this point, our results are more general.

Remark 10. In Theorem 8, the sufficient conditions are derived to achieve the adaptive synchronization for the proposed competitive neural networks. Compared with the adaptive synchronization criteria given in [7], the conditions obtained in Theorem 8 depend on not only the timescale ε but also the controller parameter μ. It is beneficial to design an adaptive controller to realize the adaptive synchronization for the neural networks. Therefore, the criteria derived in this paper have wider application.

4. Numerical Simulations

In this section, some numerical simulation examples demonstrate the main results in Theorem 8.

In system (5), we choose $n=2$. Then system (5) takes the form

$$\begin{aligned}\text{STM: }\varepsilon\frac{\partial y_i(t,x)}{\partial t} &= D_i\Delta y_i(t,x)-c_iy_i(t-\delta(t),x)\\ &\quad+\sum_{j=1}^{n}a_{ij}f_j\left(y_j(t,x)\right)\\ &\quad+\sum_{j=1}^{n}b_{ij}f_j\left(y_j(t-\tau(t),x)\right)\\ &\quad+\sum_{j=1}^{n}d_{ij}\int_{t-\tau^*(t)}^{t}f_j\left(y_j(s,x)\right)\mathrm{d}s\\ &\quad+H_is_i(t,x),\\ \text{LTM: }\frac{\partial s_i(t,x)}{\partial t} &= -K_is_i(t,x)+f_i\left(y_i(t,x)\right),\end{aligned} \tag{49}$$

where $f_j(u_j(t,x)) = \tanh(u_j(t,x))$, $\delta(t) = 0.4+0.3\cos t$, $\tau(t) = 0.5+0.4\cos t$, and $\tau^*(t) = 0.3+0.2\sin t$. The parameters of (49) are assumed as follows: $\varepsilon = 0.5$, $D_1 = 0.025$, $D_2 = 0.025$, $c_1 = 0.35$, $c_2 = 0.25$, $a_{11} = 0.625$, $a_{12} = -0.3$, $a_{21} = -0.225$, $a_{22} = -0.3$, $b_{11} = -0.475$, $b_{12} = -0.425$, $b_{21} = 0.375$, $b_{22} = 0.475$, $d_{11} = -0.375$, $d_{12} = -0.325$, $d_{21} = 0.6$, $d_{22} = 0.8$, $H_1 = 0.375$, $H_2 = 0.375$, $K_1 = 1$, $K_2 = 1$, and $x \in \Omega = [-5,5]$. The initial conditions of system (49) are chosen as

$$\begin{aligned}y_1(s,x) &= 0.1\cos\left(\frac{x+5}{10}\pi\right),\\ y_2(s,x) &= 0.2\cos\left(\frac{x+5}{10}\pi\right),\\ s_1(s,x) &= 0.3\cos\left(\frac{x+5}{10}\pi\right),\\ s_2(s,x) &= 0.4\cos\left(\frac{x+5}{10}\pi\right),\end{aligned} \tag{50}$$

where $(s,x) \in [-0.9,0]\times\Omega$.

Numerical simulation illustrates that the reaction-diffusion neural network (49) with boundary condition (6) and the initial condition (50) exhibits a chaotic behavior (see Figure 1).

The noise-perturbed response system is described by

$$\begin{aligned}\text{STM: }\mathrm{d}z_i(t,x) &= \frac{1}{\varepsilon}\Bigg[D_i\Delta z_i(t,x)-c_iz_i(t-\delta(t),x)\\ &\quad+\sum_{j=1}^{n}a_{ij}f_j\left(z_j(t,x)\right)+\sum_{j=1}^{n}b_{ij}f_j\left(z_j(t-\tau(t),x)\right)\\ &\quad+\sum_{j=1}^{n}d_{ij}\int_{t-\tau^*(t)}^{t}f_j\left(z_j(s,x)\right)\mathrm{d}s+H_ih_i(t,x)+u_i(t,\\ &\quad x)\Bigg]\mathrm{d}t+\sum_{j=1}^{n}\sigma_{ij}\left(e_j(t,x),e_j(t-\delta_i(t),x),\right.\\ &\quad\left.e_j\left(t-\tau_{ij}(t),x\right),e_j\left(t-\tau_{ij}^*(t),x\right)\right)\mathrm{d}\omega_j(t),\\ \text{LTM: }\frac{\partial h_i(t,x)}{\partial t} &= -K_ih_i(t,x)+f_i\left(z_i(t,x)\right),\end{aligned} \tag{51}$$

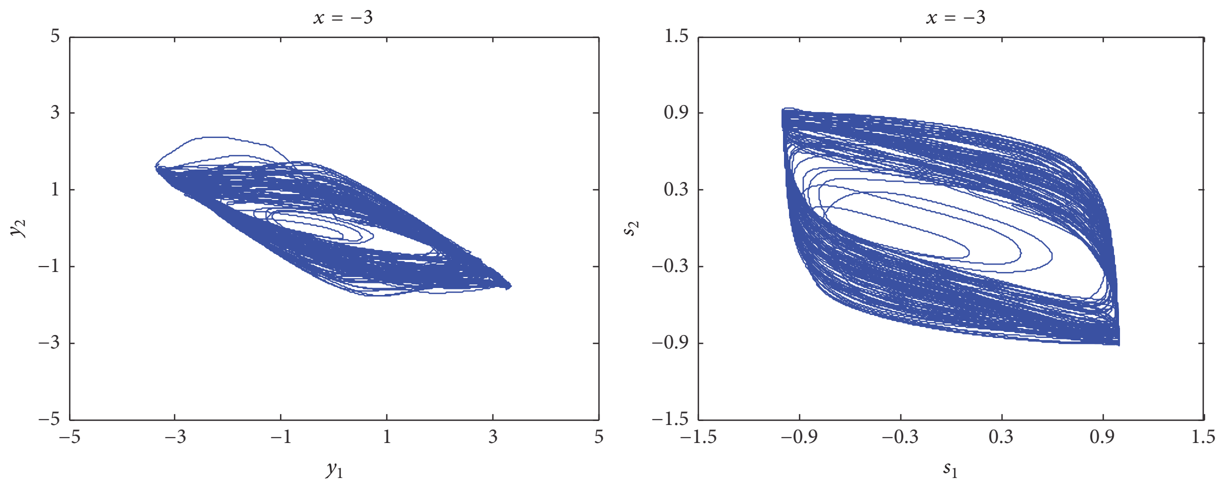

Figure 1: Chaotic behaviors of competitive neural networks (49).

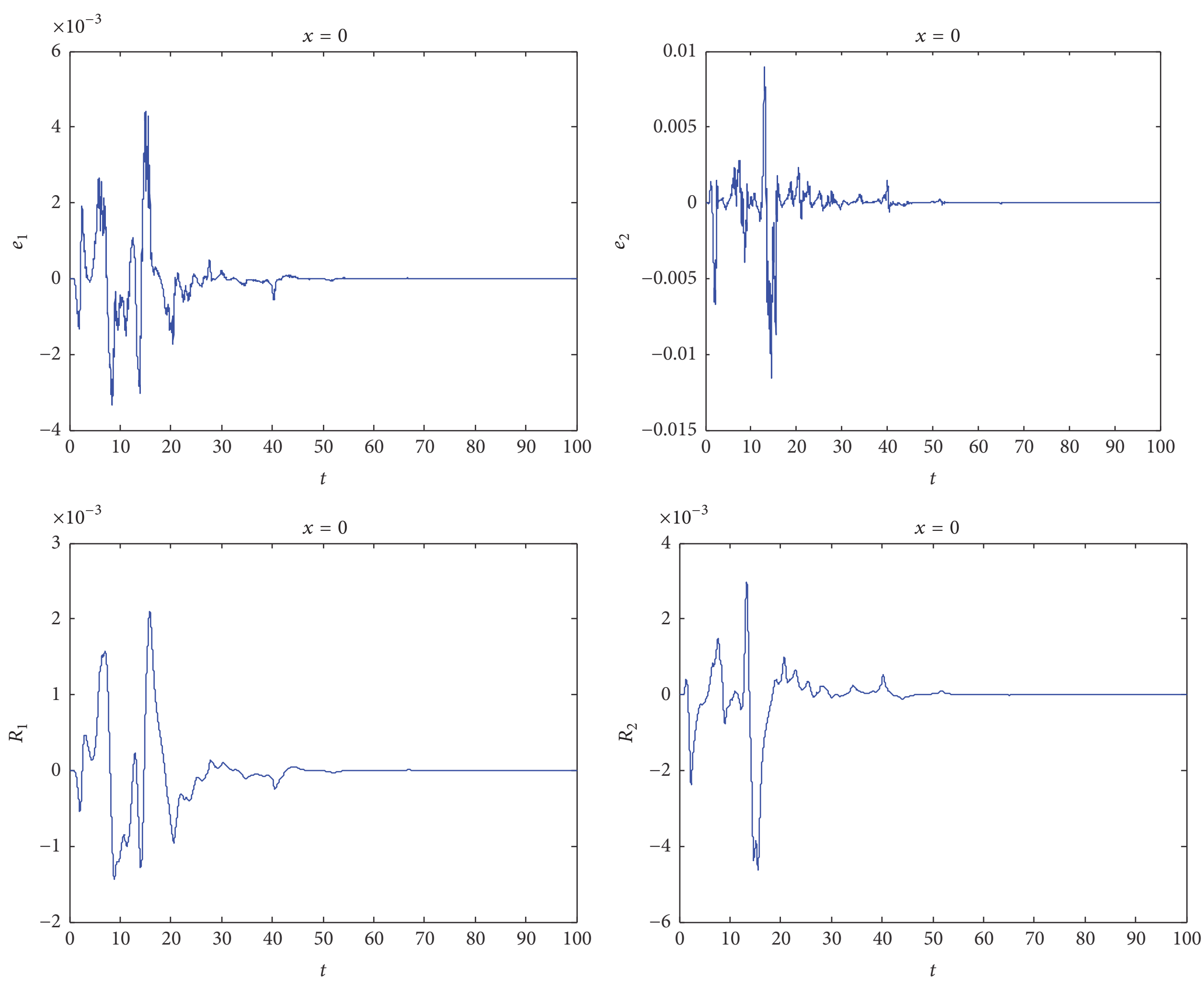

Figure 2: Asymptotical behaviors of the synchronization errors between systems (49) and (51).

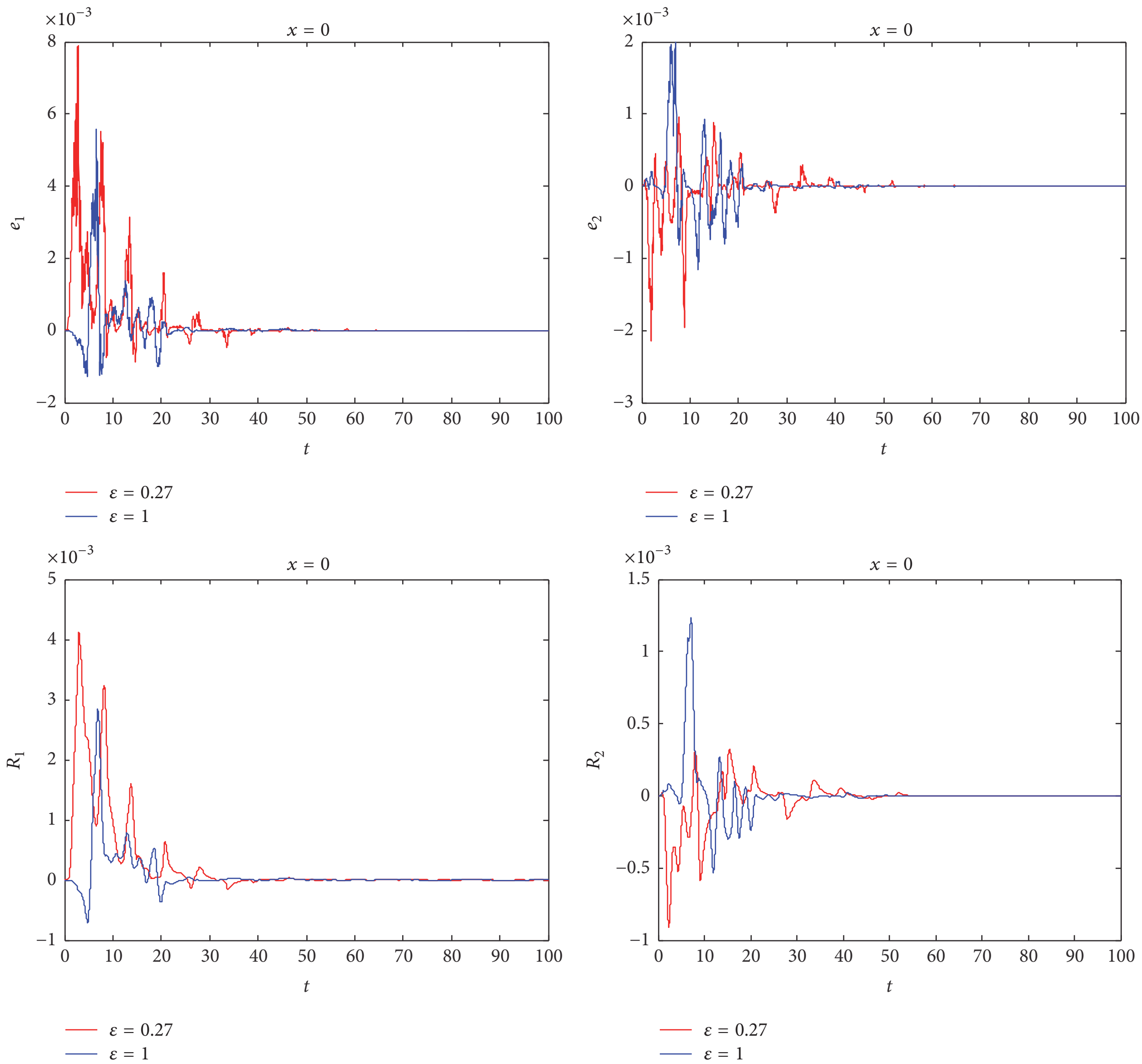

FIGURE 3: Asymptotical behaviors of the synchronization errors with differential timescale ε.

where

$$\begin{aligned}
\sigma_{11} &= 0.1e_1(t,x) + 0.2e_1(t-\tau(t),x) \\
&\quad + 0.1e_1(t-\delta(t),x) + 0.3e_1(t-\tau^*(t),x), \\
\sigma_{12} &= 0, \\
\sigma_{21} &= 0, \\
\sigma_{22} &= 0.2e_2(t,x) + 0.1e_2(t-\tau(t),x) \\
&\quad + 0.3e_2(t-\delta(t),x) + 0.1e_2(t-\tau^*(t),x).
\end{aligned} \tag{52}$$

The adaptive controller is

$$u_i(t,x) = \epsilon_i e_i(t,x),$$

$$\frac{\partial \epsilon_i}{\partial t} = -\left|e_i(t,x)\right|^2 e_i^{0.1t}. \tag{53}$$

The initial conditions for the response system (51) are chosen as

$$\begin{aligned}
z_1(s,x) &= 0.1\cos\left(\frac{x+5}{10}\pi\right), \\
z_2(s,x) &= 0.2\cos\left(\frac{x+5}{10}\pi\right), \\
h_1(s,x) &= 0.3\cos\left(\frac{x+5}{10}\pi\right), \\
h_2(s,x) &= 0.4\cos\left(\frac{x+5}{10}\pi\right),
\end{aligned} \tag{54}$$

where $(s,x) \in [-0.9,0] \times \Omega$.

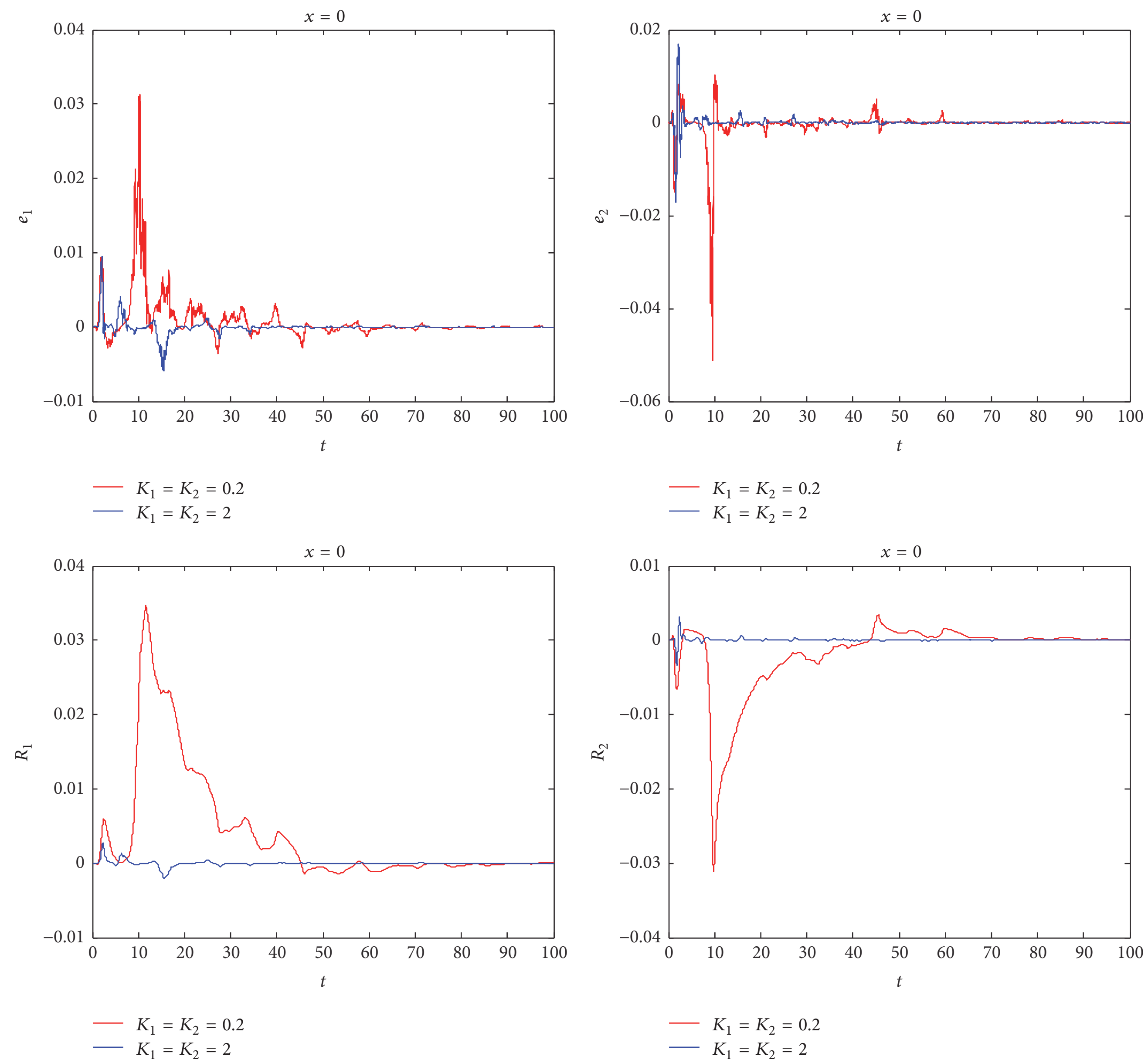

FIGURE 4: Asymptotical behaviors of the synchronization errors with differential disposable scaling constants K_i.

Evidently, $L_1 = L_2 = 1$, $\rho' = 0.3 < 1$, $\varrho' = 0.4 < 1$, and $\varrho^{*\prime} = 0.2 < 1$. Let $p = 2$, $\varepsilon = 0.5$, $\xi_1 = \xi_2 = 0.1$, and $\xi_1^* = \xi_2^* = 1$. By simple computation, it is easy to verify that assumptions (H_1)–(H_6) are satisfied. According to Theorem 8, the drive system (49) and the response system (51) are exponentially synchronized based on p-norm. Numerical simulation illustrates our results (see Figure 2).

Remark 11. The conclusions given in Theorem 8 show that the adaptive synchronization criteria for competitive neural networks are dependent on the timescale ε, the disposable scaling constant K_i, and the external stimulus H_i. When ε increases, and K_i increases or H_i decreases, respectively, assumption (H_6) can be satisfied more easily, and the adaptive synchronization of the competitive neural networks is more easily realized. Dynamical behaviors of the synchronization errors between systems (49) and (51) with the differential timescale, disposable scaling constant, and external stimulus, respectively, are shown in Figures 3–5.

Remark 12. By (48), it is clear to see that the controller parameter μ denotes the rate of the synchronization. That is, the larger the controller parameter μ is, the faster systems (49) and (51) realize synchronization. Hence, our results are consistent with the practical situation. Dynamical behaviors of the synchronization errors between systems (49) and (51) with differential controller parameter μ are shown in Figure 6.

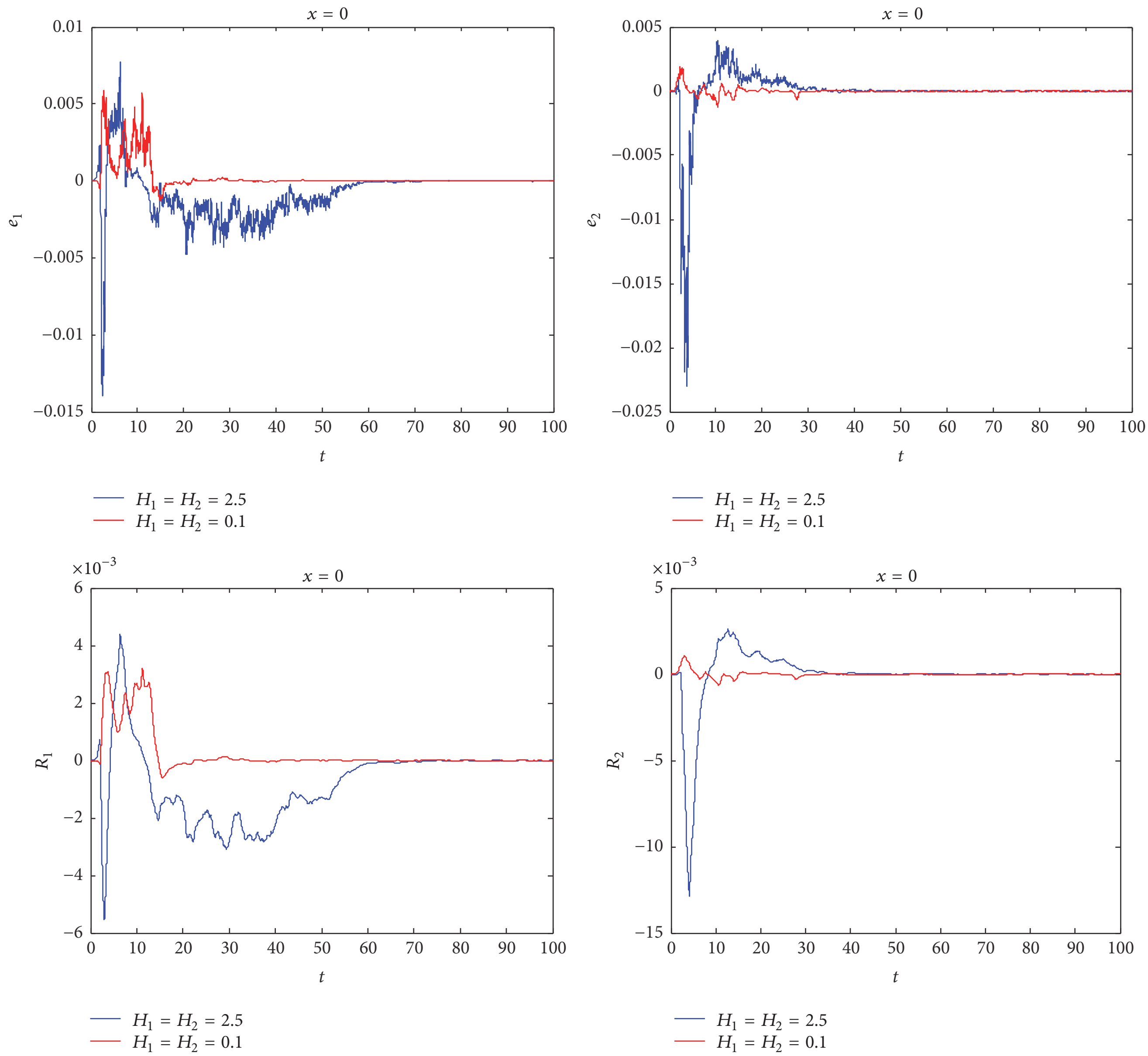

FIGURE 5: Asymptotical behaviors of the synchronization errors with differential external stimulus constants H_i.

The parameter ν_i is another controller parameter in the feedback controller (11). By numerical simulations, we can see that it is beneficial for competitive neural networks to realize the synchronization by increasing controller parameter ν_i. Dynamical behaviors of the synchronization errors between systems (49) and (51) with differential controller parameter ν_i are shown in Figure 7. However, we cannot prove it. It is an interesting open problem to research.

Remark 13. In many cases, two-neuron networks show the same behavior as large-size networks and many research methods used in two-neuron networks can be applied to large-size networks. Therefore, a two-neuron network can be used as an example to improve our understanding of our theoretical results. In addition, the parameter values are selected randomly to ensure that neural networks (49) exhibit a chaotic behavior.

5. Conclusion

In this paper, an adaptive feedback controller was designed to achieve the exponential synchronization for stochastic competitive neural networks with spatial diffusion, time-varying leakage delays, and discrete and distributed time-varying delays based on p-norm. Evidently, the model discussed in this paper is more general than those correspondent models when the delays are constant delays. By constructing the Lyapunov functional and using and the stochastic analysis

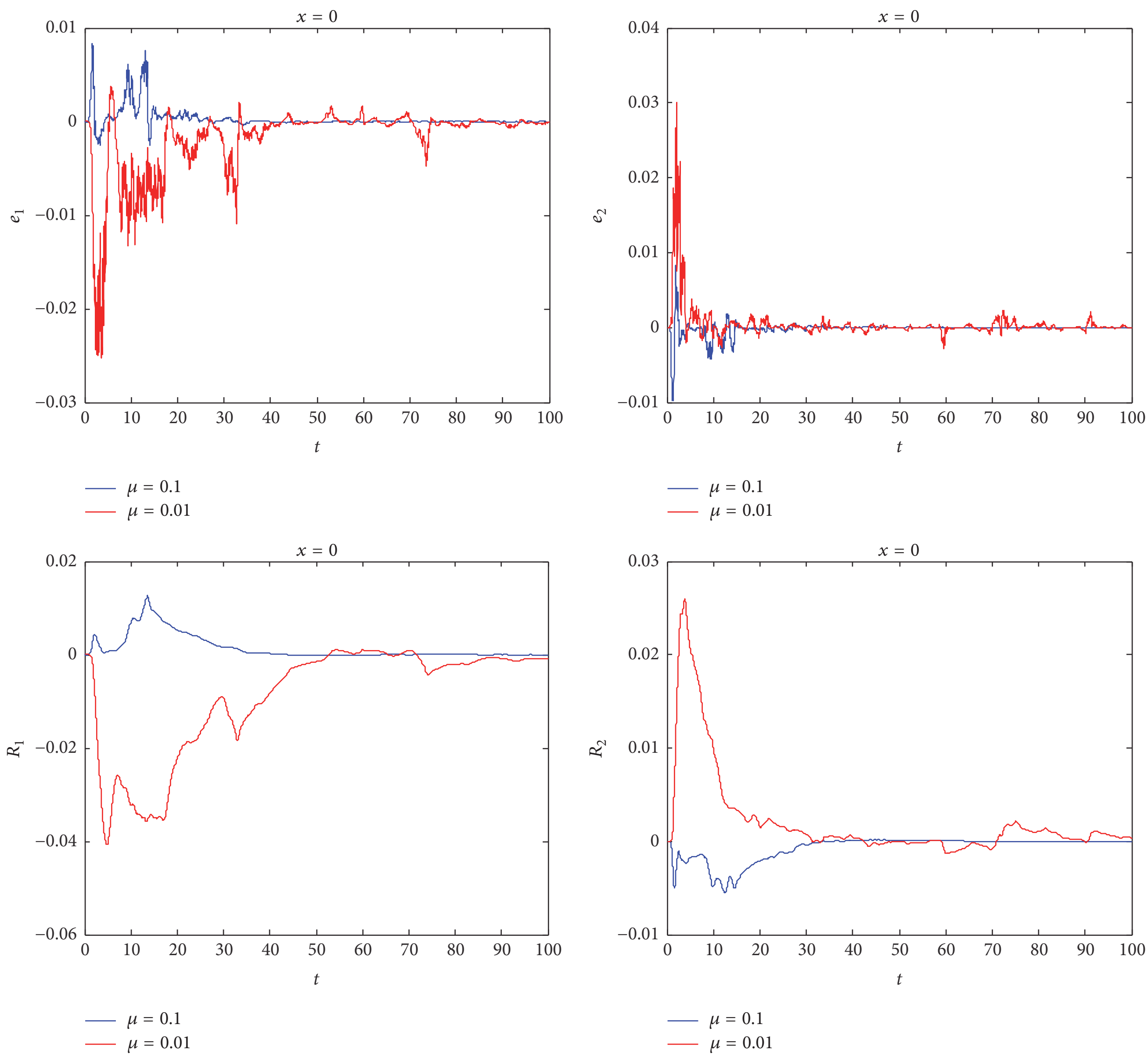

Figure 6: Asymptotical behaviors of the synchronization errors with differential controller parameter μ.

theory, the novel exponential synchronization criteria dependent on the timescale ε, external stimulus constants H_i, disposable scaling constants K_i, and controller parameter μ were obtained. By theory analysis, it was shown that competitive neural networks can achieve exponential synchronization more easily by increasing the timescale and disposable scaling constants or reducing disposable scaling constants, respectively. Numerical examples and their simulations are given to show the effectiveness of the obtained results.

Figures 8 and 9 show that it is beneficial for competitive neural networks with reaction-diffusion terms to realize the synchronization by increasing diffusion coefficients D_i or decreasing diffusion space x, respectively. However, the exponential synchronization criteria obtained in this paper are independent of the diffusion coefficients and the diffusion space. They cannot reflect the influence of the diffusion coefficients and diffusion space on synchronization, which limits the application scopes of the results. Therefore, we will investigate that in our future work.

Appendix

Proof of Lemma 6. According to the eigenvalue theory of elliptic operators, the Laplacian $-\Delta$ on Ω with the Neumann boundary conditions is a self-adjoint operator with compact inverse, so there exists a sequence of nonnegative eigenvalues $0 = \lambda_0 < \lambda_1 < \lambda_2 < \cdots$, $(\lim_{i\to\infty}\lambda_i = +\infty)$ as well as a sequence of corresponding eigenfunctions $\vartheta_0(x)$,

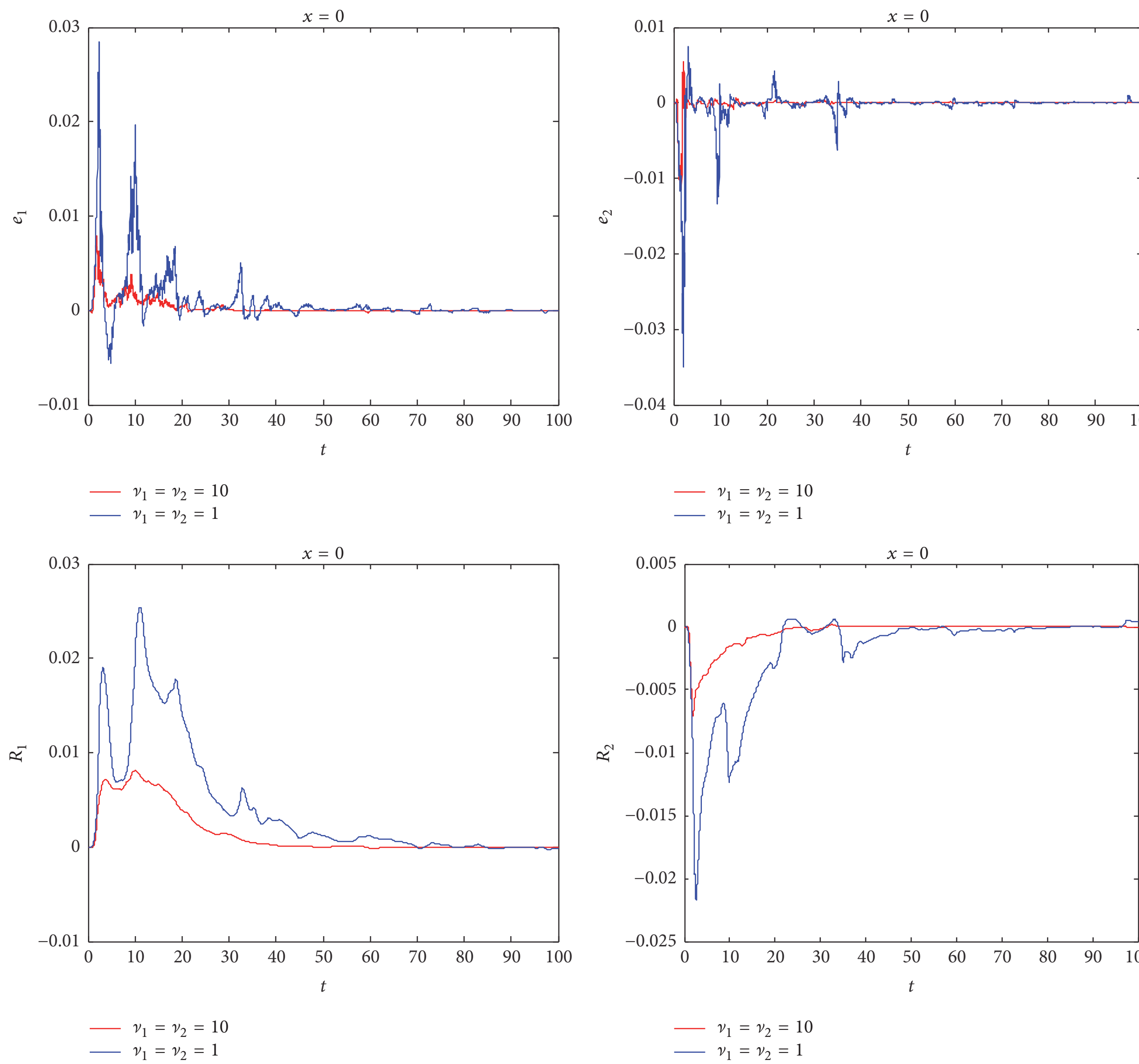

FIGURE 7: Asymptotical behaviors of the synchronization errors with differential controller parameter ν_i.

$\vartheta_1(x), \vartheta_2(x), \ldots$ for the Neumann boundary problem (29); that is,

$$\begin{gathered}\lambda_0 = 0, \quad \vartheta_0(x) = 1, \\ -\Delta\vartheta_m(x) = \lambda_m \vartheta_m(x), \quad \text{in } \Omega, \\ \frac{\partial \vartheta_m(x)}{\partial \mathbf{n}} = 0, \quad \text{on } \partial\Omega.\end{gathered} \tag{A.1}$$

Multiply the second equation of (A.1) by $\vartheta_m^{p-1}(x)$ ($m = 1, 2, \ldots$) and integrate Ω. By Green formula (27), we obtain

$$\begin{aligned}\lambda_m \int_\Omega \vartheta_m^p(x)\,dx &= -\int_\Omega \vartheta_m^{p-1}(x)\,\Delta\vartheta_m(x)\,dx \\ &= -\int_{\partial\Omega} \vartheta_m^{p-1}(x)\,\frac{\partial\vartheta_m(x)}{\partial \mathbf{n}}\,ds \\ &\quad + \int_\Omega \left(\nabla\vartheta_m^{p-1}(x)\right)^T \nabla\vartheta_m(x)\,dx = \int_\Omega (p-1) \\ &\quad \cdot \vartheta_m^{p-2}(x)\left[\left(\frac{\partial\vartheta_m(x)}{\partial x_1}\right)^2 + \left(\frac{\partial\vartheta_m(x)}{\partial x_2}\right)^2 + \cdots \right. \\ &\quad \left. + \left(\frac{\partial\vartheta_m(x)}{\partial x_{l^*}}\right)^2\right] dx = (p-1)\int_\Omega \vartheta_m^{p-2}(x) \\ &\quad \cdot \left|\nabla\vartheta_m(x)\right|^2 dx.\end{aligned} \tag{A.2}$$

It is easy to show that (A.2) is also true for $m = 0$.

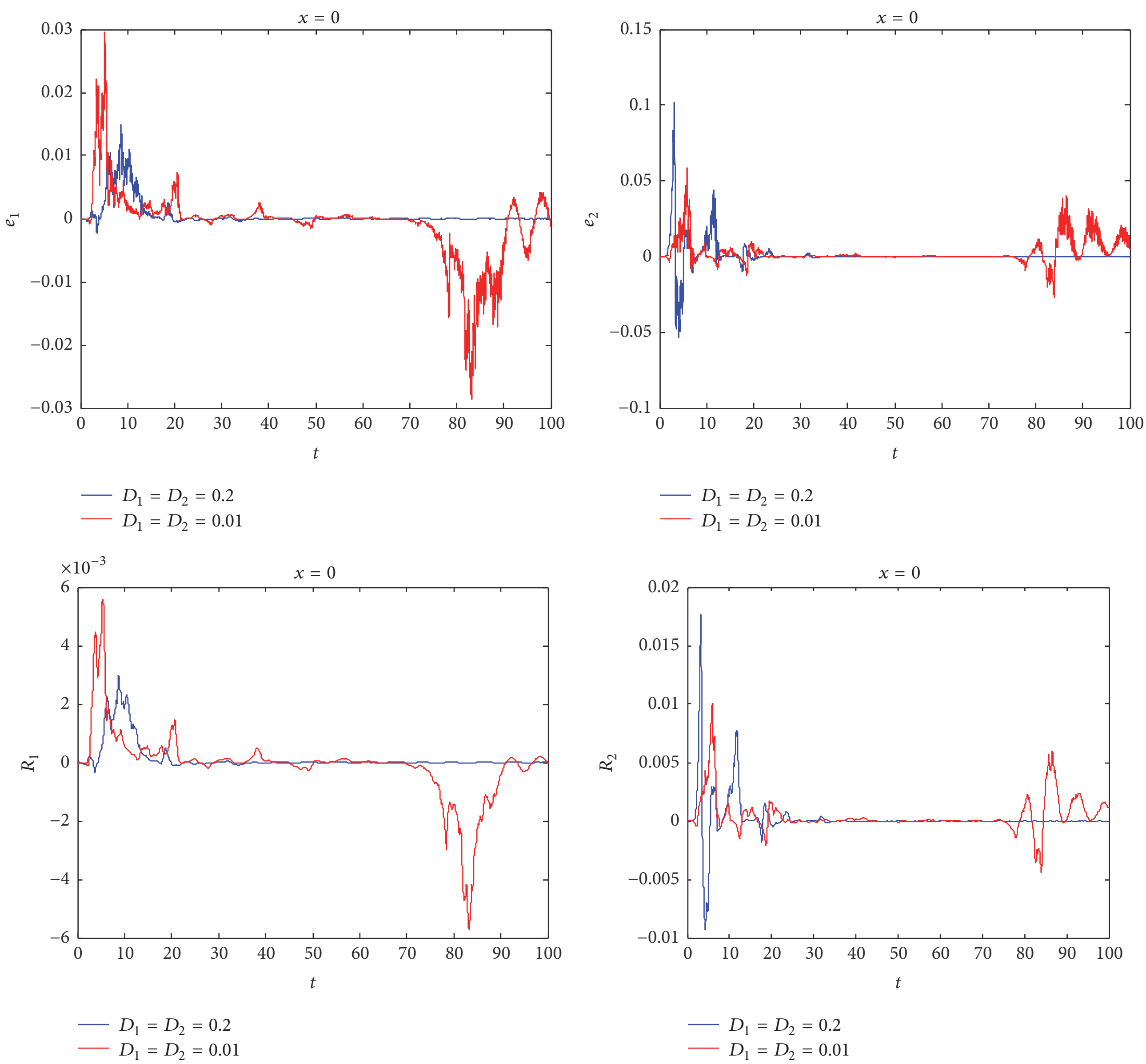

FIGURE 8: Asymptotical behaviors of the synchronization errors with differential diffusion coefficients D_i.

The sequence of eigenfunctions $\{\vartheta_i(x)\}_{i\geq 0}$ generates an orthonormal basis of $L^2(\Omega)$. Hence, for any $\varphi(x) \in L^2(\Omega)$, there exists a sequence of constant $\{c_m\}_{m\geq 0}$ such that

$$\varphi(x) = \sum_{m=0}^{\infty} c_m \vartheta_m(x). \tag{A.3}$$

It follows from (A.2) and (A.3) that

$$\begin{aligned}\int_\Omega |\varphi(x)|^p \, dx &\leq \int_\Omega \sum_{m=0}^{\infty} |c_m \vartheta_m(x)|^p \, dx \\ &\leq \frac{p-1}{\lambda_1} \int_\Omega \sum_{m=0}^{\infty} |c_m \vartheta_m(x)|^{p-2} |c_m \nabla \vartheta_m(x)|^2 \, dx \\ &\leq \frac{p-1}{\lambda_1} \int_\Omega \sum_{m=0}^{\infty} |c_m \vartheta_m(x)|^{p-2} \sum_{m=0}^{\infty} |c_m \nabla \vartheta_m(x)|^2 \, dx \\ &= \frac{p-1}{\lambda_1} \int_\Omega |\varphi(x)|^{p-2} |\nabla \varphi(x)|^2 \, dx.\end{aligned} \tag{A.4}$$

The proof of Lemma 6 is complete. □

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

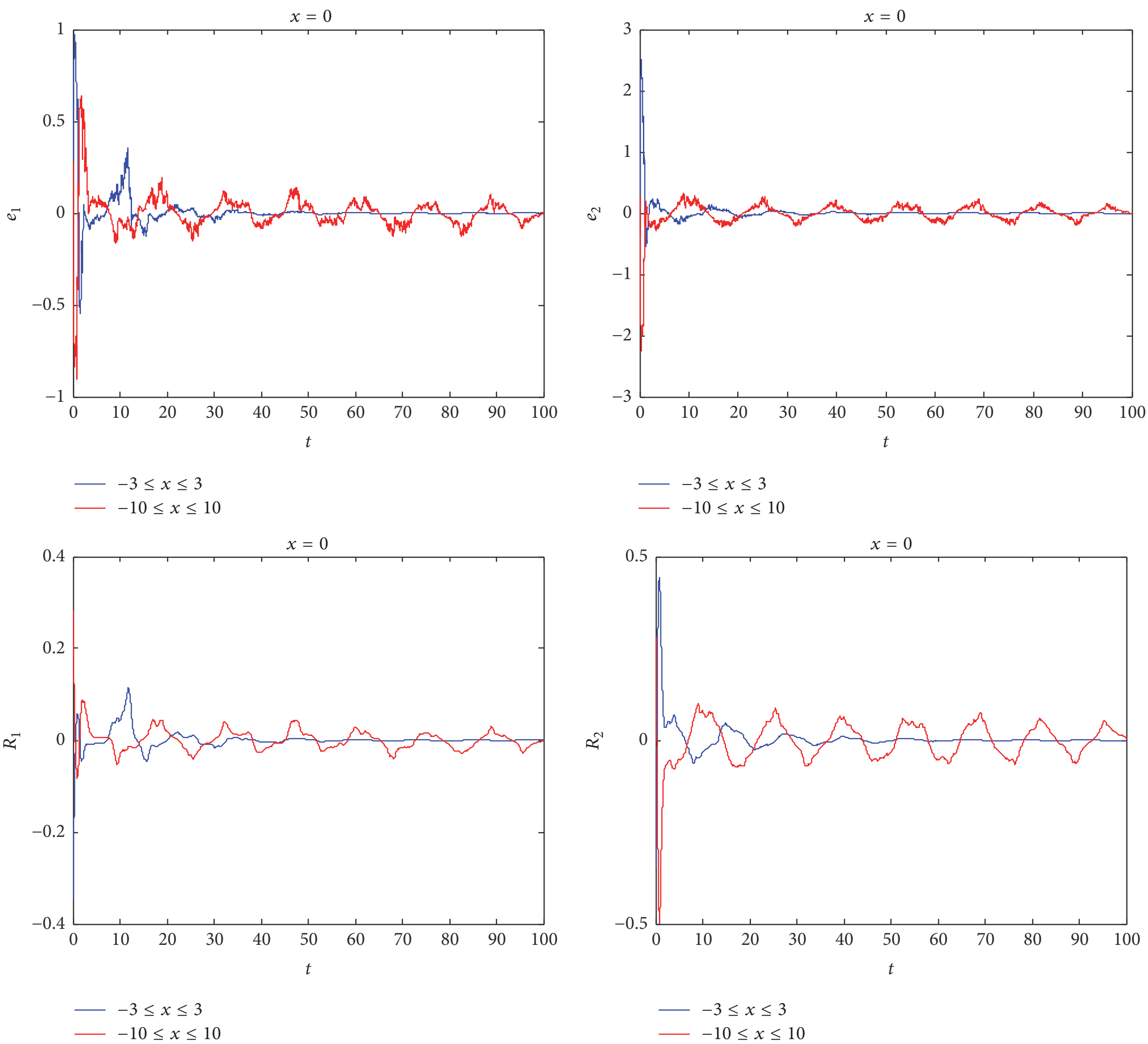

FIGURE 9: Asymptotical behaviors of the synchronization errors with differential diffusion space.

Acknowledgments

This work was supported by the Science and Technology Support Program of Hebei Academy of Sciences (17606), the Science and Technology Support Program of Hebei Province (16290106D), and the National Natural Science Foundation of China (61305076).

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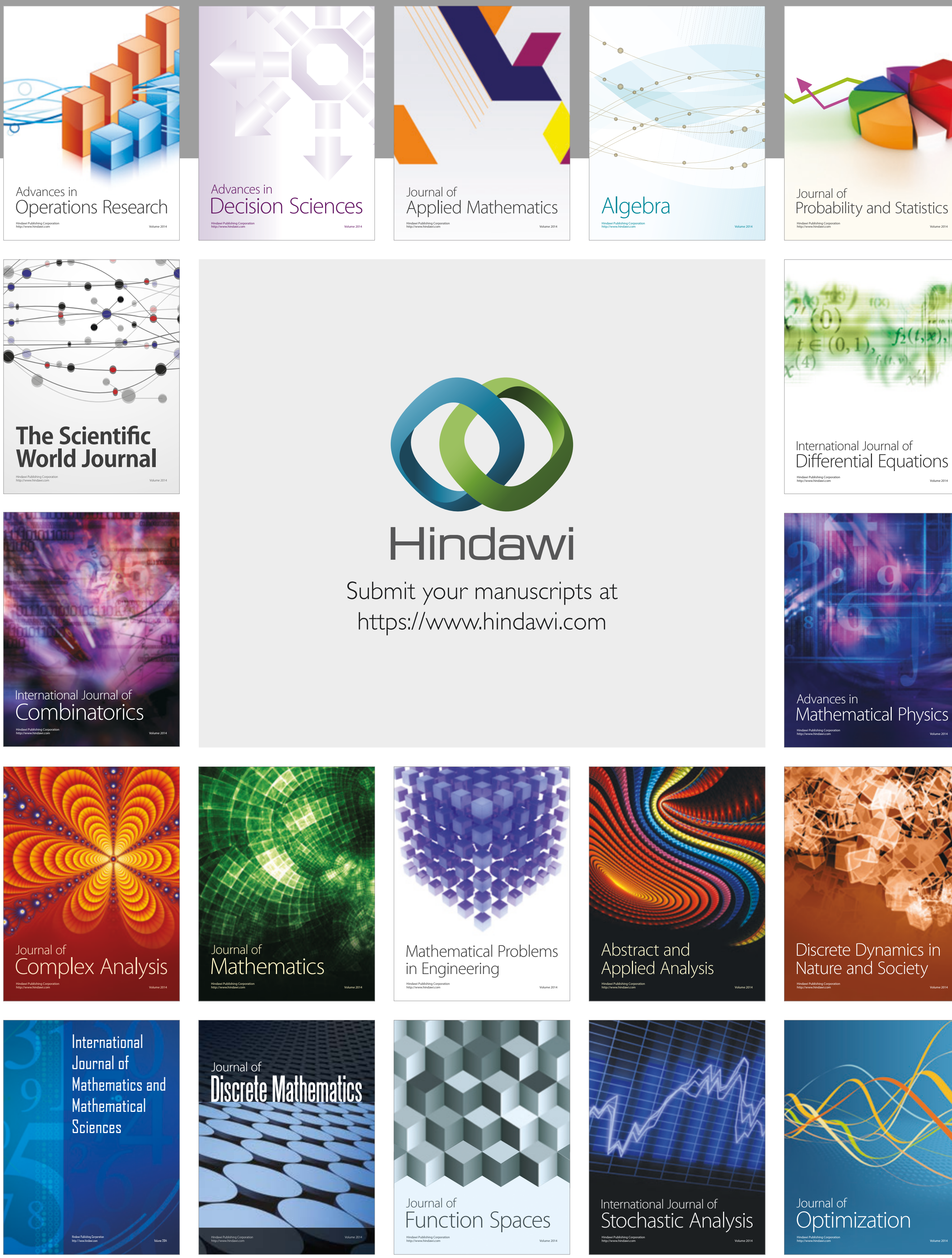
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