

Research Article

A New Nonlinear Diffusion Equation Model for Noisy Image Segmentation

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Image segmentation and image denoising are two important and fundamental topics in the field of image processing. Geometric active contour model based on level set method can deal with the problem of image segmentation, but it does not consider the problem of image denoising. In this paper, a new diffusion equation model for noisy image segmentation is proposed by incorporating some classical diffusion equation denoising models into the segmental process. An assumption about the connection between the image intensity and level set function is given firstly. Some classical denoising models are employed to describe the evolution of level set function secondly. The final nonlinear diffusion equation model for noisy image segmentation is built thirdly. Then image segmentation and image denoising are combined in a united framework. The segmental results can be presented by level set function. Experimental results show that the new model has the advantage of noise resistance and is superior to traditional segmentation model.

1. Introduction

Image segmentation and image denoising are basic topics in the fields of image processing. The results of image segmentation and denoising can influence the following image processing directly. They also play an important role in the fields of computer vision, target tracking, and so forth.

The target of image denoising is to recover the true image from the noisy one. Because noise and boundary of image both share high frequency, it is difficult to reduce the noise while containing the boundary of image. For the sake of solving this difficulty, a series of nonlinear denoising models [1–5] are proposed and total variation (TV) model [1] is an excellent one among them. TV model is a famous denoising model proposed by Rudin et al. and it has the ability to preserve image boundary. At the same time, the target of image segmentation is to find a curve which coincides with the true boundary of image object. Thousands of segmentation

methods were proposed before. Active contour model (ACM) is the focus recently. ACM can be categorized as two classes. One is parameterized model [6, 7] and another is geometrical model [8–12]. Geometric active contour model also has two classes, gradient-based model and region-based model. Chan-Vese (CV) model [10] is a famous region-based geometric active contour model. It is more flexible for initial curve and can deal with images which have weak boundaries because it does not rely on the gradient information.

Image segmentation and image denoising are independent most of the time. In this paper, a new diffusion equation model for noisy image segmentation is proposed by incorporating some classical diffusion equation denoising models into the segmental process. An assumption about the connection between the image intensity and level set function is given firstly. Some classical denoising models are employed to describe the evolution of level set function secondly. The final nonlinear diffusion equation model for noisy image

segmentation is built thirdly. Then image segmentation and image denoising are combined in a united framework. The segmental results can be presented by level set function. Experimental results show that the new model is available for noisy image segmentation.

This paper is organized as follows. The next section will introduce the related models. The new nonlinear diffusion equation model for noisy image segmentation is described in the third section. Experimental results are shown in the fourth section and we draw a conclusion in the fifth section.

2. Related Models

Four models will be introduced in this section. One is segmentation model and the other is denoising models.

2.1. CV Model. CV model is an important segmentation model and it is suitable for the image with intensity homogeneity. CV model assumes that the image is homogeneous and different objects in the image have different intensities. The corresponding energy function is as follows:

$$\begin{aligned} E(C, c_1, c_2) &= \mu_0 \text{Length}(C) \\ &+ \lambda_1 \int_{\text{inside}(C)} |u_0(x, y) - c_1|^2 dx dy \\ &+ \lambda_2 \int_{\text{outside}(C)} |u_0(x, y) - c_2|^2 dx dy, \end{aligned} \quad (1)$$

where μ_0 is the nonnegative weight for length of curve C , u_0 is the image intensity, and λ_1, λ_2 are the weights of external energy term. c_1 and c_2 are the mean of gray values inside and outside the moving contour. In order to solve the topological changes of the curve C easily, we introduce the level set function into (1):

$$\begin{aligned} \phi(x, y) &< 0, \quad (x, y) \in \text{outside}(C), \\ \phi(x, y) &> 0, \quad (x, y) \in \text{inside}(C), \\ \phi(x, y) &= 0, \quad (x, y) \in C. \end{aligned} \quad (2)$$

Then (1) also can be represented as follows:

$$\begin{aligned} E(\phi, c_1, c_2) &= \mu_0 \int_{\Omega} |\nabla H(\phi)| dx dy \\ &+ \lambda_1 \int_{\Omega} |u_0(x, y) - c_1|^2 H(\phi) dx dy \\ &+ \lambda_2 \int_{\Omega} |u_0(x, y) - c_2|^2 (1 - H(\phi)) dx dy, \end{aligned} \quad (3)$$

where Ω denotes the field of image. $H(\phi)$ is the Heaviside function. For making Heaviside function $H(\phi)$ differentiable, we replace $H(\phi)$ by $H_\varepsilon(\phi)$ as follows:

$$\begin{aligned} H_\varepsilon(\phi) &= \frac{1}{2} \left(1 + \frac{2}{\pi} \arctan \left(\frac{\phi}{\varepsilon} \right) \right), \\ \delta_\varepsilon(\phi) &= H'_\varepsilon(\phi) = \frac{1}{\pi} \frac{\varepsilon}{\varepsilon^2 + \phi^2}. \end{aligned} \quad (4)$$

Then (3) can be written as follows:

$$\begin{aligned} E(\phi, c_1, c_2) &= \mu_0 \int_{\Omega} |\nabla H_\varepsilon(\phi)| dx dy \\ &+ \lambda_1 \int_{\Omega} |u_0(x, y) - c_1|^2 H_\varepsilon(\phi) dx dy \\ &+ \lambda_2 \int_{\Omega} |u_0(x, y) - c_2|^2 (1 - H_\varepsilon(\phi)) dx dy. \end{aligned} \quad (5)$$

By taking partial derivations for c_1 and c_2 in (5), we can get the expressions of c_1 and c_2 as follows:

$$\begin{aligned} c_1 &= \frac{\int_{\Omega} u_0(x, y) H_\varepsilon(\phi) dx dy}{\int_{\Omega} H_\varepsilon(\phi) dx dy}, \\ c_2 &= \frac{\int_{\Omega} u_0(x, y) (1 - H_\varepsilon(\phi)) dx dy}{\int_{\Omega} (1 - H_\varepsilon(\phi)) dx dy}. \end{aligned} \quad (6)$$

The corresponding Euler-Lagrange equation of (5) is as follows:

$$\begin{aligned} -\delta_\varepsilon(\phi) &\cdot \left[\mu_0 \text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \right] \\ &= 0. \end{aligned} \quad (7)$$

The final level set evolution equation can be written as follows:

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= \delta_\varepsilon(\phi) \\ &\cdot \left[\mu_0 \text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \right]. \end{aligned} \quad (8)$$

2.2. TV Model. TV model is a classical model for image denoising and it utilizes L_1 norm of image gradient magnitude to replace H_1 norm of image gradient magnitude as the smooth term of image. TV model can protect the boundary of image while reducing the noise. The energy function of TV model is established as follows:

$$E(u) = \int_{\Omega} |\nabla u| dx dy + \frac{\lambda}{2} \cdot \int_{\Omega} (u - u_0)^2 dx dy, \quad (9)$$

where the first term $\int_{\Omega} |\nabla u| dx dy$ is the smooth term of image and the second term $\int_{\Omega} (u - u_0)^2 dx dy$ is the fidelity term of image. u denotes the denoising image. Ω denotes the field of image. $|\nabla u|$ is the gradient magnitude of image. $\lambda \geq 0$ is the coefficient of fidelity term.

The Euler-Lagrange equation of (9) is as follows:

$$-\text{div} \left(\frac{\nabla u}{|\nabla u|} \right) + \lambda \cdot (u - u_0) = 0. \quad (10)$$

Utilizing the gradient decent flow, we can get the partial difference equation as follows:

$$\frac{\partial u}{\partial t} = \text{div} \left(\frac{\nabla u}{|\nabla u|} \right) + \lambda \cdot (u_0 - u). \quad (11)$$

2.3. *Adaptive TV Model.* Blomgren and Chan present a new norm which is different from the L_1 norm in [2]:

$$R_p(u) = \int_{\Omega} |\nabla u|^{p(|\nabla u|)} dx dy, \quad (12)$$

where the exponent p depends on the module value $|\nabla u|$ of image gradient and satisfies these constrains as follows:

- (1) decreasing monotonously,
- (2) $p(x) \rightarrow 2$ when $x \rightarrow 0$ and $p(x) \rightarrow 1$ when $x \rightarrow \infty$.

Therefore, $R_p(u) \approx \int_{\Omega} |\nabla u|^2 dx dy$ while lying in the flat region of image; namely, $|\nabla u|$ is very small. $|\nabla u|$ is very big when the pixel is near the boundary of object. At the same time, $p(|\nabla u|) \approx 1$ when $|\nabla u| \rightarrow \infty$. Therefore, $R_p(u) \approx \int_{\Omega} |\nabla u| dx dy$ according to (12). That is to say, the new norm $R_p(u)$ would choose L_1 norm and H_1 norm automatically.

We can define $p(x)$ as follows:

$$p(x) = \frac{2+x}{1+x}. \quad (13)$$

The corresponding energy function can be expressed as follows:

$$E(u) = \int_{\Omega} |\nabla u|^{p(|\nabla u|)} dx dy + \frac{\lambda}{2} \cdot \int_{\Omega} (u - u_0)^2 dx dy. \quad (14)$$

Finally, the partial difference equation of image u about time parameter t can be written as follows:

$$\frac{\partial u}{\partial t} = \operatorname{div} \left(q(|\nabla u|) \frac{\nabla u}{|\nabla u|} \right) + \lambda \cdot (u_0 - u), \quad (15)$$

where

$$q(|\nabla u|) = p(|\nabla u|) |\nabla u|^{p(|\nabla u|)-1} + \ln |\nabla u| \cdot p'(|\nabla u|) \cdot |\nabla u|^{p(|\nabla u|)}. \quad (16)$$

Compared with (11), (15) has more computation complexities. For the sake of reducing the computation complexities, we replace $q(|\nabla u|)$ in (14) by the term $q(|\nabla(G_{\sigma} * u_0)|)$, where $G_{\sigma} * u_0$ denotes that the image u_0 is filtered by Gaussian filter of variation σ . Therefore, we do not need to update $q(|\nabla u|)$ every time when updating the denoising image u .

2.4. *Anisotropic Diffusion Model.* Catté et al. propose a denoising model based on anisotropic diffusion [13]. The final partial difference equation is as follows:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \operatorname{div} (g(|\nabla(G_{\sigma} * u_0)|) \nabla u), \\ u(x, 0) &= u_0, \end{aligned} \quad (17)$$

where

$$g(s) = \frac{1}{1 + s^2/K}, \quad (18)$$

where K is a nonnegative number. The diffusion term can protect the boundary information and so the anisotropic diffusion model can obtain better results than the denoising model based on Laplace Operation.

3. The New Nonlinear Diffusion Equation Model for Noisy Image Segmentation

Inspired by CV model, we can know that the image with two objects can be described as follows:

$$u = c_1 \cdot H_{\varepsilon}(\phi) + c_2 \cdot (1 - H_{\varepsilon}(\phi)). \quad (19)$$

From (19), we can establish the connection between the image intensity and the level set function. Therefore, we can introduce the level set function into the energy function of denoising models. Averagely, we establish the energy function of denoising models as follows:

$$E(u) = \operatorname{smooth}(u) + \lambda \cdot \operatorname{fidelity}(u), \quad (20)$$

where $\operatorname{smooth}(u)$ is the smooth term, $\operatorname{fidelity}(u)$ is the fidelity term, and λ is the weight of the fidelity term.

Now we introduce (19) into (20) and the corresponding Euler-Lagrange equation can be written as follows:

$$\frac{\partial E(u(\phi))}{\partial \phi} = \frac{\partial E}{\partial u} \cdot \frac{\partial u}{\partial \phi}, \quad (21)$$

where

$$\frac{\partial u}{\partial \phi} = (c_1 - c_2) \cdot \delta_{\varepsilon}(\phi). \quad (22)$$

Finally, the corresponding level set equation is written as follows:

$$\frac{\partial \phi}{\partial t} = -\frac{\partial E(u(\phi))}{\partial \phi} = -\frac{\partial E}{\partial u} \cdot \frac{\partial u}{\partial \phi} = \frac{\partial u}{\partial t} \cdot \frac{\partial u}{\partial \phi}. \quad (23)$$

We will present three examples of segmentation model based on the related denoising models mentioned before.

3.1. *TV-Based Segmentation Model.* According to (23), we can formulate the level set equation based on TV model. The Euler-Lagrange equation of TV model is as follows:

$$\frac{\partial E}{\partial u} = -\operatorname{div} \left(\frac{\nabla u}{|\nabla u|} \right) + \lambda \cdot (u - u_0) = 0. \quad (24)$$

According to (11), (19), (23), and (24), we can obtain the final level set equation as follows:

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= \delta_{\varepsilon}(\phi) \cdot (c_1 - c_2) \left[\operatorname{div} \left(\frac{(c_1 - c_2) \cdot \delta_{\varepsilon}(\phi) \cdot \nabla \phi}{|c_1 - c_2| \cdot \delta_{\varepsilon}(\phi) \cdot |\nabla \phi|} \right) \right. \\ &\quad \left. + \lambda (u_0 - c_1 \cdot H_{\varepsilon}(\phi) + c_2 \cdot (1 - H_{\varepsilon}(\phi))) \right] = \delta_{\varepsilon}(\phi) \\ &\quad \cdot (c_1 - c_2) \cdot \left[\operatorname{div} \left(\frac{(c_1 - c_2) \cdot \nabla \phi}{|c_1 - c_2| \cdot |\nabla \phi|} \right) \right. \\ &\quad \left. + \lambda (u_0 - c_1 \cdot H_{\varepsilon}(\phi) + c_2 \cdot (1 - H_{\varepsilon}(\phi))) \right]. \end{aligned} \quad (25)$$

3.2. Adaptive TV-Based Segmentation Model. Similar to the TV-based segmentation model, we can find the corresponding partial difference equation of image u firstly and it is described in (15). Then utilizing (15), (19), and (23), we can obtain the Euler-Lagrange equation of adaptive TV-based segmentation model as follows:

$$\begin{aligned} \frac{\partial E(u(\phi))}{\partial \phi} &= \frac{\partial E}{\partial u} \cdot \frac{\partial u}{\partial \phi} \\ &= \left(\operatorname{div} \left(q(|\nabla(G_\sigma * u_0)|) \frac{\nabla u}{|\nabla u|} \right) + \lambda \cdot (u_0 - u) \right) \\ &\quad \cdot (c_1 - c_2) \cdot \delta_\varepsilon(\phi) = 0. \end{aligned} \quad (26)$$

The partial different equation of level set function is written as follows:

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= \delta_\varepsilon(\phi) \cdot (c_1 - c_2) \\ &\quad \cdot \left[\operatorname{div} \left(q(|\nabla(G_\sigma * u_0)|) \cdot \frac{(c_1 - c_2) \cdot \delta_\varepsilon(\phi) \cdot \nabla \phi}{|c_1 - c_2| \cdot \delta_\varepsilon(\phi) \cdot |\nabla \phi|} \right) \right. \\ &\quad \left. + \lambda (u_0 - c_1 \cdot H_\varepsilon(\phi) + c_2 \cdot (1 - H_\varepsilon(\phi))) \right]. \end{aligned} \quad (27)$$

3.3. Anisotropic Diffusion-Based Segmentation Model. As described in Section 2.4, we can acquire the corresponding partial difference equation of image u described in (17). For preserving the important image information, we introduce the fidelity term to this model and the final partial difference equation of image u can be written as follows:

$$\frac{\partial u}{\partial t} = \operatorname{div} (g(|\nabla(G_\sigma * u_0)|) \nabla u) + \lambda \cdot (u - u_0). \quad (28)$$

According to (17), (19), (23), and (29), we can obtain the final level set function as follows:

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= \delta_\varepsilon(\phi) \cdot (c_1 - c_2) \cdot [\operatorname{div} (g(|\nabla \phi|) \cdot \nabla \phi) \\ &\quad + \lambda (u_0 - c_1 \cdot H_\varepsilon(\phi) + c_2 \cdot (1 - H_\varepsilon(\phi)))]. \end{aligned} \quad (29)$$

3.4. Reinitialization of Level Set Function. A signed distance penalizing energy functional is presented by Li et al. [14] as follows:

$$P(\phi) = \frac{1}{2} \cdot \int_{\Omega} (|\nabla \phi| - 1)^2 dx dy. \quad (30)$$

The corresponding gradient descent flow of $P(\phi)$ is as follows:

$$\frac{\partial \phi}{\partial t} = \operatorname{div} \left((|\nabla \phi| - 1) \cdot \frac{\nabla \phi}{|\nabla \phi|} \right) = \Delta \phi - \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right). \quad (31)$$

Literatures [15, 16] show that a function with Laplace evolution is equivalent to using a Gaussian filter. Then we filter the initial conditions of the level set function with Gaussian kernel. So we can remove the curvature term $\operatorname{div}(\nabla \phi / |\nabla \phi|)$.

We can conclude the procedures of the presented model as follows:

- (1) Initialize the level set function.
- (2) If necessary, compute $q(|\nabla(G_\sigma * u_0)|)$, $g(|\nabla(G_\sigma * u_0)|)$ by (16) and (18).
- (3) Calculate c_1, c_2 by (6).
- (4) Obtain the level set function ϕ by (25), (27), or (29).
- (5) Smooth the level set function ϕ with Gaussian filter. If the level set function ϕ is not satisfied, return to procedure (3).

4. Experimental Results

In this section, we compare the segmental results of four models, CV model, TV-based segmentation model, adaptive TV-based segmentation model, and anisotropic diffusion-based segmentation model. In CV model, we set time step as 1, space step as 1, iteration number as 1500, the length parameter as μ_0 , and the radius of initial circle as different value in different case. In the other models, we set time step and space step the same as CV model, $\sigma = 0.5$, $K = 500$, and iteration number as 1500. Besides, we choose ε as 1.5 for all models. Their initial curve is set as blue curves. The parameters of external energy λ_1, λ_2 are set as 1 for all models. The parameter λ depends on the degree of noise.

Figure 1 shows the segmental results of a noisy image contaminated by Gaussian noise with variation 0.05. (a, e), (b, f), (c, g), and (d, h) represent the segmentation results of CV model, TV-based segmentation model, adaptive TV-based model, and anisotropic diffusion-based segmentation model, respectively. The initial curve of all models is a circle as Figure 1(a) shows and its radius is equal to 20. In TV-based segmentation model, adaptive TV-based model, and anisotropic diffusion-based segmentation model, we choose $\lambda = 0.01$. Obviously, the new segmentation model incorporating with denoising models is better than the original CV model.

Figure 2 shows the segmental results of a noisy image contaminated by Gaussian noise with variation as 0.1. (a, e), (b, f), (c, g), and (d, h) represent the segmentation result of CV model, TV-based segmentation model, adaptive TV-based model, and anisotropic diffusion-based segmentation model, respectively. The initial curve of all models is a circle as in Figure 2(a) and its radius is set as 20. In TV-based segmentation model, adaptive TV-based model, and anisotropic diffusion-based segmentation model, we choose $\lambda = 0.01$. Obviously, the new segmentation model based on denoising algorithm can get satisfied contours for noisy image.

Figure 3 gives the segmental results of a galaxy image contaminated by Gaussian noise with variation of 0.05. (a, e), (b, f), (c, g), and (d, h) represent the segmentation result of CV model, TV-based segmentation model, adaptive TV-based model, and anisotropic diffusion-based segmentation model, respectively. The initial curve of all models is a circle as in Figure 3(a) and its radius is set as 50. In TV-based segmentation model and adaptive TV-based model, we choose $\lambda = 0.02$. In anisotropic diffusion-based segmentation

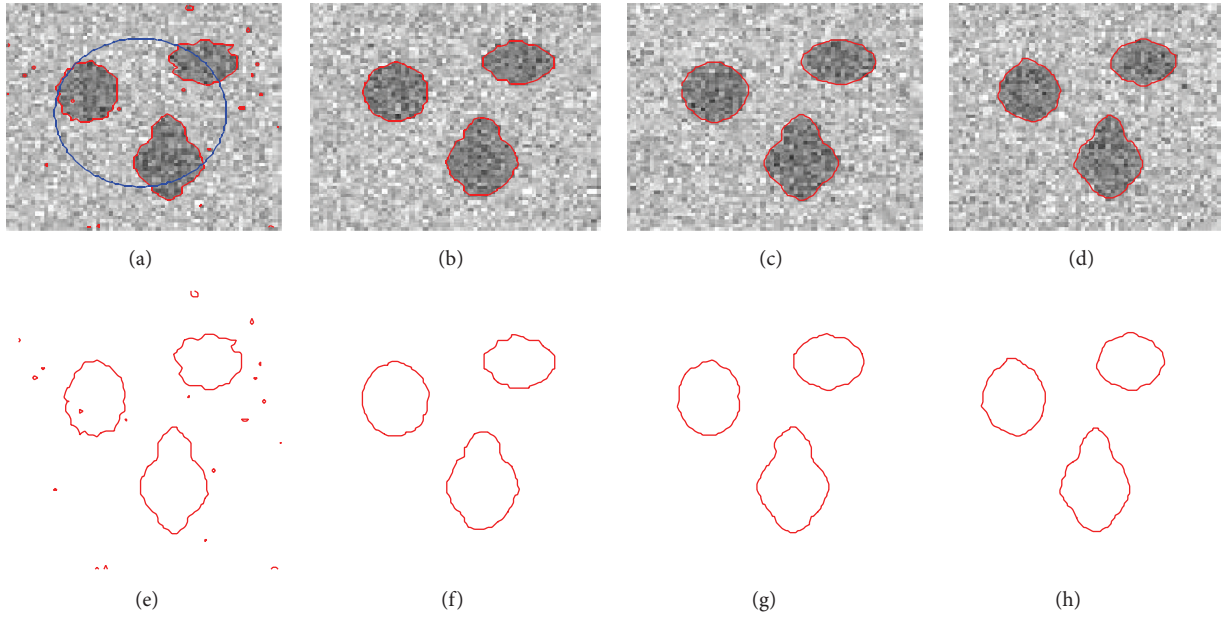


FIGURE 1: Segmental results of a noisy image with Gaussian variance 0.05. (a) Segmental result with CV model and the blue curve is the initial contour ($\mu_0 = 0.02 * 255 * 255$). (b) Segmental result with TV-based segmentation model. (c) Segmental result with adaptive TV-based model. (d) Anisotropic diffusion-based segmentation model. (e), (f), (g), and (h) are the corresponding contours to (a), (b), (c), and (d), respectively.

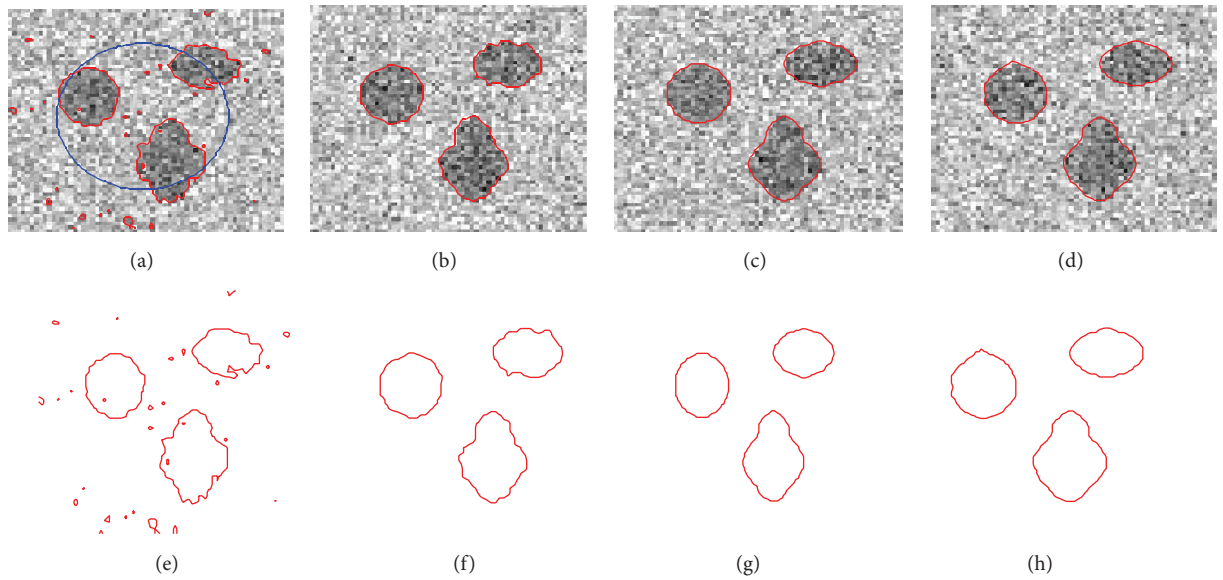


FIGURE 2: Segmental results of a noisy image with Gaussian variance 0.1. (a) Segmental result with CV model and the blue curve is the initial contour ($\mu_0 = 0.03 * 255 * 255$). (b) Segmental result with TV-based segmentation model. (c) Segmental result with adaptive TV-based model. (d) Anisotropic diffusion-based segmentation model. (e), (f), (g), and (h) are the corresponding contours to (a), (b), (c), and (d), respectively.

model, we choose $\lambda = 0.005$. As shown in Figure 3(a), the result of CV model obviously extracts the spots produced by Gaussian noise. Our models can obtain the satisfied contour without noisy spots.

Figure 4 shows the segmental results of a galaxy image contaminated by Gaussian noise with variation of 0.1. (a, e), (b, f), (c, g), and (d, h) represent the segmentation results of CV model, TV-based segmentation model, adaptive

TV-based segmentation model, and anisotropic diffusion-based segmentation model, respectively. The initial curve of all models is a circle shown in Figure 4(a) and its radius is set as 70. In TV-based segmentation model and adaptive TV-based segmentation model, we choose $\lambda = 0.02$. In anisotropic diffusion-based segmentation model, we choose $\lambda = 0.005$. Obviously, our models can get satisfied contour of objects in galaxy image.

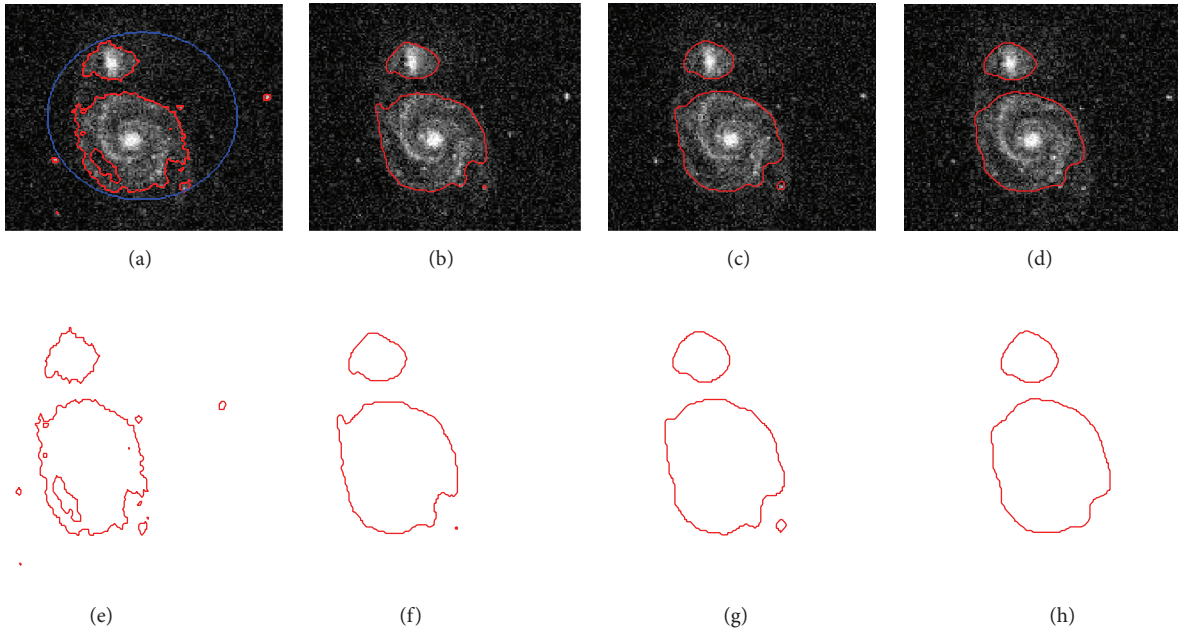


FIGURE 3: Segmental results of a galaxy image with Gaussian variance 0.05. (a) Segmental result with CV model and the blue curve is the initial contour ($\mu_0 = 0.03 * 255 * 255$). (b) Segmental result with TV-based segmentation model. (c) Segmental result with adaptive TV-based model. (d) Anisotropic diffusion-based segmentation model. (e), (f), (g), and (h) are the corresponding contours to (a), (b), (c), and (d), respectively.

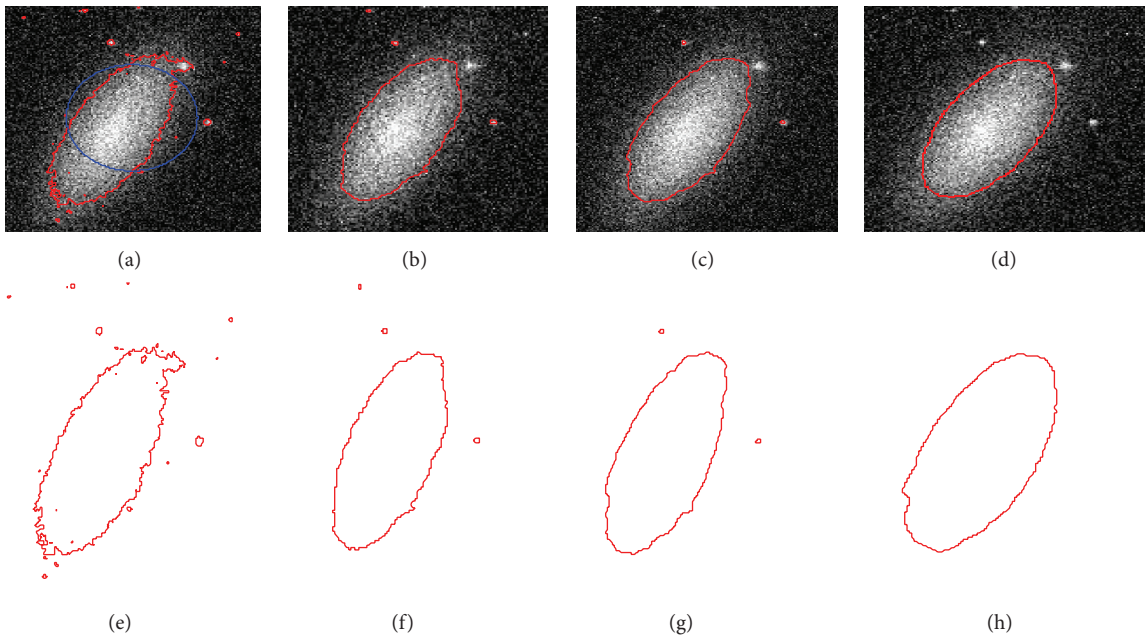


FIGURE 4: Segmental results of a galaxy image with Gaussian variance 0.1. (a) Segmental result with CV model and the blue curve is the initial contour ($\mu_0 = 0.04 * 255 * 255$). (b) Segmental result with TV-based segmentation model. (c) Segmental result with adaptive TV-based model. (d) Anisotropic diffusion-based segmentation model. (e), (f), (g), and (h) are the corresponding contours to (a), (b), (c), and (d), respectively.

5. Conclusion

Image segmentation and image denoising are all important in the field of image processing, but they are independent most of the time. In this paper, a new diffusion equation model

for noisy image segmentation is proposed by incorporating some classical diffusion equation denoising models into the segmental process. An assumption about the connection between the image intensity and level set function is given. Some classical denoising models are employed. Image

segmentation and image denoising are combined in a united framework. The segmental results can be presented by level set function. Experimental results show that the new segmentation model has the advantage of noise resistance and is superior to traditional segmentation model.

Competing Interests

The authors declare that they have no competing interests.

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