Research Article

Growth of Solutions with Positive Initial Energy to Systems of Nonlinear Wave Equations with Damping and Source Terms

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We consider initial-boundary conditions for coupled nonlinear wave equations with damping and source terms. We prove that the solutions of the problem are unbounded when the initial data are large enough in some sense.

1. Introduction

In this work, we consider the following initial-boundary value problem:

$$u_{tt} - \Delta u_t + |u_t|^{p-1} u_t = \text{div}\left(\rho\left(|\nabla u|^2\right)\nabla u\right) + f_1(u,v),$$
$$(x,t) \in \Omega \times (0,T),$$

$$v_{tt} - \Delta v_t + |v_t|^{q-1} v_t = \text{div}\left(\rho\left(|\nabla v|^2\right)\nabla v\right) + f_2(u,v),$$
$$(x,t) \in \Omega \times (0,T),$$

$$u(x,t) = v(x,t) = 0, \quad (x,t) \in \partial\Omega \times (0,T),$$

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega,$$

$$v(x,0) = v_0(x), \quad v_t(x,0) = v_1(x), \quad x \in \Omega,$$
$$\tag{1}$$

where Ω is a bounded domain with smooth boundary $\partial\Omega$ in R^n, $n = 1, 2, 3$; $p, q \geq 1$; $f_i : R^2 \rightarrow R$ are given functions to be specified later.

Throughout this paper, we define ρ by

$$\rho(s) = b_1 + b_2 s^m, \quad s \geq 0, \tag{2}$$

where b_1, b_2 are nonnegative constants and $b_1 + b_2 > 0$.

This type of problems not only is important from the theoretical point of view, but also arises in material science and physics that deal with system of nonlinear wave equations.

Ye [1] obtained the local existence and the blowup of the solution of problem (1), for $\rho(s) = s^m$. In the absence of the strong damping (Δu_t and Δv_t) terms, problem (1) becomes

$$u_{tt} + |u_t|^{p-1} u_t = \text{div}\left(\rho\left(|\nabla u|^2\right)\nabla u\right) + f_1(u,v),$$
$$v_{tt} + |v_t|^{q-1} v_t = \text{div}\left(\rho\left(|\nabla v|^2\right)\nabla v\right) + f_2(u,v). \tag{3}$$

Wu et al. [2] obtained the global existence and blowup of the solution of problem (3) under some suitable conditions. Fei and Hongjun [3] considered problem (3) and improved the blowup result obtained in [2], for a large class of initial data in positive initial energy, using the same techniques as in Payne and Sattinger [4] and some estimates used firstly by Vitillaro [5]. Recently, Pişkin and Polat [6, 7] studied the local and global existence, energy decay, and blowup of the solution of problem (3). Also, for more information about (1) and (3), see [2, 3, 7].

The many problems associated with (1) are studied from various aspects in many papers [8–13].

In this work, we will consider the blowup property in infinity time, that is, exponential growth.

This work is organized as follows. In Section 2, we state the local existence result. In Section 3, we establish that the energy will grow up as an exponential as time goes to infinity, provided that the initial data are large enough or $E(0) < E_1$, where $E(0)$ and E_1 are defined in (9) and (15).

2. Preliminaries

In this section, we introduce some notations and lemmas and local existence theorem needed in the proof of our main results. Let $\| \cdot \|$ and $\| \cdot \|_p$ denote the usual $L^2(\Omega)$ norm and $L^p(\Omega)$ norm, respectively.

Concerning the functions $f_1(u, v)$ and $f_2(u, v)$, we take

$$f_1(u, v) = a|u + v|^{2(r+1)}(u + v) + b|u|^r u|v|^{r+2},$$
$$f_2(u, v) = a|u + v|^{2(r+1)}(u + v) + b|v|^r v|u|^{r+2}, \tag{4}$$

where $a, b > 0$ are constants and r satisfies

$$-1 < r \quad \text{if } n \leq 2,$$
$$-1 < r \leq 1 \quad \text{if } n = 3. \tag{5}$$

According to the above equalities they can easily verify that

$$uf_1(u, v) + vf_2(u, v) = 2(r + 2)F(u, v), \quad \forall (u, v) \in R^2, \tag{6}$$

where

$$F(u, v) = \frac{1}{2(r+2)} \left[a|u + v|^{2(r+2)} + 2b|uv|^{r+2} \right]. \tag{7}$$

We have the following result.

Lemma 1 (see [14]). *There exist two positive constants c_0 and c_1 such that*

$$c_0 \left(|u|^{2(r+2)} + |v|^{2(r+2)} \right) \leq 2(r+2)F(u, v)$$
$$\leq c_1 \left(|u|^{2(r+2)} + |v|^{2(r+2)} \right) \tag{8}$$

is satisfied.

We define the energy function as follows:

$$E(t) = \frac{1}{2} \left(\|u_t\|^2 + \|v_t\|^2 \right)$$
$$+ \frac{1}{2} \int_\Omega \left(P\left(|\nabla u|^2 \right) + P\left(|\nabla v|^2 \right) \right) dx \tag{9}$$
$$- \int_\Omega F(u, v) \, dx,$$

where $P(s) = \int_0^s \rho(\xi) d\xi, s \geq 0$.

Lemma 2 (see [7]). *$E(t)$ is a nonincreasing function for $t \geq 0$ and*

$$E'(t) = -\left(\|\nabla u_t\|^2 + \|\nabla v_t\|^2 + \|u_t\|_{p+1}^{p+1} + \|v_t\|_{q+1}^{q+1} \right) \leq 0. \tag{10}$$

Lemma 3 (Sobolev-Poincare inequality [15]). *Let p be a number with $2 \leq p < \infty$ ($n = 1, 2$) or $2 \leq p \leq 2n/(n-2)$ ($n \geq 3$), then there is a constant $C_* = C_*(\Omega, p)$ such that*

$$\|u\|_p \leq C_* \|\nabla u\| \quad \text{for } u \in H_0^1(\Omega); \tag{11}$$

Next, we state the local existence theorem [1, 7].

Theorem 4 (local existence). *Suppose that (5) holds. Then there exist p, q satisfying*

$$1 \leq p, q \quad \text{if } n \leq 2,$$
$$1 \leq p, q \leq 5 \quad \text{if } n = 3, \tag{12}$$

and further $(u_0, v_0) \in (W_0^{1,2(m+1)}(\Omega) \cap L^{2(r+2)}(\Omega)) \times (W_0^{1,2(m+1)}(\Omega) \cap L^{2(r+2)}(\Omega)), (u_1, v_1) \in L^2(\Omega) \times L^2(\Omega)$. Then problem (1) has a unique local solution

$$u, v \in C\left([0, T) ; W_0^{1,2(m+1)}(\Omega) \cap L^{2(r+2)}(\Omega) \right),$$
$$u_t \in C\left([0, T) ; L^2(\Omega) \right) \cap L^{p+1}(\Omega \times [0, T)), \tag{13}$$
$$v_t \in C\left([0, T) ; L^2(\Omega) \right) \cap L^{q+1}(\Omega \times [0, T)).$$

3. Exponential Growth

In this section, we will prove that the energy is unbounded when the initial data are large enough in some sense. Our techniques of proof follow very carefully the techniques used in [16].

Lemma 5 (see [3]). *Suppose that (5) holds. Then there exists $\eta > 0$ such that for any $(u, v) \in (W_0^{1,2(m+1)}(\Omega) \cap L^{2(r+2)}(\Omega)) \times (W_0^{1,2(m+1)}(\Omega) \cap L^{2(r+2)}(\Omega))$ the inequality*

$$\|u + v\|_{2(r+2)}^{2(r+2)} + 2\|uv\|_{r+2}^{r+2}$$
$$\leq \eta \left(\int_\Omega \left(P\left(|\nabla u|^2 \right) + P\left(|\nabla v|^2 \right) \right) dx \right)^{r+2} \tag{14}$$

holds.

For the sake of simplicity and to prove our result, we take $a = b = 1$ and introduce

$$B = \eta^{1/2(r+2)}, \qquad \alpha_1 = B^{-((r+2)/(r+1))},$$
$$E_1 = \left(\frac{1}{2} - \frac{1}{2(r+2)} \right) \alpha_1^2, \tag{15}$$

where η is the optimal constant in (14). Next, we will state a lemma which is similar to the one introduced firstly by Vitillaro in [5] to study a class of a single wave equation.

Lemma 6 (see [3]). *Suppose that (5) holds. Let (u, v) be the solution of problem (1). Assume further that $E(0) < E_1$ and*

$$\left(\int_\Omega \left(P\left(|\nabla u_0|^2 \right) + P\left(|\nabla v_0|^2 \right) \right) dx \right)^{1/2} > \alpha_1. \tag{16}$$

Then there exists a constant $\alpha_2 > \alpha_1$ such that

$$\left(\int_\Omega \left(P\left(|\nabla u||^2 \right) + P\left(|\nabla v|^2 \right) \right) dx \right)^{1/2} \geq \alpha_2, \qquad (17)$$

$$\left(\|u + v\|_{2(r+2)}^{2(r+2)} + 2 \|uv\|_{r+2}^{r+2} \right)^{1/2(r+2)} \geq B\alpha_2, \qquad (18)$$

for all $t \in [0, T)$.

Theorem 7. *Suppose that (5) and $2(r + 2) > \max\{p + 1, q + 1\}$ hold. Then any solution of problem (1) with initial data satisfying*

$$\left(\int_\Omega \left(P\left(|\nabla u_0|^2 \right) + P\left(|\nabla v_0|^2 \right) \right) dx \right)^{1/2} > \alpha_1, \qquad E(0) < E_1, \qquad (19)$$

grows exponentially.

Proof. We set

$$H(t) = E_1 - E(t), \quad t \geq 0. \qquad (20)$$

From (10) and (20) we get

$$H'(t) = -E'(t) = \|u_t\|_{p+1}^{p+1} + \|v_t\|_{q+1}^{q+1} \geq 0; \qquad (21)$$

hence we have $H(t) \geq H(0) = E_1 - E(0) > 0$.

We consider the following functional:

$$L(t) = H(t) + \varepsilon \int_\Omega (uu_t + vv_t)\, dx + \frac{\varepsilon}{2} \left(\|\nabla u\|^2 + \|\nabla v\|^2 \right) \qquad (22)$$

for small ε to be specified later.

Our goal is to show that $L(t)$ satisfies a differential inequality of the form

$$L'(t) \geq CL(t) \quad \forall t \geq 0. \qquad (23)$$

This, of course, will lead to exponential growth.

By taking a derivative of (22) and using (1), it follows that

$$\begin{aligned} L'(t) &= H'(t) + \varepsilon \left(\|u_t\|^2 + \|v_t\|^2 \right) \\ &\quad + \varepsilon \int_\Omega (uu_{tt} + vv_{tt})\, dx \\ &\quad + \varepsilon \int_\Omega (\nabla u \nabla u_t + \nabla v \nabla v_t)\, dx \\ &= \left(\|u_t\|_{p+1}^{p+1} + \|v_t\|_{q+1}^{q+1} \right) + \varepsilon \left(\|u_t\|^2 + \|v_t\|^2 \right) \\ &\quad - \varepsilon b_1 \left(\|\nabla u\|^2 + \|\nabla v\|^2 \right) \\ &\quad - \varepsilon b_2 \left(\|\nabla u\|_{2(m+1)}^{2(m+1)} + \|\nabla v\|_{2(m+1)}^{2(m+1)} \right) \\ &\quad - \varepsilon \int_\Omega (uf_1(u,v) + vf_2(u,v))\, dx \\ &\quad - \varepsilon \int_\Omega \left(u\, |u_t|^{p-1}\, u_t + v\, |v_t|^{q-1}\, v_t \right) dx. \end{aligned} \qquad (24)$$

From (9) and (20), it follows that

$$\begin{aligned} &- b_2 \left(\|\nabla u\|_{2(m+1)}^{2(m+1)} + \|\nabla v\|_{2(m+1)}^{2(m+1)} \right) \\ &= 2(m+1) H(t) - 2(m+1) E_1 \\ &\quad + (m+1) \left(\|u_t\|^2 + \|v_t\|^2 \right) \\ &\quad + b_1 (m+1) \left(\|\nabla u\|^2 + \|\nabla v\|^2 \right) \\ &\quad - 2(m+1) \int_\Omega F(u,v)\, dx. \end{aligned} \qquad (25)$$

Inserting (25) into (24), we get

$$\begin{aligned} L'(t) &= \left(\|u_t\|_{p+1}^{p+1} + \|v_t\|_{q+1}^{q+1} \right) \\ &\quad + \varepsilon (m+2) \left(\|u_t\|^2 + \|v_t\|^2 \right) \\ &\quad + \varepsilon b_1 m \left(\|\nabla u\|^2 + \|\nabla v\|^2 \right) \\ &\quad + \varepsilon \left(1 - \frac{m+1}{r+2} \right) \left(\|u + v\|_{2(r+2)}^{2(r+2)} + 2 \|uv\|_{r+2}^{r+2} \right) \\ &\quad + 2\varepsilon (m+1) H(t) - 2\varepsilon (m+1) E_1 \\ &\quad - \varepsilon \int_\Omega \left(u\, |u_t|^{p-1}\, u_t + v\, |v_t|^{q-1}\, v_t \right) dx. \end{aligned} \qquad (26)$$

Then using (18), we obtain

$$\begin{aligned} L'(t) &\geq \left(\|u_t\|_{p+1}^{p+1} + \|v_t\|_{q+1}^{q+1} \right) \\ &\quad + \varepsilon (m+2) \left(\|u_t\|^2 + \|v_t\|^2 \right) \\ &\quad + \varepsilon b_1 m \left(\|\nabla u\|^2 + \|\nabla v\|^2 \right) + 2\varepsilon (m+1) H(t) \\ &\quad + \varepsilon c' \left(\|u + v\|_{2(r+2)}^{2(r+2)} + 2 \|uv\|_{r+2}^{r+2} \right) \\ &\quad - \varepsilon \int_\Omega \left(u\, |u_t|^{p-1}\, u_t + v\, |v_t|^{q-1}\, v_t \right) dx, \end{aligned} \qquad (27)$$

where $c' = 1 - (m+1)/(r+2) - 2(m+1)E_1(B\alpha_2)^{-2(r+2)}$. It is clear that $c' > 0$, since $\alpha_2 > B^{-((r+2)/(r+1))}$. In order to estimate the last two terms in (27), we use the following Young inequality:

$$XY \leq \frac{\delta^k X^k}{k} + \frac{\delta^{-l} Y^l}{l}, \qquad (28)$$

where $X, Y \geq 0$, $\delta > 0$, $k, l \in R^+$ such that $1/k + 1/l = 1$. Consequently, applying the above inequality we have

$$\int_\Omega uu_t\, |u_t|^{p-1}\, dx \leq \frac{\delta_1^{p+1}}{p+1} \|u\|_{p+1}^{p+1} + \frac{p\delta_1^{-((p+1)/p)}}{p+1} \|u_t\|_{p+1}^{p+1},$$

$$\int_\Omega vv_t\, |v_t|^{q-1}\, dx \leq \frac{\delta_2^{q+1}}{q+1} \|v\|_{q+1}^{q+1} + \frac{q\delta_2^{-((q+1)/q)}}{q+1} \|v_t\|_{q+1}^{q+1}. \qquad (29)$$

Inserting estimates (29) into (27), we have

$$
\begin{aligned}
L'(t) \geq\ & \varepsilon(m+2)\left(\|u_t\|^2 + \|v_t\|^2\right) \\
& + \varepsilon b_1 m\left(\|\nabla u\|^2 + \|\nabla v\|^2\right) + 2\varepsilon(m+1)H(t) \\
& + \varepsilon c_2\left(\|u\|_{2(r+2)}^{2(r+2)} + \|v\|_{2(r+2)}^{2(r+2)}\right) \\
& - \varepsilon\frac{\delta_1^{p+1}}{p+1}\|u\|_{p+1}^{p+1} + \left(1 - \varepsilon\frac{p\delta_1^{-((p+1)/p)}}{p+1}\right)\|u_t\|_{p+1}^{p+1} \\
& - \varepsilon\frac{\delta_2^{q+1}}{q+1}\|v\|_{q+1}^{q+1} + \left(1 - \varepsilon\frac{q\delta_2^{-((q+1)/q)}}{q+1}\right)\|v_t\|_{q+1}^{q+1},
\end{aligned}
\tag{30}
$$

where $c_2 = c'c_0$.

Since $2(r+2) > \max\{p+1, q+1\}$, from the embedding $L^{2(r+2)}(\Omega) \hookrightarrow L^{p+1}(\Omega)$ and embedding $L^{2(r+2)}(\Omega) \hookrightarrow L^{p+1}(\Omega)$, we have

$$
\begin{aligned}
\|u\|_{p+1}^{p+1} &\leq c_3 \|u\|_{2(r+2)}^{p+1}, \\
\|v\|_{q+1}^{q+1} &\leq c_4 \|v\|_{2(r+2)}^{q+1}
\end{aligned}
\tag{31}
$$

for some positive constants c_3 and c_4. Using the algebraic inequality

$$
z^\upsilon \leq z + 1 \leq \left(1 + \frac{1}{a}\right)(z+a),
\tag{32}
$$

$$
\forall z \geq 0, \quad 0 < \upsilon \leq 1, \quad a \geq 0,
$$

and since $H(t) \geq H(0)$, we get

$$
\begin{aligned}
\|u\|_{2(r+2)}^{p+1} &\leq d\left(\|u\|_{2(r+2)}^{2(r+2)} + H(0)\right) \\
&\leq d\left(\|u\|_{2(r+2)}^{2(r+2)} + H(t)\right),
\end{aligned}
\tag{33}
$$

where $d = 1 + 1/H(0)$. Similarly

$$
\|v\|_{2(r+2)}^{q+1} \leq d\left(\|v\|_{2(r+2)}^{2(r+2)} + H(t)\right).
\tag{34}
$$

Inserting (33) and (34) into (30), we have

$$
\begin{aligned}
L'(t) \geq\ & \varepsilon(m+2)\left(\|u_t\|^2 + \|v_t\|^2\right) \\
& + \varepsilon b_1 m\left(\|\nabla u\|^2 + \|\nabla v\|^2\right) \\
& + \varepsilon\left(2(m+1) - d\frac{\delta_1^{p+1}}{p+1} - d\frac{\delta_2^{q+1}}{q+1}\right)H(t) \\
& + \varepsilon\left(c_2 - d\frac{\delta_1^{p+1}}{p+1} - d\frac{\delta_2^{q+1}}{q+1}\right)\left(\|u\|_{2(r+2)}^{2(r+2)} + \|v\|_{2(r+2)}^{2(r+2)}\right) \\
& + \left(1 - \varepsilon\frac{p\delta_1^{-((p+1)/p)}}{p+1}\right)\|u_t\|_{p+1}^{p+1} \\
& + \left(1 - \varepsilon\frac{q\delta_2^{-((q+1)/q)}}{q+1}\right)\|v_t\|_{q+1}^{q+1}.
\end{aligned}
\tag{35}
$$

Now, once δ_1 and δ_2 are fixed, we can choose ε small enough such that

$$
1 - \varepsilon\frac{p\delta_1^{-((p+1)/p)}}{p+1} > 0, \qquad 1 - \varepsilon\frac{q\delta_2^{-((q+1)/q)}}{q+1} > 0,
\tag{36}
$$

$$
L(0) = H(0) + \varepsilon\int_\Omega\left(u_0 u_1 + v_0 v_1\right)dx > 0.
$$

Consequently (35) takes the form

$$
\begin{aligned}
L'(t) \geq\ & \varepsilon(m+2)\left(\|u_t\|^2 + \|v_t\|^2\right) \\
& + \varepsilon b_1 m\left(\|\nabla u\|^2 + \|\nabla v\|^2\right) \\
& + \varepsilon\left(2(m+1) - d\frac{\delta_1^{p+1}}{p+1} - d\frac{\delta_2^{q+1}}{q+1}\right)H(t) \\
& + \varepsilon\left(c_2 - d\frac{\delta_1^{p+1}}{p+1} - d\frac{\delta_2^{q+1}}{q+1}\right)\left(\|u\|_{2(r+2)}^{2(r+2)} + \|v\|_{2(r+2)}^{2(r+2)}\right) \\
\geq\ & \theta\left(\|u_t\|^2 + \|v_t\|^2 + H(t) + \|\nabla u\|^2 \right. \\
& \left. + \|\nabla v\|^2 + \|u\|_{2(r+2)}^{2(r+2)} + \|v\|_{2(r+2)}^{2(r+2)}\right),
\end{aligned}
\tag{37}
$$

where $\theta = \min\{\varepsilon(m+2), \varepsilon b_1 m, (2(m+1) - d(\delta_1^{p+1}/(p+1)) - d(\delta_2^{q+1}/(q+1))), \varepsilon(c_2 - d(\delta_1^{p+1}/(p+1)) - d(\delta_2^{q+1}/(q+1)))\}$.

Then we have

$$
L(t) \geq L(0) > 0, \quad \forall t \geq 0.
\tag{38}
$$

On the other hand, applying Hölder inequality, we obtain

$$
\begin{aligned}
\left|\int_\Omega uu_t\,dx + \int_\Omega vv_t\,dx\right| & \\
&\leq \|u\|\,\|u_t\| + \|v\|\,\|v_t\| \\
&\leq C\left(\|u\|_{2(r+2)}\,\|u_t\| + \|v\|_{2(r+2)}\,\|v_t\|\right).
\end{aligned}
\tag{39}
$$

Young inequality gives

$$
\begin{aligned}
\left|\int_\Omega uu_t\,dx + \int_\Omega vv_t\,dx\right| & \\
&\leq \frac{C}{2}\left(\|u\|_{2(r+2)}^2 + \|u_t\|^2 + \|v\|_{2(r+2)}^2 + \|v_t\|^2\right) \\
&\leq \frac{C}{2}\left(\left(\|u\|_{2(r+2)}^{2(r+2)}\right)^{1/(r+2)} + \|u_t\|^2 \right. \\
&\quad \left. + \left(\|v\|_{2(r+2)}^{2(r+2)}\right)^{1/(r+2)} + \|v_t\|^2\right).
\end{aligned}
\tag{40}
$$

Since $r > -1$, algebraic inequality (32) yields

$$
\begin{aligned}
\left|\int_\Omega uu_t\,dx + \int_\Omega vv_t\,dx\right| & \\
&\leq \frac{C}{2}\left(\left(1 + \frac{1}{H(0)}\right)\left(\|u\|_{2(r+2)}^{2(r+2)} + H(t)\right) + \|u_t\|^2 \right. \\
&\quad \left. + \left(1 + \frac{1}{H(0)}\right)\left(\|v\|_{2(r+2)}^{2(r+2)} + H(t)\right) + \|v_t\|^2\right).
\end{aligned}
\tag{41}
$$

Note that

$$L(t) \leq H(t) + \varepsilon \int_\Omega (uu_t + vv_t)\, dx$$

$$\leq H(t) + \left| \int_\Omega uu_t dx + \int_\Omega vv_t dx \right|$$

$$\leq C' \left(\|u_t\|^2 + \|v_t\|^2 + H(t) + \|\nabla u\|^2 + \|\nabla v\|^2 \right.$$

$$\left. + \|u\|_{2(r+2)}^{2(r+2)} + \|v\|_{2(r+2)}^{2(r+2)} \right). \tag{42}$$

Combining with (37) and (42), we arrive at

$$L'(t) \geq CL(t) \quad \forall t \geq 0. \tag{43}$$

Integrating differential inequality (43) between 0 and t gives the following estimate for $L(t)$:

$$L(t) \geq L(0)\, e^{Ct}. \tag{44}$$

The proof of Theorem 7 is completed. $\qquad\square$

Remark 8. When $E(0) < 0$, by setting $H(t) = -E(t)$, the similar result is obtained by applying the same arguments in the proof of Theorem 7.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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